Question 1

A skydiver jumps from a moving aircraft at a point directly above a fixed point, O, on the ground. The trajectory of the skydiver is then modelled by the function

$$h(x) = 3200 - 0.5x^2$$

where h m is the height of the skydiver above the ground and x m is the horizontal distance along the ground from point O.

(a) (i) Explain the significance of the value 3200 in the model.

(ii) Calculate the horizontal distance the skydiver covered upon landing.

$h = 0$

[2]

(b) Sketch a graph of h against x.

[2]

(c) Explain why the model is not suitable for values of x larger than 80 m.

[1]

(a)(i) $\boxed{3200 \text{ m is the initial height of the skydiver.}}$

(ii) Solve for $h = 0$

$3200 - 0.5x^2 = 0$

$0.5x^2 - 3200 = 0$

$x = 80$ or $x = -80$

Reject as $x \geq 0$

$\boxed{\text{Horizontal distance} = 80 \text{ m}}$

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(ii) Calculate the horizontal distance the skydiver covered upon landing.

[2]

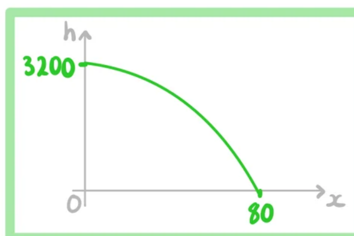
(b) Sketch a graph of h against x.

[2]

(c) Explain why the model is not suitable for values of x larger than 80 m.

[1]

(b) This will be a negative parabola with a maximum at $(0, 3200)$ and a root at $(80, 0)$.



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[2]

(b) Sketch a graph of h against x.

[2]

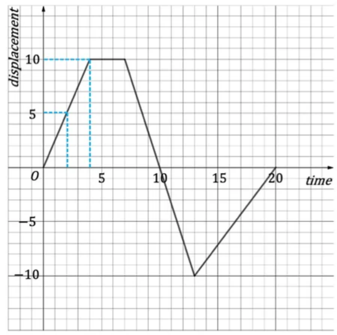
(c) Explain why the model is not suitable for values of x larger than 80 m.

[1]

(c) The skydiver lands on the ground when $x = 80$, so for values of x larger than 80 the height above ground would be 0 m. However the model gives negative values when $x > 80$.

Question 2

A particle moves along a horizontal line starting at the point O. The displacement-time graph for the first 20 seconds of its motion is shown below. Displacement is measured in metres.



(a) (i) Write down the displacement of the particle after 2 seconds.

(ii) Write down the displacement of the particle after 4 seconds.

[2]

(b) Find the velocity of the particle between 13 and 20 seconds.

[1]

(c) Find the speed of the particle between 7 and 10 seconds.

[1]

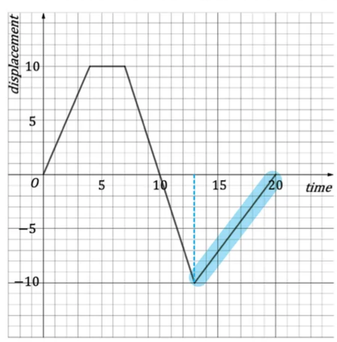
(d) Find the total distance travelled by the particle after 20 seconds.

[2]

(a)(i) $\boxed{5\text{m}}$

(ii) $\boxed{10\text{m}}$

A particle moves along a horizontal line starting at the point O. The displacement-time graph for the first 20 seconds of its motion is shown below. Displacement is measured in metres.



(a) (i) Write down the displacement of the particle after 2 seconds.

(ii) Write down the displacement of the particle after 4 seconds.

[2]

(b) Find the velocity of the particle between 13 and 20 seconds.

[1]

(c) Find the speed of the particle between 7 and 10 seconds.

[1]

(d) Find the total distance travelled by the particle after 20 seconds.

[2]

(b) Velocity is the gradient of the displacement-time graph.

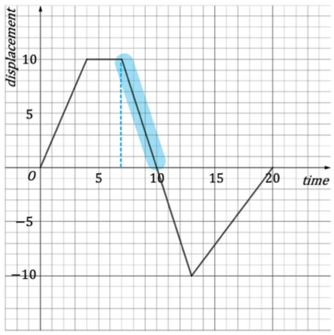
$m = \frac{y_2 - y_1}{x_2 - x_1}$ (from formula booklet)

$m = \frac{0-(-10)}{20-13}$

$= \frac{10}{7}$

$\boxed{\text{Velocity} = \frac{10}{7}\ \text{ms}^{-1}}$

A particle moves along a horizontal line starting at the point O. The displacement-time graph for the first 20 seconds of its motion is shown below. Displacement is measured in metres.



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[2]

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[1]

(c) Find the speed of the particle between 7 and 10 seconds.

