Applied Math

Lesson 7

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Measures of central tendency and dispersion

The statistical term used for 'average' is the **mean**. Other measures of central tendency may be used and these include the **median** and the **modal** values.

In general, the mean of the set: $\{x_1, x_2, x_3, \ldots, x_n\}$ is

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}, \text{ written as } \frac{\sum x}{n}$$

where $\sum$ is the Greek letter 'sigma' and means 'the sum of', and $\bar{x}$ (called x-bar) is used to signify a mean value.

Determine the mean for the following set: {2, 3, 7, 5, 5, 13, 1, 7, 4, 8, 3, 4, 3}

$$\bar{x} = \frac{\begin{array}{c} 2+3+7+5+5+13+1 \\ +7+4+8+3+4+3 \end{array}}{13} = \frac{65}{13} = \mathbf{5}$$

The median value often gives a better indication of the general size of a set containing extreme values.
For example, the set: {7, 5, 74, 10} is ranked as {5, 7, 10, 74}, and since it contains an even number of members (four in this case), the mean of 7 and 10 is taken, giving a median value of 8.5.

Standard deviation

(a) Discrete data

The standard deviation of a set of data gives an indication of the amount of dispersion, or the scatter, of members of the set from the measure of central tendency. Its value is the root-mean-square value of the members of the set and for discrete data is obtained as follows:

(a) determine the measure of central tendency, usually the mean value, (occasionally the median or modal values are specified),

(b) calculate the deviation of each member of the set from the mean, giving

$$(x_1-\overline{x}), (x_2-\overline{x}), (x_3-\overline{x}), \ldots,$$

(c) determine the squares of these deviations, i.e.

$$(x_1-\overline{x})^2, (x_2-\overline{x})^2, (x_3-\overline{x})^2, \ldots,$$

(d) find the sum of the squares of the deviations, that is

$$(x_1-\overline{x})^2 + (x_2-\overline{x})^2 + (x_3-\overline{x})^2, \ldots,$$

(e) divide by the number of members in the set, n, giving

$$\frac{(x_1-\overline{x})^2 + (x_2-\overline{x})^2 + (x_3-\overline{x})^2 + \cdots}{n}$$

(f) determine the square root of (e).

The standard deviation is indicated by σ (the Greek letter small 'sigma') and is written mathematically as:

$$\textbf{Standard deviation, } \sigma = \sqrt{\left\{\frac{\sum(x-\overline{x})^2}{n}\right\}}$$

where x is a member of the set, $\overline{x}$ is the mean value of the set and n is the number of members in the set. The value of standard deviation gives an indication of the distance of the members of a set from the mean value. The set: $\{1, 4, 7, 10, 13\}$ has a mean value of 7 and a standard deviation of about 4.2.

Problem 5. Determine the standard deviation from the mean of the set of numbers: $\{5, 6, 8, 4, 10, 3\}$ correct to 4 significant figures.

The arithmetic mean,

$$\overline{x} = \frac{\sum x}{n} = \frac{5+6+8+4+10+3}{6} = 6$$

Standard deviation, $\sigma = \sqrt{\left\{\frac{\sum (x-\overline{x})^2}{n}\right\}}$

The $(x-\overline{x})^2$ values are: $(5-6)^2$, $(6-6)^2$, $(8-6)^2$, $(4-6)^2$, $(10-6)^2$ and $(3-6)^2$.

The sum of the $(x-\overline{x})^2$ values,

i.e. $\sum (x-\overline{x})^2 = 1+0+4+4+16+9 = 34$

and $\frac{\sum (x-\overline{x})^2}{n} = \frac{34}{6} = 5.\dot{6}$

since there are 6 members in the set.

Hence, **standard deviation**,

$$\sigma = \sqrt{\left\{\frac{\sum (x-\overline{x})^2}{n}\right\}} = \sqrt{5.\dot{6}}$$

$$= \mathbf{2.380}, \text{ correct to 4 significant figures}$$

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Sample variance

When you collect data from a sample, the sample variance is used to make estimates or inferences about the population variance.

