GCE

Further Mathematics B MEI

Y436/01: Further pure with technology

A Level

Mark Scheme for June 2025

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1.   Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training: OCR Essential Guide to Marking*.

2.   Make sure that you have read and understood the mark scheme and the question paper for this unit. These are available in RM Assessor

3.   Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **required number** of standardisation responses.

MARKING

1.   Mark strictly to the mark scheme.

2.   Marks awarded must relate directly to the marking criteria.

3.   The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4.   If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone, email or via the RM Assessor messaging system.

5.   **Crossed-Out Responses**
Where a candidate has crossed out a response and provided a clear alternative then the crossed-out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed-out response where legible.

Rubric Error Responses – Optional Questions
Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM Assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple-Choice Question Responses
When a multiple-choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).
When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.

Contradictory Responses
When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only one mark per response)
Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth two or more marks)
If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space).

Longer Answer Questions (requiring a developed response)
Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6.   Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there, then add the annotation 'SEEN' to confirm that the work has been seen and mark any responses using the annotations in section 11.

7.   There is a NR (**No Response**) option. Award NR (No Response):

   -      if there is nothing written at all in the answer space

   -      OR if there is a comment which does not in any way relate to the question (e.g., 'can't do', 'don't know')

   -      OR if there is a mark (e.g., a dash, a question mark) which is not an attempt at the question.

   Note: Award 0 marks – for an attempt that earns no credit (including copying out the question).

8.   The RM Assessor **comments box** is used by your Team Leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**

9.   Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. For answers marked by levels of response: Not applicable in F501
   To determine the level – start at the highest level and work down until you reach the level that matches the answer
   To determine the mark within the level, consider the following

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

11. **Annotations**

Annotation	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject Specific Marking Instructions

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is

sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all

responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
-        if there is nothing written at all in the answer space and no attempt elsewhere in the script
-        OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
-        OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.
Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of
doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in
marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer
alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and
anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following
an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark
scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually
lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
 • When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
 • When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
   NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.
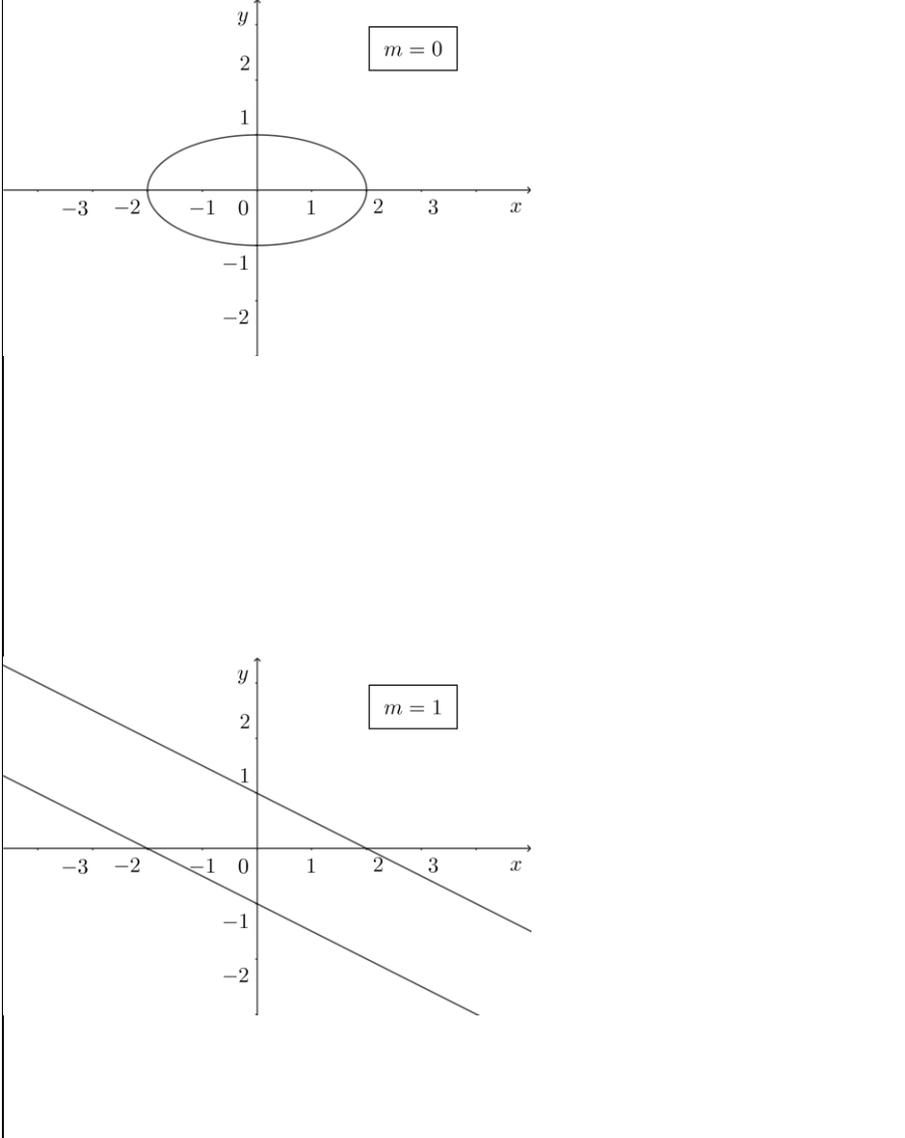
g. Rules for replaced work and multiple attempts:
 • If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 • If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 • if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)	(i)		B1	1.1	An ellipse with the values of points of intersection with the axes indicated.
						SC. If the values of x and y intercepts are **not** indicated then penalise **once** with **B0**. Then do not penalise in subsequent plots provided the graph are symmetrical and meet all other guidance conditions with no other errors. For example, the lines when $m = 1$ must be clearly parallel.
				B1	1.1	Two parallel lines with values of points of intersection with the axes indicated.

