Applied Math

Lesson 2

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Differential calculus

1- The gradient of a curve

If a tangent is drawn at a point *P* on a curve, then the gradient of this tangent is said to be the gradient of the curve at *P*. In next figure, the gradient of the curve at *P* is equal to the gradient of the tangent *PQ*.

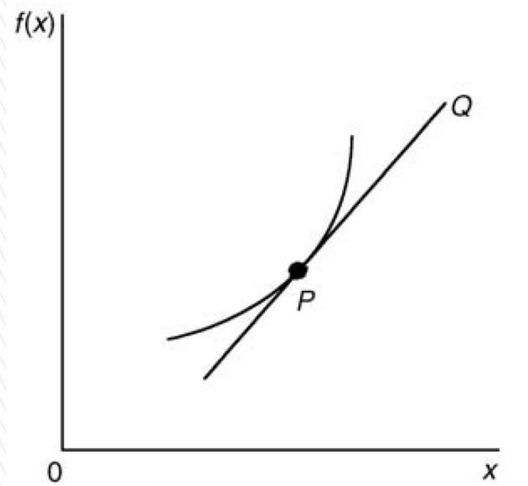

Figure 1

For the curve shown in next figure let the points *A* and *B* have co-ordinates (x_1, y_1) and (x_2, y_2), respectively. In functional notation, $y_1 = f(x_1)$ and $y_2 = f(x_2)$ as shown.

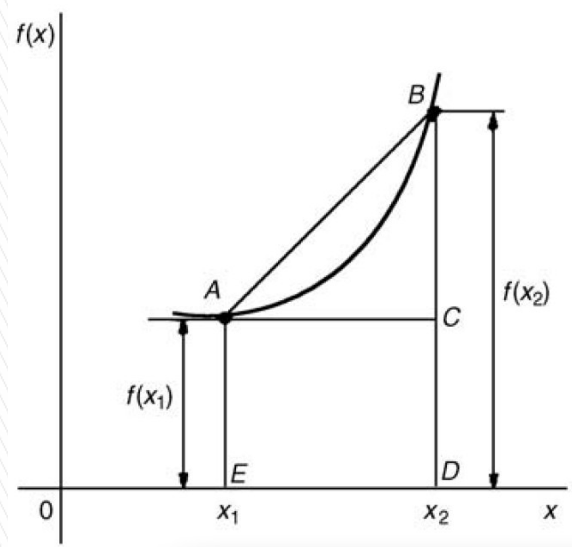

Figure 2

(Obtained from J. O. Bird 2017)