The **sample variance** formula looks like this:

Formula	Explanation
$s^2 = \frac{\Sigma (X - \bar{x})^2}{n - 1}$	• s^2 = sample variance • Σ = sum of... • X = each value • $\bar{x}$ = sample mean • n = number of values in the sample

Let’s calculate the variance of the follow data set: 2, 7, 3, 12, 9.

$=[(2-6.6)^2 + (7-6.6)^2 +(3-6.6)^2 +(12-6.6)^2 +(9-6.6)^2]/5=69.20/5$
$=13.84$

The variance is 13.84. To get the standard deviation, you calculate the square root of the variance, which is 3.72.

EXAM PAPERS PRACTICE

Probability

The probability of something happening is the likelihood or chance of it happening. Values of probability lie between 0 and 1, where 0 represents an absolute impossibility and 1 represents an absolute certainty. The probability of an event happening usually lies somewhere between these two extreme values and is expressed either as a proper or decimal fraction. Examples of probability are:

that a length of copper wire has zero resistance at 100°C	0
that a fair, six-sided dice will stop with a 3 upwards	$\frac{1}{6}$ or 0.1667
that a fair coin will land with a head upwards	$\frac{1}{2}$ or 0.5
that a length of copper wire has some resistance at 100°C	1

If p is the probability of an event happening and q is the probability of the same event not happening, then the total probability is $p+q$ and is equal to unity, since it is an absolute certainty that the event either does or does not occur, i.e. $p+q=1$

Expectation

The expectation, E, of an event happening is defined in general terms as the product of the probability p of an event happening and the number of attempts made, n, i.e. $E=pn$.

Thus, since the probability of obtaining a 3 upwards when rolling a fair dice is $\frac{1}{6}$, the expectation of getting a 3 upwards on four throws of the dice is $\frac{1}{6} \times 4$, i.e. $\frac{2}{3}$

Thus expectation is the average occurrence of an event.

Laws of probability

The addition law of probability is recognized by the word 'or' joining the probabilities. If pA is the probability of event A happening and pB is the probability of event B happening, the probability of event A or event B happening is given by $pA+pB$. In general: $pA + pB + pC + \cdots + pN$

The multiplication law of probability can be shown as following equation

$$pA \times pB \times pC \times \cdots \times pN$$

Problem 1. Determine the probabilities of selecting at random (a) a man, and (b) a woman from a crowd containing 20 men and 33 women?

(a) The probability of selecting at random a man, p, is given by the ratio

$$\frac{\text{number of men}}{\text{number in crowd}},$$

i.e. $p = \frac{20}{20+33} = \mathbf{\frac{20}{53}}$ or **0.3774**

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(b) The probability of selecting at random a women, q, is given by the ratio

$$\frac{\text{number of women}}{\text{number in crowd}},$$

i.e. $$q = \frac{33}{20+33} = \mathbf{\frac{33}{53}} \text{ or } \mathbf{0.6226}$$

(Check: the total probability should be equal to 1;

$$p = \frac{20}{53} \text{ and } q = \frac{33}{53},$$

thus the total probability,

$$p + q = \frac{20}{53} + \frac{33}{53} = 1$$

hence no obvious error has been made).

Problem2. The probability of a component failing in one year due to excessive temperature is 1/20 , due to excessive vibration is 1/25 and due to excessive humidity is1/50 . Determine the probabilities that during a one-year period a component: (a) fails due to excessive temperature and excessive vibration, (b) fails due to excessive vibration or excessive humidity, and (c) will not fail because of both excessive temperature and excessive humidity?

Let p_A be the probability of failure due to excessive temperature, then

$$p_A = \frac{1}{20} \text{ and } \overline{p_A} = \frac{19}{20}$$

(where $\overline{p_A}$ is the probability of not failing).

