Question 1

(a) State the value of $\arctan(\sqrt{3})$. [1]

(b) If $\arccos x = \frac{\pi}{6}$ find

(i) the exact value of $\arcsin x$.

(ii) the exact value of $\sec(\arccos x)$. [6]

(a) $\tan\frac{\pi}{3} = \sqrt{3}$

$\boxed{\arctan(\sqrt{3}) = \frac{\pi}{3}}$

Note: range for $\arctan x$ is $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$

(a) State the value of $\arctan(\sqrt{3})$. [1]

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(ii) the exact value of $\sec(\arccos x)$. [6]

(b) $\arccos x = y \Rightarrow x = \cos y$

$x = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

(i) $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\boxed{\arcsin x = \frac{\pi}{3}}$

Note: range for $\arcsin x$ is $-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

$\sec(\arccos x) = \frac{1}{\cos(\arccos x)}$

$\cos(\arccos\theta) = \theta$

$\sec(\arccos x) = \frac{1}{x}$

$\boxed{\sec(\arccos x) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}}$

Question 2

Find the exact values of the following expressions:

(i) $\operatorname{cosec}\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{6}\right)$

(ii) $3\sin\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{3}\right)$

[6]

You can use these 'magic' triangles to remember exact values.

i) $\operatorname{cosec}\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} + \tan\left(\frac{\pi}{6}\right)$

$\operatorname{cosec}\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{6}\right) = \frac{1}{\frac{\sqrt{3}}{2}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$

$$\boxed{\operatorname{cosec}\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{6}\right) = \sqrt{3}}$$

ii) $3\sin\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{3}\right) = 3\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}} = \frac{3\sqrt{3} - \sqrt{2}}{\sqrt{6}}$

$$\boxed{3\sin\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{3}\right) = \frac{9\sqrt{2} - 2\sqrt{3}}{6}}$$

Question 3

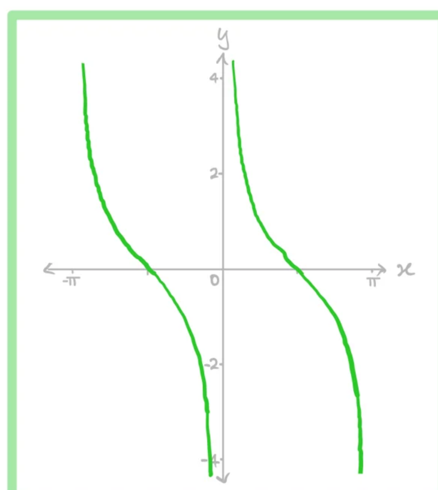
(a) Sketch the graph of $y = \cot x$ for $-\pi \le x \le \pi$.

[2]

(b) Given that $\cot\theta = \frac{9}{7}$ and $\pi \le \theta \le \frac{3\pi}{2}$, find the values of $\cos\theta$, $\sin\theta$ and $\tan\theta$.

[5]

a) $y = \cot x$



(a) Sketch the graph of $y = \cot x$ for $-\pi \le x \le \pi$.

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(b) Given that $\cot\theta = \frac{9}{7}$ and $\pi \le \theta \le \frac{3\pi}{2}$, find the values of $\cos\theta$, $\sin\theta$ and $\tan\theta$.

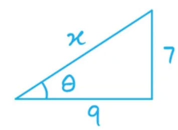
[5]

b) θ is in the 3rd quadrant, so $\sin\theta$ and $\cos\theta$ will be negative and $\tan\theta$ will be positive.

$\cot\theta = \frac{1}{\tan\theta} = \frac{9}{7}$

$\boxed{\tan\theta = \frac{7}{9}}$

Draw a diagram



$x = \sqrt{9^2 + 7^2} = \sqrt{81 + 49} = \sqrt{130}$

$\boxed{\cos\theta = -\frac{9}{\sqrt{130}}, \quad \sin\theta = -\frac{7}{\sqrt{130}}}$

Question 4

Solve $\tan^2 x = \sec x + 11$ for $0 \le x \le \pi$.