[1]

(d) Find the total distance travelled by the particle after 20 seconds.

[2]

(c) Speed is the magnitude of velocity

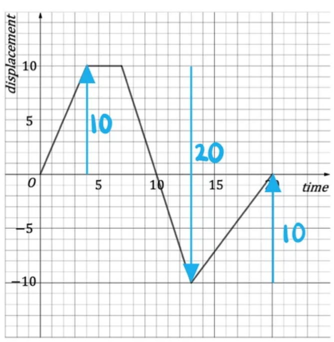
$$\text{Velocity} = \frac{0-10}{10-7}$$

$$= -\frac{10}{3}$$

$$\left|-\frac{10}{3}\right| = \frac{10}{3}$$

$$\boxed{\text{Speed} = \frac{10}{3}\ ms^{-1}}$$

A particle moves along a horizontal line starting at the point O. The displacement-time graph for the first 20 seconds of its motion is shown below. Displacement is measured in metres.



(a) (i) Write down the displacement of the particle after 2 seconds.

(ii) Write down the displacement of the particle after 4 seconds.

[2]

(b) Find the velocity of the particle between 13 and 20 seconds.

[1]

(c) Find the speed of the particle between 7 and 10 seconds.

[1]

(d) Find the total distance travelled by the particle after 20 seconds.

[2]

(d) Look at the distance for each section:

0-4s 4-7s 7-13s 13-20s

10m + 0m + 20m + 10m

$$\boxed{\text{Total distance} = 40m}$$

Question 3

A cricket ball is projected directly upwards from ground level. The motion of the cricket ball is modelled by the function

$$h(t) = 13t - 4.9t^2 \qquad t > 0$$

where h metres is the height of the cricket ball above ground level after t seconds.

(a) Find the times at which the cricket ball is exactly 3 m above the ground.

[2]

(b) For how long is the cricket ball at least 3 m above the ground?

[1]

A player catches the cricket ball (on its way down) at a height of 0.8 m above the ground.

(c) Find the length of time the ball was in the air.

[2]

(d) Find the total distance travelled by the ball.

[2]

(e) Find the velocity of the cricket ball at $t = 1$ second.

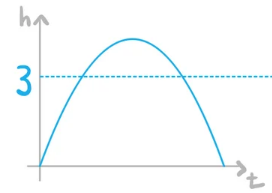
[2]

(a) Solve $h = 3$

$13t - 4.9t^2 = 3$

$4.9t^2 - 13t + 3 = 0$

$t = 0.2553\ldots \quad t = 2.3977\ldots$



$\boxed{t = 0.255\text{s } (3\text{sf}) \quad t = 2.40\text{s } (3\text{sf})}$

A cricket ball is projected directly upwards from ground level. The motion of the cricket ball is modelled by the function

$$h(t) = 13t - 4.9t^2 \qquad t > 0$$

where h metres is the height of the cricket ball above ground level after t seconds.

(a) Find the times at which the cricket ball is exactly 3 m above the ground.

$\boxed{t = 0.255\text{s } (3\text{sf}) \quad t = 2.40\text{s } (3\text{sf})}$

[2]

(b) For how long is the cricket ball at least 3 m above the ground?

[1]

A player catches the cricket ball (on its way down) at a height of 0.8 m above the ground.

(c) Find the length of time the ball was in the air.

[2]

(d) Find the total distance travelled by the ball.

[2]

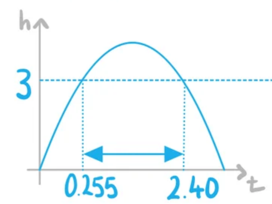
(e) Find the velocity of the cricket ball at $t = 1$ second.

[2]

(b) Subtract the times when its at 3m.

$2.3977\ldots - 0.2553\ldots$

$= 2.142\ldots$



$\boxed{2.14 \text{ seconds}}$

A cricket ball is projected directly upwards from ground level. The motion of the cricket ball is modelled by the function

$$h(t) = 13t - 4.9t^2 \qquad t > 0$$

where h metres is the height of the cricket ball above ground level after t seconds.

(a) Find the times at which the cricket ball is exactly 3 m above the ground.

[2]

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A player catches the cricket ball (on its way down) at a height of 0.8 m above the ground.

(c) Find the length of time the ball was in the air.

[2]

(d) Find the total distance travelled by the ball.

[2]

(e) Find the velocity of the cricket ball at $t = 1$ second.

