Oxford Cambridge and RSA

GCE

Further Mathematics B MEI

Y420/01: Core Pure

A Level

Mark Scheme for June 2025

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training: OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are available in RM Assessor

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **required number** of standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone, email or via the RM Assessor messaging system.

5. **Crossed-Out Responses**
   Where a candidate has crossed out a response and provided a clear alternative then the crossed-out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed-out response where legible.

Rubric Error Responses – Optional Questions
Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM Assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple-Choice Question Responses
When a multiple-choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate).
When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.

Contradictory Responses
When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only one mark per response)
Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth two or more marks)
If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space).

Longer Answer Questions (requiring a developed response)
Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there, then add the annotation 'SEEN' to confirm that the work has been seen and mark any responses using the annotations in section 11.

7. There is a NR (**No Response**) option. Award NR (No Response):

   - if there is nothing written at all in the answer space
   - OR if there is a comment which does not in any way relate to the question (e.g., 'can't do', 'don't know')
   - OR if there is a mark (e.g., a dash, a question mark) which is not an attempt at the question.

   Note: Award 0 marks – for an attempt that earns no credit (including copying out the question).

8. The RM Assessor **comments box** is used by your Team Leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**

9. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. For answers marked by levels of response: Not applicable in F501
    To determine the level – start at the highest level and work down until you reach the level that matches the answer
    To determine the mark within the level, consider the following

Descriptor	Award mark
On the borderline of this level and the one below	At bottom of level
Just enough achievement on balance for this level	Above bottom and either below middle or at middle of level (depending on number of marks available)
Meets the criteria but with some slight inconsistency	Above middle and either below top of level or at middle of level (depending on number of marks available)
Consistently meets the criteria for this level	At top of level

11. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject Specific Marking Instructions

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

   For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

   It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   Award NR (No Response)
   - if there is nothing written at all in the answer space and no attempt elsewhere in the script
   - OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
   - OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

   Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

   If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

   If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

   M
   A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

g

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1			$z^* = x - \mathrm{i}y$	**B1**	**1.2**	seen or used
			$x + \mathrm{i}y + 2\mathrm{i}x + 2y + 1 - 4\mathrm{i}$ [$= 0$]	**M1**	**1.1**	$\mathrm{i}^2 = -1$ used with their z^*; soi by correct equating of real and imaginary parts
			$\Rightarrow x + 2y + 1 = 0$, $y + 2x - 4 = 0$	**M1**	**3.1a**	equating their real and imaginary parts (imaginary component may still be in terms of i), allow a slip including a missing term
			$\Rightarrow x = 3$, $y = -2$	**A1**	**1.1**	www. Accept [$z =$]$3 - 2\mathrm{i}$. An M1 step must be seen before final answer.
				[4]		

Question			Answer	Marks	AO	Guidance
2			**DR**			
			Normal vectors are 2**i** – **j** + 2**k** and **i** + 2**j** + **k**	**B1**	**1.1**	both soi
			$\cos\theta = \dfrac{(2\mathbf{i}-\mathbf{j}+2\mathbf{k}).(\mathbf{i}+2\mathbf{j}+\mathbf{k})}{\sqrt{2^2+(-1)^2+2^2}\sqrt{1^2+2^2+1^2}}$	**M1**	**1.1**	scalar product formula with $\cos\theta$, allow a slip
			$= \dfrac{2}{3\sqrt{6}}$	**A1**	**1.1**	with scalar product = 2 soi
			θ = 74.2° or better	**A1**	**1.1**	or 1.30 radians or better, www. Mark final answer. Answer with no supporting working scores 0 marks.
			Alternative method 1			
			Normal vectors are 2**i** – **j** + 2**k** and **i** + 2**j** + **k**	**B1**		both soi
			$\sin\theta = \dfrac{\lvert(2\mathbf{i}-\mathbf{j}+2\mathbf{k})\times(\mathbf{i}+2\mathbf{j}+\mathbf{k})\rvert}{\sqrt{2^2+(-1)^2+2^2}\sqrt{1^2+2^2+1^2}}$	**M1**		vector product formula with $\sin\theta$, allow a slip
			$= \dfrac{\sqrt{50}}{3\sqrt{6}}$	**A1**		with |vector product| = $\sqrt{50}$ soi
			θ = 74.2° or better	**A1**		or 1.30 radians or better, www. Mark final answer. Answer with no supporting working scores 0 marks.
			Alternative method 2			
			Normal vectors are 2**i** – **j** + 2**k** and **i** + 2**j** + **k**	**B1**	**1.1**	both soi
			$\sin\alpha = \dfrac{(2\mathbf{i}-\mathbf{j}+2\mathbf{k})\cdot(\mathbf{i}+2\mathbf{j}+\mathbf{k})}{\sqrt{2^2+(-1)^2+2^2}\sqrt{1^2+2^2+1^2}}$	**M1**	**1.1**	scalar product formula with $\sin\theta$, allow a slip
			$= \dfrac{2}{3\sqrt{6}}$	**A1**	**1.1**	with scalar product = 2 soi
			$\theta = 90-\alpha = 74.2°$ or better	**A1**	**1.1**	or 1.30 radians or better, www. Mark final answer. Answer with no supporting working scores 0 marks.
				[4]		

Question			Answer	Marks	AO	Guidance
3			$1\times3+2\times4+\ldots+n\times(n+2)=\sum_{r=1}^{n}r(r+2)$	**M1**	**2.1**	converting series correctly to sigma notation
			$=\sum_{r=1}^{n}r^2+2\sum_{r=1}^{n}r$	**M1**	**2.5**	writing their sum in terms of $\sum_{r=1}^{n}r^2$ and $\sum_{r=1}^{n}r$ soi
			$=\frac{1}{6}n(n+1)(2n+1)+2\times\frac{1}{2}n(n+1)$	**M1**	**1.1**	substituting standard formulae for $\sum_{r=1}^{n}r^2$ and $\sum_{r=1}^{n}r$. r^2+2r or $r(r+2)$ must have been seen.
			$=\frac{1}{6}n(n+1)(2n+1+6)$	**M1**	**1.1**	correctly taking out a factor n, $n+1$ or both from their expression; soi by correct final answer
			$=\frac{1}{6}n(n+1)(2n+7)$ or $a=2, b=7$	**A1** **[5]**	**2.1**	www but condone errors with sigma notation

