Applied Math

Lesson 3

Lecturer: Dr. Mahdi Jalali

Logarithms and exponential functions

If a number y can be written in the form a^x, then the index x is called the 'logarithm of y to the base of a',

i.e. $$\boxed{\textbf{if } y = a^x \textbf{ then } x = \log_a y}$$

(a) Logarithms having a base of 10 are called **common logarithms** and $\log_{10}$ is usually abbreviated to lg. The following values may be checked by using a calculator:

lg 17.9 = 1.2528 . . ., lg 462.7 = 2.6652 . . . and lg 0.0173 = − 1.7619 . . .

(b) Logarithms having a base of e (where 'e' is a mathematical constant approximately equal to 2.7183) are called **hyperbolic, Napierian** or **natural logarithms**, and $\log_e$ is usually abbreviated to ln. The following values may be checked by using a calculator:

ln 3.15 = 1.1474 . . ., ln 362.7 = 5.8935 . . . and ln 0.156 = −1.8578 . . . .

Laws of logarithms

(i) To multiply two numbers:

$$\boxed{\log(A \times B) = \log A + \log B}$$

The following may be checked by using a calculator:

$$\lg 10 = 1, \text{also} \lg 5 + \lg 2$$

$$= 0.69897\ldots + 0.301029\ldots = 1$$

Hence $\lg(5 \times 2) = \lg 10 = \lg 5 + \lg 2$

(ii) To divide two numbers:

$$\boxed{\log\left(\frac{A}{B}\right) = \log A - \log B}$$

Problem 7. Solve the equation:
$\log(x-1)+\log(x+1)=2\log(x+2)$.

$$\log(x-1)+\log(x+1)=\log(x-1)(x+1),$$

from the first law of logarithms

$$=\log(x^2-1)$$

$$2\log(x+2)=\log(x+2)^2$$

$$=\log(x^2+4x+4)$$

Hence if $\log(x^2-1)=\log(x^2+4x+4)$

then $x^2-1=x^2+4x+4$

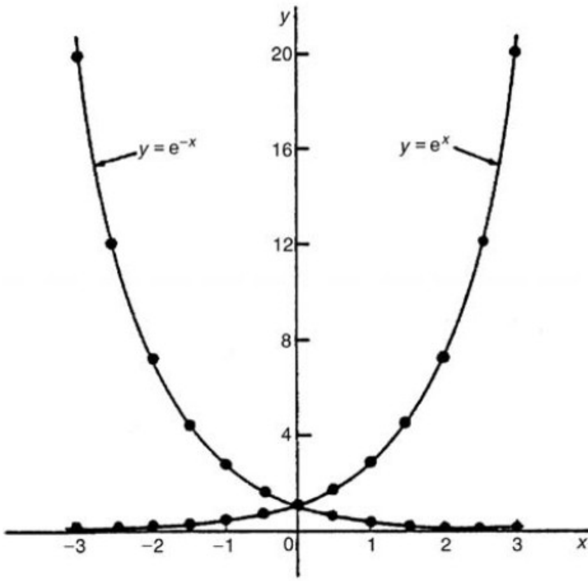
i.e. $-1=4x+4$

i.e. $-5=4x$

i.e. $\boldsymbol{x=-\frac{5}{4}}$ or $\boldsymbol{-1\frac{1}{4}}$

The exponential function

An exponential function is one which contains e^x, e being a constant called the exponent and having an approximate value of 2.7183. The exponent arises from the natural laws of growth and decay and is used as a base for natural or Napierian logarithms.



The value of e^x can be calculated to any required degree of accuracy since it is defined in terms of the following **power series:**

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

(where $3! = 3 \times 2 \times 1$ and is called 'factorial 3')
The series is valid for all values of x.

EXAM PAPERS PRACTICE

Hyperbolic functions

Functions which are associated with the geometry of the conic section called a hyperbola are called **hyperbolic functions**. Applications of hyperbolic functions include transmission line theory and catenary problems. By definition:

(i) Hyperbolic sine of x,

$$\mathbf{\sinh x = \frac{e^x - e^{-x}}{2}} \qquad (1)$$

'$\sinh x$' is often abbreviated to '$\mathrm{sh}\, x$' and is pronounced as 'shine x'

(ii) Hyperbolic cosine of x,

$$\mathbf{\cosh x = \frac{e^x + e^{-x}}{2}} \qquad (2)$$

'$\cosh x$' is often abbreviated to '$\mathrm{ch}\, x$' and is pronounced as 'kosh x'

