Question 1

A curve $y = f(x)$ passes through point A(4, 2) and has a gradient of $f'(x) = 5x - 2$.

(a) Find the gradient of the curve at point A. [2]

(b) Find the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$. [2]

(c) Determine the equation of the curve $y = f(x)$. [3]

a) Substitute x-coordinate into $f'(x)$

$f'(4) = 5(4) - 2 = \boxed{18}$

A curve $y = f(x)$ passes through point A(4, 2) and has a gradient of $f'(x) = 5x - 2$.

(a) Find the gradient of the curve at point A.

$\boxed{18}$

[2]

(b) Find the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$. [2]

(c) Determine the equation of the curve $y = f(x)$. [3]

b) We want the line through (4, 2) with gradient 18.

Use $y - y_1 = m(x - x_1)$

$y - 2 = 18(x - 4)$

$y - 2 = 18x - 72$

$\boxed{y = 18x - 70}$

A curve $y = f(x)$ passes through point A(4, 2) and has a gradient of $f'(x) = 5x - 2$.

(a) Find the gradient of the curve at point A. [2]

(b) Find the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$. [2]

(c) Determine the equation of the curve $y = f(x)$. [3]

c) $f(x) = \int f'(x)\,dx$

constant of integration

$f(x) = \int (5x - 2)\,dx = \frac{5}{2}x^2 - 2x + c$

But $y = f(x)$ goes through (4, 2), so $f(4) = 2$.

$\frac{5}{2}(4)^2 - 2(4) + c = 2$

$\frac{5}{2}(16) - 2(4) + c = 2$

$40 - 8 + c = 2$

$32 + c = 2 \implies c = -30$

$\boxed{y = f(x) = \frac{5}{2}x^2 - 2x - 30}$

Powers of x integration formulae

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ $(n \neq -1)$

$\int k\,dx = kx + c$ (k is a constant)

Question 2

A point P(3, 8) lies on the curve $y = f(x)$ that has a gradient of $f'(x) = -2x^2 + 11$.

(a) Find the gradient of the curve at point P. [2]

(b) Find the equation of the tangent to the curve at point P.
Give your answer in the form $y = mx + c$. [2]

(c) Determine the equation of the curve $y = f(x)$. [3]

a) Substitute x-coordinate into $f'(x)$

$$f'(3) = -2(3)^2 + 11 = -18 + 11 = \boxed{-7}$$

A point P(3, 8) lies on the curve $y = f(x)$ that has a gradient of $f'(x) = -2x^2 + 11$.

(a) Find the gradient of the curve at point P. [2]

$\boxed{-7}$

(b) Find the equation of the tangent to the curve at point P.
Give your answer in the form $y = mx + c$. [2]

(c) Determine the equation of the curve $y = f(x)$. [3]

b) We want the line through (3, 8) with gradient −7.

Use $y - y_1 = m(x - x_1)$

$$y - 8 = -7(x - 3)$$

$$y - 8 = -7x + 21$$

$$\boxed{y = -7x + 29}$$

A point P(3, 8) lies on the curve $y = f(x)$ that has a gradient of $f'(x) = -2x^2 + 11$.

(a) Find the gradient of the curve at point P. [2]

(b) Find the equation of the tangent to the curve at point P.
Give your answer in the form $y = mx + c$. [2]

(c) Determine the equation of the curve $y = f(x)$. [3]

c) $f(x) = \int f'(x)\,dx$

constant of integration

$$\int (-2x^2 + 11)\,dx = -\frac{2}{3}x^3 + 11x + c$$

But $y = f(x)$ goes through (3, 8), so $f(3) = 8$.

$$-\frac{2}{3}(3)^3 + 11(3) + c = 8$$

$$-\frac{2}{3}(27) + 11(3) + c = 8$$

$$-18 + 33 + c = 8$$

$$15 + c = 8 \implies c = -7$$

$$\boxed{y = f(x) = -\frac{2}{3}x^3 + 11x - 7}$$

Powers of x integration formulae

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$\int k\,dx = kx + c \quad (k \text{ is a constant})$$

EXAM PAPERS PRACTICE

Question 3

The following table shows the x and y coordinates of five points that lie on a curve $y = f(x)$.

x	0	0.25	0.5	0.75	1
$y = f(x)$	1	2.25	4	6.25	9
	y_0	y_1	y_2	y_3	y_4

$n = 4$

(a) Estimate the area under the curve over the interval $0 \le x \le 1$.