[2]

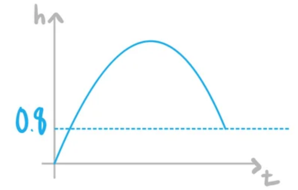
(c) Solve $h = 0.8$

$13t - 4.9t^2 = 0.8$

$4.9t^2 - 13t + 0.8 = 0$

$t = 0.0630\ldots \quad t = 2.5900\ldots$

Reject as this is on the way up



$\boxed{2.59 \text{ seconds}}$

A cricket ball is projected directly upwards from ground level. The motion of the cricket ball is modelled by the function

$$h(t) = 13t - 4.9t^2 \qquad t > 0$$

where h metres is the height of the cricket ball above ground level after t seconds.

(a) Find the times at which the cricket ball is exactly 3 m above the ground.

[2]

(b) For how long is the cricket ball at least 3 m above the ground?

[1]

A player catches the cricket ball (on its way down) at a height of 0.8 m above the ground.

(c) Find the length of time the ball was in the air.

$\boxed{2.59 \text{ seconds}}$

[2]

(d) Find the total distance travelled by the ball.

[2]

(e) Find the velocity of the cricket ball at $t = 1$ second.

[2]

(d) Method 1 - Find the maximum height

Draw the graph on the calculator and find the coordinates of the maximum.

Distance up + Distance down

$8.622\ldots + (8.622\ldots - 0.8)$

$= 16.44\ldots$

$\boxed{16.4\text{m}}$

Method 2 - Calculus

Distance $= \int_{t_1}^{t_2} |v(t)|\,dt$ (from formula booklet)

$v(t) = h'(t)$

$v(t) = 13 - 9.8t$

$\int_0^{2.59\ldots} |13 - 9.8t|\,dt = 16.444\ldots$

$\boxed{16.4\text{m}}$

A cricket ball is projected directly upwards from ground level. The motion of the cricket ball is modelled by the function

$$h(t) = 13t - 4.9t^2 \qquad t > 0$$

where h metres is the height of the cricket ball above ground level after t seconds.

(a) Find the times at which the cricket ball is exactly 3 m above the ground.

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A player catches the cricket ball (on its way down) at a height of 0.8 m above the ground.

(c) Find the length of time the ball was in the air.

[2]

(d) Find the total distance travelled by the ball.

[2]

(e) Find the velocity of the cricket ball at $t = 1$ second.

[2]

(e) Velocity is the derivative of displacement.

$v(t) = h'(t)$

$v(t) = 13 - 9.8t$

Substitute $t = 1$

$v(1) = 13 - 9.8(1)$

$= 3.2$

$\boxed{v(1) = 3.2 \text{ ms}^{-1}}$

Question 4

A soft ball is thrown upwards from the top of a 10 m tall building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = H + 7.8t - 4.9t^2 \qquad t > 0$$

(a) Write down the value of H.

[1]

(b) Find the height of the ball after 2 seconds.

[2]

(c) Find the time at which the ball is at the same height as it was when thrown.

[2]

(d) Find the time the ball first hits the ground.

[2]

(e) Find $h''(t)$ and hence show that the acceleration at any time is -9.8 m/s^2.

[3]

(a) $t = 0 \Rightarrow h = H$

H is the initial height of the ball.

$\boxed{H = 10}$

A soft ball is thrown upwards from the top of a 10 m tall building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = H + 7.8t - 4.9t^2 \qquad t > 0$$

(a) Write down the value of H.

[1]

(b) Find the height of the ball after 2 seconds.

[2]

(c) Find the time at which the ball is at the same height as it was when thrown.

[2]

(d) Find the time the ball first hits the ground.

[2]

(e) Find $h''(t)$ and hence show that the acceleration at any time is $-9.8\ \text{m/s}^2$.

[3]

(b) Substitute $t=2$.

$h(2) = 10 + 7.8(2) - 4.9(2)^2$

$\boxed{h = 6\text{m}}$

A soft ball is thrown upwards from the top of a 10 m tall building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = H + 7.8t - 4.9t^2 \qquad t > 0$$

(a) Write down the value of H.

[1]

(b) Find the height of the ball after 2 seconds.

[2]

(c) Find the time at which the ball is at the same height as it was when thrown.

[2]

(d) Find the time the ball first hits the ground.

[2]

(e) Find $h''(t)$ and hence show that the acceleration at any time is $-9.8\ \text{m/s}^2$.

[3]

(c) Solve $h(t) = 10$

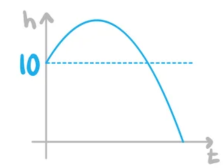
$10 + 7.8t - 4.9t^2 = 10$

$4.9t^2 - 7.8t = 0$

$t = 0 \qquad t = \frac{78}{49} = 1.5918...$

Reject as this was when thrown

$\boxed{t = 1.59\text{s}\ (3\text{sf})}$



A soft ball is thrown upwards from the top of a 10 m tall building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = H + 7.8t - 4.9t^2 \qquad t > 0$$

(a) Write down the value of H.

[1]

(b) Find the height of the ball after 2 seconds.