Question			Answer	Marks	AO	Guidance
4	(a)		$(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$	**B1** **[1]**	**1.2**	do not allow misread of **NM** for **MN**
4	(b)		det $\mathbf{M} = a$	**B1**	**1.1**	it must be unambiguous which matrix is which for each mark to be awarded
			$\mathbf{M}^{-1} = \frac{1}{a}\begin{pmatrix}1 & -1\\0 & a\end{pmatrix}$	**B1**	**2.1**	
			$\mathbf{N}^{-1} = \begin{pmatrix}b & 1\\-1 & 0\end{pmatrix}$	**B1**	**1.1**	
			$\mathbf{MN} = \begin{pmatrix}1 & b-a\\1 & b\end{pmatrix}$	**B1**	**1.1**	Allow $\mathbf{NM} = \begin{pmatrix}0 & -1\\a & b+1\end{pmatrix}$ if $(\mathbf{NM})^{-1} = \mathbf{M}^{-1}\mathbf{N}^{-1}$ given in part (a)
			$(\mathbf{MN})^{-1} = \frac{1}{a}\begin{pmatrix}b & a-b\\-1 & 1\end{pmatrix}$	**B1**	**1.1**	may be unsimplified. Allow $(\mathbf{NM})^{-1} = \frac{1}{a}\begin{pmatrix}b+1 & 1\\-a & 0\end{pmatrix}$ if $(\mathbf{NM})^{-1} = \mathbf{M}^{-1}\mathbf{N}^{-1}$ given in part (a)
			$\mathbf{N}^{-1}\mathbf{M}^{-1} = \frac{1}{a}\begin{pmatrix}b & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}1 & -1\\0 & a\end{pmatrix} = \frac{1}{a}\begin{pmatrix}b & a-b\\-1 & 1\end{pmatrix}$	**B1** **[6]**	**2.2a**	product of two matrices must be seen. Allow $\mathbf{M}^{-1}\mathbf{N}^{-1} = \frac{1}{a}\begin{pmatrix}1 & -1\\0 & a\end{pmatrix}\begin{pmatrix}b & 1\\-1 & 0\end{pmatrix} = \frac{1}{a}\begin{pmatrix}b+1 & 1\\-a & 0\end{pmatrix}$ if $(\mathbf{NM})^{-1} = \mathbf{M}^{-1}\mathbf{N}^{-1}$ given in part (a).

Question			Answer	Marks	AO	Guidance
5			let $y = ½ (x + 1) \Rightarrow x = 2y - 1$	**M1**	1.1	correct substitution chosen and rearranged, condone a rearrangement slip
			$2(2y - 1)^3 - 3(2y - 1) + 4 = 0$	**M1**	1.1	substitution made, condone one slip only
			$2(8y^3 - 12y^2 + 6y - 1) - 6y + 3 + 4 = 0$	**B1**	1.1	correctly expanding cubic, soi. Do not ft.
			$16y^3 - 24y^2 + 6y + 5 = 0$	**A1**	1.1	must be an equation with integer coefficients
			Alternative method			
			$\sum\alpha = 0, \sum\alpha\beta = -\frac{3}{2}, \alpha\beta\gamma = -2$	**B1**		Soi
			$\sum\frac{1}{2}(\alpha+1) = \frac{3}{2}$, $\sum\frac{1}{2}(\alpha+1)\frac{1}{2}(\beta+1) = \frac{3}{8}$	**M1**		using sum and products to correctly find at least one of sum of roots, products of pairs or triple product for transformed equation
			$\frac{1}{2}(\alpha+1)\frac{1}{2}(\beta+1)\frac{1}{2}(\gamma+1) = -\frac{5}{16}$	**A1**		two of $\frac{3}{2}, \frac{3}{8}, -\frac{5}{16}$
			$\Rightarrow 16y^3 - 24y^2 + 6y + 5 = 0$	**A1**		must be an equation with integer coefficients
				[4]		

Question			Answer	Marks	AO	Guidance
6	(a)		$x^2 + y^2 = r^2$, $x = r\cos\theta$, $y = r\sin\theta$	M1	1.1	substituting for all x and y terms correctly. Implied by a correct unsimplified equation for r^4 or $(r^2)^2$
			$\Rightarrow r^4 = r^2\cos\theta\sin\theta$	A1	1.1	RHS could be unsimplified; r^4 could be left as $(r^2)^2$
			$\Rightarrow r^2 = ½\sin 2\theta$	A1	1.1	
			[so $a = ½$ and $b = 2$]	**[3]**		
6	(b)		maximum when $\sin 2\theta = 1$	B1FT	3.1a	or r^2 maximum $= \frac{1}{2}$ or correct differentiation of their r^2 (or r), equating to zero and getting $\cos 2\theta = 0$, or $\theta = \frac{\pi}{4}$. soi by correct value for their r. **FT** from part (a)
			$\Rightarrow r = \frac{1}{\sqrt{2}}$	B1FT	1.1	o.e. must be exact. www. **FT** from part (a)
				[2]		
6	(c)		$A = \frac{1}{4}\int_0^{\frac{\pi}{2}} \sin 2\theta \, d\theta$	M1	1.1	$\frac{1}{2}\int r^2 \, d\theta$ used with their r^2 provided it is in the form $a\sin b\theta$. Must be seen. Condone missing $d\theta$ and missing or incorrect limits
			$= \left[-\frac{1}{8}\cos 2\theta\right]_0^{\frac{\pi}{2}}$	A1FT	1.1	FT their integral (i.e. $-\frac{a}{2b}\cos b\theta$). Must be seen. Note $\int \sin\theta\cos\theta \, d\theta = \frac{1}{2}\sin^2\theta$ or $-\frac{1}{2}\cos^2\theta$,
			$= -\frac{1}{8}\left(\cos 2\left(\frac{\pi}{2}\right) - \cos 0\right)$	M1	1.1	substituting correct limits after attempt to integrate, soi by e.g. correct answer or $\frac{1}{4}\left(\frac{1}{2}+\frac{1}{2}\right)$ if correct integral found. Accept correct alternative limits, e.g. π to $\frac{3\pi}{2}$ Requires an initial integral of the form $k\int a\sin b\theta \, d\theta$
			$= \frac{1}{4}$	A1	1.1	cao www
				[4]		

Question			Answer	Marks	AO	Guidance
7			**DR**			
			$\dfrac{1}{x^2-4}=\dfrac{1}{(x-2)(x+2)}$	**B1**	**1.1**	rewriting $\dfrac{1}{x^2-4}$ as $\dfrac{1}{(x-2)(x+2)}$ soi
			$\dfrac{1}{(x-2)(x+2)}=\dfrac{A}{x-2}+\dfrac{B}{x+2}$	**M1**	**1.1**	rewriting as partial fractions
			$1=A(x+2)+B(x-2)$	**M1**	**1.1**	evaluating their A and B using substitution, or equating coeffs
			$x=2\Rightarrow A=¼$, $x=-2\Rightarrow B=-¼$	**A1**	**1.1**	$A=¼$ and $B=-¼$ or $\frac{1}{4(x-2)}-\frac{1}{4(x+2)}$
			$\displaystyle\int_3^\infty \frac{1}{x^2-4}\,\mathrm{d}x=\frac{1}{4}\int_3^\infty\left(\frac{1}{x-2}-\frac{1}{x+2}\right)\mathrm{d}x$			
			$=\dfrac{1}{4}\lim\limits_{k\to\infty}\left[\ln(x-2)-\ln(x+2)\right]_3^k$	**M1***	**2.1**	integrate their $\left(\frac{1}{x-2}-\frac{1}{x+2}\right)$ correctly (condone missing $\frac{1}{4}$ or incorrect multiples). $\frac{1}{4}$ could be incorporated into logarithms.
			$=\dfrac{1}{4}\lim\limits_{k\to\infty}\left[\ln\left(\dfrac{x-2}{x+2}\right)\right]_3^k$	**A1**	**1.1**	$\frac{1}{4}\left[\ln\left(\frac{x-2}{x+2}\right)\right]$. Fraction could be unsimplified.
			As $k\to\infty$ $\dfrac{k-2}{k+2}\to 1$ or $\ln(\dfrac{k-2}{k+2})\to 0$	**A1**	**2.5**	clear limit argument used to evaluate limit as $k\to\infty$ or $\lim\limits_{k\to\infty}(\ln\frac{k-2}{k+2})=0$. Must work with a single term. $\to$ or $=$ must be used correctly, e.g. do not condone $\lim\limits_{k\to\infty}\left[\ln\frac{k-2}{k+2}\right]\to 0$ or "$k\to\infty, \frac{k-2}{k+2}=1$"
			so $\dfrac{1}{4}\lim\limits_{k\to\infty}\left[\ln\left(\dfrac{x-2}{x+2}\right)\right]_3^k=-\dfrac{1}{4}\ln\dfrac{1}{5}=\dfrac{1}{4}\ln 5$	**B1dep** **[8]**	**2.1**	Or $m=4, n=5$.