The gradient of the chord *AB*

$$= \frac{BC}{AC} = \frac{BD - CD}{ED} = \frac{f(x_2) - f(x_1)}{(x_2 - x_1)}$$

For the curve $f(x) = x^2$ shown in the following figure

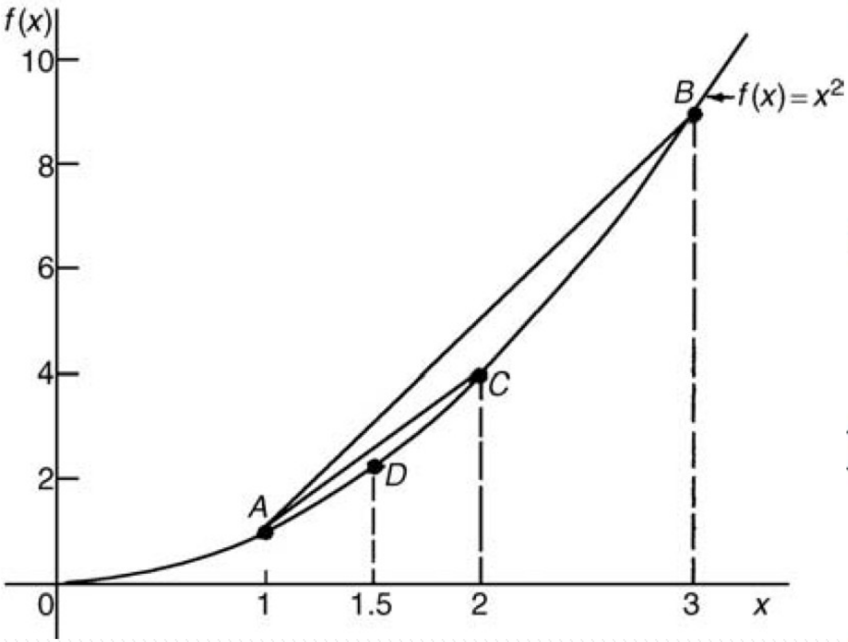

Figure 3

(Obtained from J. O. Bird 2017)

(i) the gradient of chord AB

$$= \frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = \mathbf{4}$$

(ii) the gradient of chord AC

$$= \frac{f(2) - f(1)}{2 - 1} = \frac{4 - 1}{1} = \mathbf{3}$$

(iii) the gradient of chord AD

$$= \frac{f(1.5) - f(1)}{1.5 - 1} = \frac{2.25 - 1}{0.5} = \mathbf{2.5}$$

(iv) if E is the point on the curve $(1.1,\ f(1.1))$ then the gradient of chord AE

$$= \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{1.21 - 1}{0.1} = \mathbf{2.1}$$

(v) if F is the point on the curve $(1.01, f(1.01))$ then the gradient of chord AF

$$= \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{1.0201 - 1}{0.01} = \mathbf{2.01}$$

Thus as point B moves closer and closer to point A the gradient of the chord approaches nearer and nearer to the value 2. This is called the limiting value of the gradient of the chord AB and when B coincides with A the chord becomes the tangent to the curve.

Differentiation from first principles

In following Figure, A and B are two points very close together on a curve, δx (delta x) and δy (delta y) representing small increments in the x and y directions, respectively.

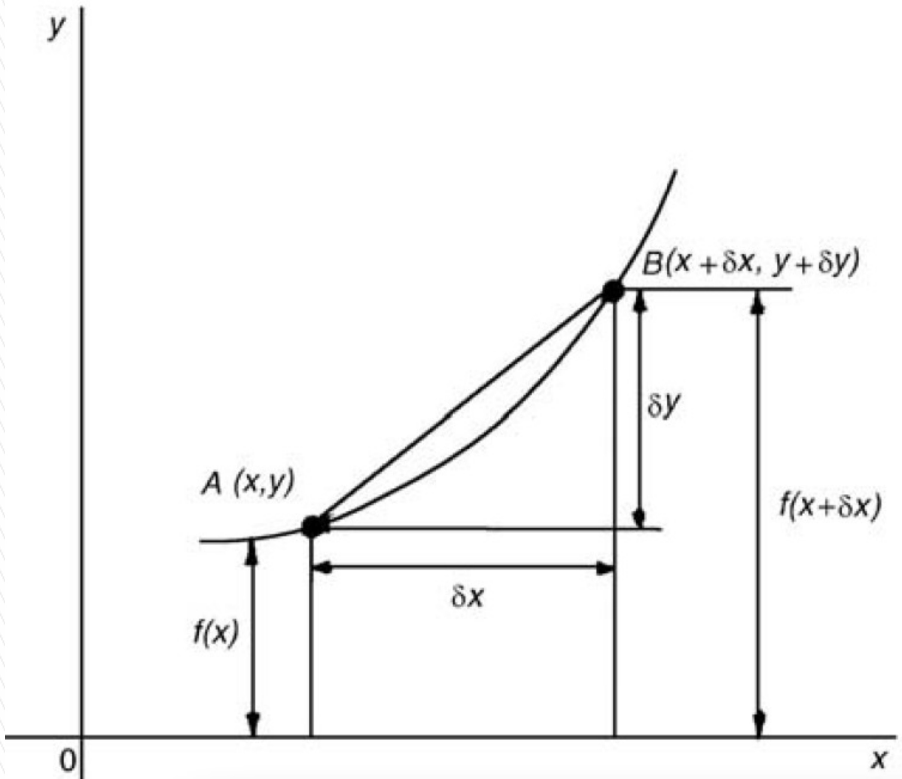

Figure 4

(Obtained from J. O. Bird 2017)

Gradient of chord $AB = \frac{\delta y}{\delta x}$; however, $\delta y = f(x + \delta x) - f(x)$.