Let p_B be the probability of failure due to excessive vibration, then

$$p_B = \frac{1}{25} \text{ and } \overline{p_B} = \frac{24}{25}$$

Let p_C be the probability of failure due to excessive humidity, then

$$p_C = \frac{1}{50} \text{ and } \overline{p_C} = \frac{49}{50}$$

(a) The probability of a component failing due to excessive temperature **and** excessive vibration is given by:

$$p_A \times p_B = \frac{1}{20} \times \frac{1}{25} = \mathbf{\frac{1}{500}} \text{ or } \mathbf{0.002}$$

(b) The probability of a component failing due to excessive vibration **or** excessive humidity is:

$$p_B + p_C = \frac{1}{25} + \frac{1}{50} = \mathbf{\frac{3}{50}} \text{ or } \mathbf{0.06}$$

(c) The probability that a component will not fail due to excessive temperature **and** will not fail due to excess humidity is:

$$\overline{p_A} \times \overline{p_C} = \frac{19}{20} \times \frac{49}{50} = \mathbf{\frac{931}{1000}} \text{ or } \mathbf{0.931}$$

The binomial distribution

The binomial distribution deals with two numbers only, these being the probability that an event will happen, *p*, and the probability that an event will not happen, *q*. Thus, when a coin is tossed, if p is the probability of the coin landing with a head upwards, *q* is the probability of the coin landing with a tail upwards. $p + q$ must always be equal to unity. A binomial distribution can be used for finding, say, the probability of getting three heads in seven tosses of the coin.

In other words if *p* is the probability that an event will happen and *q* is the probability that the event will not happen, then the probabilities that the event will happen 0, 1, 2, 3, . . . , n times in n trials are given by the successive terms of the expansion of $(q + p)^n$, taken from left to right

The binomial expansion of $(q + p)^n$ is:

$$q^n + nq^{n-1}p + \frac{n(n-1)}{2!}q^{n-2}p^2 + \frac{n(n-1)(n-2)}{3!}q^{n-3}p^3 + \cdots$$

(Obtained from J. O. Bird 2017)

Problem 1. Determine the probabilities of having (a) at least 1 girl and (b) at least 1 girl and 1 boy in a family of 4 children, assuming equal probability of male and female birth.

Solution: The probability of a girl being born, *p*, is 0.5 and the probability of a girl not being born (male birth), *q*, is also 0.5. The number in the family, *n*, is 4. From above, the probabilities of 0, 1, 2, 3, 4 girls in a family of 4 are given by the successive terms of the expansion of $(q+p)^4$ taken from left to right. From the binomial expansion:

$$(q+p)^4 = q^4 + 4q^3p + 6q^2p^2 + 4qp^3 + p^4$$

Hence the probability of no girls is q^4,

i.e. $0.5^4 = 0.0625$

the probability of 1 girl is $4q^3p$,

i.e. $4 \times 0.5^3 \times 0.5 = 0.2500$

the probability of 2 girls is $6q^2p^2$,

i.e. $6 \times 0.5^2 \times 0.5^2 = 0.3750$

the probability of 3 girls is $4qp^3$,

i.e. $4 \times 0.5 \times 0.5^3 = 0.2500$

the probability of 4 girls is p^4,

i.e. $0.5^4 = 0.0625$

Total probability, $(q+p)^4 = 1.0000$

Solution: (a) The probability of having at least one girl is the sum of the probabilities of having 1, 2, 3 and 4 girls, i.e.
$0.2500 + 0.3750 + 0.2500 + 0.0625 = 0.9375$

(Alternatively, the probability of having at least 1 girl is: 1 − (the probability of having no girls), i.e. $1-0.0625$, giving 0.9375, as obtained previously.)