(iii) Hyperbolic tangent of x,

$$\mathbf{\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}} \qquad (3)$$

Prove the hyperbolic identities

$$\text{ch}^2 x - \text{sh}^2 x = 1$$

(a) $$\text{ch}\, x + \text{sh}\, x = \left(\frac{e^x + e^{-x}}{2}\right) + \left(\frac{e^x - e^{-x}}{2}\right) = e^x$$

$$\text{ch}\, x - \text{sh}\, x = \left(\frac{e^x + e^{-x}}{2}\right) - \left(\frac{e^x - e^{-x}}{2}\right) = e^{-x}$$

$$(\text{ch}\, x + \text{sh}\, x)(\text{ch}\, x - \text{sh}\, x) = (e^x)(e^{-x}) = e^0 = 1$$

i.e. $\mathbf{ch^2 x - sh^2 x = 1}$ (1)

Trigonometric identity	Corresponding hyperbolic identity
$\cos^2 x + \sin^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \text{cosec}^2 x$	$\text{ch}^2 x - \text{sh}^2 x = 1$ $1 - \text{th}^2 x = \text{sech}^2 x$ $\text{coth}^2 x - 1 = \text{cosech}^2 x$

Solve the following equation $2.6 \text{ ch } x + 5.1 \text{ sh } x = 8.73$

(i) $2.6 \operatorname{ch} x + 5.1 \operatorname{sh} x = 8.73$

i.e. $2.6\left(\dfrac{e^x + e^{-x}}{2}\right) + 5.1\left(\dfrac{e^x - e^{-x}}{2}\right) = 8.73$

(ii) $1.3e^x + 1.3e^{-x} + 2.55e^x - 2.55e^{-x} = 8.73$

i.e. $3.85e^x - 1.25e^{-x} - 8.73 = 0$

(iii) $3.85(e^x)^2 - 8.73e^x - 1.25 = 0$

(iv) e^x

$$= \frac{-(-8.73) \pm \sqrt{[(-8.73)^2 - 4(3.85)(-1.25)]}}{2(3.85)}$$

$$= \frac{8.73 \pm \sqrt{95.463}}{7.70} = \frac{8.73 \pm 9.7705}{7.70}$$

Hence $e^x = 2.4027$ or $e^x = -0.1351$

(v) $x = \ln 2.4027$ or $x = \ln(-0.1351)$ which has no real solution.

Hence $\boldsymbol{x = 0.8766}$, correct to 4 decimal places.

Solving linear equations graphically

Solve the simultaneous equations $x + y = 5$ and $y = x + 1$ using graphs.

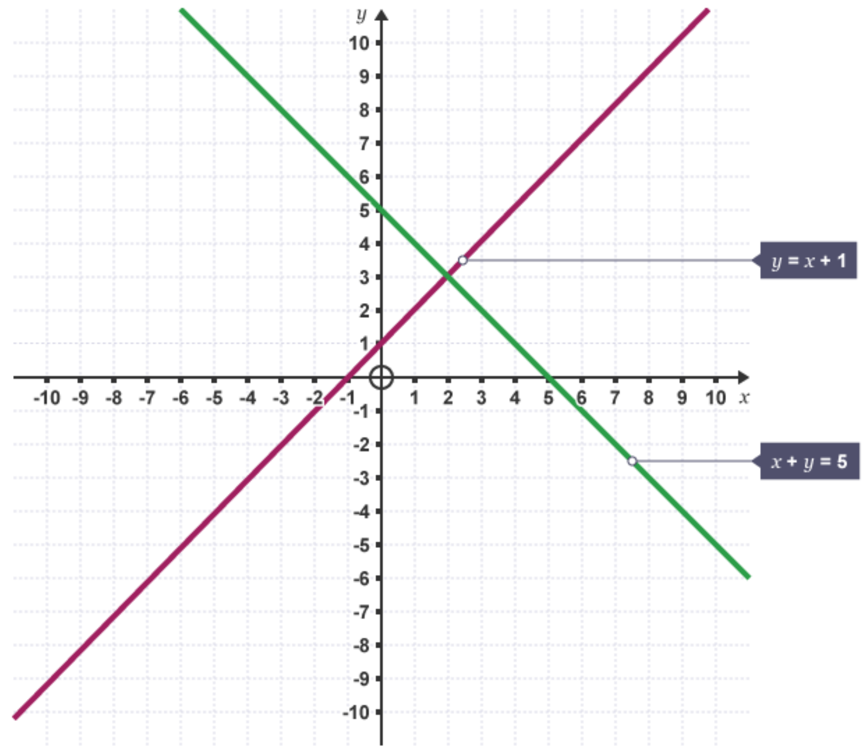
$y = x + 1$

x	-1	0	1	2	3
y	0	1	2	3	4

$x + y = 5$

x	-1	0	1	2	3
y	6	5	4	3	2

Plot these graphs onto the axes and label each graph.