$a = 0 \quad b = 1$

[2]

The equation of the curve was found to be $y = (2x+1)^2$.

(b) Find the exact value of the area under the curve over the interval $0 \le x \le 1$.

[2]

(c) Find the percentage error between the estimation in part (a) and the exact value in part (b). Provide a reason for the difference.

[2]

a) Use the Trapezoidal Rule

$$\int_0^1 y\,dx \approx \frac{1}{2}\left(\frac{1-0}{4}\right)\left((1+9)+2(2.25+4+6.25)\right)$$

$$= \frac{1}{2}\left(\frac{1}{4}\right)(35) = \frac{35}{8}$$

$\boxed{\text{Area} \approx \frac{35}{8} \text{ units}^2} \boxed{= 4.375 \text{ units}^2}$

Either form will get the marks!

Trapezoidal Rule

$$\int_a^b y\,dx \approx \frac{1}{2}h\left((y_0+y_n)+2(y_1+y_2+\dots+y_{n-1})\right)$$

where $h = \frac{b-a}{n}$

The following table shows the x and y coordinates of five points that lie on a curve $y = f(x)$.

x	0	0.25	0.5	0.75	1
$y = f(x)$	1	2.25	4	6.25	9

(a) Estimate the area under the curve over the interval $0 \le x \le 1$.

[2]

The equation of the curve was found to be $y = (2x+1)^2$.

(b) Find the exact value of the area under the curve over the interval $0 \le x \le 1$.

[2]

(c) Find the percentage error between the estimation in part (a) and the exact value in part (b). Provide a reason for the difference.

[2]

b) $\int_0^1 (2x+1)^2\,dx = \frac{13}{3}$ from GDC

$\boxed{\text{Area} = \frac{13}{3} \text{ units}^2}$

The following table shows the x and y coordinates of five points that lie on a curve $y = f(x)$.

x	0	0.25	0.5	0.75	1
$y = f(x)$	1	2.25	4	6.25	9

(a) Estimate the area under the curve over the interval $0 \le x \le 1$.

[2]

The equation of the curve was found to be $y = (2x+1)^2$.

(b) Find the exact value of the area under the curve over the interval $0 \le x \le 1$.

[2]

(c) Find the percentage error between the estimation in part (a) and the exact value in part (b). Provide a reason for the difference.

[2]

Percentage error

$$\varepsilon = \left| \frac{V_A - V_E}{V_E} \right| \times 100\ \%$$

V_A is the approximate value

V_E is the exact value

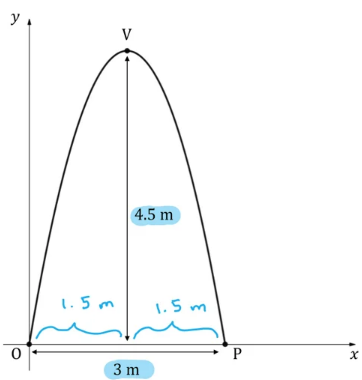
c) $\varepsilon = \left| \dfrac{\frac{35}{8} - \frac{13}{3}}{\frac{13}{3}} \right| \times 100 = \dfrac{25}{26} = 0.961538\ldots$

$\boxed{\varepsilon = 0.96\ \%\ (2\text{ d.p.})}$

The approximation is inexact because it is based on adding up the areas of straight-sided trapezoids. The actual area is bounded on top by a curve (in this case a parabola).

Question 4

The following diagram shows an arch that is 4.5 m tall and 3 m wide. The arch crosses the x-axis at the origin, O, and at point P, and its vertex is at point V. The arch may be represented by a curve with an equation of the form $y = x(ax + 6)$, where all units are measured in metres.