(b) The probability of having at least 1 girl and 1 boy is given by the sum of the probabilities of having: 1 girl and 3 boys, 2 girls and 2 boys and 3 girls and 2 boys, i.e. $0.2500 + 0.3750 + 0.2500=0.8750$

(Alternatively, this is also the probability of having 1 − (probability of having no girls + probability of having no boys), i.e. $1-2\times0.0625=0.8750$, as obtained previously.)

(Obtained from J. O. Bird 2017)

Problem 2. A dice is rolled 9 times. Find the probabilities of having a 4 upwards (a) 3 times and (b) less than 4 times.

Solution: Let p be the probability of having a 4 upwards. Then p = 1/6, since dice have six sides.
Let q be the probability of not having a 4 upwards. Then q=5/6. The probabilities of having a 4 upwards 0, 1, 2, . . . , n times are given by the successive terms of the expansion of $(q+p)^n$ taken from left to right. From the binomial expansion:

$$(q+p)^9 = q^9 + 9q^8p + 36q^7p^2 + 84q^6p^3 + \cdots$$

The probability of having a 4 upwards no times is

$$q^9 = (5/6)^9 = 0.1938$$

The probability of having a 4 upwards once is

$$9q^8p = 9(5/6)^8(1/6) = 0.3489$$

The probability of having a 4 upwards twice is

$$36q^7p^2 = 36(5/6)^7(1/6)^2 = 0.2791$$

The probability of having a 4 upwards 3 times is

$$84q^6p^3 = 84(5/6)^6(1/6)^3 = 0.1302$$

(a) The probability of having a 4 upwards 3 times is **0.1302**.

(b) The probability of having a 4 upwards less than 4 times is the sum of the probabilities of having a 4 upwards 0, 1, 2, and 3 times, i.e.

$$0.1938 + 0.3489 + 0.2791 + 0.1302 = \mathbf{0.9520}$$

Normal distribution

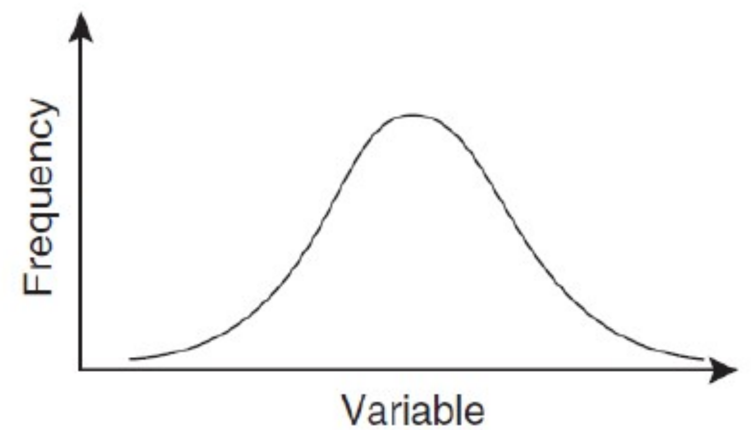
An extremely important symmetrical distribution curve is called the normal curve and is as shown in the below Figure. This curve can be described by a mathematical equation. Many natural occurrences such as the heights or weights of a group of people, the sizes of components produced by a particular machine and the life length of certain components approximate to a normal distribution.

A normal distribution curve is **standardized** as follows:

(a) The mean value of the unstandardized curve is made the origin, thus making the mean value, $\overline{x}$, zero.

(b) The horizontal axis is scaled in standard deviations. This is done by letting $z = \frac{x - \overline{x}}{\sigma}$, where z is called the **normal standard variate**, x is the value of the variable, $\overline{x}$ is the mean value of the distribution and σ is the standard deviation of the distribution.

(c) The area between the normal curve and the horizontal axis is made equal to unity.



