EXAM PAPERS PRACTICE

- Boost your performance and confidence with these topic-based exam questions
- Practice questions created by actual examiners and assessment experts
- Detailed mark scheme
- Suitable for all boards
- Designed to test your ability and thoroughly prepare you


Time allowed

| Score | Percentage |
| --- | --- |
| / | % |

# Maths

## Mark Scheme

## AQA
## AS & A LEVEL

## 3.11 J: Vectors

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|---|---|---|---|---|
| 7(a)(i) | $\overrightarrow{AB}=\begin{bmatrix}6\\5\\3\end{bmatrix}-\begin{bmatrix}2\\1\\3\end{bmatrix}=\begin{bmatrix}4\\4\\0\end{bmatrix}$ | M1<br>A1 | <br>2 | Penalise use of co-ordinates at first occurrence only |
| (ii) | $\begin{bmatrix}4\\4\\0\end{bmatrix}=4\begin{bmatrix}1\\1\\0\end{bmatrix}\Rightarrow$ parallel | E1 | 1 | Needs comment "same direction" Or "same gradient" (**Or** by scalar product) |
| (iii) | $\begin{bmatrix}2\\-3\\-1\end{bmatrix}=\begin{bmatrix}6\\1\\-1\end{bmatrix}+\lambda\begin{bmatrix}1\\1\\0\end{bmatrix}$<br>is satisfied by $\lambda=-4$ | M1<br>A1 | <br>2 | $\lambda=-4$ satisfies 2 equations |
| (b)(i) | $l_2$ has equation<br>$r=\begin{bmatrix}4\\1\\1\end{bmatrix}+\lambda\left[\begin{bmatrix}4\\1\\1\end{bmatrix}-\begin{bmatrix}2\\-3\\-1\end{bmatrix}\right]=\begin{bmatrix}4\\1\\1\end{bmatrix}+\lambda\begin{bmatrix}2\\4\\2\end{bmatrix}$ | M1A1 | 2 | **Or**<br>$r=\begin{bmatrix}2\\-3\\-1\end{bmatrix}+t\begin{bmatrix}2\\4\\2\end{bmatrix}$ M1 calculate and use direction vector A1 all correct |
| (ii) | $\begin{bmatrix}1\\2\\1\end{bmatrix}\bullet\begin{bmatrix}4\\0\\-4\end{bmatrix}=4-4=0$<br>$\Rightarrow 90°$ (or perpendicular) | M1A1<br>A1F | <br>3 | Clear attempt to use directions of $AC$ and $l_2$ in scalar product<br>Accept a correct ft value of $\cos\theta$ |
| | **Total** | | **10** | |

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| 6(a)(i) | $\overrightarrow{OC} = 2\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ -2 \end{bmatrix}$ | B1 | 1 | (Penalise coordinates once only) |
| (ii) | $\overrightarrow{AB} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$ | M1<br>A1 | <br>2 | $\overrightarrow{OA} - \overrightarrow{OB}$ or $\overrightarrow{OB} - \overrightarrow{OA}$ or 2/3 correct cpts.<br>A0 for line $AB$ |
| (b)(i) | $AC^2 = (6-2)^2 + (4-4)^2 + (-1-2)^2 = 25$<br>$AC = 5$ | M1<br>A1 | <br>2 | \|Components of AC\|<br>AG |
| (ii) | $\overrightarrow{AB} \bullet \overrightarrow{AC} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} \bullet \begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix} = 4 + 6 = 10$ | M1<br>A1F | | Clear attempt to use $\overrightarrow{AB}$ and $\overrightarrow{AC}$<br>ft $\overrightarrow{AB}$ from a(ii) and/or $\overrightarrow{AC}$ from b(i) |
| | $3 \times 5 \times \cos\theta = 10$ | M1 | | Use of $\|a\|\ \|b\|\cos\theta = \mathbf{a.b}$<br>with one correct $\|\ \|$ and **a.b** evaluated |
| | $\theta = 48.189 \approx 48°$ | A1 | 4 | CAO (AWRT) |
| | **Alternative:** use of cos rule<br>Find 3rd side + use cos rule | (M2)<br>(A1F)<br>(A1) | | <br>ft on previously found vectors<br>CAO (AWRT) |
| (c) | $\overrightarrow{BP} = \begin{bmatrix} \alpha - 3 \\ \beta - 2 \\ \gamma - -1 \end{bmatrix}$ | B1 | | |
| | $\begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix} \bullet \overrightarrow{BP} = 0$ | M1 | | Their $\overrightarrow{BP}$ |
| | $4\alpha - 3\gamma - 15 = 0$ | A1 | 3 | AG convincingly obtained |
| | **Total** | | **12** | |