(a) Find

(i) the coordinates of P

(ii) the coordinates of V

(iii) the value of a.

[4]

(b) Find the cross-sectional area under the arch.

[2]

a) (i) $\boxed{\text{P is the point } (3, 0)}$

(ii) By symmetry, V's x-coordinate is halfway between O's and P's x-coordinates.

$\boxed{\text{V is the point } (1.5, 4.5)}$

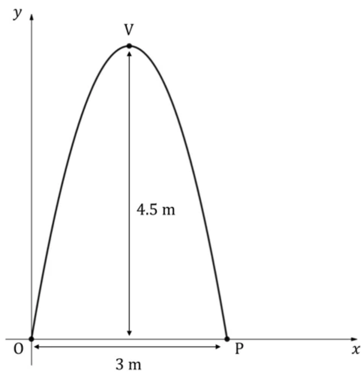
(iii) The curve goes through point P(3, 0), so $y = 0$ when $x = 3$.

$$3(a(3) + 6) = 0$$

$$9a + 18 = 0$$

$$9a = -18 \implies \boxed{a = -2}$$

The following diagram shows an arch that is 4.5 m tall and 3 m wide. The arch crosses the x-axis at the origin, O, and at point P, and its vertex is at point V. The arch may be represented by a curve with an equation of the form $y = x(ax + 6)$, where all units are measured in metres.



(a) Find

(i) the coordinates of P $\boxed{\text{P is the point } (3, 0)}$

(ii) the coordinates of V

(iii) the value of a. $\boxed{a = -2}$

[4]

(b) Find the cross-sectional area under the arch.

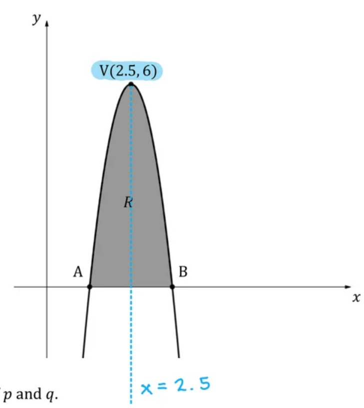
[2]

b) Area $= \int_0^3 x(-2x+6)\,dx = 9$ from GDC

$\boxed{9 \text{ m}^2}$

Question 5

The diagram below shows a part of the curve $y = -4x^2 + px + q$. Points A and B represent the x-intercepts, point $V(2.5, 6)$ represents the vertex of the curve, and the shaded region R represents the area between the curve and the x-axis.



(a) Find the values of p and q.

[2]

(b) Find the coordinates of points A and B.

[4]

(c) Find the area of region R.

[2]

a) For $f(x) = ax^2 + bx + c$, the axis of symmetry is $x = -\frac{b}{2a}$

$-\frac{p}{2(-4)} = 2.5 \implies \frac{p}{8} = 2.5 \implies \boxed{p = 20}$

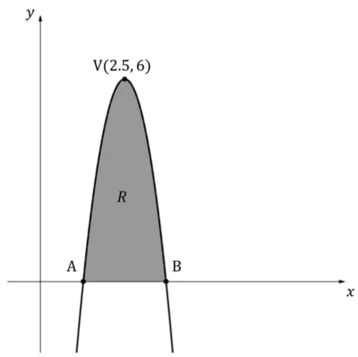
The curve goes through $(2.5, 6)$, so $y = 6$ when $x = 2.5$.

$-4(2.5)^2 + 20(2.5) + q = 6$

$-25 + 50 + q = 6$

$q + 25 = 6 \implies \boxed{q = -19}$

The diagram below shows a part of the curve $y = -4x^2 + px + q$. Points A and B represent the x-intercepts, point $V(2.5, 6)$ represents the vertex of the curve, and the shaded region R represents the area between the curve and the x-axis.



(a) Find the values of p and q.

$\boxed{p = 20}$ $\boxed{q = -19}$

[2]

(b) Find the coordinates of points A and B.