Question			Answer	Marks	AO	Guidance
8	(a)		Check result for $n = 1$: $\mathrm{f}'(x) = \frac{1}{1+x} = \frac{(-1)^{1+1}(1-1)!}{(1+x)^1}$ [so true for $n = 1$]	**B1**	2.1	no more simplified in initial step than $\frac{(-1)^2(0!)}{1+x}$; condone missing brackets
			Assume true for $n = k$: $\mathrm{f}^{(k)}(x) = \dfrac{(-1)^{k+1}(k-1)!}{(1+x)^k}$ $\mathrm{f}^{(k+1)}(x) = \dfrac{(-k)(-1)^{k+1}(k-1)!}{(1+x)^{k+1}}$	**M1**	2.1	differentiating expression for $\mathrm{f}^{(k)}(x)$; condone bracketing errors or a sign slip only
			$= \frac{(-1)^{(k+1)+1}((k+1)-1)!}{(1+x)^{k+1}}$ [which is the result for $n = k + 1$]	**A1**	2.2a	condone $\frac{(-1)^{(k+2)}k!}{(1+x)^{k+1}}$. Do not condone bracketing errors. M1 step must be seen.
			As true for $n = 1$, and if true for $n = k$ then true for $n = k+1$, true for all n	**A1** **[4]**	2.4	www. $\frac{(-1)^{(k+1)+1}((k+1)-1)!}{(1+x)^{k+1}}$ must have been seen to score this mark. $n = 1$ must have been considered. Cannot score A0A1.

Question			Answer	Marks	AO	Guidance
8	(b)		$\mathrm{f}^{(1)}(0)=\frac{(-1)^2(0!)}{1}$, $\mathrm{f}^{(2)}(0)=\frac{(-1)^3(1!)}{1^2}$, $\mathrm{f}^{(3)}(0)=\frac{(-1)^4(2!)}{1^3}$, $\mathrm{f}^{(n)}(0)=\frac{(-1)^{n+1}(n-1)!}{1^n}$	M1	3.1a	using result from (a) to find at least two derivatives (can include general term). May be embedded in coefficients. Must include powers of -1 and factorials in each term; condone missing division by 1. Allow one slip.
			n^{th} term coefficient $=\frac{(-1)^{n+1}(n-1)!}{n!}=\frac{(-1)^{n+1}}{n}$	M1	3.1	considering coefficient of general term and simplifying using $\frac{(n-1)!}{n!}$
			$\mathrm{f}(0)=\ln 1=0$ $\mathrm{f}(x)=[0]+x-\frac{x^2}{2}+\frac{x^3}{3}-\ldots+\frac{(-1)^{n+1}x^n}{n}+\ldots$	A1	2.2a	**AG** complete argument, www. Must clearly be a sum of infinitely many terms. Must have considered $\mathrm{f}(0)=0$.
			Alternative method			
			$\mathrm{f}(x)=\mathrm{f}(0)+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n-1)!x^n}{n!}$	M1		use of Maclaurin series with result from (a) used for all terms other than f(0)
			$=\ln(1)+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n}$	A1		$\frac{(n-1)!}{n!}=\frac{1}{n}$ seen or used
			$=[0]+x-\frac{x^2}{2}+\frac{x^3}{3}-\ldots+\frac{(-1)^{n+1}x^n}{n}+\ldots$	A1		**AG**
				[3]		

Question			Answer	Marks	AO	Guidance
9	(a)	(i)	w^2, w^3 and w^4	B1 [1]	1.2	or $we^{\frac{2\pi}{5}\mathrm{i}}$, $we^{\frac{4\pi}{5}\mathrm{i}}$ and $we^{\frac{-4\pi}{5}\mathrm{i}}$ (or $we^{\frac{6\pi}{5}\mathrm{i}}$ or w^*) oe. Allow misattribution between C, D, E.
9	(a)	(ii)	$z^5 = 1$ oe	B1 [1]	1.2	allow any variable for z or $z^5 = \cos 2\pi + \mathrm{i}\sin 2\pi$ or $z^5 = \cos 2k\pi + \mathrm{i}\sin 2k\pi$ (or $\mathrm{e}^{2k\pi\mathrm{i}}$) provided $k \in \mathbb{Z}$ seen
9	(a)	(iii)	$[1 + w + w^2 + w^3 + w^4 =] \frac{1-w^5}{1-w}$ $= 0$ as $w^5 = 1$	M1 A1	2.4 2.1	correct use of geometric series formula in terms of w or exponentials. Cannot be implied. $w^5 = 1$ or $1 - \mathrm{e}^{2\pi\mathrm{i}}$ seen before completion.
			Alternative method Sum of roots = coefficient of z^4 [in $z^5 - 1 = 0$] This coefficient is zero so sum of roots is zero	M1 A1		"sum of roots $= -\frac{b}{a} = 0$" alone is insufficient
				[2]		
9	(b)	(i)	$[w =] \cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}$	B1 [1]	1.1	

Question			Answer	Marks	AO	Guidance
9	(b)	(ii)	$[w-1=]\cos\frac{2\pi}{5}-1+\mathrm{i}\sin\frac{2\pi}{5}$	M1	3.1a	or $\begin{pmatrix}\cos\frac{2\pi}{5}-1\\ \sin\frac{2\pi}{5}\end{pmatrix}$; or equivalent for $1-w$. Soi by correct modulus calculation.
			$\sqrt{\left(\cos\frac{2\pi}{5}-1\right)^2+\sin^2\frac{2\pi}{5}}$	M1	1.1	finding the modulus or modulus^2 of their $w-1$ (or $1-w$)
			$=\sqrt{\cos^2\frac{2\pi}{5}-2\cos\frac{2\pi}{5}+1+\sin^2\frac{2\pi}{5}}$	A1	1.1	or $\sqrt{2-2\cos\frac{2\pi}{5}}$ or correct expression for AB^2
			$=\sqrt{2-2(1-2\sin^2\frac{\pi}{5})}$	M1	3.1a	correct use of double angle formula, must be seen. Accept $2\left(1-\cos\frac{2\pi}{5}\right)=2(2\sin^2\frac{\pi}{5})$ without intermediate step but not $2-2\cos\frac{2\pi}{5}=2\left(2\sin^2\frac{\pi}{5}\right)$.
			$=2\sin\frac{\pi}{5}$	A1	2.1	**AG**
			Alternative method 1			
			$\mathrm{AB}^2=1^2+1^2-2(1)(1)\cos\frac{2\pi}{5}$	M2		correct use of cosine rule; or correct expression for AB
			$=2-2\cos\frac{2\pi}{5}$	A1		simplified expression for AB^2 or AB
			$=2-2(1-2\sin^2\frac{\pi}{5})$	M1		correct use of double angle formula, must be seen. Accept $2\left(1-\cos\frac{2\pi}{5}\right)=2(2\sin^2\frac{\pi}{5})$ without intermediate step but not $2-2\cos\frac{2\pi}{5}=2\left(2\sin^2\frac{\pi}{5}\right)$.
			$\mathrm{AB}=2\sin\frac{\pi}{5}$	A1		**AG**
			Alternative method 2			
			Considering triangle AOB	M1		may be stated or seen in diagram; condone missing labels if the triangle is clearly isosceles
			Considering right angled triangle OAM or OBM where M is the midpoint of AB	M1		may be stated or seen in diagram; condone missing labels
			$\frac{1}{2}\mathrm{AB}=\sin\frac{\pi}{5}$ or $\mathrm{AM}=\sin\frac{\pi}{5}$	A1		must be seen
			$\mathrm{AB}=2\sin\frac{\pi}{5}$	A2		**AG**