Hence $\frac{\delta y}{\delta x} = \frac{f(x + \delta x) - f(x)}{\delta x}$.

As δx approaches zero, $\frac{\delta y}{\delta x}$ approaches a limiting value and the gradient of the chord approaches the gradient of the tangent at A.

When determining the gradient of a tangent to a curve there are two notations used. The gradient of the curve at *A* in Figure 4 can either be written as

$$f'(x) = \lim_{\delta x \to 0} \left\{ \frac{f(x + \delta x) - f(x)}{\delta x} \right\}$$

$\frac{\mathrm{d}y}{\mathrm{d}x}$ is the same as $f'(x)$ and is called the **differential coefficient** or the **derivative**. The process of finding the differential coefficient is called **differentiation**.

Problem 1. Differentiate from first principle $f(x) = x^2$ and determine the value of the gradient of the curve at $x = 2$.

To 'differentiate from first principles' means 'to find $f'(x)$' by using the expression

$$f'(x) = \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} \right\}$$

$$f(x) = x^2$$

Substituting $(x + \delta x)$ for x gives $f(x+\delta x) = (x+\delta x)^2 = x^2 + 2x\delta x + \delta x^2$, hence

$$f'(x) = \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{(x^2 + 2x\delta x + \delta x^2) - (x^2)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{(2x\delta x + \delta x^2)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \, [2x + \delta x]$$

As $\delta x \to 0$, $[2x + \delta x] \to [2x + 0]$. Thus $\boldsymbol{f'(x) = 2x}$, i.e. the differential coefficient of x^2 is $2x$. At $x = 2$, the gradient of the curve, $f'(x) = 2(2) = \mathbf{4}$.

(Obtained from J. O. Bird 2017)

Differentiation of common functions

From differentiation by first principles of a number of examples such as in Problem 1 above, a general rule for differentiating $y = ax^n$ emerges, where *a* and *n* are constants. The rule is:

$$\textbf{if } y = ax^n \textbf{ then } \frac{dy}{dx} = anx^{n-1}$$

(or, **if** $f(x) = ax^n$ **then** $f'(x) = anx^{n-1}$) and is true for all real values of a and n.

For example, if $y = 4x^3$ then $a = 4$ and $n = 3$, and

$$\frac{dy}{dx} = anx^{n-1} = (4)(3)x^{3-1} = 12x^2$$

If $y = ax^n$ and $n = 0$ then $y = ax^0$ and

$$\frac{dy}{dx} = (a)(0)x^{0-1} = 0,$$

Figure 5(a) shows a graph of y= sin x. The gradient is continually changing as the curve moves from 0 to A to B to C to D.
The gradient, given. $\frac{dy}{dx}$, may be plotted in a corresponding position below y= sin x, as shown in Fig 5(b).