[2]

(c) Find the time at which the ball is at the same height as it was when thrown.

[2]

(d) Find the time the ball first hits the ground.

[2]

(e) Find $h''(t)$ and hence show that the acceleration at any time is $-9.8\ \text{m/s}^2$.

[3]

(d) Solve $h(t) = 0$

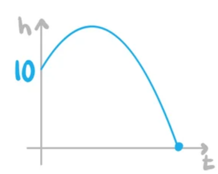
$10 + 7.8t - 4.9t^2 = 0$

$4.9t^2 - 7.8t - 10 = 0$

$t = -0.8394... \qquad t = 2.4312...$

Reject as time can't be negative

$\boxed{t = 2.43\text{s}}$



A soft ball is thrown upwards from the top of a 10 m tall building.
The height, h m of the ball above the ground after t seconds is modelled by the function

$$h(t) = H + 7.8t - 4.9t^2 \qquad t > 0$$

(a) Write down the value of H. [1]

(b) Find the height of the ball after 2 seconds. [2]

(c) Find the time at which the ball is at the same height as it was when thrown. [2]

(d) Find the time the ball first hits the ground. [2]

(e) Find $h''(t)$ and hence show that the acceleration at any time is $-9.8\ \mathrm{m/s^2}$. [3]

(e) Differentiate twice

$h'(t) = 7.8 - 9.8t$

$\boxed{h''(t) = -9.8}$

Acceleration $a = \frac{d^2s}{dt^2}$ (from formula booklet)

$a(t) = h''(t)$

$\boxed{a(t) = -9.8 \text{ for all } t}$

Question 5

A particle moves along a straight line with a velocity, v $\mathrm{ms^{-1}}$, given by $v = 2^t - 2$ where t is measured in seconds such that $0 \leq t \leq 4$.

(a) Find the acceleration of the particle at time $t = 2$. [2]

(b) State the time when the particle comes to rest. [1]

(c) Find the total distance travelled by the particle. [3]

(a) $a = \frac{dv}{dt}$ (from formula booklet)

Using the derivative function on the calculator

$a = \frac{d}{dt}(2^t - 2)\Big|_{t=2}$

$= 2.7725...$

$\boxed{a(2) = 2.77\mathrm{ms^{-2}} \ (3sf)}$

A particle moves along a straight line with a velocity, v $\mathrm{ms^{-1}}$, given by $v = 2^t - 2$ where t is measured in seconds such that $0 \leq t \leq 4$.

(a) Find the acceleration of the particle at time $t = 2$. [2]

(b) State the time when the particle comes to rest.
$v = 0$
[1]

(c) Find the total distance travelled by the particle. [3]

(b) Rest $\Rightarrow v = 0$

$2^t - 2 = 0$

$2^t = 2$

$\boxed{t = 1\ s}$

A particle moves along a straight line with a velocity, v ms^{-1}, given by $v = 2^t - 2$ where t is measured in seconds such that $0 \le t \le 4$.

(a) Find the acceleration of the particle at time $t = 2$.

[2]

(b) State the time when the particle comes to rest.

[1]

(c) Find the total distance travelled by the particle.

[3]

(c) Method 1 - Using formula booklet

Distance $= \int_{t_1}^{t_2} |v(t)|\,dt$ (from formula booklet)

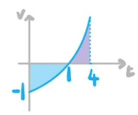
Distance $= \int_0^4 |2^t - 2|\,dt$

$= 14.755...$

$\boxed{\text{Distance} = 14.8\text{ m (3sf)}}$

Method 2 - Using a velocity-time graph

Displacement $= \int_{t_1}^{t_2} v(t)\,dt$ (from formula booklet)



0-1 s $\int_0^1 2^t - 2dt = -0.5573...$

1-4 s $\int_1^4 2^t - 2dt = 14.1977...$

Distance is magnitude of displacement

Total distance $= |-0.5573...| + 14.1977...$

$= 14.755...$

$\boxed{\text{Distance} = 14.8\text{ m (3sf)}}$

Question 6

A particle is found to have an acceleration, a ms^{-2}, according to the function

$$a = \frac{1}{t^2} + \sin t, \text{ where } t \ge 1$$

(a) Find an expression for the velocity, v, of the particle given that $v(1) = 1$.

[4]

(b) Find the velocity of the particle at $t = 2$.

[2]

(a) $a = \frac{dv}{dt}$ (from formula booklet)

$v = \int a\,dt$

$v = \int \frac{1}{t^2} + \sin t\,dt$

$v = \int t^{-2} + \sin t\,dt$

$v = -t^{-1} - \cos t + c$

$v(1) = 1$

$1 = -(1)^{-1} - \cos 1 + c$

$c = 1 + 1 + \cos 1$ ← Make sure calculator is in radians.