[5]

Pythagorean identity: $1 + \tan^2\theta = \sec^2\theta$ (in formula booklet)

$\tan^2 x = \sec x + 11$

$\sec^2 x - 1 = \sec x + 11$

$\sec^2 x - \sec x - 12 = (\sec x + 3)(\sec x - 4) = 0$

$\therefore \sec x = -3$ or 4 $\quad \therefore \cos x = -\frac{1}{3}$ or $\frac{1}{4}$

$\boxed{x = 1.318... = 1.32 \text{ (3 s.f.) or } x = 1.910... = 1.91 \text{ (3 s.f.)}}$

Question 5

(a) Show that the equation

$$\sec\theta - 5\cos\theta = 2\sqrt{2}$$

can be rewritten as

$$5\cos^2\theta + 2\sqrt{2}\cos\theta - 1 = 0$$

[3]

(b) Hence, solve the equation $\sec\theta - 5\cos\theta = 2\sqrt{2}$ for all values of θ in the interval $-\pi \le \theta \le \frac{\pi}{2}$.

[3]

a) $\sec\theta - 5\cos\theta = 2\sqrt{2}$

$\frac{1}{\cos\theta} - 5\cos\theta = 2\sqrt{2}$ $\quad \times \cos\theta$

$1 - 5\cos^2\theta = 2\sqrt{2}\cos\theta$ $\quad$ rearrange

$\boxed{5\cos^2\theta + 2\sqrt{2}\cos\theta - 1 = 0}$

(a) Show that the equation

$$\sec\theta - 5\cos\theta = 2\sqrt{2}$$

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[3]

(b) Hence, solve the equation $\sec\theta - 5\cos\theta = 2\sqrt{2}$ for all values of θ in the interval $-\pi \le \theta \le \frac{\pi}{2}$.

[3]

b) Let $x = \cos\theta$

$\therefore 5x^2 + 2\sqrt{2}x - 1 = 0$

use the quadratic solver on your GDC

$\therefore x = \cos\theta = -0.8119\ldots$ or $0.2463\ldots$

$\therefore \theta = -2.518\ldots, -1.321\ldots, 1.321\ldots$

$\boxed{\theta = -2.52, -1.32, 1.32 \text{ (3 s.f.)}}$

Note: reject $\theta = 2.518\ldots$ since $2.518\ldots > \frac{\pi}{2}$

Question 6

A function f can be defined by $f(x) = 3x - 5x\arcsin(x)$, where $-1 \le x \le 1$.

(a) Sketch the graph of f indicating clearly any intercepts with the coordinate axes and the coordinates of any local maximum or minimum points.

[3]

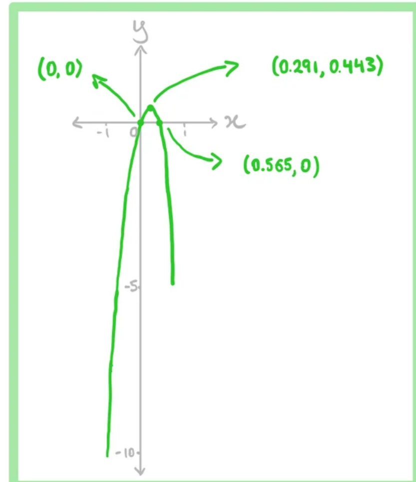
(b) State the domain and range of f.

[2]

(c) Solve the inequality $3x - 5x\arcsin(x) > -2$.

[3]

a) sketch $y = f(x) = 3x - 5x\arcsin(x)$ on your GDC



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(a) Sketch the graph of f indicating clearly any intercepts with the coordinate axes and the coordinates of any local maximum or minimum points.

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(b) State the domain and range of f.

[2]

(c) Solve the inequality $3x - 5x\arcsin(x) > -2$.

[3]

b) The domain is given in the question and f_{min} occurs when $x = -1$ and f_{max} occurs when $x = 0.2913...$

$f(-1) = 3(-1) - 5(-1)\arcsin(-1) = -10.85...$

$f(0.2913...) = 3(0.2913...) - 5(0.2913...)\arcsin(0.2913...) = 0.4433...$

$$\boxed{\begin{aligned}&\text{Domain: } \{x : -1 \le x \le 1\}\\ &\text{Range: } \{y : -10.9 \le y \le 0.443\}\end{aligned}}$$

A function f can be defined by $f(x) = 3x - 5x\arcsin(x)$, where $-1 \le x \le 1$.