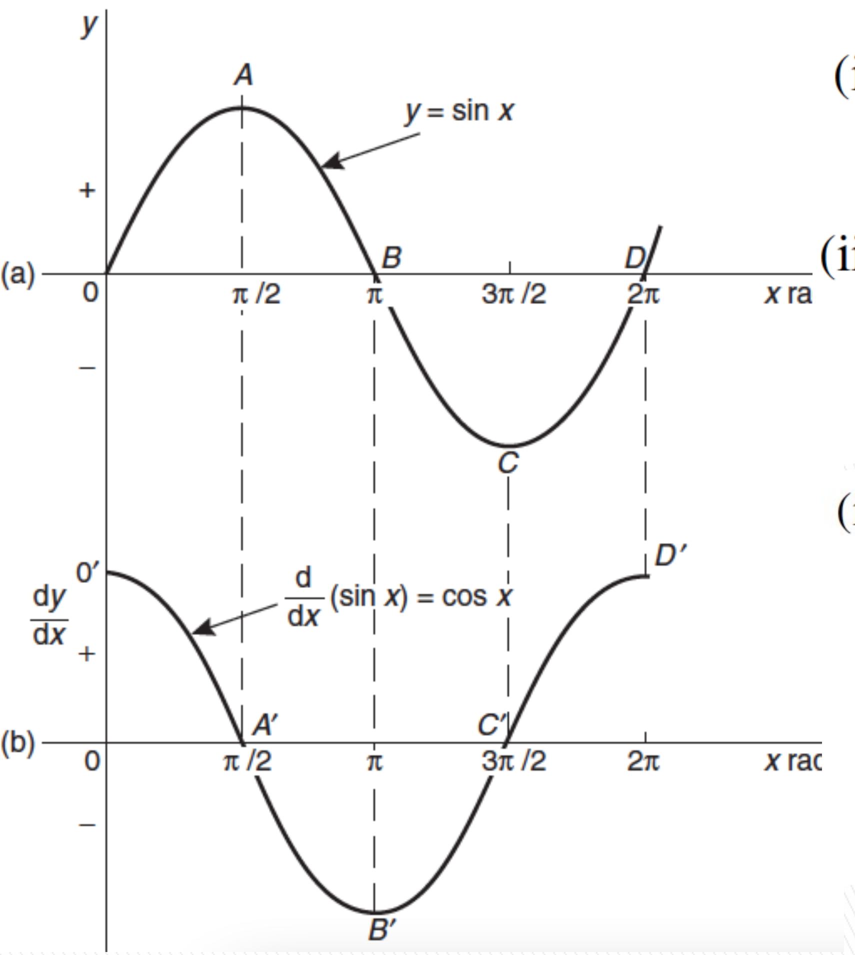

Figure 5

(i) At 0, the gradient is positive and is at its steepest. Hence $0'$ is a maximum positive value.

(ii) Between 0 and A the gradient is positive but is decreasing in value until at A the gradient is zero, shown as A'.

(iii) Between A and B the gradient is negative but is increasing in value until at B the gradient is at its steepest negative value. Hence B' is a maximum negative value.

(iv) If the gradient of $y = \sin x$ is further investigated between B and D then the resulting graph of $\frac{dy}{dx}$ is seen to be a cosine wave. Hence the rate of change of $\sin x$ is $\cos x$,

i.e. **if** $\boldsymbol{y = \sin x}$ **then** $\boldsymbol{\frac{dy}{dx} = \cos x}$

if $\boldsymbol{y = \sin ax}$ **then** $\boldsymbol{\frac{dy}{dx} = a \cos ax}$

If graphs of $y = \cos x$, $y = e^x$ and $y = \ln x$ are plotted and their gradients investigated, their differential coefficients may be determined in a similar manner

to that shown for $y = \sin x$. The rate of change of a function is a measure of the derivative. The standard derivatives summarised below may be proved theoretically and are true for all real values of x

y or $f(x)$	$\frac{dy}{dx}$ or $f'(x)$
ax^n	anx^{n-1}
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$
e^{ax}	ae^{ax}
$\ln ax$	$\frac{1}{x}$

Note:

The **differential coefficient of a sum or difference** is the sum or difference of the differential coefficients of the separate terms.

Thus, if $f(x) = p(x) + q(x) - r(x)$,
(where f, p, q and r are functions),

then $f'(x) = p'(x) + q'(x) - r'(x)$

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Problem 2. Find the differential coefficients of (a) $y = 12x^3$ (b) $y = \frac{12}{x^3}$.

If $y = ax^n$ then $\frac{dy}{dx} = anx^{n-1}$

(a) Since $y = 12x^3$, $a = 12$ and $n = 3$ thus $\frac{dy}{dx} = (12)(3)x^{3-1} = \mathbf{36x^2}$

(b) $y = \frac{12}{x^3}$ is rewritten in the standard ax^n form as $y = 12x^{-3}$ and in the general rule $a = 12$ and $n = -3$.