Question			Answer	Marks	AO	Guidance
				B1	**1.1**	Shape including position of curves in quadrants relative to axes; values of points of intersection with the axes indicated.
				[3]		
1	(a)	(ii)	Bounded curve.	**B1**	**1.2**	Alternatives include closed.
				[1]		
1	(b)		$\dfrac{(a\cos(t))^2}{a^2} + \dfrac{(b\sin(t))^2}{b^2}$ $= \dfrac{a^2\cos^2(t)}{a^2} + \dfrac{b^2\sin^2(t)}{b^2}$ $= \cos^2(t) + \sin^2(t) = 1.$	**B1**	**1.1**	Clear working to show simplification.
				[1]		
1	(c)		$\dfrac{dx}{dt} = -a\sin(t)$ $\dfrac{dy}{dt} = b\cos(t)$	**M1** **soi**	**2.1**	Differentiation of at least one of x or y equations. Can be implied by full working as written for **A1** mark below. Would not be implied if negative sign is not clearly in the denominator of fraction. Award **M0** if $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ is missing from subsequent working.
			$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{b\cos(t)}{-a\sin(t)} = -\dfrac{b}{a}\cot(t)$ **AG**	**A1**	**1.1**	Should see at least one intermediate step. Must be two steps of working if **M1** is implied.
				[2]		