(a) Sketch the graph of f indicating clearly any intercepts with the coordinate axes and the coordinates of any local maximum or minimum points. [3]

(b) State the domain and range of f. [2]

(c) Solve the inequality $3x - 5x\arcsin(x) > -2$. [3]

c) Plot the line $y = -2$ and find the intersection points it has with the graph of f.

intersection points: $(-0.3968..., -2)$ and $(0.8717..., -2)$

$\boxed{\therefore -0.397 < x < 0.872 \text{ (3 s.f.)}}$

Question 7

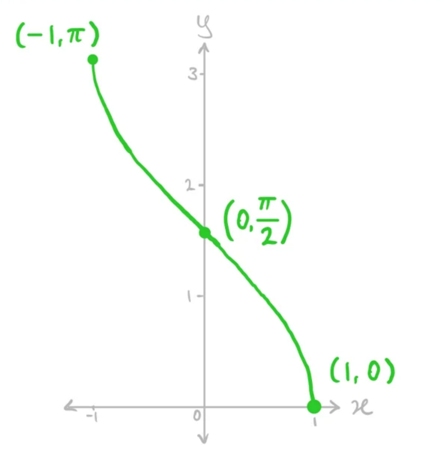
The function f is defined as $f(x) = \arccos x,\ -1 \le x \le 1$, and the function g is such that $g(x) = f(3x)$.

(a) Sketch the graph of $y = f(x)$ and state the range of f. [3]

(b) Sketch the graph of $y = g(x)$ and state the domain of g. [3]

(c) Find the inverse function $g^{-1}(x)$ and state its domain. [2]

a) $y = f(x) = \arccos x$



Range: $\{f(x) : 0 \le f(x) \le \pi\}$

A function f can be defined by $f(x) = 3x - 5x\arcsin(x)$, where $-1 \le x \le 1$.

(a) Sketch the graph of f indicating clearly any intercepts with the coordinate axes and

c) Plot the line $y = -2$ and find the intersection points

The function f is defined as $f(x) = \arccos x$, $-1 \le x \le 1$, and the function g is such that $g(x) = f(3x)$.

(a) Sketch the graph of $y = f(x)$ and state the range of f.

[3]

(b) Sketch the graph of $y = g(x)$ and state the domain of g.

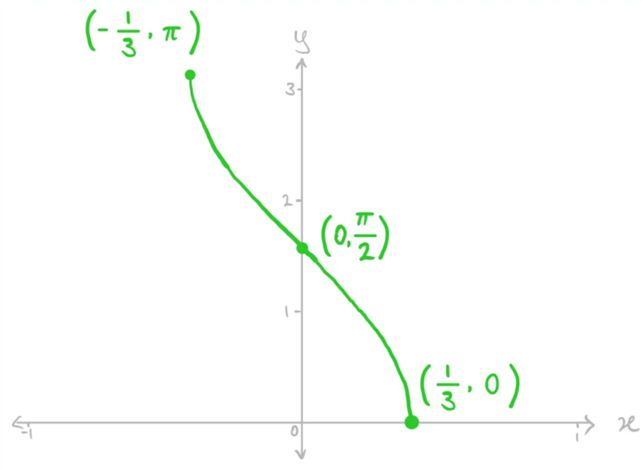
[3]

(c) Find the inverse function $g^{-1}(x)$ and state its domain.

[2]

b) $y = g(x) = f(3x) = \arccos 3x$

Horizontal stretch by s.f $\frac{1}{3}$



Domain: $\{x : -\frac{1}{3} \le x \le \frac{1}{3}\}$

The function f is defined as $f(x) = \arccos x$, $-1 \le x \le 1$, and the function g is such that $g(x) = f(3x)$.

(a) Sketch the graph of $y = f(x)$ and state the range of f.

[3]

(b) Sketch the graph of $y = g(x)$ and state the domain of g.

[3]

(c) Find the inverse function $g^{-1}(x)$ and state its domain.