The point of intersection is (2, 3) which means $x = 2$ and $y = 3$

Solving linear equations graphically

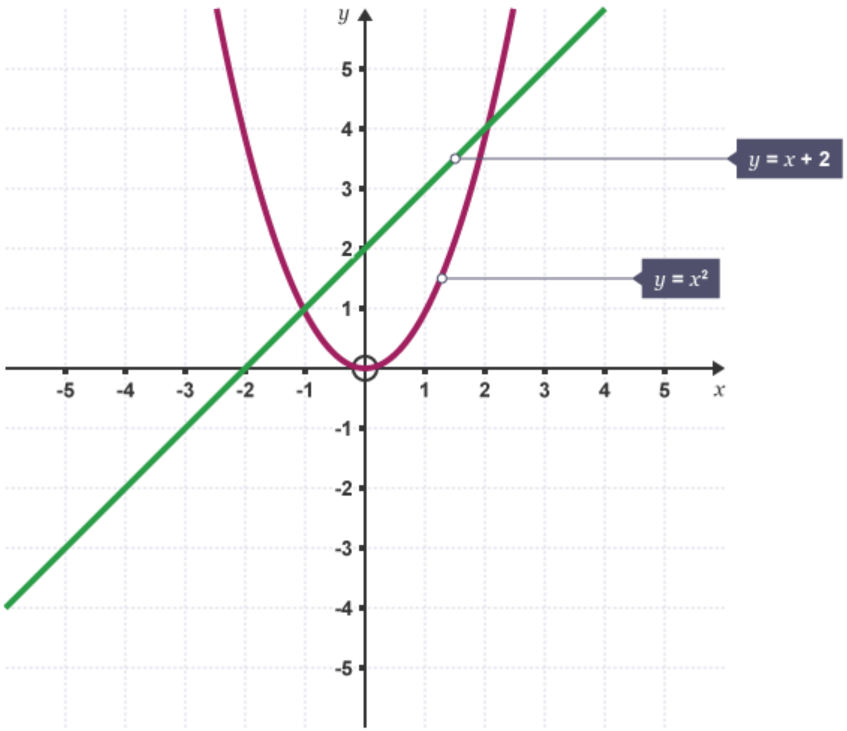
Solve the simultaneous equations $y = x^2$ and $y = x + 2$.

$y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

$y = x + 2$

x	-1	0	1	2	3
y	1	2	3	4	5



The two points of intersection are at (2, 4) and (-1, 1) so $x = 2$ and $y = 4$, and $x = -1$ and $y = 1$.

EXAM PAPERS PRACTICE

Matrices and determinants

Matrices and determinants are mainly used for the solution of linear simultaneous equations. The coefficients of the variables for linear simultaneous equations may be shown in matrix form. The coefficients of x and y in the simultaneous equations

$$x + 2y = 3$$

$$4x - 5y = 6$$

become $\begin{pmatrix} 1 & 2 \\ 4 & -5 \end{pmatrix}$ in matrix notation.

The number of rows in a matrix is usually specified by *m* and the number of columns by *n* and a matrix referred to as an ‘*m* by *n*’ matrix. Thus, the following matrix

$\begin{pmatrix} 2 & 3 & 6 \\ 4 & 5 & 7 \end{pmatrix}$ is a ‘2 by 3’ matrix

Addition, subtraction and multiplication of matrices

LITE Regal EDUCATION

Add the matrices (a) $\begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}$ and $\begin{pmatrix} -3 & 0 \\ 7 & -4 \end{pmatrix}$

Adding the corresponding elements gives:

$$\begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 7 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} 2+(-3) & -1+0 \\ -7+7 & 4+(-4) \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