Question			Answer	Marks	AO	Guidance
			Alternative method 3			
			$\dfrac{\text{AB}}{\sin\frac{2\pi}{5}} = \dfrac{1}{\sin\frac{3\pi}{10}}$	**M1***		using sine rule correctly
			$\text{AB} = \dfrac{\sin\frac{2\pi}{5}}{\sin\frac{3\pi}{10}}$	**A1**		correct expression for AB
			$\sin\frac{2\pi}{5} = 2\sin\frac{\pi}{5}\cos\frac{\pi}{5}$	**M1dep**		correct use of double angle formula
			$\sin\frac{3\pi}{10} = \cos\frac{\pi}{5}$	**M1dep**		using $\sin\theta = \cos(\frac{\pi}{2} - \theta)$
			$\text{AB} = 2\sin\frac{\pi}{5}$	**A1**		**AG**
				[5]		

Question			Answer	Marks	AO	Guidance
10			**DR**			
			$= 2\int_0^{\frac{1}{2}} \frac{1}{(x-\frac{1}{2})^2+\frac{3}{4}}\,\mathrm{d}x$	**M1***	**3.1a**	completing the square correctly on $x^2 - x + 1$; need not be in integral but must be seen
			$= \left[\frac{4}{\sqrt{3}}\arctan\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right]_0^{\frac{1}{2}}$ or $\left[\frac{4}{\sqrt{3}}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right]_0^{\frac{1}{2}}$	**A1**	**2.1**	oe; condone missing or incorrect limits. Could be in terms of another variable if substitution used for integration, e.g. if $u = x - \frac{1}{2}$ then $\left[\frac{4}{\sqrt{3}}\arctan\left(\frac{u}{\frac{\sqrt{3}}{2}}\right)\right]_{-\frac{1}{2}}^{0}$ or if $\frac{\sqrt{3}}{2}\tan u = x - \frac{1}{2}$ then $\left[\frac{4}{\sqrt{3}}u\right]_{-\frac{\pi}{6}}^{0}$. Condone $\left[\frac{4}{\sqrt{3}}\arctan\left(\frac{x}{\frac{\sqrt{3}}{2}}\right)\right]_{-\frac{1}{2}}^{0}$ if the correct substitution has been clearly made. Must be seen.
			$= \frac{4}{\sqrt{3}}\left(\arctan(0) - \arctan\left(\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right)$	**M1dep**	**2.1**	using correct limits correctly, including following any substitution. Substitution into (or evaluation of) any non-zero terms must be seen
			$= \frac{2\pi}{3\sqrt{3}}$	**A1cao**	**1.1**	or simplified, exact equivalent. www.
				[4]		

Question			Answer	Marks	AO	Guidance
11	**(a)**		l_1 has direction vector $a\mathbf{i} + b\mathbf{j} + \mathbf{k}$	**M1**	**3.1a**	direction vector for line l_1 soi
			$(-\mathbf{i} + 2\mathbf{k}).(a\mathbf{i} + b\mathbf{j} + \mathbf{k}) = 0$ or $-a + 2 = 0$	**M1**	**1.1**	scalar product = 0
			$\Rightarrow a = 2$	**A1**	**1.1**	
			$2 - \lambda = 3 + 2\mu$, $1 = -1 + \mu b$, $3 + 2\lambda = 2 + \mu$	**M1**	**2.1**	equating coordinates of two lines to get two equations in λ and μ or three equations in λ, μ and b. Could still be in terms of a.
			$\Rightarrow \lambda + 2\mu = -1$, $2\lambda - \mu = -1$			
			$\Rightarrow \lambda = -0.6$ or $\mu = -0.2$	**A1**	**1.1**	finding λ or μ
			Point of intersection is $\left(\frac{13}{5}, 1, \frac{9}{5}\right)$	**A1**	**3.2a**	or $x = \frac{13}{5}, y = 1, z = \frac{9}{5}$ but do not accept a position vector
				[6]		
11	**(b)**	**(i)**	parallel if $a\mathbf{i} + b\mathbf{j} + \mathbf{k} = \lambda(-\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$	**M1**	**3.1a**	soi
			$\lambda = 0.5$	**A1**	**1.1**	or one correct value of a or b (not from incorrect working)
			$\Rightarrow a = -\frac{1}{2}$, $b = 2$	**A1**	**1.1**	
				[3]		

Question			Answer	Marks	AO	Guidance
11	(b)	(ii)	distance between lines = $\frac{\lvert\mathbf{u}\times\mathbf{v}\rvert}{\lvert\mathbf{v}\rvert}$ where			
			$\mathbf{u} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} - (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	M1*	3.1a	vector between one point on each line, e.g. $\mathbf{i} - 2\mathbf{j} - \mathbf{k}$; could use any such points. Must be seen or implied by correct vector product. Do not condone use of direction vector.
			$\mathbf{v} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ $\mathbf{u} \times \mathbf{v} = \mathbf{j} - 2\mathbf{k}$	M1dep	1.1	vector product with their $\mathbf{u}$ and a multiple of the direction vector; allow a sign slip only
				A1	1.1	correct vector product for their vectors. Cannot be implied.
			$\lvert \mathbf{u} \times \mathbf{v} \rvert = \sqrt{5}$ $\quad \lvert \mathbf{v} \rvert = \sqrt{21}$	M1dep	1.1	modulus formula for both of their vectors
			distance = $\sqrt{\frac{5}{21}}$	A1	1.1	oe 0.49 or better, www
			Alternative method 1 Minimise distance between $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $(2-\lambda)\mathbf{i} + (1+4\lambda)\mathbf{j} + (3+2\lambda)\mathbf{k}$			
			$(1+\lambda)\mathbf{i} - (2+4\lambda)\mathbf{j} - (1+2\lambda)\mathbf{k}$	M1		considering the vector between a point on one line and a general point on the other; could use any such points.
			$\lvert(1+\lambda)\mathbf{i} - (2+4\lambda)\mathbf{j} - (1+2\lambda)\mathbf{k}\rvert$			
			$21\lambda^2 + 22\lambda + 6$	A1		or $\sqrt{21\lambda^2 + 22\lambda + 6}$, correct for their distance or distance squared
			$42\lambda + 22 = 0$ or $21\left(\lambda + \frac{11}{21}\right)^2 + \frac{5}{21}$	M1		minimising their distance or distance squared; must be seen
			$\lambda = -\frac{11}{21}$	A1		soi by correct answer and completed square form
			distance = $\sqrt{\frac{5}{21}}$	A1		oe 0.49 or better, www