[2]

c) $y = g(x) = f(3x) = \arccos(3x)$

$y = \arccos(3x)$ — take cos of both sides

$\cos y = 3x$ — $\div 3$

$\frac{1}{3}\cos y = x$

$\therefore g^{-1}(x) = \frac{1}{3}\cos x$

Domain: $\{x : 0 \le x \le \pi\}$

Question 8

(a) Show that $\sec\theta\cot\theta \equiv \operatorname{cosec}\theta$. [2]

(b) Hence solve in the range $0 \le \theta \le 2\pi$, the equation $\sec\theta\cot\theta = -2$ [3]

a) $\sec\theta = \dfrac{1}{\cos\theta}$, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$$\boxed{\begin{array}{l} \sec\theta\cot\theta \equiv \operatorname{cosec}\theta \\ \rightarrow \dfrac{1}{\cancel{\cos\theta}} \times \dfrac{\cancel{\cos\theta}}{\sin\theta} = \dfrac{1}{\sin\theta} \\ \rightarrow \dfrac{1}{\sin\theta} = \dfrac{1}{\sin\theta} = \operatorname{cosec}\theta \end{array}}$$

(a) Show that $\sec\theta\cot\theta \equiv \operatorname{cosec}\theta$. [2]

(b) Hence solve in the range $0 \le \theta \le 2\pi$, the equation $\sec\theta\cot\theta = -2$ [3]

b) $\sec\theta\cot\theta = \operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = -2$

$\therefore \sin\theta = -\dfrac{1}{2}$

$\sin\theta$ is negative in the 3rd and 4th quadrants.

$$\boxed{\theta = \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}}$$

Question 9

(a) Show that the equation

$$\tan^2 x = 6\sec x - 10$$

can be rewritten in the form

$$(\sec x - 3)^2 = 0.$$

[3]

(b) Hence, solve the equation $\tan^2 x = 6\sec x - 10$ in the range $0 \le x \le 2\pi$. [3]

a) Pythagorean identity: $1 + \tan^2\theta = \sec^2\theta$ (in formula booklet)

$\tan^2 x = 6\sec x - 10$

$\sec^2 x - 1 = 6\sec x - 10$

$\sec^2 x - 6\sec x + 9 = 0$

$$\boxed{(\sec x - 3)^2 = 0}$$

(a) Show that the equation

$$\tan^2 x = 6\sec x - 10$$

can be rewritten in the form

$$(\sec x - 3)^2 = 0\,.$$

[3]

(b) Hence, solve the equation $\tan^2 x = 6\sec x - 10$ in the range $0 \le x \le 2\pi$.

[3]

b) $(\sec x - 3)^2 = 0$

$\sec x = \dfrac{1}{\cos x} = 3$

$\therefore \cos x = \dfrac{1}{3} \longrightarrow x = \cos^{-1}\left(\dfrac{1}{3}\right)$

$x = 1.2309\ldots$ and $2\pi - 1.2309\ldots$

$\boxed{x = 1.23,\ 5.05 \ (3\text{ s.f.})}$

Question 10

(a) Show that the equation

$$\cot^2 x = 9 - 3\operatorname{cosec} x$$

can be rewritten in the form

$$(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 5) = 0\,.$$

[3]

(b) Hence, solve the equation $\cot^2 x = 9 - 3\operatorname{cosec} x$ in the interval $-180° \le x \le 180°$.
Give your answers correct to 1 decimal place.

[3]

a) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ (in formula booklet)

$\cot^2 x = 9 - 3\operatorname{cosec} x$

$\operatorname{cosec}^2 x - 1 = 9 - 3\operatorname{cosec} x$

$\operatorname{cosec}^2 x + 3\operatorname{cosec} x - 10 = 0$

$\boxed{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 5) = 0}$

(a) Show that the equation

$$\cot^2 x = 9 - 3\operatorname{cosec} x$$

can be rewritten in the form

$$(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 5) = 0\,.$$

[3]

(b) Hence, solve the equation $\cot^2 x = 9 - 3\operatorname{cosec} x$ in the interval $-180° \le x \le 180°$.
Give your answers correct to 1 decimal place.

[3]

b) $(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 5) = 0$

$\therefore \operatorname{cosec} x = 2$ and $\operatorname{cosec} x = -5$

$\sin x = \dfrac{1}{2}$ and $\sin x = -\dfrac{1}{5}$

$\therefore x = 30°, 150°$ and $x = -0.2013\ldots, -179.79\ldots$

$\boxed{\therefore x = 30.0°, 150.0° \text{ and } x = -0.2°, -179.8°}$