Thus $\frac{dy}{dx} = (12)(-3)x^{-3-1} = -36x^{-4} = \mathbf{-\frac{36}{x^4}}$

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Problem 3. Find the derivatives of (a) $y = 3\sqrt{x}$ (b) $y = \dfrac{5}{\sqrt[3]{x^4}}$.

Solution:

(a) $y = 3\sqrt{x}$ is rewritten in the standard differential form as $y = 3x^{\frac{1}{2}}$.

In the general rule, $a = 3$ and $n = \dfrac{1}{2}$

Thus $$\frac{dy}{dx} = (3)\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} = \frac{3}{2}x^{-\frac{1}{2}}$$

$$= \frac{3}{2x^{\frac{1}{2}}} = \mathbf{\frac{3}{2\sqrt{x}}}$$

(b) $y = \dfrac{5}{\sqrt[3]{x^4}} = \dfrac{5}{x^{\frac{4}{3}}} = 5x^{-\frac{4}{3}}$ in the standard differential form.

In the general rule, $a = 5$ and $n = -\frac{4}{3}$

Thus $$\frac{dy}{dx} = (5)\left(-\frac{4}{3}\right)x^{-\frac{4}{3}-1} = \frac{-20}{3}x^{-\frac{7}{3}}$$

$$= \frac{-20}{3x^{\frac{7}{3}}} = \mathbf{\frac{-20}{3\sqrt[3]{x^7}}}$$

Problem 4. Differentiate, with respect to x,

$$y = 5x^4 + 4x - \frac{1}{2x^2} + \frac{1}{\sqrt{x}} - 3.$$

Solution:

$y = 5x^4 + 4x - \dfrac{1}{2x^2} + \dfrac{1}{\sqrt{x}} - 3$ is rewritten as

$$y = 5x^4 + 4x - \frac{1}{2}x^{-2} + x^{-\frac{1}{2}} - 3$$

When differentiating a sum, each term is differentiated in turn.

Thus
$$\frac{dy}{dx} = (5)(4)x^{4-1} + (4)(1)x^{1-1} - \frac{1}{2}(-2)x^{-2-1} + (1)\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} - 0$$

$$= 20x^3 + 4 + x^{-3} - \frac{1}{2}x^{-\frac{3}{2}}$$

i.e. $$\mathbf{\frac{dy}{dx} = 20x^3 + 4 + \frac{1}{x^3} - \frac{1}{2\sqrt{x^3}}}$$

(Obtained from J. O. Bird 2017)

Problem 5. Find the differential coefficients of *(a) y=3 sin 4x* *(b) f (t)=2 cos 3t* with respect to the variable. Solution:

(a) When $y = 3\sin 4x$ then $\dfrac{dy}{dx} = (3)(4\cos 4x)$

$$= \mathbf{12\cos 4x}$$

(b) When $f(t) = 2\cos 3t$ then

$f'(t) = (2)(-3\sin 3t) = \mathbf{-6\sin 3t}$

Problem 6. Determine the derivatives of (a) $y = 3e^{5x}$ (b) $f(\theta) = \dfrac{2}{e^{3\theta}}$ (c) $y = 6\ln 2x$.

Solution:

(a) When $y = 3e^{5x}$ then $\dfrac{dy}{dx} = (3)(5)e^{5x} = \mathbf{15e^{5x}}$

(b) $f(\theta) = \dfrac{2}{e^{3\theta}} = 2e^{-3\theta}$, thus

$$f'(\theta) = (2)(-3)e^{-30} = -6e^{-3\theta} = \mathbf{\frac{-6}{e^{3\theta}}}$$

(c) When $y = 6\ln 2x$ then $\dfrac{dy}{dx} = 6\left(\dfrac{1}{x}\right) = \mathbf{\dfrac{6}{x}}$

(Obtained from J. O. Bird 2017)

Problem 7. Determine the co-ordinates of the point on the graph y=3x2 −7x +2 where the gradient is −1.
Solution:

The gradient of the curve is given by the derivative.