Question	Answer	Marks	AO	Guidance
1 (d)	Gradient of normal $= -\dfrac{1}{\frac{dy}{dx}}$	M1soi	2.1	Seen or implied by answer.
	$= \dfrac{a}{b}\tan(t)$	A1	1.1	
	Using $y - y_1 = m(x - x_1)$ $y - y_1 = \dfrac{a}{b}\tan(t)\,(x - x_1)$	M1	1.1a	Apply equation of straight line with their gradient.
	Use $x_1 = a\cos(t)$ and $y_1 = b\sin(t)$ to get $y - b\sin(t) = \dfrac{a}{b}\tan(t)\,(x - a\cos(t))$	A1	1.1	Substitution of point on curve.
	$y = \dfrac{a}{b}\tan(t)x - a\cos(t) \times \dfrac{a}{b}\tan(t) + b\sin(t)$ $y = \dfrac{a}{b}\tan(t)x - \dfrac{a^2}{b}\sin(t) + b\sin(t)$			
	$y = \dfrac{a}{b}\tan(t)x - \left(\dfrac{a^2 - b^2}{b}\right)\sin(t)$. **AG**	A1	2.1	Must see at least one intermediate step.
		[5]		
1 (e)	$\mathrm{f}(x, y, t) = y - \dfrac{a}{b}\tan(t)x + \dfrac{a^2 - b^2}{b}\sin(t)$	M1	3.1a	$\mathrm{f}(x, y, t)$ must be seen or implied. Do not accept incorrect parameters. For example, $\mathrm{f}(x, y, a)$.
	$\dfrac{\partial f}{\partial t} = -\dfrac{a}{b}\sec^2(t)x + \dfrac{a^2 - b^2}{b}\cos(t)$ **oe**	A1	1.1	$= \dfrac{(a^2 - b^2)\cos(t) - ax\sec^2 t}{b}$
	$\dfrac{\partial f}{\partial t} = 0 \Rightarrow$ $x = \dfrac{(a^2 - b^2)\cos(t)}{a\sec^2(t)}$	M1	1.1	
	$= \dfrac{(a^2 - b^2)}{a}\cos^3(t)$ **AG**	A1	2.1	
	Substitute into equation for y			
	$y = \dfrac{a}{b}\tan(t)\dfrac{(a^2 - b^2)}{a}\cos^3(t) - \left(\dfrac{a^2 - b^2}{b}\right)\sin(t)$	M1	2.1	

Question			Answer	Marks	AO	Guidance
			$=\left(\frac{a^2-b^2}{b}\right)\cos^2(t)\sin(t)-\left(\frac{a^2-b^2}{b}\right)\sin(t)$ $=\left(\frac{a^2-b^2}{b}\right)(\cos^2(t)-1)\,\sin(t)$ $=\left(\frac{b^2-a^2}{b}\right)\sin^3(t)$ **AG**	**A1** **[6]**	2.1	
1	(f)			**B1** **[1]**	1.1	Shape including position of curves in quadrants relative to axes; points on axes clearly indicated. x intercepts are ± 1.5. y intercepts are ± 3.
1	(g)		$\left(\mathrm{a}x(t)\right)^{\left(\frac{2}{3}\right)}+\left(\mathrm{b}y(t)\right)^{\left(\frac{2}{3}\right)}$ $=(a^2-b^2)^{\frac{2}{3}}\cos^2(t)+(b^2-a^2)^{\frac{2}{3}}\sin^2(t)$	**M1**	1.1a	Simplified expressions for $\left(\mathrm{a}x(t)\right)^{\left(\frac{2}{3}\right)}+\left(\mathrm{b}y(t)\right)^{\left(\frac{2}{3}\right)}$ seen.
			$=(a^2-b^2)^{\frac{2}{3}}(\sin^2(t)+\cos^2(t))$ $=(a^2-b^2)^{\frac{2}{3}}.$ $(ax)^{\frac{2}{3}}+(by)^{\frac{2}{3}}=(a^2-b^2)^{\frac{2}{3}}.$ **oe**	**A1** **[2]**	1.1	ISW Must see some intermediate working. Accept any equivalent equation. For example, may see $(b^2-a^2)^{\frac{2}{3}}$ on right hand side. **A0** if no equation given.

