



EXAM PAPERS PRACTICE

GCSE OCR Math J560

Iteration

Answers

*"We will help you to
achieve A Star "*



Answer 1

- (b) Explain the relationship between the values of x_1 , x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

$$x^3 + 2x^2 + 4 = 0$$

CAN BE REARRANGED TO GIVE

$$x = -2 - \frac{4}{x^2}$$

AND SO THE ITERATIVE FORMULA

$$x_{n+1} = -2 - \frac{4}{x_n^2}$$

GIVE SUCCESSIVE ESTIMATES x_1, x_2, x_3, \dots
FOR A ROOT OF $x^3 + 2x^2 + 4 = 0$

Answer 2

- (b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

$$x^3 + x = 7$$

$$-x \quad -x$$

$$\sqrt[3]{\quad} \quad x^3 = 7 - x$$

$$x = \sqrt[3]{7 - x}$$



Answer 3

(a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

$$x^3 + 4x - 1 = 0$$

LET $F(x) = x^3 + 4x - 1$

$x = 0$: $F(0) = 0^3 + 4 \times 0 - 1 = -1$

$x = 1$: $F(1) = 1^3 + 4 \times 1 - 1 = 4$

CHANGE OF SIGN RULE
THE EQUATION $F(x) = 0$ WILL
HAVE AT LEAST ONE SOLUTION
BETWEEN $x = a$ AND $x = b$
IF $F(a)$ AND $F(b)$ HAVE
DIFFERENT SIGNS

THE CHANGE OF SIGN INDICATES AT LEAST ONE SOLUTION
IN THE INTERVAL $x = 0$ TO $x = 1$

Answer 4

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice,
to find an estimate for the solution of $x^3 + 4x = 1$

$$x_0 = 0 \quad \text{so} \quad x_1 = \frac{1}{4} - \frac{0^3}{4} = \frac{1}{4} \rightarrow \boxed{\text{Ans}}$$
$$x_1 = \frac{1}{4} \quad \text{so} \quad x_2 = \frac{1}{4} - \frac{\text{Ans}^3}{4} = \frac{63}{256}$$



Answer 5

The number of slugs in a garden t days from now is p_t where

$$p_0 = 100$$

$$p_{t+1} = 1.06p_t \quad \rightarrow \text{ITERATIVE FORMULA}$$

Work out the number of slugs in the garden 3 days from now.

WE WANT p_3

TYPE "100 [=]"

TYPE "1.06 x [Ans]"

PRESS [=] FOR p_1 ETC

$$p_1 = 106$$

$$p_2 = 112.36$$

$$p_3 = 119.1016$$

No OF SLUGS = 119

ITERATION BY CALCULATOR

- TYPE IN " x_0 " THEN [=]
(THIS PUTS x_0 IN [Ans] MEMORY)
- TYPE IN THE ITERATIVE FORMULA WITH [Ans] INSTEAD OF x_n
- PRESS [=] FOR x_1
- PRESS [=] AGAIN FOR x_2
- ETC.!



Answer 6

Using $x_{n+1} = -2 - \frac{4}{x_n^2}$

with $x_0 = -2.5$

(a) find the values of x_1, x_2 and x_3

$x_0 = -2.5 \rightarrow \boxed{\text{ANS}}$

TYPE IN $-2 - \frac{4}{\boxed{\text{Ans}}^2}$

(PRESS $\boxed{\text{AC}}$ FIRST)

$x_1 = -2.64$

$x_2 = -2.573921\dots$

$x_3 = -2.603767\dots$

ITERATION BY CALCULATOR

- TYPE IN " x_0 " THEN $\boxed{=}$
(THIS PUTS x_0 IN $\boxed{\text{Ans}}$ MEMORY)
- TYPE IN THE ITERATIVE FORMULA WITH $\boxed{\text{Ans}}$ INSTEAD OF x_n
- PRESS $\boxed{=}$ FOR x_1
- PRESS $\boxed{=}$ AGAIN FOR x_2
- ETC. !