$c = 2.5403...$

$\boxed{v = -\frac{1}{t} - \cos t + 2.54}$

A particle is found to have an acceleration, a ms^{-2}, according to the function

$$a = \frac{1}{t^2} + \sin t, \text{ where } t \geq 1$$

(a) Find an expression for the velocity, v, of the particle given that $v(1) = 1$. [4]

$\boxed{v = -\frac{1}{t} - \cos t + 2.54}$

(b) Find the velocity of the particle at $t = 2$. [2]

(b) Substitute $t = 2$.

$v(2) = -\frac{1}{2} - \cos 2 + 2.54\ldots$

$= 2.456\ldots$

Make sure calculator is in radians.

$\boxed{v(2) = 2.46 \text{ ms}^{-1} \ (3sf)}$

Question 7

A particle, moving in a straight line, is found to have a velocity $v = \sin t + \cos 2t$ where v is measured in ms^{-1} and time t is measured in seconds such that $0 \leq t \leq 5$.

(a) Find the time(s) when the particle is instantaneously at rest. [2]

$v = 0$

(b) Find the time(s) when the particle changes direction. [1]

(c) Find the distance travelled in the first second of motion. [3]

(d) Find the acceleration of the particle at the instant it first changes direction. [3]

(e) Find the displacement of the particle from its starting point to the point when $t = 5$. [4]

(a) Rest $\Rightarrow$ $v = 0$

$\sin t + \cos 2t = 0$

Solve using calculator (in radians).

$t = 1.5707\ldots$ $\quad t = 3.6651\ldots$

$\boxed{t = 1.57 \text{ s} \ (3sf) \quad t = 3.67 \text{ s} \ (3sf)}$

A particle, moving in a straight line, is found to have a velocity $v = \sin t + \cos 2t$ where v is measured in ms^{-1} and time t is measured in seconds such that $0 \leq t \leq 5$.

(a) Find the time(s) when the particle is instantaneously at rest. [2]

$\boxed{t = 1.57 \text{ s} \quad t = 3.67 \text{ s}}$

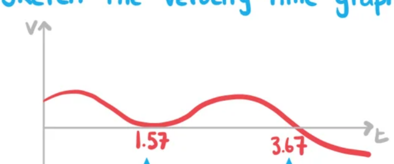
(b) Find the time(s) when the particle changes direction. [1]

(c) Find the distance travelled in the first second of motion. [3]

(d) Find the acceleration of the particle at the instant it first changes direction. [3]

(e) Find the displacement of the particle from its starting point to the point when $t = 5$. [4]

(b) Sketch the velocity-time graph



Still moves forward

Changes direction

$\boxed{t = 3.67 \text{ s} \ (3sf)}$

EXAM PAPERS PRACTICE

A particle, moving in a straight line, is found to have a velocity $v = \sin t + \cos 2t$ where v is measured in ms^{-1} and time t is measured in seconds such that $0 \le t \le 5$.

(a) Find the time(s) when the particle is instantaneously at rest. [2]

(b) Find the time(s) when the particle changes direction. [1]

(c) Find the distance travelled in the first second of motion. [3]

(d) Find the acceleration of the particle at the instant it first changes direction. [3]

(e) Find the displacement of the particle from its starting point to the point when $t = 5$. [4]

(c) Distance $= \int_{t_1}^{t_2} |v(t)|\, dt$ (from formula booklet)

Distance $= \int_0^1 |\sin t + \cos 2t|\, dt$

$= 0.9143...$

$\boxed{\text{Distance} = 0.914 \text{ (3sf)}}$

A particle, moving in a straight line, is found to have a velocity $v = \sin t + \cos 2t$ where v is measured in ms^{-1} and time t is measured in seconds such that $0 \le t \le 5$.

(a) Find the time(s) when the particle is instantaneously at rest. [2]

(b) Find the time(s) when the particle changes direction. [1]

(c) Find the distance travelled in the first second of motion. [3]

(d) Find the acceleration of the particle at the instant it first changes direction.

$t = 3.665...$ [3]

(e) Find the displacement of the particle from its starting point to the point when $t = 5$. [4]

(d) $a = \frac{dv}{dt}$ (from formula booklet)

Method 1 - Using calculator

$a = \frac{d}{dt}(\sin t + \cos 2t)\Big|_{t=3.665...}$

$= -2.5977...$

$\boxed{a = -2.60 \text{ ms}^{-2} \text{ (3sf)}}$

Method 2 - By hand

$a(t) = \cos t - 2\sin 2t$

$a(3.665...) = -2.5977...$

$\boxed{a = -2.60 \text{ ms}^{-2} \text{ (3sf)}}$

A particle, moving in a straight line, is found to have a velocity $v = \sin t + \cos 2t$ where v is measured in ms^{-1} and time t is measured in seconds such that $0 \le t \le 5$.