(b) $\begin{pmatrix} 3 & 1 & -4 \\ 4 & 3 & 1 \\ 1 & 4 & -3 \end{pmatrix}$ and $\begin{pmatrix} 2 & 7 & -5 \\ -2 & 1 & 0 \\ 6 & 3 & 4 \end{pmatrix}$ $\Rightarrow$ $\begin{pmatrix} 3 & 1 & -4 \\ 4 & 3 & 1 \\ 1 & 4 & -3 \end{pmatrix} + \begin{pmatrix} 2 & 7 & -5 \\ -2 & 1 & 0 \\ 6 & 3 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 3+2 & 1+7 & -4+(-5) \\ 4+(-2) & 3+1 & 1+0 \\ 1+6 & 4+3 & -3+4 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{5} & \mathbf{8} & \mathbf{-9} \\ \mathbf{2} & \mathbf{4} & \mathbf{1} \\ \mathbf{7} & \mathbf{7} & \mathbf{1} \end{pmatrix}$$

EXAM PAPERS PRACTICE

Problem 4. If $A = \begin{pmatrix} -3 & 0 \\ 7 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ -2 & -4 \end{pmatrix}$ find $2A - 3B + 4C$.

$$2A = 2\begin{pmatrix} -3 & 0 \\ 7 & -4 \end{pmatrix} = \begin{pmatrix} -6 & 0 \\ 14 & -8 \end{pmatrix}$$

$$3B = 3\begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ -21 & 12 \end{pmatrix}$$

and $4C = 4\begin{pmatrix} 1 & 0 \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ -8 & -16 \end{pmatrix}$

Hence $2A - 3B + 4C$

$$= \begin{pmatrix} -6 & 0 \\ 14 & -8 \end{pmatrix} - \begin{pmatrix} 6 & -3 \\ -21 & 12 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ -8 & -16 \end{pmatrix}$$

$$= \begin{pmatrix} -6 - 6 + 4 & 0 - (-3) + 0 \\ 14 - (-21) + (-8) & -8 - 12 + (-16) \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-8} & \mathbf{3} \\ \mathbf{27} & \mathbf{-36} \end{pmatrix}$$

Matrices and determinants are mainly used for the solution of linear simultaneous equations. The coefficients of the variables for linear simultaneous equations may be shown in matrix form. The coefficients of x and y in the simultaneous equations

$$x + 2y = 3$$

$$4x - 5y = 6$$

become $\begin{pmatrix} 1 & 2 \\ 4 & -5 \end{pmatrix}$ in matrix notation.

The number of rows in a matrix is usually specified by *m* and the number of columns by *n* and a matrix referred to as an '*m* by *n*' matrix. Thus, the following matrix

$\begin{pmatrix} 2 & 3 & 6 \\ 4 & 5 & 7 \end{pmatrix}$ is a '2 by 3' matrix

In general terms, when multiplying a matrix of dimensions (m by n) by a matrix of dimensions (n by r), the resulting matrix has dimensions (m by r). Thus a 2 by 3 matrix multiplied by a 3 by 1 matrix gives a matrix of dimensions 2 by 1

Problem 5. If $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 7 \\ -3 & 4 \end{pmatrix}$ find $A \times B$.

Let $A \times B = C$ where $C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$

$C11 = (2 \times (-5)) + (3 \times (-3)) = -19$

$C12 = (2 \times 7) + (3 \times 4) = 26$

$C21 = (1 \times (-5)) + (-4 \times (-3)) =$
7

$C22 = (1 \times 7) + ((-4) \times 4) = -9$

Thus, $\boldsymbol{A \times B} = \begin{pmatrix} \mathbf{-19} & \mathbf{26} \\ \mathbf{7} & \mathbf{-9} \end{pmatrix}$

LITE Regal EDUCATION

Problem 7. If $A = \begin{pmatrix} 3 & 4 & 0 \\ -2 & 6 & -3 \\ 7 & -4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -5 \\ 5 & -6 \\ -1 & -7 \end{pmatrix}$, find $A \times B$.

$$\begin{pmatrix} 3 & 4 & 0 \\ -2 & 6 & -3 \\ 7 & -4 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & -5 \\ 5 & -6 \\ -1 & -7 \end{pmatrix}$$

$$= \begin{pmatrix} [(3 \times 2) + (4 \times 5) + (0 \times (-1))] & [(3 \times (-5)) + (4 \times (-6)) + (0 \times (-7))] \\ [(-2 \times 2) + (6 \times 5) + (-3 \times (-1))] & [(-2 \times (-5)) + (6 \times (-6)) + (-3 \times (-7))] \\ [(7 \times 2) + (-4 \times 5) + (1 \times (-1))] & [(7 \times (-5)) + (-4 \times (-6)) + (1 \times (-7))] \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{26} & \mathbf{-39} \\ \mathbf{29} & \mathbf{-5} \\ \mathbf{-7} & \mathbf{-18} \end{pmatrix}$$

EXAM PAPERS PRACTICE

The unit matrix

A **unit matrix, *I***, is one in which all elements of the leading diagonal (\) have a value of 1 and all other elements have a value of 0. Multiplication of a matrix by *I* is the equivalent of multiplying by 1 in arithmetic.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; \ldots, \; I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

The determinant of a 2 by 2 matrix

The **determinant** of a 2 by 2 matrix, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is defined as $(ad - bc)$.