[4]

(c) Find the area of region R.

[2]

b) The x-coordinates of A and B are the solutions to:

$-4x^2 + 20x - 19 = 0$

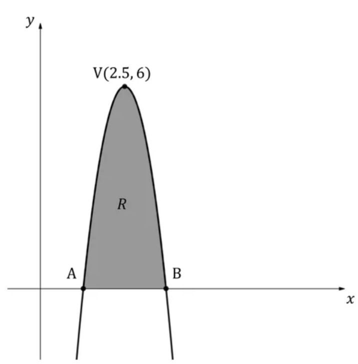
$\Rightarrow x = \frac{5-\sqrt{6}}{2}$ or $x = \frac{5+\sqrt{6}}{2}$ from GDC

$= 1.275\ldots$ $\quad = 3.274\ldots$

A is $\left(\frac{5-\sqrt{6}}{2}, 0\right)$

B is $\left(\frac{5+\sqrt{6}}{2}, 0\right)$

The diagram below shows a part of the curve $y = -4x^2 + px + q$. Points A and B represent the x-intercepts, point $V(2.5, 6)$ represents the vertex of the curve, and the shaded region R represents the area between the curve and the x-axis.



(a) Find the values of p and q.

$\boxed{p = 20}$ $\boxed{q = -19}$

[2]

(b) Find the coordinates of points A and B.

A is $\left(\frac{5-\sqrt{6}}{2}, 0\right)$

B is $\left(\frac{5+\sqrt{6}}{2}, 0\right)$

[4]

(c) Find the area of region R.

[2]

c) Area of $R = \int_{\frac{5-\sqrt{6}}{2}}^{\frac{5+\sqrt{6}}{2}} (-4x^2 + 20x - 19)\, dx$

$= 4\sqrt{6} = 9.797958\ldots$

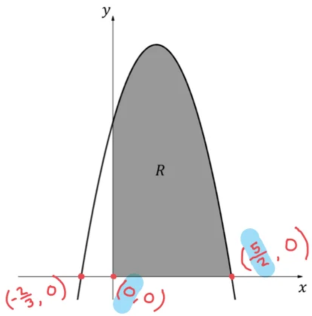
from GDC

Area of $R = 4\sqrt{6}$ units^2

$= 9.80$ units^2 (3 s.f.)

Question 6

The following diagram shows part of the graph of $f(x) = (5 - 2x)(2 + 3x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, the y-axis and the graph of f.



(a) Write down an integral for the area of region R.

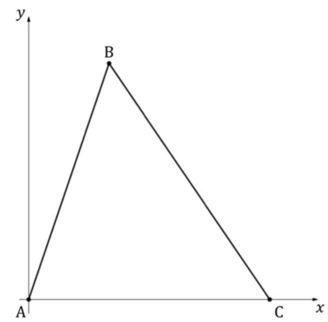
[2]

(b) Find the area of region R.

[1]

(This question continues on the following page)

The three points A(0, 0), B(4, h) and C(9, 0) define the vertices of a triangle.



(c) Find the value of h, the y-coordinate of B, given that the area of the triangle is equal to the area of region R.

[2]

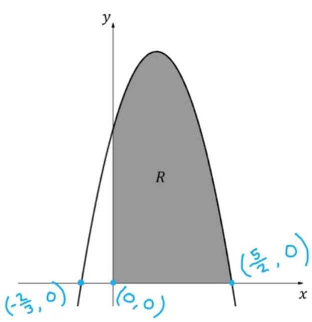
a) $f(x) = 0$ when $(5-2x) = 0$ or $(2+3x) = 0$

$2 + 3x = 0 \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$

$5 - 2x = 0 \Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2}$

$$\boxed{\text{Area of } R = \int_0^{5/2} (5-2x)(2+3x)\,dx}$$

The following diagram shows part of the graph of $f(x) = (5 - 2x)(2 + 3x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, the y-axis and the graph of f.