Question			Answer	Marks	AO	Guidance	
2	(a)	(i)	249	B1 [1]	1.1		
2	(a)	(ii)	13 is prime and 2 and 13 are co-prime. (So Fermat's little theorem implies $2^{12} = 2^{13-1} \equiv 1 \pmod{13}$.)	B1 [1]	2.4	Sufficient to state that 13 is prime **and** both 2 and 13 are co-prime. Award **B0** if either condition omitted.	
2	(a)	(iii)	$2^{(12q+1)} = (2^{12})^q \times 2$ $2^{(12q+1)} \pmod{13}$ $\equiv \left(2^{12} \pmod{13}\right)^q \times 2 \pmod{13}$ $\equiv \left(1 \pmod{13}\right)^q \times 2 \pmod{13}$ $\equiv 1 \pmod{13} \times 2 \pmod{13}$ $\equiv 2 \pmod{13}$	M1 A1 [2]	1.1 1.1	Suitable decomposition of the expression. This mark can be implied by the observation of correct later decomposition. Consider the equation modulo 13 and simplify.	
2	(b)	(i)	3	B1 [1]	1.1		

Question			Answer	Marks	AO	Guidance	

2	**(b)**	**(ii)**				Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors where answer to **(b)(iii)** is correct.	Allow use of import math or manual computation of highest common factor. Example hcf program
							def hcf (m,n):
							  for i in range (1, m + 1):
							   if n % i == 0 and m % i == 0:
							    g = i
							  return (g)

			Appropriate program structure.	**M1**	3.3	Use of a loop, if condition, seed setting for function return value.	
			Loops with correct range.	**M1**	2.1	Range could be n or $n + 1$ depending on implementation. For example, in Python range(1, n+1) is a list from 1 to n inclusive. For example, i in range (1, n):	
			Correct check required condition on n and i with if statement.	**M1**	2.1	For example, math.gcd(i,n) == 1:	

			Fully correct program.	**A1**	2.5	Example program with import math.
						import math
						def totient(n):
						  t = 0
						  for i in range (1, n):
						   if math.gcd(i,n) == 1:
						    t = t + 1
						  return (t)
							Example program without import math where hcf function is as defined above in guidance preamble.
							def etf(n):
							  t = 0
							  for i in range (1, n):
							   if hcf(i,n) == 1:
							    t = t + 1
							  return (t)

				[4]			

Question			Answer	Marks	AO	Guidance
2	(b)	(iii)	$\varphi(1000) = 400$.	**B1** [1]	3.4	
2	(b)	(iv)	7 and 1000 are coprime.	**B1**	3.1a	
			$7^{400}(\text{mod } 1000) \equiv 1(\text{mod } 1000)$.	**B1**	2.2a	From **(b)(iii)** $\varphi(1000) = 400$ so Euler's theorem applies. Accept $7^{\varphi(1000)} \equiv 1 \ (\text{mod } 1000)$.
			So $7^{(400r+10)} \ (\text{mod } 1000)$ $\equiv 7^{400r}(\text{mod } 1000) \times 7^{10}(\text{mod } 1000)$	**M1**	1.1	Decomposition of powers.
			$\equiv 1(\text{mod}1000) \times 7^{10}(\text{mod}1000)$ $\equiv 7^{10}(\text{mod}1000) = 249$.	**A1** [4]	1.1	
2	(b)	(v)	$2010 = 400 \times 5 + 10$ so $7^{2010}(\text{mod } 1000) = 249$.	**M1 soi**	2.2a	Can be implied by the correct answer **and** either $r = 5$ or $2010 = 400 \times 5 + 10$.
			So, the tens digit is 4.	**A1** [2]	3.2a	249 is the unit (9), tens (4) and hundreds (2) digits of 7^{2010}.
2	(c)		Since p is an odd prime, $p \equiv 1(\text{mod } 4)$ or $p \equiv 3(\text{mod } 4)$.	**B1**	3.1a	Alternatively, check conditions of Euler's Theorem to set up the deduction that $p^2 \equiv 1(\text{mod } 4)$.
			So $p^2 \equiv 1(\text{mod } 4)$.	**B1**	2.1	
			So, $p^{2x} + 1 \equiv 2(\text{mod } 4)$.	**B1**	2.2a	
			Since $y \geq 1$, $2^{2y} = 4^y \equiv 0(\text{mod } 4)$.	**B1**	2.4	
			Since $0 \neq 2$, the two sides of the equation are not equal modulo 4 and there are no solutions.	**B1** [5]	2.2a	Must have **correct** working to obtained 0 and 2 to award mark for this deduction.
			Alternative (Proof by contradiction.) Assume there is a solution (x, y) to $p^{2x} + 1 = 2^{2y}$.	**B1**	3.1a	Set up for contradiction can be implied by a correct contradiction argument.