(a) Find the time(s) when the particle is instantaneously at rest. [2]

(b) Find the time(s) when the particle changes direction. [1]

(c) Find the distance travelled in the first second of motion. [3]

(d) Find the acceleration of the particle at the instant it first changes direction. [3]

(e) Find the displacement of the particle from its starting point to the point when $t = 5$. [4]

$t=0$

(e) Displacement $= \int_{t_1}^{t_2} v(t)\,dt$ (from formula booklet)

Displacement $= \int_0^5 \sin t + \cos 2t \, dt$

Method 1 - Using calculator

$\int_0^5 \sin t + \cos 2t\,dt = 0.4443...$

$\boxed{\text{Displacement} = 0.444\text{m (3sf)}}$

Method 2 - By hand

$$\int_0^5 \sin t + \cos 2t\,dt = \left[-\cos t + \tfrac{1}{2}\sin 2t\right]_0^5$$
$$= (-\cos 5 + \tfrac{1}{2}\sin 10) - (-\cos 0 + \tfrac{1}{2}\sin 0)$$
$$= -0.5556... \quad -(-1)$$
$$= 0.4443...$$

$\boxed{\text{Displacement} = 0.444\text{m (3sf)}}$

Question 8

A particle is moving along a straight line. The position of the particle at time t seconds, measured in metres relative to a fixed origin point, is denoted by $x(t)$.

The particle starts at the origin at time $t = 0$, and its motion over the next eight seconds is described by the equation

$$\dot{x}(t) = \frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3, \quad 0 \le t \le 8$$

(a) Find an expression for $x(t)$. [4]

(b) Hence determine the maximum distance of the particle from the origin during the first eight seconds of its movement. [3]

(c) Find the change in displacement of the particle during the first eight seconds of its movement. [2]

(d) Find the total distance travelled by the particle during the first eight seconds of its movement. [2]

(e) Find an expression for the particle's acceleration $\ddot{x}(t)$. [3]

(a) Displacement $x(t)$ is the integral of velocity $\dot{x}(t)$

$x(t) = \int \dot{x}(t)\,dt$

$x(t) = \int \left(\frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3\right)dt$

Using formula booklet $\int \frac{1}{\cos^2 x}dx = \tan x + c$

$x(t) = \frac{1}{\frac{\pi}{20}} \tan\left(\frac{\pi}{20}t\right) - 3t + c$

$x(t) = \frac{20}{\pi} \tan\left(\frac{\pi}{20}t\right) - 3t + c$

Apply boundary condition $x(0) = 0$

$0 = \frac{20}{\pi}\tan(0) - 0 + c$

$\therefore c = 0$

$\boxed{x(t) = \frac{20}{\pi}\tan\left(\frac{\pi}{20}t\right) - 3t}$

Note that there is no "c" in the final answer.
This is only because $c = 0$ in this case.
In most cases c won't be zero.

A particle is moving along a straight line. The position of the particle at time t seconds, measured in metres relative to a fixed origin point, is denoted by $x(t)$.

The particle starts at the origin at time $t = 0$, and its motion over the next eight seconds is described by the equation

$$\dot{x}(t) = \frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3, \quad 0 \le t \le 8$$

(a) Find an expression for $x(t)$. $\boxed{x(t) = \frac{20}{\pi}\tan\left(\frac{\pi}{20}t\right) - 3t}$

[4]

(b) Hence determine the maximum distance of the particle from the origin during the first eight seconds of its movement.

[3]

(c) Find the change in displacement of the particle during the first eight seconds of its movement.

[2]

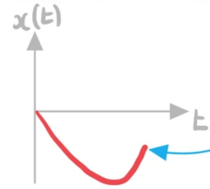
(d) Find the total distance travelled by the particle during the first eight seconds of its movement.

[2]

(e) Find an expression for the particle's acceleration $\ddot{x}(t)$.

[3]

(b) Use GDC to sketch the graph for $0 \le t \le 8$



This is the point when it is furthest from the origin

$(6.081..., -9.242...)$

Distance $= |-9.242...|$

$\boxed{\text{Maximum distance from origin} = 9.24 \text{ m (3sf)}}$

We could also solve analytically

Maximum distance $\Rightarrow$ velocity $= 0$

$\dot{x}(t) = 0$

$\frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3 = 0$

$\cos\left(\frac{\pi}{20}t\right) = \frac{1}{\sqrt{3}}$

$t = \frac{20}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 6.0817...$ Need to use radians

$x(6.0817...) = -9.2420...$

$|x(6.0817...)| = 9.2420...$ $\boxed{9.24 \text{ m (3sf)}}$

A particle is moving along a straight line. The position of the particle at time t seconds, measured in metres relative to a fixed origin point, is denoted by $x(t)$.