			Alternative method 2			
			$\begin{pmatrix}-1-t\\2+4t\\1+2t\end{pmatrix}$	**M1**		considering the vector between a general point on one line and a general point on the other; could be in terms of any variable, e.g. $(-1-(\lambda-\mu))\mathbf{i}+(2+4(\lambda-\mu))\mathbf{j}+(1+2(\lambda-\mu))\mathbf{k}$
			$\begin{pmatrix}-1-t\\2+4t\\1+2t\end{pmatrix}\cdot\begin{pmatrix}-1\\4\\2\end{pmatrix}$	**M1**		considering the scalar product between this vector and any multiple of the direction vector
			$-(-1-t)+4(2+4t)+2(1+2t)=0$	**M1**		writing scalar product as a sum and equating to zero. Soi by $t=-\frac{11}{21}$.
			$t=-\frac{11}{21}$	**A1**		solving their correct equation correctly
			distance = $\sqrt{\frac{5}{21}}$	**A1**		oe 0.49 or better, www
			Alternative method 3			
			u = 2**i** + **j** + 3**k** – (3**i** – **j** + 2**k**) = –**i** +2**j** + **k**	**M1***		vector between one point on each line, e.g. **i** – 2**j** – **k**; could use any such points. Must be seen or implied by correct scalar product. Do not condone use of direction vector.
			$\cos\theta=\frac{(-\mathbf{i}+4\mathbf{j}+2\mathbf{k}).(-\mathbf{i}+2\mathbf{j}+\mathbf{k})}{\sqrt{(-1)^2+4^2+2^2}\sqrt{(-1)^2+2^2+1^2}}$	**M1dep**		scalar product with their **u** and a multiple of the direction vector; allow a sign slip only. $\cos\theta$ must be seen.
			$\cos\theta=\frac{11}{\sqrt{6}\sqrt{21}}$	**A1**		implied by $\theta=11.49°$ or $\theta=0.2005$ rad
			distance = $\sqrt{(-1)^2+2^2+1^2}\sin\theta$	**M1dep**		$\lvert\mathbf{u}\rvert\sin\theta$ ft their values; dependent on M1M1 having been awarded
			distance = $\sqrt{\frac{5}{21}}$	**A1**		oe 0.49 or better, www
				[5]		

Question	Answer	Marks	AO	Guidance
12	**DR**			
	$\alpha + \frac{2}{\alpha} + \beta + (-\beta) = 1$	**M1**	**3.1a**	sum of roots, allow a sign error. No missing terms except for those which have cancelled.
	$\Rightarrow \alpha^2 - \alpha + 2 = 0$	**A1**	**1.1**	any correct quadratic equation e.g. $\alpha^2 + 2 = \alpha$; soi by correct α
	$\frac{1}{2} + \mathrm{i}\frac{\sqrt{7}}{2},\ \frac{1}{2} - \mathrm{i}\frac{\sqrt{7}}{2}$	**A1**	**2.2a**	
	$\alpha . \frac{2}{\alpha} . \beta . (-\beta) = 18$	**M1**	**3.1a**	product of roots, allow a sign error. No missing terms except for those which have cancelled.
	$\Rightarrow 3\mathrm{i}, -3\mathrm{i}$	**A1**	**1.1**	
	$c = 2 + \alpha\beta - \alpha\beta + \frac{2\beta}{\alpha} - \frac{2\beta}{\alpha} - \beta^2$	**M1**	**3.1a**	using product of roots two at a time, soi by correct numerical work. Allow only a sign error on RHS, not on c. No missing terms except for those which have cancelled. Condone missing brackets. Or for substituting a root correctly into equation if d already found.
	$-d = 2\beta - 2\beta - \alpha\beta^2 - \frac{2\beta^2}{\alpha}$	**M1**	**1.1**	using product of roots three at a time, soi by correct numerical work. Allow only a sign error on RHS, not on d. No missing terms except for those which have cancelled. Condone missing brackets. Or for substituting a root correctly into equation if c already found.
	so $c = 11$ and $d = -9$.	**A1**	**2.1**	
	Alternative method 1			
	$(z-\alpha)\left(z - \frac{2}{\alpha}\right)(z-\beta)(z+\beta) = 0$			
	$\Rightarrow \left(z^2 + \left(-\frac{2}{\alpha} - \alpha\right)z + 2\right)\left(z^2 - \beta^2\right) = 0$	**M1** **M1**		expanding comparing coefft of x^3 and constants
	$\Rightarrow (z^2 - z + 2)(z^2 + 9) = 0$	**A1**		
	so $c = 11$ and $d = -9$	**A1,A1**		
	$z^2 - z + 2 = 0 \Rightarrow z = \frac{1 \pm \sqrt{-7}}{2} = \frac{1}{2} \pm \frac{7}{2}\mathrm{i}$	**M1A1**		
	$z^2 + 9 = 0 \Rightarrow z = \pm 3\mathrm{i}$	**B1**		

Question			Answer	Marks	AO	Guidance
			Alternative method 2			
			$\alpha + \frac{2}{\alpha} + \beta + (-\beta) = 1$	**M1**		sum of roots, allow a sign error
			$\Rightarrow \alpha^2 - \alpha + 2 = 0$	**A1**		any correct quadratic equation e.g. $\alpha^2 + 2 = \alpha$
			$\alpha = \frac{1}{2} + \mathrm{i}\frac{\sqrt{7}}{2},\ \frac{2}{\alpha} = \frac{1}{2} - \mathrm{i}\frac{\sqrt{7}}{2}$	**A1**		
			$\alpha.\frac{2}{\alpha}.\beta.(-\beta) = 18$	**M1**		product of roots, allow a sign error
			$\Rightarrow \beta = 3\mathrm{i},\ -\beta = -3\mathrm{i}$	**A1**		
			$(z^2 - z + 2)(z^2 + 9) = 0$	**M1**		at least one quadratic factor identified
			$z^4 - z^3 + 11z^2 - 9z + 18 = 0$	**M1**		multiplying their quadratic factors
			so $c = 11$ and $d = -9$.	**A1**		
			Alternative method 3			
			$\alpha + \frac{2}{\alpha} + \beta + (-\beta) = 1$	**M1**		sum of roots, allow a sign error
			$\Rightarrow \alpha^2 - \alpha + 2 = 0$	**A1**		Any correct quadratic equation e.g. $\alpha^2 + 2 = \alpha$
			$\frac{1}{2} + \mathrm{i}\frac{\sqrt{7}}{2},\ \frac{1}{2} - \mathrm{i}\frac{\sqrt{7}}{2}$	**A1**		
			$\alpha.\frac{2}{\alpha}.\beta.(-\beta) = 18$	**M1**		product of roots, allow a sign error
			$\Rightarrow 3\mathrm{i}, -3\mathrm{i}$	**A1**		
			e.g. $(3\mathrm{i})^4 - (3\mathrm{i})^3 + c(3\mathrm{i})^2 + d(3\mathrm{i}) + 18 = 0$	**M1**		substituting any root into equation correctly
			$81 - 9c + 18 = 0$ and $27 + 3d = 0$	**M1**		equating real and imaginary parts leading to two equations
			so $c = 11$ and $d = -9$.	**A1**		
				[8]		