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| **6(a)(i)** | $\overrightarrow{BA}=\begin{bmatrix}3\\-2\\4\end{bmatrix}-\begin{bmatrix}5\\4\\0\end{bmatrix}=\begin{bmatrix}-2\\-6\\4\end{bmatrix}$ | M1A1 | 2 | Attempt $\pm\overrightarrow{BA}$ $(OA-OB$ or $OB-OA)$ |
| **(ii)** | $\overrightarrow{BC}=\begin{bmatrix}6\\2\\-4\end{bmatrix}$ | B1 | | Allow $\overrightarrow{CB}$ ; or $\begin{bmatrix}-6\\-2\\4\end{bmatrix}=\overrightarrow{BC}$ or $\overrightarrow{CB}=\begin{bmatrix}6\\2\\-4\end{bmatrix}$ May not see explicitly |
| | $\lvert\overrightarrow{BA}\rvert\left(=\sqrt{(-2)^2+(-6)^2+(4)^2}\right)=\sqrt{56}$ | B1F | | Calculate modulus of $\overrightarrow{BA}$ or $\overrightarrow{BC}$ ; for finding modulus of one of vectors they have used |
| | $\overrightarrow{BA}\bullet\overrightarrow{BC}=\begin{bmatrix}-2\\-6\\4\end{bmatrix}\bullet\begin{bmatrix}6\\2\\-4\end{bmatrix}=-12-12-16$ | M1 | | Attempt at $\overrightarrow{BA}\bullet\overrightarrow{BC}$ with numerical answer; or $\overrightarrow{AB}\bullet\overrightarrow{CB}$ |
| | | A1 | | for –40, or correct if done with multiples of vectors |

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| $\cos ABC=\dfrac{-40}{\sqrt{56}\sqrt{56}}=-\dfrac{5}{7}$ | A1 | 5 | AG (convincingly obtained)<br>Cosine rule: M1 attempt to find 3 sides<br>A1 lengths of sides<br>M1 cosine rule<br>A1F correct<br>A1 rearrange to get<br>$\cos ABC=\dfrac{-5}{7}$<br>(ft on length of sides) |

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| **6 (cont) (b)(i)** | $\begin{bmatrix}8\\-3\\2\end{bmatrix}+\lambda\begin{bmatrix}1\\3\\-2\end{bmatrix}=\begin{bmatrix}11\\6\\-4\end{bmatrix}$ $(\lambda=3)$ | M1A1 | 2 | $\lambda=3$ verified in three equations<br>M1 for $\begin{cases}11=8+\lambda\\6=-3+3\lambda\\-4=2-2\lambda\end{cases}$<br>A1 for $\lambda=3$ shown for all three equations<br>$\lambda\begin{bmatrix}1\\3\\-2\end{bmatrix}=\begin{bmatrix}11\\6\\-4\end{bmatrix}-\begin{bmatrix}8\\-3\\2\end{bmatrix}\therefore\ \lambda=3$ M1A1<br>SC: $\lambda=3$ written and nothing else: SC1 |
| **(ii)** | $\begin{bmatrix}2\\6\\-4\end{bmatrix}=2\begin{bmatrix}1\\3\\-2\end{bmatrix}$<br>$\therefore$ same direction or same gradient or parallel | E1 | 1 | |

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| (c) | $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$ | B1 | | PI; $\overrightarrow{OD}$ = correct vector expression which may involve $\overrightarrow{AD}$ |
| | $= \begin{bmatrix} 11 \\ 6 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ -6 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$ $D$ is $(9,0,0)$ | M1A1 | 3 | M1 for substituting into vector expression for $\overrightarrow{OD}$<br>NMS 3/3 |
| | **Total** | | **13** | |

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| **7(a)** | $\begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = 3 - 6 + 3 = 0$ | M1 | | attempt at sp, 3 terms, added |
| | $= 0 \Rightarrow$ perpendicular | A1 | 2 | $= 0 \Rightarrow$ perpendicular seen<br>$(\text{or } \cos\theta = 0 \Rightarrow \theta = 90^\circ)$<br>Allow $\begin{matrix} 3 \\ -6 \\ 3 \\ \hline 0 \end{matrix}$ but not $\begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix} = 0$ |
| **(b)** | $8 + 3\lambda = -4 + \mu$<br>$6 - 3\lambda = 2\mu$<br>$-9 - \lambda = 11 - 3\mu$ | M1 | | set up any two equations |
| | $\lambda = -2,\ \mu = 6$ | m1 A1 | | solve for $\lambda$ and $\mu$ |
| | verify third equation | m1 | | substitute $\lambda, \mu$ in third equation |
| | intersect at $(2, 12, -7)$ | A1 | 5 | CAO |
| | **Alt (for last two marks)**<br>substitute $\lambda$ into $l_1$ and $\mu$ into $l_2$ | (m1) | | |
| | intersect at $(2, 12, -7)$, condone $\begin{pmatrix} 2 \\ 12 \\ -7 \end{pmatrix}$ | (A1) | | $(2, 12, -7)$ found from both lines<br>Note: working for (b) done in (a): award marks in (b) |
| **7(c)** | $\overrightarrow{AP} = \begin{pmatrix} 6 \\ 12 \\ -18 \end{pmatrix}$ | M1 | | $\overrightarrow{AP} = \pm\left\{\text{their } \overrightarrow{OP} - \begin{pmatrix} -4 \\ 0 \\ 11 \end{pmatrix}\right\}$ |
| | $AP^2 = 504$ | A1F | | ft on $P$ |
| | $AB^2 = 2AP^2$ | M1 | | Calculate $AB^2$ |
| | $AB = 12\sqrt{7}$ | A1 | 4 | OE accept 31.7 or better |
| | **Total** | | **11** | |