When $y = 3x^2 - 7x + 2$ then $\dfrac{dy}{dx} = 6x - 7$

Since the gradient is -1 then $6x - 7 = -1$, from which, $x = 1$

When $x = 1$, $y = 3(1)^2 - 7(1) + 2 = -2$

Hence the gradient is –1 at the point (1, –2).

Differentiation of a product

When $y=uv$, and u and v are both functions of x, then

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

This is known as the **product rule.**

Problem 8. Find the differential coefficient of $y = 3x^2 \sin 2x$

$3x^2 \sin 2x$ is a product of two terms $3x^2$ and $\sin 2x$

Let $u = 3x^2$ and $v = \sin 2x$

Using the product rule:

$$\frac{dy}{dx} = \underset{\downarrow}{u}\ \underset{\downarrow}{\frac{dv}{dx}} + \underset{\downarrow}{v}\ \underset{\downarrow}{\frac{du}{dx}}$$

gives: $$\frac{dy}{dx} = (3x^2)(2\cos 2x) + (\sin 2x)(6x)$$

i.e. $$\frac{dy}{dx} = 6x^2 \cos 2x + 6x \sin 2x$$

$$= \mathbf{6x(x\cos 2x + \sin 2x)}$$

Problem 9. Find the rate of change of y with respect to *x* given *y=3√x ln 2x.*

Solution: The rate of change of y with respect to x is given by $\frac{dy}{dx}$

$y = 3\sqrt{x}\ln 2x = 3x^{\frac{1}{2}}\ln 2x$, which is a product.

Let $u = 3x^{\frac{1}{2}}$ and $v = \ln 2x$

$$\text{Then}\quad \frac{dy}{dx} = \underset{\downarrow}{u}\;\underset{\downarrow}{\frac{dv}{dx}} + \underset{\downarrow}{v}\;\underset{\downarrow}{\frac{du}{dx}}$$

$$= \left(3x^{\frac{1}{2}}\right)\left(\frac{1}{x}\right) + (\ln 2x)\left[3\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}\right]$$

$$= 3x^{\frac{1}{2}-1} + (\ln 2x)\left(\frac{3}{2}\right)x^{-\frac{1}{2}}$$

$$= 3x^{-\frac{1}{2}}\left(1 + \frac{1}{2}\ln 2x\right)$$

i.e. $$\boldsymbol{\frac{dy}{dx} = \frac{3}{\sqrt{x}}\left(1 + \frac{1}{2}\ln 2x\right)}$$

Problem 10. Differentiate $y = x^3 \cos 3x \ln x$.

Solution:

Let $u = x^3 \cos 3x$ (i.e. a product) and $v = \ln x$

Then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

where $\dfrac{du}{dx} = (x^3)(-3 \sin 3x) + (\cos 3x)(3x^2)$

and $\dfrac{dv}{dx} = \dfrac{1}{x}$

Hence $\dfrac{dy}{dx} = (x^3 \cos 3x)\left(\dfrac{1}{x}\right) + (\ln x)[-3x^3 \sin 3x + 3x^2 \cos 3x]$

$= x^2 \cos 3x + 3x^2 \ln x(\cos 3x - x \sin 3x)$

i.e. $\mathbf{\dfrac{dy}{dx} = x^2\{\cos 3x + 3 \ln x(\cos 3x - x \sin 3x)\}}$

Differentiation of a quotient

When $y = \dfrac{u}{v}$, and u and v are both functions of x

then $$\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}}$$

This is known as the **quotient rule**.