The particle starts at the origin at time $t = 0$, and its motion over the next eight seconds is described by the equation

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(a) Find an expression for $x(t)$.

[4]

(b) Hence determine the maximum distance of the particle from the origin during the first eight seconds of its movement.

[3]

(c) Find the change in displacement of the particle during the first eight seconds of its movement.

[2]

(d) Find the total distance travelled by the particle during the first eight seconds of its movement.

[2]

(e) Find an expression for the particle's acceleration $\ddot{x}(t)$.

[3]

(c) Method 1

Displacement $= \int_{t_1}^{t_2} v(t)\,dt$ Given in formula booklet

Displacement $= \int_0^8 \dot{x}(t)\,dt$

$= \int_0^8 \left(\frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3\right)dt$

$= -4.40685...$

Method 2

Displacement = Final position − Initial position

Displacement $= x(8) - x(0)$

$= (-4.40685...) - (0)$

$= -4.40685...$

$\boxed{\text{Displacement} = -4.41 \text{ m (3sf)}}$

A particle is moving along a straight line. The position of the particle at time t seconds, measured in metres relative to a fixed origin point, is denoted by $x(t)$.

The particle starts at the origin at time $t = 0$, and its motion over the next eight seconds is described by the equation

$$\dot{x}(t) = \frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3, \quad 0 \le t \le 8$$

(a) Find an expression for $x(t)$.

[4]

(b) Hence determine the maximum distance of the particle from the origin during the first eight seconds of its movement.

[3]

(c) Find the change in displacement of the particle during the first eight seconds of its movement.

[2]

(d) Find the total distance travelled by the particle during the first eight seconds of its movement.

[2]

(e) Find an expression for the particle's acceleration $\ddot{x}(t)$.

[3]

(d) Distance travelled $= \int_{t_1}^{t_2} |v(t)|\,dt$ Given in formula booklet

Distance $= \int_0^8 |\dot{x}(t)|\,dt$

$= \int_0^8 \left| \frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3 \right| dt$

$= 14.0772\ldots$

$\boxed{\text{Distance} = 14.1\text{ m } (3\text{ sf})}$

A particle is moving along a straight line. The position of the particle at time t seconds, measured in metres relative to a fixed origin point, is denoted by $x(t)$.

The particle starts at the origin at time $t = 0$, and its motion over the next eight seconds is described by the equation

$$\dot{x}(t) = \frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3, \quad 0 \le t \le 8$$

(a) Find an expression for $x(t)$.

[4]

(b) Hence determine the maximum distance of the particle from the origin during the first eight seconds of its movement.

[3]

(c) Find the change in displacement of the particle during the first eight seconds of its movement.

[2]

(d) Find the total distance travelled by the particle during the first eight seconds of its movement.

[2]

(e) Find an expression for the particle's acceleration $\ddot{x}(t)$.

[3]

(e) Acceleration $\ddot{x}(t)$ is the derivate of velocity $\dot{x}(t)$

$\ddot{x}(t) = \frac{d}{dt}(\dot{x}(t))$

$\ddot{x}(t) = \frac{d}{dt}\left(\frac{1}{\cos^2\left(\frac{\pi}{20}t\right)} - 3\right)$

$= \frac{d}{dt}\left(\left(\cos\left(\frac{\pi}{20}t\right)\right)^{-2} - 3\right)$

Differentiate $\left(\cos\left(\frac{\pi}{20}t\right)\right)^{-2}$ using the chain rule

$u = \cos\left(\frac{\pi}{20}t\right) \Rightarrow \frac{du}{dt} = -\frac{\pi}{20}\sin\left(\frac{\pi}{20}t\right)$

$y = u^{-2} \Rightarrow \frac{dy}{du} = -2u^{-3}$

$\ddot{x}(t) = -2\left(\cos\left(\frac{\pi}{20}t\right)\right)^{-3}\left(-\frac{\pi}{20}\sin\left(\frac{\pi}{20}t\right)\right)$

$\boxed{\ddot{x}(t) = \frac{\pi}{10}\left(\cos\left(\frac{\pi}{20}t\right)\right)^{-3}\left(\sin\left(\frac{\pi}{20}t\right)\right)}$

This expression can be rewritten as

$\ddot{x}(t) = \frac{\pi\left(\tan\left(\frac{\pi}{20}t\right)\right)}{10\left(\cos\left(\frac{\pi}{20}t\right)\right)^2}$

Question 9

A particle is moving along a straight line. The position of the particle at any given time, measured in metres relative to a fixed origin point, is denoted by x.