The area under part of a normal probability curve can be determined by:

$$\int \frac{1}{\sqrt{(2\pi)}} e^{\left(\frac{z^2}{2}\right)} dz, \text{ where } z = \frac{x - \overline{x}}{\sigma}$$

Normal distribution

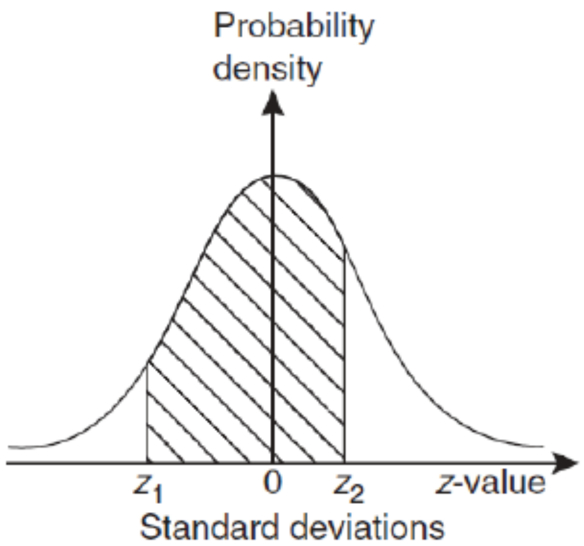


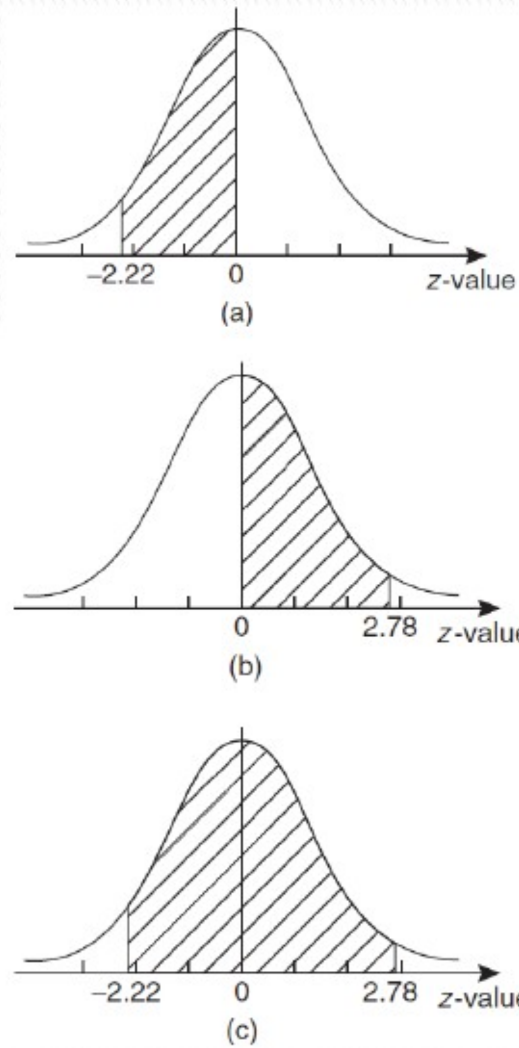
Figure 58.2

To save repeatedly determining the values of this function, tables of partial areas under the standardized normal curve are available in many mathematical books, and such a table is shown in the following figure

$z=\frac{x-\overline{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Problem. The mean height of 500 people is 170 cm and the standard deviation is 9 cm. Assuming the heights are normally distributed, determine the number of people likely to have heights between 150 cm and 195 cm.

The mean value, $\bar{x}$, is 170 cm and corresponds to a normal standard variate value, z, of zero on the standardized normal curve. A height of 150 cm has a z-value given by $z = \frac{x - \bar{x}}{\sigma}$ standard deviations, i.e. $\frac{150 - 170}{9}$ or -2.22 standard deviations. Using a table of partial areas beneath the standardized normal curve (see previous Table), a z-value of −2.22 corresponds to an area of 0.4868 between the mean value and the ordinate z=−2.22. The negative z-value shows that it lies to the left of the z=0 ordinate. This area is shown shaded in Fig (a). Similarly, 195 cm has a z-value of (195−170)/9 that is 2.78 standard deviations. From Table, this value of z corresponds to an area of 0.4973, the positive value of z showing that it lies to the right of the z=0 ordinate. This area is shown shaded in Fig. (b). The total area shaded in Figs. (a) and (b) is shown in Fig. (c) and is 0.4868+0.4973, i.e. 0.9841 of the total area beneath the curve.