Question			Answer	Marks	AO	Guidance
			Since $y \geq 1$, $2^{2y} = 4^y \equiv 0 (\text{mod } 4)$.	B1	2.4	
			Then $p^{2x} + 1 \equiv 0 (\text{mod } 4)$. So $(p^x)^2 \equiv -1 (\text{mod } 4)$ $(\equiv 3 (\text{mod } 4))$.	B1	2.2a	
			We have $n^2 \equiv 0$ or $1 \ (\text{mod } 4)$ for $n = 0,1,2,3$. So, the quadratic residues mod 4 are 0 and 1.	B1	2.1	
			Therefore, there is no integer p^x such that $(p^x)^2 \equiv -1 (\text{mod } 4)$. This is a contradiction, therefore there are no integers x and y such that $p^{2x} + 1 = 2^{2y}$.	B1	2.4a	
				[5]		

Question			Answer	Marks	AO	Guidance
3	(a)	(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - 2e^{-x}$	M1	2.1	
			$x - y = x - (2e^{-x} + x - 1)$ $= 1 - 2e^{-x}$ $= \dfrac{\mathrm{d}y}{\mathrm{d}x}$	A1	1.1	Substitution of y into equation and simplification. Condone not including final derivative verification if $\frac{dy}{dx} =$ is already stated. **SC.** Use of CAS permitted to find general solution if its use is clearly stated, for example, "Using CAS". Award **M1A1** for fully correct CAS application only.
			When $x = 0$, $y = 2e^0 + 0 - 1 = 1$.	B1 **[3]**	1.1	Verify initial condition.
			Alternative (Show that rather than verify).			
			When $a = 0, \frac{dy}{dx} + y = x$. Integrating factor is e^x.	M1	2.1	Correct integrating factor.
			$e^x \dfrac{dy}{dx} + e^x y = x e^x$. $y e^x = x e^x - e^x + A$. $y = A e^{-x} + x - 1$.	A1	1.1	Must correctly derive general solution.
			Using $x = 0$, $y = 1$ gives $A - 1 = 1$. So $A = 2$.	B1 **[3]**	1.1	Verify initial condition.
3	(a)	(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - 2e^{-x} = 0$.	M1 (soi)	1.1	Can be implied by correct final answer. May see equivalents such as $\frac{dy}{dx} = x - y = 0$ following by substitution of equation for y.