(a) Write down an integral for the area of region R.

$$\boxed{\text{Area of } R = \int_0^{5/2} (5-2x)(2+3x)\,dx}$$

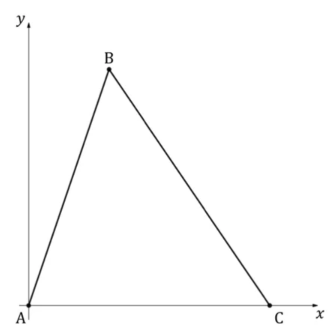
[2]

(b) Find the area of region R.

[1]

(This question continues on the following page)

The three points A(0, 0), B(4, h) and C(9, 0) define the vertices of a triangle.



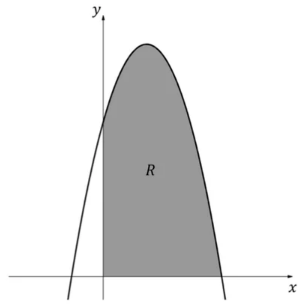
(c) Find the value of h, the y-coordinate of B, given that the area of the triangle is equal to the area of region R.

[2]

b) $\int_0^{5/2} (5-2x)(2+3x)\,dx = \frac{225}{8}$ from GDC

$$\boxed{\text{Area of } R = \frac{225}{8} \text{ units}^2} = 28.125$$

The following diagram shows part of the graph of $f(x) = (5 - 2x)(2 + 3x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, the y-axis and the graph of f.



(a) Write down an integral for the area of region R.

[2]

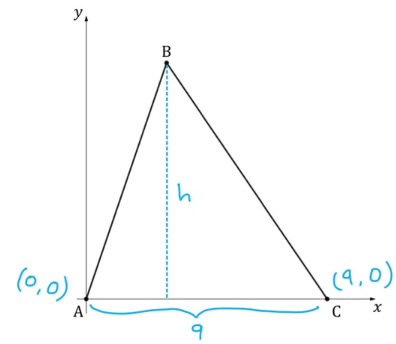
(b) Find the area of region R.

$\boxed{\text{Area of } R = \frac{225}{8} \text{ units}^2}$

[1]

(This question continues on the following page)

The three points $A(0, 0)$, $B(4, h)$ and $C(9, 0)$ define the vertices of a triangle.



(c) Find the value of h, the y-coordinate of B, given that the area of the triangle is equal to the area of region R.

[2]

c) Area of triangle $= \frac{1}{2} \times$ base $\times$ height

$\frac{1}{2}(9)(h) = \frac{9}{2}h = \frac{225}{8}$

$h = \frac{225}{8} \div \frac{9}{2} = \frac{225}{8} \times \frac{2}{9}$

$\boxed{h = \frac{25}{4}}$ $= 6.25$

Question 7

A rice farm sells x kg of rice every week.

It is known that $\frac{dP}{dx} = -0.02x + 6, x \geq 0$, where P is the weekly profit, in dollars ($), from the sale of x kg of rice.

(a) Find the amount of rice, in kg, that should be sold each week to maximise the profit.

[3]

The profit from selling 250 kg of rice is $480.

(b) Find $P(x)$.

[5]

a) P has a stationary point where $\frac{dP}{dx} = 0$

$-0.02x + 6 = 0 \implies 0.02x = 6 \implies x = 300$

Check that that is a maximum!

When $x = 299$, $-0.02(299) + 6 = 0.02 > 0$

When $x = 301$, $-0.02(301) + 6 = -0.02 < 0$

So $x = 300$ is a maximum

Because derivative goes from positive to negative

$\boxed{\text{Selling 300 kg of rice each week will maximise profit.}}$

A rice farm sells x kg of rice every week.

It is known that $\frac{\mathrm{d}P}{\mathrm{d}x} = -0.02x + 6, x \geq 0,$ where P is the weekly profit, in dollars ($), from the sale of x kg of rice.

(a) Find the amount of rice, in kg, that should be sold each week to maximise the profit.

[3]

The profit from selling 250 kg of rice is $480.