Problem6 For the group of people given in pervious problem, find the number of people likely to have heights of less than 165 cm.

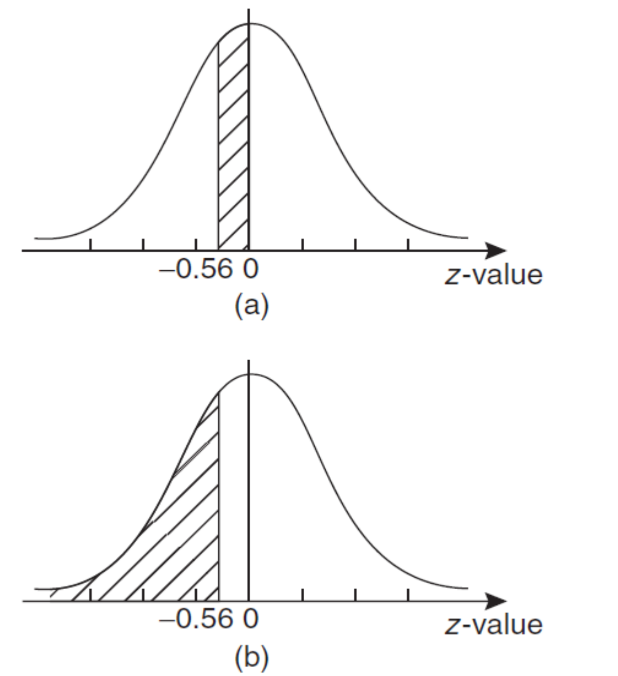
Solution:

A height of 165 cm corresponds to $\dfrac{165-170}{9}$ i.e. -0.56 standard deviations.

The area between $z=0$ and $z=-0.56$ (from next figure (a) and pervious table.

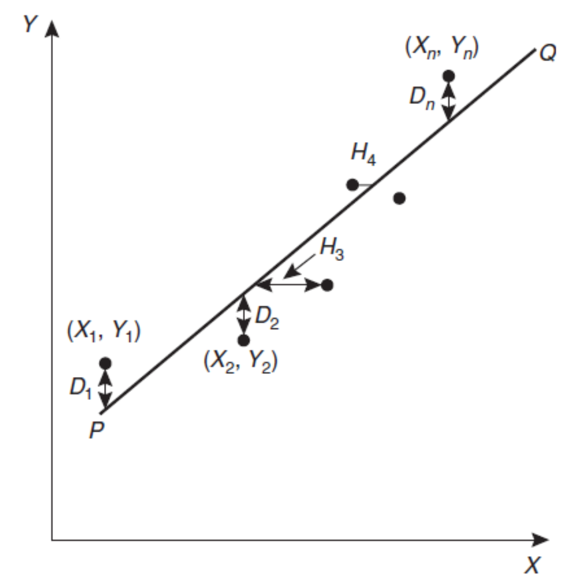
The total area under the standardized normal curve is unity and since the curve is symmetrical, it follows that the total area to the left of the z=0 ordinate is 0.5000. Thus the area to the left of the $z=-0.56$ ordinate ('left' means 'less than', 'right' means 'more than') is 0.5000−0.2123, i.e. 0.2877 of the total area, which is shown shaded in figure (b).

The area is directly proportional to probability and since the total area beneath the standardized normal curve is unity, the probability of a person's height being less than 165 cm is 0.2877. For a group of 500 people, 500×0.2877, i.e. 144 people are likely to have heights of less than 165 cm.



