EXAM PAPERS PRACTICE

Boost your performance and confidence with these topic-based exam questions

Practice questions created by actual examiners and assessment experts

Detailed mark scheme

Suitable for all boards

Designed to test your ability and thoroughly prepare you

Time allowed

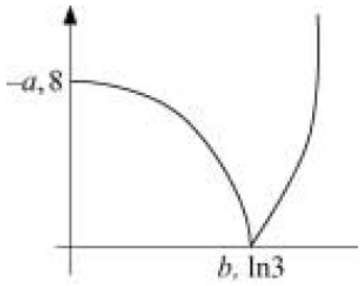
Score	Percentage
/	%

Maths

Mark Scheme

AQA
AS & A LEVEL

3.7 F: Exponentials and logarithms

5(a)	$a=-8$	B1		
	$e^{2x}-9=0$	M1		
	$e^{2x}=9$			
	$2x=\ln 9$			
	$x=\ln 3$	A1	3	AG Condone verification
(b)	$\left(e^{2x}-9\right)^2=e^{4x}-18e^{2x}+81$	B1	1	AG
(c)	$V=\pi\int y^2\,(dx)$	B1		
	$=(\pi)\int e^{4x}-18e^{2x}+81\ dx$	M1		
	$=(\pi)\left[\frac{e^{4x}}{4}-9e^{2x}+81x\right]_0^{\ln 3}$	M1 A1		1^{ST} or 2^{nd} term correct All correct
	$=(\pi)\left[\frac{e^{4x}}{4}-9e^{2x}+81x\right]_0^{\ln 3}$	M1 A1		1^{ST} or 2^{nd} term correct All correct
	$=(\pi)\left[\left(\frac{e^{\ln 81}}{4}-9e^{\ln 9}+81\ln 3\right)-\left(\frac{1}{4}-9\right)\right]$	m1		Attempt at limits with ln3
	$=\pi[81\ln 3-52]$	A1	6	
(d)		M1		Modulus graph
		A1F	2	All correct
	Total		**12**	

9(a)	$y=x^{-2}\ln x$			
	$\frac{dy}{dx}=x^{-2}\frac{1}{x}-2x^{-3}\ln x$	M1 A1 A1		Use of product or quotient each term
	$=\frac{1-2\ln x}{x^3}$	A1	4	Convincing argument $x^{-2}\times\frac{1}{x}=x^{-3}$ AG

(c)(i)	At A, $\frac{dy}{dx}=0$			
	$1-2\ln x=0$			
	$\ln x=\frac{1}{2}$	M1		Attempt at $\ln x = k$
	$x=e^{\frac{1}{2}}$	A1	2	

5(a)	$y=e^{2x}-10e^{x}+12x$			
(i)	$\frac{dy}{dx}=2e^{2x}-10e^{x}+12$	B1 B1	 2	$2e^{2x}$ remaining terms correct, no extras
(ii)	$\frac{d^2y}{dx^2}=4e^{2x}-10e^{x}$	B1F	1	ft 1 slip
(b)(i)	$2e^{2x}-10e^{x}+12=0$ $e^{2x}-5e^{x}+6=0$	B1	1	AG (be convinced)
(ii)	$z^2-5z+6=0$	M1		use of $z=e^{x}$ oe
	$z=2,3$ $z=2,\ e^{x}=2$ $x=\ln 2$ $z=3,\ e^{x}=3$ $x=\ln 3$	M1 A1	 3	finding $e^{x}=$ their 2,3 all correct AG SC: verification
				$\ln 2$ (B1) $\ln 3$ (B1)
(iii)	$x=\ln 2$: $y=e^{2\ln 2}-10e^{\ln 2}+12\ln 2$ or $2^2-10\times 2+12\ln 2$ $=4-20+12\ln 2$ $=-16+12\ln 2$ $x=\ln 3$: $y=e^{2\ln 3}-10e^{\ln 3}+12\ln 3$ $=9-30+12\ln 3$ $=-21+12\ln 3$	M1 A1 A1	 3	either substitution of their $x=\ln 2$ $\left(e^{x}=2\right)$ or their $x=\ln 3$ $\left(e^{x}=3\right)$
(iv)	$x=\ln 2$: $\frac{d^2y}{dx^2}=4e^{2\ln 2}-10e^{\ln 2}$	M1		use of; in either of their $e^{x}=2,3$ into their $\frac{d^2y}{dx^2}$

	$= 16 - 20 = -4$ $\therefore$ maximum $x = \ln 3$: $\frac{d^2y}{dx^2} = 4e^{2\ln 3} - 10e^{\ln 3}$ $= 36 - 30 = 6$ $\therefore$ minimum	A1 A1	 3	CSO CSO
	Total		**13**	