Question			Answer	Marks	AO	Guidance
13	(a)		$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{2}{x}y = \frac{2+x^2}{x}$	M1	3.1a	division by x seen. Condone errors on RHS.
			$\text{IF} = \mathrm{e}^{\int -\frac{2}{x}\mathrm{d}x}$	M1	1.1	integrating factor
			$= \mathrm{e}^{-2\ln x} = x^{-2}$	A1	1.1	**AG** An intermediate step must be seen before the final answer. Condone errors on RHS. Requires M1M1.
				[3]		
	(b)		$\frac{\mathrm{d}}{\mathrm{d}x}(yx^{-2}) = \frac{2+x^2}{x^3}$ or $yx^{-2} = \int \frac{2+x^2}{x^3}\mathrm{d}x$	M1	2.1	multiplying through by x^{-2} (allow FT from an incorrect rearrangement in (a) if clear) and writing as an exact derivative or setting up integral
			$yx^{-2} = -\frac{1}{x^2} + \ln x$ $[+c]$	A1	1.1	
			$y = -1 + x^2 \ln x + cx^2$			
			$0 = -1 + c \Rightarrow c = 1$	M1	1.1	substituting $x = 1, y = 0$ in leading to a value for c
			$y = x^2 \ln x + x^2 - 1$	A1cao	2.1	oe
			$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\ln x + x + 2x$	M1*		differentiating their y, allow a slip
			$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\ln x + 3x$	A1		correctly (do not FT), need not be simplified
			$2x\ln x + 3x = 0 \Rightarrow x(2\ln x + 3) = 0$	M1dep		setting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and meaningfully attempting to solve; soi by correct final answer
			$x = \mathrm{e}^{-\frac{3}{2}}$ only	A1		www, do not condone $\frac{\mathrm{d}y}{\mathrm{d}x}$ used for $\frac{\mathrm{d}}{\mathrm{d}x}$. Must dismiss $x = 0$ if previously given
			Alternative method for last 4 marks			
			$\left[\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right] - 2y = 2 + x^2$	B1	3.1a	could be stated anywhere or implied by next line
			$-2(x^2\ln x + x^2 - 1) = 2 + x^2$	M1	1.1	setting up an equation in x only, cannot be implied
			$x^2(3 + 2\ln x) = 0 \Rightarrow x = \cdots$	M1	1.1	re-arranging and attempting to solve; soi by correct final answer
			$x = \mathrm{e}^{-\frac{3}{2}}$ only	A1	2.2a	www, do not condone $\frac{\mathrm{d}y}{\mathrm{d}x}$ used for $\frac{\mathrm{d}}{\mathrm{d}x}$. Must dismiss $x = 0$ if previously given
				[8]		

Question			Answer	Marks	AO	Guidance
14	(a)		$\cosh^2 x + \sinh^2 x = \frac{1}{4}(e^x + e^{-x})^2 + \frac{1}{4}(e^x - e^{-x})^2$	M1	2.1	LHS correctly written in exponential form. Must be seen.
			$= \frac{1}{4}(e^{2x} + 2 + e^{-2x} + e^{2x} - 2 + e^{-2x})$			
			$= \frac{1}{2}(e^{2x} + e^{-2x}) = \cosh 2x$	A1	2.2a	**AG** expanded and simplified to $\frac{1}{2}(e^{2x} + e^{-2x})$ must be seen before final answer. Complete argument required, www.
				[2]		
14	(b)		$\begin{pmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$	M1	2.1	finding images by matrix multiplication or at least two correct images
			$= \begin{pmatrix} 0 & \cosh x & \sinh x & \cosh x + \sinh x \\ 0 & \sinh x & \cosh x & \cosh x + \sinh x \end{pmatrix}$	A1	1.1	or all correct images found
			Suppose vertices are O, P, Q, R respectively $OP^2 = OQ^2 = \cosh^2 x + \sinh^2 x$ $PR^2 = (\cosh x + \sinh x - \sinh x)^2 + (\cosh x + \sinh x - \cosh x)^2 = \cosh^2 x + \sinh^2 x$ $QR^2 = (\cosh x + \sinh x - \sinh x)^2 + (\cosh x + \sinh x - \cosh x)^2 = \cosh^2 x + \sinh^2 x$	M1	3.1a	explicitly giving two side lengths of image or showing that there are two pairs of parallel sides. Lengths or their squares could be used.
			So $OP = OQ = PR = QR$ so rhombus	A1	2.2a	Complete method to show image is rhombus and concluding, e.g. showing all four sides are equal (working must be seen for PR and QR). Could also show there are two pairs of parallel sides and two adjacent sides are equal length; that P and Q are reflections in $y = x$, R lies on $y = x$ and $OP = PR$; that two diagonals are perpendicular bisectors of each other. Results must be shown not just stated.
			$\det \mathbf{M} = \cosh^2 x - \sinh^2 x$	M1	2.1	must be seen; or for a complete method to find area of image
			$=1 \Rightarrow$ transformation preserves area, so area of rhombus is 1	A1	3.2a	$= 1$, and some comment about the effect of the transformation if det **M** used. Accept "[area scale] factor = 1", "area stays same", "area = $1 \times 1 = 1$" etc., but not just "so area = 1".
				[6]		

Question			Answer	Marks	AO	Guidance
14	(c)		$2 = \sqrt{\cosh^2 x + \sinh^2 x}$ or $2 = \sqrt{\cosh 2x}$	M1	3.1a	or $4 = \cosh^2 x + \sinh^2 x$ or $4 = \cosh 2x$ oe
			$x = \frac{1}{2}\ln(4 + \sqrt{15})$	A1	1.1	oe e.g. $x = \ln\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}\right)$. Do not condone missing brackets. Answer must be supported by some working. ISW if error manipulating logs only.
				[2]		

Question			Answer	Marks	AO	Guidance
15	(a)		$(3-\mathrm{e}^{4\mathrm{i}\theta})(3-\mathrm{e}^{-4\mathrm{i}\theta}) = 9 - 3\mathrm{e}^{4\mathrm{i}\theta} - 3\mathrm{e}^{-4\mathrm{i}\theta} + 1$	M1	2.1	expanding correctly and fully to give at least three terms, allow $10 - 3(\mathrm{e}^{4\mathrm{i}\theta} + \mathrm{e}^{-4\mathrm{i}\theta})$. Must be seen. Condone $\mathrm{e}^0 = 1$.
			$= 10 - 6\cos 4\theta$	A1	1.1	www. Condone only incorrect values quoted for a and b. No intermediate step required.
				[2]		
15	(b)		$C + \mathrm{i}S = \mathrm{e}^{\mathrm{i}\theta} + \frac{1}{3}\mathrm{e}^{5\mathrm{i}\theta} + \frac{1}{9}\mathrm{e}^{9\mathrm{i}\theta} + \frac{1}{27}\mathrm{e}^{13\mathrm{i}\theta} + \ldots$	M1	2.1	at least two terms of series in exponential form soi by correct GP formula
			$= \mathrm{e}^{\mathrm{i}\theta} + \frac{1}{3}(\mathrm{e}^{\mathrm{i}\theta})^5 + \frac{1}{9}(\mathrm{e}^{\mathrm{i}\theta})^9 + \frac{1}{27}(\mathrm{e}^{\mathrm{i}\theta})^{13} + \ldots$	A1	1.1	writing series as powers of $\mathrm{e}^{\mathrm{i}\theta}$ or identifying geometric series with correct first term and common ratio soi by correct GP formula
			This is a GP with $a = \mathrm{e}^{\mathrm{i}\theta}, r = \frac{1}{3}(\mathrm{e}^{\mathrm{i}\theta})^4$			
			so $C + \mathrm{i}S = \dfrac{\mathrm{e}^{\mathrm{i}\theta}}{1 - \frac{1}{3}(\mathrm{e}^{\mathrm{i}\theta})^4}$	M1	3.1a	correct use of sum to infinity formula. Must be seen.
			$= \dfrac{3\mathrm{e}^{\mathrm{i}\theta}}{3 - \mathrm{e}^{4\mathrm{i}\theta}}$	A1	2.1	**AG** www
				[4]		
15	(c)		$C + \mathrm{i}S = \dfrac{3\mathrm{e}^{\mathrm{i}\theta}(3 - \mathrm{e}^{-4\mathrm{i}\theta})}{(3 - \mathrm{e}^{4\mathrm{i}\theta})(3 - \mathrm{e}^{-4\mathrm{i}\theta})}$	M1	3.1a	multiplying numerator and denominator by a multiple of $3 - \mathrm{e}^{-4\mathrm{i}\theta}$; multiplication of numerator must be shown but denominator could be given immediately as $10 - 6\cos 4\theta$.
			$C + \mathrm{i}S = \dfrac{9(\cos\theta + \mathrm{i}\sin\theta) - 3(\cos 3\theta - \mathrm{i}\sin 3\theta)}{10 - 6\cos 4\theta}$	M1	2.1	converting to sine and cosine form after denominator has been simplified to a real expression. Must be seen. No errors allowed FT their expression. Condone only missing brackets around (-3θ).
			$C = \dfrac{9\cos\theta - 3\cos 3\theta}{10 - 6\cos 4\theta}$	A1	2.2a	**AG** www. Condone only missing brackets around (-3θ).
				[3]		