(b) Find $P(x)$.

[5]

Powers of x integration formulae

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ $(n \neq -1)$

$\int k \, dx = kx + c$ (k is a constant)

b) $P(x) = \int \frac{dP}{dx} \, dx$

constant of integration

$P(x) = \int (-0.02x + 6) \, dx = \frac{-0.02}{2} x^2 + 6x + c$

$= -0.01x^2 + 6x + c$

But $P = 480$ when $x = 250$, so

$-0.01(250)^2 + 6(250) + c = 480$

$-0.01(62500) + 6(250) + c = 480$

$-625 + 1500 + c = 480$

$c + 875 = 480 \Rightarrow c = -395$

$\boxed{P(x) = -0.01x^2 + 6x - 395}$

Question 8

A paint company sells x hundred of litres of paint every week.

It is known that $\frac{\mathrm{d}P}{\mathrm{d}x} = -1.9x + 145, x \geq 0,$ where P is the weekly profit, in euros (€), from the sale of x hundred litres of paint.

(a) Find the number of litres that should be sold each week to maximise the profit.

[3]

The profit from selling 7000 litres of paint is €5000.

(b) Find $P(x)$.

[5]

a) P has a stationary point where $\frac{dP}{dx} = 0$

$-1.9x + 145 = 0 \Rightarrow 1.9x = 145$

$\Rightarrow x = \frac{1450}{19} = 76.315789\ldots$

Check that that is a maximum!

When $x = 76$, $-1.9(76) + 145 = 0.6 > 0$

When $x = 77$, $-1.9(77) + 145 = -1.3 < 0$

So $x = \frac{1450}{19}$ is a maximum

Because derivative goes from positive to negative

$100\left(\frac{1450}{19}\right) = \frac{145000}{19} = 7631.578947\ldots$

Don't forget that x is hundreds of litres!

To the nearest litre, selling 7632 litres of paint each week will maximise profit.

A paint company sells x hundred of litres of paint every week.

It is known that $\frac{dP}{dx} = -1.9x + 145, x \geq 0$, where P is the weekly profit, in euros (€), from the sale of x hundred litres of paint.

(a) Find the number of litres that should be sold each week to maximise the profit.

[3]

The profit from selling 7000 litres of paint is €5000.

(b) Find $P(x)$.

[5]

Powers of x integration formulae

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ $(n \neq -1)$

$\int k \, dx = kx + c$ (k is a constant)

b) $P(x) = \int \frac{dP}{dx} \, dx$

constant of integration

$P(x) = \int (-1.9x + 145) \, dx = \frac{-1.9}{2} x^2 + 145x + c$

$= -0.95x^2 + 145x + c$

$\frac{7000}{100} = 70$

Don't forget that x is hundreds of litres!

So $P = 5000$ when $x = 70$

$-0.95(70)^2 + 145(70) + c = 5000$

$-0.95(4900) + 145(70) + c = 5000$

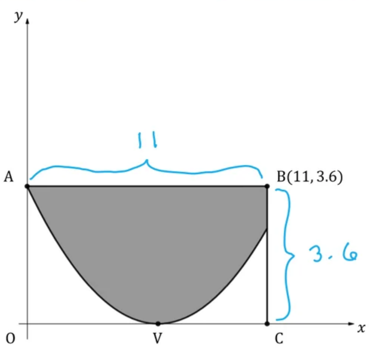
$-4655 + 10150 + c = 5000$

$c + 5495 = 5000 \Rightarrow c = -495$

$\boxed{P(x) = -0.95x^2 + 145x - 495}$

Question 9

A river has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in metres. The riverbed (the curved part of the region shown) has an equation of the form $y = q(x-6)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the riverbed, is situated on the x-axis.



(a) Find

(i) the coordinates of V

(ii) the area of the rectangle OABC.

[3]

(b) Determine the value of q.

[2]

(c) Find the cross-sectional area of the riverbed.