(b)(i)	$y = x \ln x$ $\frac{dy}{dx} = x \times \frac{1}{x} + \ln x$ $= \ln x + 1$	 M1 A1	 2	 use of product rule (only differentiating, 2 terms with + sign)
(ii)	$\int (\ln x + 1)\,dx = x \ln x$ $\int \ln x\,dx = x \ln x - x (+c)$	M1 A1	 2	OE; attempt at parts with $u = \ln x$
(iii)	$\int_1^5 \ln x\,dx = [x \ln x - x]_1^5$ $= (5\ln 5 - 5) - (1\ln 1 - 1)$ $5\ln 5 - 4$	 M1 A1	 2	 correct substitution of limits into their (ii) provided $\ln x$ is involved ISW
	Total		**9**	

(b)(i)	$x = 2y^3 + \ln y$ $\frac{dx}{dy} = 6y^2 + \frac{1}{y}$	 B1	 1	
(ii)	At $(2,1)$ $\frac{dx}{dy} = 6 + 1 = 7$ $\frac{dy}{dx} = \frac{1}{7}$ $(y-1) = \frac{1}{7}(x-2)$	 M1 A1√ A1	 3	 May be implied OE

9(a)(i)	$\int\left(4-e^{2x}\right)dx$			
	$=4x-\frac{1}{2}e^{2x}\ (+c)$	B1 B1	2	$4x$ $-\frac{1}{2}e^{2x}$
(ii)	$\int_0^{\ln 2}=\left[4x-\frac{1}{2}e^{2x}\right]_0^{\ln 2}$			
	$=\left[4\ln 2-\frac{1}{2}e^{2\ln 2}\right]-\left[(0)-\frac{1}{2}\left(e^0\right)\right]$	M1		Substitute both ln 2 and 0 correctly into an integrated expression
	$=4\ln 2-2+\frac{1}{2}$			Convincing
	$=4\ln 2-\frac{3}{2}$	A1	2	**AG**
(b)(i)	$x=0$ $y=4-1=3$	B1	1	
(ii)	At B, $y=0$ $4-e^{2x}=0$ $e^{2x}=4$	M1		Or reverse argument
	$x=\ln 2$	A1	2	**AG**
(c)	$\frac{dy}{dx}=-2e^{2x}$	B1		
	$x=\ln 2$, Gradient $=-2e^{2\ln 2}$ $=-8$	M1		$x=\ln 2$ into ke^{2x}
	Gradient normal $=\frac{1}{8}=\frac{1}{2e^{2\ln 2}}$	A1		OE
	Equation $y=\frac{1}{8}x-\frac{1}{8}\ln 2$	A1	4	OE
(d)	When $x=0$ $y=-\frac{1}{8}\ln 2$	M1		Attempt to integrate their line and substitute $x=0, \ln 2$
	Area $\Delta=\frac{1}{16}(\ln 2)^2$ condone – ve sign $=0.03$	A1√		$\frac{1}{2}(\text{their } y)\times\ln 2$
	Total area $=4\ln 2-\frac{3}{2}+\frac{1}{16}(\ln 2)^2=1.30$ AWRT	A1	3	CSO
	Total		**14**	

1(a)	$y = \ln x$ $\frac{dy}{dx} = \frac{1}{x}$	B1	1	penalise + c once on 1(a) or 2(a)
(b)	$y = (x+1)\ln x$ $\frac{dy}{dx} = (x+1)\times\frac{1}{x} + \ln x$	M1 A1	2	product rule
(c)	$y = (x+1)\ln x$ $\frac{dy}{dx} = \frac{1}{x} + 1 + \ln x$			
	$x = 1$: $\frac{dy}{dx} = 1+1 = 2$	M1		substitute $x = 1$ into their $\frac{dy}{dx}$
	Grad normal $= -\frac{1}{2}$	M1 A1		use of $m_1 m_2 = -1$ CSO
	$y = -\frac{1}{2}(x-1)$	A1	4	OE
	Total		**7**	

7(a)(i)	$y = (x^2-3)e^x$ $\frac{dy}{dx} = (x^2-3)e^x + 2xe^x$	M1 A1	2	product rule
(ii)	$\frac{d^2y}{dx^2} = (x^2-3)e^x + 2xe^x + 2xe^x + 2e^x$	M1 A1	2	product rule from their $\frac{dy}{dx}$
(b)(i)	$\frac{dy}{dx} = 0$ $\Rightarrow e^x(x^2+2x-3) = 0$	M1		$e^x f(x) = 0$ from $\frac{dy}{dx} = 0$
	$e^x(x+3)(x-1) = 0$	m1		attempt at factorising or use of formula
	$\therefore x = -3, 1$	A1 A1	4	first correct solution second correct solution, and no others SC No working shown: $x = -3$ B2, $x = 1$ B2
(ii)	$x = -3$ $y'' = -4e^x$ max (−0.2)	M1		Condone slip
	$x = 1$ $y'' = 4e^x$ min (10.9)	A1	2	
	Total		**10**	