Linear regression

Regression analysis, usually termed regression, is used to draw the line of 'best fit' through co-ordinates on a graph. The techniques used enable a mathematical equation of the straight line form $y=mx+c$ to be deduced for a given set of co-ordinate values, the line being such that the sum of the deviations of the co-ordinate values from the line is a minimum, i.e. it is the line of 'best fit'. For a given set of co-ordinate values, (X_1, Y_1), (X_2, Y_2), . . . , (X_n, Y_n) let the X values be the independent variables and the Y-values be the dependent values. Also let D_1, . . . ,D_n be the vertical distances between the line shown as PQ in next Fig and the points representing the co-ordinate values. The least squares regression line, i.e. the line of best fit, is the line which makes the value of $D_1^2+D_2^2+\cdots+D_n^2$ a minimum value.



The equation of the least-squares regression line is usually written as $Y=a_0+a_1X$, where a_0 is the Y-axis intercept value and a_1 is the gradient of the line (analogous to c and m in the equation $y=mx+c$). The values of a_0 and a_1 to make the sum of the 'deviations squared' a minimum can be obtained from the two equations:

$$\sum Y = a_0 N + a_1 \sum X \qquad (1)$$

$$\sum (XY) = a_0 \sum X + a_1 \sum X^2 \qquad (2)$$

where X and Y are the co-ordinate values, N is the number of co-ordinates and a_0 and a_1 are called the regression coefficients of Y on X. Equations (1) and (2) are called the normal equations of the regression
lines of Y on X.

Problem. In an experiment to determine the relationship between frequency and the inductive reactance of an electrical circuit, the following results were obtained:

Frequency (Hz)	Inductive reactance (ohms)
50	30
100	65
150	90
200	130
250	150
300	190
350	200

Determine the equation of the regression line of inductive reactance on frequency, assuming a linear relationship.

Since the regression line of inductive reactance on frequency is required, the frequency is the independent variable, X, and the inductive reactance is the dependent variable, Y. The equation of the regression line of Y on X is:

$$Y = a_0 + a_1 X$$

and the regression coefficients a_0 and a_1 are obtained by using the normal equations

$$\sum Y = a_0 N + a_1 \sum X$$

and $$\sum XY = a_0 \sum X + a_1 \sum X^2$$

(from equations (1) and (2))

A tabular approach is used to determine the summed quantities.

Frequency, X	Inductive reactance, Y	X^2
50	30	2500
100	65	10000
150	90	22500
200	130	40000
250	150	62500
300	190	90000
350	200	122500
$\sum X = 1400$	$\sum Y = 855$	$\sum X^2 = 350000$

XY	Y^2
1500	900
6500	4225
13500	8100
26000	16900
37500	22500
57000	36100
70000	40000
$\sum XY = 212000$	$\sum Y^2 = 128725$

The number of co-ordinate values given, N is 7. Substituting in the normal equations gives:

$$855 = 7a_0 + 1400a_1 \qquad (1)$$

$$212000 = 1400a_0 + 350000a_1 \qquad (2)$$

$1400 \times (1)$ gives:

$$1197000 = 9800a_0 + 1960000a_1 \qquad (3)$$

$7 \times (2)$ gives:

$$1484000 = 9800a_0 + 2450000a_1 \qquad (4)$$

$(4) - (3)$ gives:

$$287000 = 0 + 490000a_1$$

from which, $a_1 = \dfrac{287000}{490000} = 0.586$

Substituting $a_1 = 0.586$ in equation (1) gives:

$$855 = 7a_0 + 1400(0.586)$$

i.e. $\quad a_0 = \dfrac{855 - 820.4}{7} = 4.94$

Thus the equation of the regression line of inductive reactance on frequency is:

$$\mathbf{Y = 4.94 + 0.586\,X}$$

References:

1. J. O. Bird (2017), Higher engineering mathematics Eighth ed.
2. K. A. Stroud (1995) , Engineering mathematics Fourth ed.