Problem 11 Find the differential coefficient of $y = \dfrac{4\sin 5x}{5x^4}$

$\dfrac{4\sin 5x}{5x^4}$ is a quotient. Let $u = 4\sin 5x$ and $v = 5x^4$

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Solution:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

where $\frac{du}{dx} = (4)(5)\cos 5x = 20\cos 5x$

and $\frac{dv}{dx} = (5)(4)x^3 = 20x^3$

Hence $\frac{dy}{dx} = \frac{(5x^4)(20\cos 5x) - (4\sin 5x)(20x^3)}{(5x^4)^2}$

$$= \frac{100x^4\cos 5x - 80x^3\sin 5x}{25x^8}$$

$$= \frac{20x^3[5x\cos 5x - 4\sin 5x]}{25x^8}$$

i.e. $\mathbf{\frac{dy}{dx} = \frac{4}{5x^5}(5x\cos 5x - 4\sin 5x)}$

Problem 12. Determine the differential coefficient of *y= tan ax*
Solution:

$y = \tan ax = \dfrac{\sin ax}{\cos ax}$. Differentiation of $\tan ax$ is thus treated as a quotient with $u = \sin ax$ and $v = \cos ax$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(\cos ax)(a \cos ax) - (\sin ax)(-a \sin ax)}{(\cos ax)^2}$$

$$= \frac{a\cos^2 ax + a\sin^2 ax}{(\cos ax)^2} = \frac{a(\cos^2 ax + \sin^2 ax)}{\cos^2 ax}$$

$$= \frac{a}{\cos^2 ax}, \text{ since } \cos^2 ax + \sin^2 ax = 1$$

Hence $\boldsymbol{\dfrac{dy}{dx} = a\sec^2 ax}$ since $\sec^2 ax = \dfrac{1}{\cos^2 ax}$

Problem 13. Find the derivative of *y= sec ax*
Solution:

$y = \sec ax = \dfrac{1}{\cos ax}$ (i.e. a quotient). Let $u = 1$ and $v = \cos ax$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$= \frac{(\cos ax)(0) - (1)(-a\sin ax)}{(\cos ax)^2}$$

$$= \frac{a\sin ax}{\cos^2 ax} = a\left(\frac{1}{\cos ax}\right)\left(\frac{\sin ax}{\cos ax}\right)$$

i.e. $\boldsymbol{\dfrac{dy}{dx} = a\sec ax\tan ax}$

Problem 14. Differentiate $y = \dfrac{te^{2t}}{2\cos t}$

Solution:

The function $\dfrac{te^{2t}}{2\cos t}$ is a quotient, whose numerator is a product.

Let $u = te^{2t}$ and $v = 2\cos t$ then

$$\frac{du}{dt} = (t)(2e^{2t}) + (e^{2t})(1) \text{ and } \frac{dv}{dt} = -2\sin t$$

$$\text{Hence } \frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$= \frac{(2\cos t)[2te^{2t} + e^{2t}] - (te^{2t})(-2\sin t)}{(2\cos t)^2}$$

$$= \frac{4te^{2t}\cos t + 2e^{2t}\cos t + 2te^{2t}\sin t}{4\cos^2 t}$$

$$= \frac{2e^{2t}[2t\cos t + \cos t + t\sin t]}{4\cos^2 t}$$

i.e. $$\boldsymbol{\frac{dy}{dx} = \frac{e^{2t}}{2\cos^2 t}(2t\cos t + \cos t + t\sin t)}$$

Problem 15. Determine the gradient of the curve

$$y = \frac{5x}{2x^2+4} \text{ at the point } \left(\sqrt{3}, \frac{\sqrt{3}}{2}\right)$$

Solution: Let $y = 5x$ and $v = 2x^2 + 4$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(2x^2+4)(5) - (5x)(4x)}{(2x^2+4)^2}$$

$$= \frac{10x^2 + 20 - 20x^2}{(2x^2+4)^2} = \frac{20 - 10x^2}{(2x^2+4)^2}$$

At the point $\left(\sqrt{3}, \frac{\sqrt{3}}{2}\right)$, $x = \sqrt{3}$,

$$\text{hence the gradient} = \frac{dy}{dx} = \frac{20 - 10(\sqrt{3})^2}{[2(\sqrt{3})^2 + 4]^2}$$

$$= \frac{20-30}{100} = -\frac{1}{10}$$

Function of a function

It is often easier to make a substitution before differentiating. If *y* is a function of *x* then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

This is known as the 'function of a function' rule (or sometimes the chain rule).