The elements of the determinant of a matrix are written between vertical lines. Thus, the determinant of $\begin{pmatrix} 3 & -4 \\ 1 & 6 \end{pmatrix}$ is written as $\begin{vmatrix} 3 & -4 \\ 1 & 6 \end{vmatrix}$ and is equal to $(3 \times 6) - (-4 \times 1)$, i.e. $18 - (-4)$ or 22. Hence the determinant of a matrix can be expressed as a single numerical value, i.e. $\begin{vmatrix} 3 & -4 \\ 1 & 6 \end{vmatrix} = 22$.

The determinant of a 3 by 3 matrix

. Find the value of

$$\begin{vmatrix} 3 & 4 & -1 \\ 2 & 0 & 7 \\ 1 & -3 & -2 \end{vmatrix}$$

=(0x-2-7x-3)x3+(7x1-2x-2)x4+(2x-3-1x0)x-1=63+44+ 113

The inverse of a 2 by 2 matrix

For any matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ the inverse may be obtained by:

(i) interchanging the positions of p and s,
(ii) changing the signs of q and r, and
(iii) multiplying this new matrix by the reciprocal of the determinant of $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$

Thus the inverse of matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is

$$\frac{1}{4-6}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

Transpose of a Matrix Definition

The transpose of a matrix is found by interchanging its rows into columns or columns into rows. The transpose of the matrix is denoted by using the letter “T” in the superscript of the given matrix. For example, if “A” is the given matrix, then the transpose of the matrix is represented by A’ or A^T.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3} \qquad A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3 \times 2}$$

Example- Find the transpose of the given matrix

$$M = \begin{bmatrix} 2 & -9 & 3 \\ 13 & 11 & -17 \\ 3 & 6 & 15 \\ 4 & 13 & 1 \end{bmatrix}$$

Solution- Given a matrix of the order 4×3.

The transpose of a matrix is given by interchanging rows and columns.

$$M^T = \begin{bmatrix} 2 & 13 & 3 & 4 \\ -9 & 11 & 6 & 13 \\ 3 & -17 & 15 & 1 \end{bmatrix}$$

Addition Property of Transpose

$(A+B)' = A'+B'$

Multiplication Property of Transpose

$(AB)' = B'A'$

Example: $A = \begin{bmatrix} 9 & 8 \\ 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 1 & 0 \end{bmatrix}$

Let us find $A \times B$.

$A \times B = \begin{bmatrix} 44 & 18 \\ 5 & 4 \end{bmatrix} \Rightarrow (AB)' = \begin{bmatrix} 44 & 5 \\ 18 & 4 \end{bmatrix}$

$B'A' = \begin{bmatrix} 4 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 9 & 2 \\ 8 & -3 \end{bmatrix}$

$= \begin{bmatrix} 44 & 5 \\ 18 & 4 \end{bmatrix} = (AB)'$

$\therefore (AB)' = B'A'$

$A'B' = \begin{bmatrix} 9 & 2 \\ 8 & -3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 40 & 9 \\ 26 & 8 \end{bmatrix}$

Solution of simultaneous equations by matrices

The procedure for solving linear simultaneous equations in **two unknowns using matrices** is:

(i) write the equations in the form

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

(ii) write the matrix equation corresponding to these equations,

i.e. $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

(iii) determine the inverse matrix of $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$

i.e. $\dfrac{1}{a_1b_2 - b_1a_2}\begin{pmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{pmatrix}$

(iv) multiply each side of (ii) by the inverse matrix, and

(v) solve for x and y by equating corresponding elements.