Answer 7

(a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

$f(x) = x^3 + x - 7 = 0$

$f(1) = 1^3 + 1 - 7 = -5$

$f(2) = 2^3 + 2 - 7 = 3$

CHANGE OF SIGN SO SOLUTION BETWEEN $x=1$ AND $x=2$

CHANGE OF SIGN RULE

THE EQUATION $F(x) = 0$ WILL HAVE AT LEAST ONE SOLUTION BETWEEN $x=a$ AND $x=b$ IF $F(a)$ AND $F(b)$ HAVE DIFFERENT SIGNS



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Answer 8

- (c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

$$x_0 = 2$$

$$x_1 = 1.7099759\dots$$

$$x_2 = 1.7424188\dots$$

$$x_3 = \underline{\underline{1.7388495\dots}}$$

ITERATION BY CALCULATOR

- TYPE IN " x_0 " THEN [=] (THIS PUTS x_0 IN [Ans] MEMORY)
- TYPE IN THE ITERATIVE FORMULA WITH [Ans] INSTEAD OF x_n
- PRESS [=] FOR x_1
- PRESS [=] AGAIN FOR x_2
- ETC.!

Answer 9

- (b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

$$\begin{aligned}x^3 + 4x &= 1 \\ -x^3 &\quad -x^3 \\ \hline 4x &= 1 - x^3 \\ \frac{4x}{4} &= \frac{1 - x^3}{4} \\ x &= \frac{1 - x^3}{4} \\ x &= \frac{1}{4} - \frac{x^3}{4}\end{aligned}$$



Answer 10

The number of bees in a beehive at the start of year n is P_n .

The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05(P_n - 250) \rightarrow \text{ITERATIVE FORMULA}$$

At the start of 2015 there were 9500 bees in the beehive. $\rightarrow x_0$ (or P_0)

How many bees will there be in the beehive at the start of 2018? $\rightarrow x_3$ (or P_3)

$$2016 : x_1 = 9712.5$$

$$2017 : x_2 = 9935.625$$

$$2018 : x_3 = 10169.9...$$

10170 BEES

ITERATION BY CALCULATOR

- TYPE IN " x_0 " THEN [=] (THIS PUTS x_0 IN [Ans] MEMORY)
- TYPE IN THE ITERATIVE FORMULA WITH [Ans] INSTEAD OF x_n
- PRESS [=] FOR x_1
- PRESS [=] AGAIN FOR x_2
- ETC.!



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Answer 11

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \frac{5}{x^2 + 7}$

$$\begin{aligned}x^3 + 7x - 5 &= 0 \\x^3 + 7x &= 5 \\x(x^2 + 7) &= 5 \\ \frac{x(x^2 + 7)}{(x^2 + 7)} &= \frac{5}{(x^2 + 7)} \\x &= \frac{5}{x^2 + 7}\end{aligned}$$

Answer 12

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$, comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

$$\begin{aligned}f(0.6704483001\dots) &= -5.49 \times 10^{-3} \\ &= -0.00549\end{aligned}$$

THIS IS VERY CLOSE TO 0 SO ACCURACY IS GOOD.



Answer 13

(b) Using

$$x_{n+1} = 3 + \frac{3}{x_n^2} \quad \text{with } x_0 = 3.2,$$

find the values of x_1, x_2 and x_3

$$x_1 = \frac{843}{256} = 3.29296875$$

$$x_2 = 3.276659786$$

$$x_3 = 3.279420685$$

ITERATIVE FORMULA

CLEVER WAY:

PUT "3.2 =" INTO
OUR CALCULATOR

(PUTS 3.2 → ANS)

USE $3 + \frac{3}{\text{ANS}^2}$

Answer 14

$$f(x) = x^4 - 8x^2 + 2$$

(a) Show that the equation $f(x) = 0$ can be written as $x = \sqrt{ax^4 + b}$, $x > 0$, where a and b are constants to be found.

Let $x_0 = 1.5$.

Rearrange $f(x) = 0$ for

$$x^4 - 8x^2 + 2 = 0$$

$$8x^2 = x^4 + 2$$

$$x = \sqrt{\frac{1}{8}x^4 + \frac{1}{4}}$$



Answer 15

$$f(x) = x^3 - 2x - 5.$$

(a) Show that there is a root α of $f(x) = 0$ for x in the interval $[2, 3]$.

$$f(2) = 2^3 - 2(2) - 5$$

$$= 8 - 4 - 5$$

$$= -1$$

$$f(3) = 3^3 - 2(3) - 5$$

$$= 27 - 6 - 5$$

$$= 16$$

A change of sign in the interval hence there is a root because $f(x)$ is a continuous function.