Question			Answer	Marks	AO	Guidance
			$x = \ln(2)$	**A1** [2]	1.1	May see $x = -\ln\left(\frac{1}{2}\right)$.
3	(a)	(iii)		**B1** **B1** [2]	1.1 1.1	Curve starts at $(0,1)$ and decreases. Curve increases from a minimum point with x coordinate less than 1 to reach a point that is approximately $(3,2)$. Final coordinate can be implied by correct approximate location relative to initial point if not explicitly label. Award **B0** is $(3,2)$ is not labelled and end point is inconsistent with other graph labelling.
3	(b)			**B1** **B1** [2]	1.1 1.1	Curve showing correct shape up to location of inflection point at approximately $x = 1$. Curve showing correct shape up to $x = 3$ with solution curve tangent to tangent field.
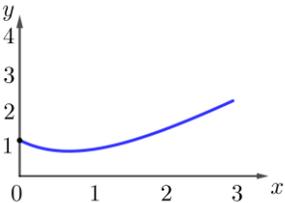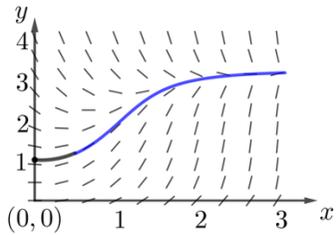
Question			Answer	Marks	AO	Guidance
3	(c)	(i)				Give reasonable BOD for possible transcription errors and consider correct answer to **3(c)(ii)** as evidence of correct formulae in the spreadsheet.
						May see use of Geogebra or Excel functions. Here, $f(x, y)$ is define and cell entries are in terms of f with arguments referring to other cells. In Geogebra may see a and/or h sliders.
			A2 contains 0 (value of x_0) B2 contains 4 (value of y_0) N1 contains 0.05 (value of h) P1 contains 1 (value of a)	**B1**	**3.1a**	Allow for a and h to be varied. Set up initial conditions. Allow any h value here, for example $h = 0.1$.
			C2 contains =A2+$N$1 $(x_n + h)$ D2 contains =A2+$N$1/2 $(x_n + \frac{h}{2})$ F2 contains =B2+(E2/2) $(y_n + \frac{k_1}{2})$ H2 contains =B2+(G2/2) $(y_n + \frac{k_2}{2})$ J2 contains =B2+I2 $(y_n + k_3)$	**B1**	**3.1a**	Auxiliary formulae for k_1, k_2, k_3 and k_4. Can be implied by correct formulae for k_1, k_2, k_3 and k_4.
			E2 contains =$N$1*(A2 -B2+$P$1*EXP(A2)*SIN(B2)) (k_1) G2 contains =$N$1*(D2-F2+$P$1*EXP(D2)*SIN(F2)) (k_2) I2 contains =$N$1*(D2-H2+$P$1*EXP(D2)*SIN(H2)) (k_3) K2 contains =$N$1*(C2-J2+$P$1*EXP(C2)*SIN(J2)) (k_4)	**B1**	**3.1a**	Formulae for k_1, k_2, k_3 and k_4.
			A3 contains =C2 $(x_{n+1} = x_n + h)$ B3 contains =B2+(1/6)*(E2+2*G2+2*I2+K2) $(y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).)$ Copy down	**B1**	**2.5**	Formulae for x_{n+1} and y_{n+1}. Indication of how spreadsheet is constructed e.g. "copy down" or equivalent.
				[4]		

Question			Answer	Marks	AO	Guidance
3	(c)	(ii)	Spreadsheet gives 2.43112005 So, 2.43 to 2 decimal places.	B1 [1]	1.1	
3	(c)	(iii)	With $h = 0.005$ the spreadsheet gives 	1.0200	2.17905556	 	1.0250	2.17897694	 	1.0300	2.17895237	 	1.0350	2.17898151	 So, $1.025 < r < 1.035$ and estimate $r = 1.03$ to 2 decimal places.	M1 A1 [2]	3.4 2.2b	Must see suitable (x, y) pairs so that minimum is clear, or an argument based on a sign change in the gradient sufficient close to 1.03. **SC1** for exactly 1.03 given clearly as final answer without working and no alternative values provided. For example, do **not** accept 1.030.
3	(c)	(iv)	With $h = 0.05$ the spreadsheet gives Suitable value of $a = 1.1$ to 1 decimal place.	M1 soi A1 [2]	3.4 2.2b	Can be implied by correct final answer. Correct value of a implies **M1A1**. Award **M1A0** if correct method is seen but no correct final value of a is given.

Table for 3(c)(iv):

Value of a	x	y	y(1) to 2 dp
1.00	1.00	2.43112005	2.43
1.05	1.00	2.45159519	2.45
1.10	1.00	2.47130782	2.47
1.15	1.00	2.49028155	2.49
1.20	1.00	2.50854029	2.51

APPENDIX

Exemplar responses for Q2(b)

Response	Mark

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