For example, if $y = (3x - 1)^9$ then, by making the substitution $u = (3x - 1)$, $y = u^9$, which is of the 'standard' form.

Hence $\frac{dy}{du} = 9u^8$ and $\frac{du}{dx} = 3$

Then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (9u^8)(3) = 27u^8$

Rewriting u as $(3x - 1)$ gives: $\mathbf{\frac{dy}{dx} = 27(3x - 1)^8}$

Since y is a function of u, and u is a function of x, then y is a function of a function of x.

Problem 16. Differentiate $y = 3\cos(5x^2 + 2)$

Solution: Let $u = 5x^2 + 2$ then $y = 3\cos u$

$$\text{Hence } \frac{du}{dx} = 10x \text{ and } \frac{dy}{du} = -3\sin u.$$

Using the function of a function rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (-3\sin u)(10x) = -30x\sin u$$

Rewriting u as $5x^2 + 2$ gives:

$$\mathbf{\frac{dy}{dx} = -30x\sin(5x^2 + 2)}$$

Problem 17. Find the derivative of $y = (4t^3 - 3t)^6$

Solution: Let $u = 4t^3 - 3t$, then $y = u^6$

Hence $\dfrac{du}{dt} = 12t^2 - 3$ and $\dfrac{dy}{du} = 6u^5$

Using the function of a function rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (6u^5)(12t^2 - 3)$$

Rewriting u as $(4t^3 - 3t)$ gives:

$$\frac{dy}{dt} = 6(4t^3 - 3t)^5(12t^2 - 3)$$

$$= \mathbf{18(4t^2 - 1)(4t^3 - 3t)^5}$$

Problem 18. Determine the differential coefficient of

$$y = \sqrt{(3x^2 + 4x - 1)}$$

Solution:

$$y = \sqrt{(3x^2 + 4x - 1)} = (3x^2 + 4x - 1)^{\frac{1}{2}}$$

Let $u = 3x^2 + 4x - 1$ then $y = u^{\frac{1}{2}}$

Hence $\dfrac{du}{dx} = 6x + 4$ and $\dfrac{dy}{du} = \dfrac{1}{2}u^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{u}}$

Using the function of a function rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \left(\frac{1}{2\sqrt{u}}\right)(6x + 4) = \frac{3x + 2}{\sqrt{u}}$$

i.e. $\mathbf{\dfrac{dy}{dx} = \dfrac{3x + 2}{\sqrt{(3x^2 + 4x - 1)}}}$

Problem 19. Differentiate $y = 3\tan^4 3x$

Solution:

Let $u = \tan 3x$ then $y = 3u^4$

Hence $\frac{du}{dx} = 3\sec^2 3x$, (from Problem 12 and

$$\frac{dy}{du} = 12u^3$$

Then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (12u^3)(3\sec^2 3x)$

$$= 12(\tan 3x)^3(3\sec^2 3x)$$

i.e. $\mathbf{\frac{dy}{dx} = 36\tan^3 3x \sec^2 3x}$

Problem 20. Find the differential coefficient of $y=\dfrac{2}{(2t^3-5)^4}$

Solution:

$$y=\frac{2}{(2t^3-5)^4}=2(2t^3-5)^{-4}.\quad \text{Let}\quad u=(2t^3-5),$$

then $y=2u^{-4}$

$$\text{Hence}\quad \frac{du}{dt}=6t^2 \text{ and } \frac{dy}{du}=-8u^{-5}=\frac{-8}{u^5}$$

$$\text{Then}\quad \frac{dy}{dt}=\frac{dy}{du}\times\frac{du}{dt}=\left(\frac{-8}{u^5}\right)(6t^2)$$

$$=\frac{-48t^2}{(2t^3-5)^5}$$

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References:

1. J. O. Bird (2017), Higher engineering mathematics Eighth ed.
2. K. A. Stroud (1995) , Engineering mathematics Fourth ed.