Question			Answer	Marks	AO	Guidance
16	(a)		**DR**			
			$A=\int_0^4 \frac{x+3}{\sqrt{x^2+9}}\,dx=\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx+\int_0^4 \frac{3}{\sqrt{x^2+9}}\,dx$	**M1***	**3.1a**	splitting fraction, allow one numerical slip only
			$=\left[\frac{1}{2}.2\sqrt{x^2+9}+3\ln(x+\sqrt{x^2+9})\right]_0^4$	**M1**	**1.1**	$k\sqrt{x^2+9}$ or substituting $u=x^2+9$ and reaching $ku^{\frac{1}{2}}$
				A1	**1.1**	$k=1$
				B1	**1.1**	$3\ln(x+\sqrt{x^2+9})$ or $3\,\text{ar}\sinh\left(\frac{x}{3}\right)$
			$=5+3\ln 9-3-3\ln 3$ or $=5+3\ln 3-3[-0]$	**M1dep**	**1.1**	substituting limits into an integrated expression and converting to lns if required. Condone missing ln 1 term.
			$=2+\ln 27$	**A1cao**	**2.1**	no acceptable equivalents. www.
			Alternative method 1			
			let $x=3\sinh u$, $dx=3\cosh u\,du$	**M1***		substituting $x=3\sinh u$ and $dx=3\cosh u\,du$
			$A=\int_0^{ar\sinh\frac{4}{3}} \frac{3\sinh u+3}{3\cosh u}3\cosh u\,du$	**A1**		$\int \frac{3\sinh u+3}{3\cosh u}3\cosh u\,du$
			$=\left[3\cosh u+3u\right]_0^{ar\sinh\frac{4}{3}}$	**A1**		$\left[3\cosh u+3u\right]$. Ignore incorrect limits until the point of substitution.
				M1dep		substituting correct limits and converting arsinh $\frac{4}{3}$ to lns
			$=3\sqrt{1+\frac{16}{9}}+3\ln\left(\frac{4}{3}+\sqrt{1+\frac{16}{9}}\right)-3$	**B1**		$\cosh(\text{arsinh}\frac{4}{3})=\sqrt{1+\frac{16}{9}}$
			$=2+\ln 27$	**A1cao**		no acceptable equivalents. www.

			Alternative method 2 $A = \int_0^4 \frac{x+3}{\sqrt{x^2+9}}\,dx$ $= \left[(x+3)\,\text{arsinh}\frac{x}{3}\right]_0^4 - \int_0^4 \text{arsinh}\frac{x}{3}\,dx$			
			$= \left[(x+3)\,\text{arsinh}\frac{x}{3} - x\,\text{arsinh}\frac{x}{3}\right]_0^4 + \int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx$	**M1***		integrating by parts twice until an integral of the form $k\int \frac{x}{\sqrt{x^2+9}}\,dx$ remains
			$= \left[3\,\text{arsinh}\frac{x}{3} + \frac{1}{2}.2\sqrt{x^2+9}\right]_0^4$	**M1** **A1**	1.1 1.1	$k\sqrt{x^2+9}$ or substituting $u = x^2+9$ $k = 1$
				B1	1.1	$3\ln(x+\sqrt{x^2+9})$ or $3\,\text{arsinh}\left(\frac{x}{3}\right)$
			$= 5 + 3\ln 9 - 3 - 3\ln 3$ or $= 5 + 3\ln 3 - 3[-0]$	**M1dep**	1.1	substituting limits into an integrated expression and converting to lns if required. Condone missing ln 1 term.
			$= 2 + \ln 27$	**A1cao**	2.1	no acceptable equivalents. www.
			Alternative method 3 let $x = 3\tan u$, $dx = 3\sec^2 u\,du$	**M1***		substituting $x = 3\tan u$ and $dx = 3\sec^2 u\,du$
			$A = \int_0^{\arctan\frac{4}{3}} \frac{3\tan u + 3}{3\sec u} 3\sec^2 u\,du$	**A1**		$\int \frac{3\tan u+3}{3\sec u} 3\sec^2 u\,du$ or $\int (3\sec u\tan u + 3\sec u)du$
			$= [3\sec u + 3\ln\lvert\sec u + \tan u\rvert]_0^{\arctan\frac{4}{3}}$	**A1**		$[3\sec u + 3\ln\lvert\sec u + \tan u\rvert]$. Ignore incorrect limits until the point of substitution
			$= 3\left(\frac{5}{3}\right) + 3\ln 3 - 3 - 3\ln 1$	**M1dep**		substituting correct limits. Condone missing ln 1 term.
				B1		$\sec\left(\arctan\frac{4}{3}\right) = \frac{5}{3}$
			$= 2 + \ln 27$	**A1cao**		no acceptable equivalents. www.
				[6]		