It is known that the velocity, v ms^{-1}, of the particle is dependent on the particle's position, and that the velocity may be described by the equation

$$v(x) = \sqrt{1-x^2}, \quad -1 \le x \le 1$$

(a) Use the chain rule to explain why the acceleration, a ms^{-2}, of the particle may be expressed in the form

$$a = v\frac{\mathrm{d}v}{\mathrm{d}x}$$

[3]

(b) Show that the derivative of $\sqrt{1-x^2}$ is $-\frac{x}{\sqrt{1-x^2}}$.

[4]

(c) Hence find an expression for the acceleration of the particle in terms of x, being sure to indicate the domain of x values for which the expression is valid.

[2]

(d) Identify the minimum and maximum values of

(i) the speed of the particle

(ii) the magnitude of the particle's acceleration

along with the values of x for which those occur.

[3]

(a) $a = \frac{dv}{dt}$ and $v = \frac{dx}{dt}$

$a = \frac{dv}{dt}$

By the chain rule

$$\boxed{\begin{aligned} a &= \frac{dv}{dx} \times \frac{dx}{dt} \\ a &= \frac{dv}{dx} \times v \\ a &= v\frac{dv}{dx} \end{aligned}}$$

A particle is moving along a straight line. The position of the particle at any given time, measured in metres relative to a fixed origin point, is denoted by x.

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[3]

(b) Show that the derivative of $\sqrt{1-x^2}$ is $-\frac{x}{\sqrt{1-x^2}}$.

[4]

(c) Hence find an expression for the acceleration of the particle in terms of x, being sure to indicate the domain of x values for which the expression is valid.

[2]

(d) Identify the minimum and maximum values of

(i) the speed of the particle

(ii) the magnitude of the particle's acceleration

along with the values of x for which those occur.

[3]

(b) Rewrite using exponents

$\sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}}$

By the chain rule

$u = 1-x^2 \Rightarrow \frac{du}{dx} = -2x$

$y = u^{\frac{1}{2}} \Rightarrow \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$

You could do it without writing down $u = \ldots$

$$\boxed{\begin{aligned} \frac{d}{dx}\left((1-x^2)^{\frac{1}{2}}\right) &= \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) \\ &= -x(1-x^2)^{-\frac{1}{2}} \\ \frac{d}{dx}\left(\sqrt{1-x^2}\right) &= -\frac{x}{\sqrt{1-x^2}} \end{aligned}}$$

A particle is moving along a straight line. The position of the particle at any given time, measured in metres relative to a fixed origin point, is denoted by x.

It is known that the velocity, v ms^{-1}, of the particle is dependent on the particle's position, and that the velocity may be described by the equation

$$v(x) = \sqrt{1 - x^2}, \quad -1 \le x \le 1$$

(a) Use the chain rule to explain why the acceleration, a ms^{-2}, of the particle may be expressed in the form

$$a = v\frac{\mathrm{d}v}{\mathrm{d}x}$$

[3]

(b) Show that the derivative of $\sqrt{1-x^2}$ is $-\frac{x}{\sqrt{1-x^2}}$.

[4]

(c) Hence find an expression for the acceleration of the particle in terms of x, being sure to indicate the domain of x values for which the expression is valid.

[2]

(d) Identify the minimum and maximum values of

(i) the speed of the particle

(ii) the magnitude of the particle's acceleration

along with the values of x for which those occur.

[3]

(c) $a = v\frac{dv}{dx}$

$a(x) = \cancel{\sqrt{1-x^2}} \times -\frac{x}{\cancel{\sqrt{1-x^2}}}$ Cancel common factors

$a(x)$ depends on v so it is valid when v is valid

$\boxed{a(x) = -x \quad \text{for } -1 \le x \le 1}$

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(c) Hence find an expression for the acceleration of the particle in terms of x, being sure to indicate the domain of x values for which the expression is valid.

[2]

(d) Identify the minimum and maximum values of

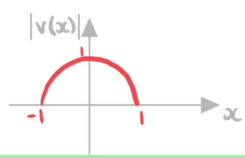
(i) the speed of the particle

(ii) the magnitude of the particle's acceleration

along with the values of x for which those occur.

[3]

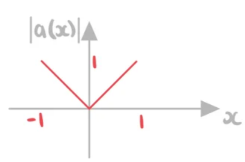
(d) (i) Sketch $|v(x)|$ in GDC



Could also do it without a graph by solving $a(x) = 0$ and looking at boundaries

Maximum speed $|v| = 1$ ms^{-1} when $x = 0$
Minimum speed $|v| = 0$ ms^{-1} when $x = -1$ and $x = 1$

(ii) Sketch $|a(x)|$ in GDC



Maximum $|a| = 1$ ms^{-2} when $x = -1$ and $x = 1$
Minimum $|a| = 0$ ms^{-2} when $x = 0$