Problem 1. Use matrices to solve the simultaneous equations:

$$3x + 5y - 7 = 0 \qquad (1)$$
$$4x - 3y - 19 = 0 \qquad (2)$$

EXAM PAPERS PRACTICE

(i) Writing the equations in the $a_1x+b_1y = c$ form gives:

$$3x + 5y = 7$$
$$4x - 3y = 19$$

(ii) The matrix equation is

$$\begin{pmatrix} 3 & 5 \\ 4 & -3 \end{pmatrix} \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 19 \end{pmatrix}$$

(iii) The inverse of matrix $\begin{pmatrix} 3 & 5 \\ 4 & -3 \end{pmatrix}$ is

$$\frac{1}{3 \times (-3) - 5 \times 4} \begin{pmatrix} -3 & -5 \\ -4 & 3 \end{pmatrix}$$

i.e. $\begin{pmatrix} \frac{3}{29} & \frac{5}{29} \\ \frac{4}{29} & \frac{-3}{29} \end{pmatrix}$

(iv) Multiplying each side of (ii) by (iii) and remembering that $A \times A^{-1} = I$, the unit matrix, gives:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{29} & \frac{5}{29} \\ \frac{4}{29} & \frac{-3}{29} \end{pmatrix} \times \begin{pmatrix} 7 \\ 19 \end{pmatrix}$$

Thus $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{21}{29} + \frac{95}{29} \\ \frac{28}{29} - \frac{57}{29} \end{pmatrix}$

i.e. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

(v) By comparing corresponding elements:

$$\boldsymbol{x = 4} \quad \text{and} \quad \boldsymbol{y = -1}$$

Solution of simultaneous equations by determinants

(a) When solving linear simultaneous equations in **two unknowns using determinants:**

(i) write the equations in the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

and then

(ii) the solution is given by

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{1}{D}$$

where $D_x = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$

i.e. the determinant of the coefficients left when the x-column is covered up,

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

i.e. the determinant of the coefficients left when the y-column is covered up,

and $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

Solve the following equation

$$3x - 4y = 12$$
$$7x + 5y = 6.5$$

LITE Regal EDUCATION

Following the above procedure:

(i) $3x - 4y - 12 = 0$
$7x + 5y - 6.5 = 0$

(ii) $$\frac{x}{\begin{vmatrix} -4 & -12 \\ 5 & -6.5 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 3 & -12 \\ 7 & -6.5 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & -4 \\ 7 & 5 \end{vmatrix}}$$

i.e. $$\frac{x}{(-4)(-6.5) - (-12)(5)}$$
$$= \frac{-y}{(3)(-6.5) - (-12)(7)}$$
$$= \frac{1}{(3)(5) - (-4)(7)}$$

i.e. $$\frac{x}{26 + 60} = \frac{-y}{-19.5 + 84} = \frac{1}{15 + 28}$$

i.e. $$\frac{x}{86} = \frac{-y}{64.5} = \frac{1}{43}$$

Since $\frac{x}{86} = \frac{1}{43}$ then $\boldsymbol{x} = \frac{86}{43} = \mathbf{2}$

and since

$$\frac{-y}{64.5} = \frac{1}{43} \text{ then } \boldsymbol{y} = -\frac{64.5}{43} = \mathbf{-1.5}$$

EXAM PAPERS PRACTICE

Solution of simultaneous equations using the Gaussian elimination method

Consider the following simultaneous equations:

$$x + y + z = 4 \quad (1)$$
$$2x - 3y + 4z = 33 \quad (2)$$
$$3x - 2y - 2z = 2 \quad (3)$$

Leaving equation (1) as it is gives:

$$x + y + z = 4 \quad (1)$$

Equation (2) − 2 × equation (1) gives:

$$0 - 5y + 2z = 25 \quad (2')$$

and equation (3) − 3 × equation (1) gives:

$$0 - 5y - 5z = -10 \quad (3')$$

Leaving equations (1) and (2′) as they are gives:

$$x + y + z = 4 \quad (1)$$
$$0 - 5y + 2z = 25 \quad (2')$$

Equation (3′) − equation (2′) gives:

$$0 + 0 - 7z = -35 \quad (3'')$$

By appropriately manipulating the three original equations we have deliberately obtained zeros in the positions shown in equations (2′) and (3″).

Working backwards, from equation (3″),

$$z = \frac{-35}{-7} = \mathbf{5},$$

from equation (2′),

$$-5y + 2(5) = 25,$$

from which,

$$\boldsymbol{y} = \frac{25 - 10}{-5} = \mathbf{-3}$$

and from equation (1),

$$x + (-3) + 5 = 4,$$

from which,

$$\boldsymbol{x} = 4 + 3 - 5 = \mathbf{2}$$

EXAM PAPERS PRACTICE

References:

K. A. Stroud (1995) , Engineering mathematics Fourth ed.

J. O. Bird (2017), Higher engineering mathematics Eighth ed.