Question			Answer	Marks	AO	Guidance
16	(b)		**DR**			
			$V=\pi\int_0^4 \frac{(x+3)^2}{x^2+9}[\mathrm{d}x]$	**B1**	**3.1a**	correct integral and limits (could be seen later)
			$=[\pi]\int_0^4\left(1+\frac{6x}{x^2+9}\right)[\mathrm{d}x]$	**M1***	**1.1**	writing as sum of proper fractions; ignore incorrect multiples of π. soi by correct integral.
			$=[\pi]\left[x+3\ln(x^2+9)\right]_0^4$	**M1**	**1.1**	inspection or substituting $u=x^2+9$, $\mathrm{d}u=2x\mathrm{d}x$; ignore incorrect multiples of π
				A1	**1.1**	$[\pi]\left[x+3\ln(x^2+9)\right]$
			$=\pi\left[4+3\ln 25-3\ln 9\right]$	**M1dep**	**1.1**	substituting limits with function fully integrated
			$=\pi\left[4+3\ln\frac{25}{9}\right]$	**A1cao**	**2.1**	oe but must be in the form $\pi\left(a+b\ln\left(\frac{c}{d}\right)\right)$. Accept e.g. $a=4$, $b=3$, $c=25$, $d=9$.
			Alternative method 1			
			$V=\pi\int_0^4\frac{(x+3)^2}{x^2+9}\mathrm{d}x$	**B1**		correct integral and limits
			let $x=3\tan u$, $\mathrm{d}x=3\sec^2 u\mathrm{d}u$	**M1***		substitution
			$=[\pi]\int_0^{\arctan\frac{4}{3}}\frac{(3\tan u+3)^2}{9\sec^2 u}3\sec^2 u\,\mathrm{d}u$			
			$=[\pi]\int_0^{\arctan\frac{4}{3}}(3\sec^2 u+6\tan u)\,\mathrm{d}u$	**M1**		expanding and using $1+\tan^2 u=\sec^2 u$
			$=[\pi]\left[3\tan u+6\ln(\sec u)\right]_0^{\arctan\frac{4}{3}}$	**A1**		$=\left[3\tan u+6\ln(\sec u)\right]$
			$=[\pi]\left(3\tan\left(\arctan\frac{4}{3}\right)+6\ln\left\|\sec\left(\arctan\frac{4}{3}\right)\right\|\right)$	**M1dep**		substituting limits
			$=\pi\left[4+6\ln\frac{5}{3}\right]$	**A1cao**		oe but must be in the form $\pi\left(a+b\ln\left(\frac{c}{d}\right)\right)$. Accept e.g. $a=4$, $b=6$, $c=5$, $d=3$.

			Alternative method 2			
			$V=\pi\int_0^4 \frac{(x+3)^2}{x^2+9}\,\mathrm{d}x$	**B1**		correct integral and limits
			Let $x = 3\sinh u$, $\mathrm{d}x = 3\cosh u\,\mathrm{d}u$	**M1***		substitution
			$= [\pi]\int_0^{\operatorname{arsinh}\frac{4}{3}} \frac{(3\sinh u+3)^2}{9\cosh^2 u} 3\cosh u\,\mathrm{du}$			
			$= [\pi]\int_0^{\operatorname{arsinh}\frac{4}{3}} (3\cosh u + 6\tanh u)\,\mathrm{du}$	**M1**		expanding and using $\sinh u \tanh u = \cosh u - \operatorname{sech} u$
			$= [\pi][3\sinh u + 6\ln(\cosh u)]_0^{\operatorname{arsinh}\frac{4}{3}}$	**A1**		$= [3\sinh u + 6\ln(\cosh u)]$
			$= 3\left(\frac{4}{3} + 2\ln\left\lvert\cosh(\operatorname{arsinh}\frac{4}{3})\right\rvert\right)$	**M1dep**		substituting limits
			$= 4 + 6\ln\frac{5}{3}$	**A1**		oe but must be in the form $\pi\left(a+b\ln\left(\frac{c}{d}\right)\right)$. Accept e.g. a =4, b = 6, $c = 5$, $d = 3$.
				[6]		

Question			Answer	Marks	AO	Guidance
17	(a)	(i)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.2(20 - h)$	B1 [1]	3.3	or any rearrangement. ISW.
17	(a)	(ii)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow h = 20$	B1 [1]	3.4	or converse. Could be in words. Do not condone $h \to 20$. Accept their expression for $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$ from (a)(i) provided it is correct or of the form $k(20 - h)$.
17	(a)	(iii)	$\int \frac{\mathrm{d}h}{20-h} = \int 0.2\ \mathrm{d}t$ or $he^{0.2t} = \int 4e^{0.2t}\mathrm{d}t$	M1*	2.1	correctly separating variables or using integrating factor for their DE, leading to an integral soi. If a CF/PI approach used award for a correct method leading to the CF and a value for the PI.
			$-\ln(20 - h) = 0.2t\ [+ c]$	A1	1.1	or $h\mathrm{e}^{0.2t} = 20\mathrm{e}^{0.2t}[+c]$ or $h = Be^{-0.2t} + 20$. Condone missing brackets if recovered.
			$20 - h = A\mathrm{e}^{-0.2t}$			
			when $t = 0$, $h = 0 \Rightarrow A = 20$	M1dep	3.4	substituting initial conditions into their equation leading to a value for A, B or c
			$\Rightarrow h = 20(1 - \mathrm{e}^{-0.2t})$	A1 [4]	2.1	**AG** www, condone missing brackets earlier if recovered but do not condone $\frac{\mathrm{d}h}{\mathrm{d}t}$ or $\frac{\mathrm{d}}{\mathrm{d}x}$ used for $\frac{\mathrm{d}}{\mathrm{d}t}$
17	(a)	(iv)	When $t = 5$, $h = 20(1 - \frac{1}{e}) = 12.64$ 25.28… [$\neq 20$], so model does not fit the data well	M1 A1 [2]	3.4 3.5a	substituting $t = 5$ into given expression for h or $12.64 \ldots \neq 10$ soi but 10 oe must be seen, e.g. $\frac{20}{2}$ or "half the maximum height". Comment of inconsistency required (must not say that the value is consistent).

Question			Answer	Mark	AO	Guidance
17	(b)	(i)	$\lambda^2 + 0.3\lambda + 0.02 = 0$ $\Rightarrow \lambda = -0.1$ or -0.2 CF: $[h =]A\,\mathrm{e}^{-0.1t} + B\mathrm{e}^{-0.2t}$ PI: $[h =]k \Rightarrow 0.02k = 0.4 \quad [k = 20]$ $\Rightarrow h = A\mathrm{e}^{-0.1t} + B\mathrm{e}^{-0.2t} + 20$	M1 A1 M1 A1 [4]	1.1 1.1 2.1 1.1	$= 0$ must be seen unless implied by correct roots correct form of particular integral (any polynomial with a constant term) and substituting into DE correctly, soi by correct k must see $h =$
17	(b)	(ii)	When $t \to \infty$, $\mathrm{e}^{-kt} \to 0$ so $h \to 20$ [so final height is 20 m]	B1 [1]	3.4	must state that each exponential term (or their sum) tends to 0. Accept general term with negative exponent, e.g. e^{-kt}, but not e^{-t}. Condone $h = 20$.
17	(b)	(iii)	$t = 0$: $A + B + 20 = 0$ $\frac{\mathrm{d}h}{\mathrm{d}t} = -0.1A\mathrm{e}^{-0.1t} - 0.2B\mathrm{e}^{-0.2t}$ $\frac{\mathrm{d}h}{\mathrm{d}t} = 2.9 : -0.1A - 0.2B = 2.9$ $A = -11, B = -9 \quad [h = 20 - 11\mathrm{e}^{-0.1t} - 9\mathrm{e}^{-0.2t}]$ When $t = 5$, $h = 10.017\ldots \approx 10$ so model does fit the data well	M1FT M1FT M1FT A1 A1 [5]	3.4 3.1b 1.1 1.1 3.5a	substituting $t = 0$, $h = 0$ correctly into their formula for h from part (b)(i) to form an equation in A and B; must be seen differentiate their h from part (b)(i) correctly, soi by correct second equation in A and B substituting $t = 0$ and $\frac{\mathrm{d}h}{\mathrm{d}t} = 2.9$ to form (a second) equation in A and B; must be seen or $10.017 \ldots \times 2 = 20.03 \ldots$ so does fit the data well. Accept "half the maximum height" oe for 10.

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