[3]

a) (i) Point V is on the x-axis, so at point V $y = 0$

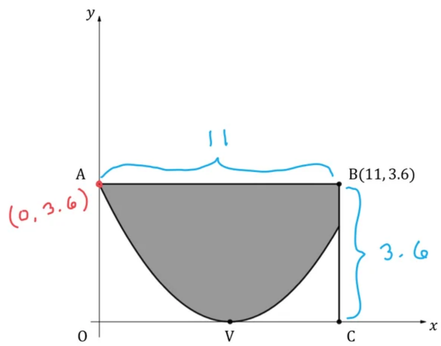
$q(x-6)^2 = 0 \Rightarrow (x-6)^2 = 0 \Rightarrow x = 6$

$\boxed{\text{V is the point } (6, 0)}$

(ii) $11 \times 3.6 = 39.6$

$\boxed{\text{Area of OABC} = 39.6 \text{ m}^2}$

A river has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in metres. The riverbed (the curved part of the region shown) has an equation of the form $y = q(x-6)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the riverbed, is situated on the x-axis.



(a) Find

(i) the coordinates of V

(ii) the area of the rectangle OABC.

[3]

(b) Determine the value of q.

[2]

(c) Find the cross-sectional area of the riverbed.

[3]

b) Because OABC is a rectangle, A must be the point $(0, 3.6)$. So when $x=0$, $y=3.6$.

$$q(0-6)^2 = 3.6$$

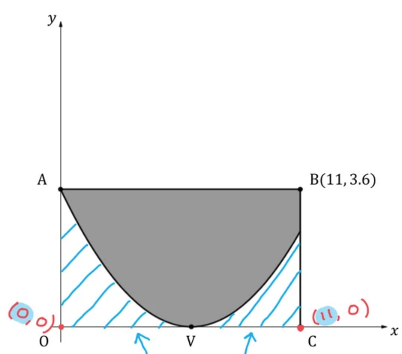
$$q(-6)^2 = 3.6$$

$$36q = 3.6$$

$$q = \frac{3.6}{36}$$

$$\boxed{q = \frac{1}{10}} = 0.1$$

A river has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in metres. The riverbed (the curved part of the region shown) has an equation of the form $y = q(x-6)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the riverbed, is situated on the x-axis.



$\int_0^{11} \frac{1}{10}(x-6)^2 \, dx$

(a) Find

(i) the coordinates of V

(ii) the area of the rectangle OABC.

$\boxed{\text{Area of OABC} = 39.6 \text{ m}^2}$

[3]

(b) Determine the value of q.

$\boxed{q = \frac{1}{10}}$

[2]

(c) Find the cross-sectional area of the riverbed.

[3]

c) The shaded area is the area of OABC minus $\int_0^{11} \frac{1}{10}(x-6)^2 \, dx$.

$$\int_0^{11} \frac{1}{10}(x-6)^2 \, dx = \frac{341}{30} \quad \text{from GDC}$$

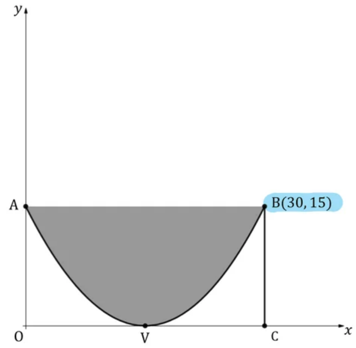
$$39.6 - \frac{341}{30} = \frac{847}{30} = 28.233333...$$

The cross-sectional area is

$$\frac{847}{30} \text{ m}^2 = 28.2 \text{ m}^2 \text{ (3 s.f.)}$$

Question 10

A trough has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in centimetres. The curved bottom of the trough has an equation in the form $y = r(x - 15)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the trough, is situated on the x-axis.



(a) Determine the value of r.

[2]

(b) Find the cross-sectional area of the trough.

[4]

The length of the trough is 1.2 m.

(c) Find the volume of the trough. Give your answer in cm^3.

[2]

a) The curve goes through (30, 15), so when $x = 30$, $y = 15$.

$$r(30-15)^2 = 15$$

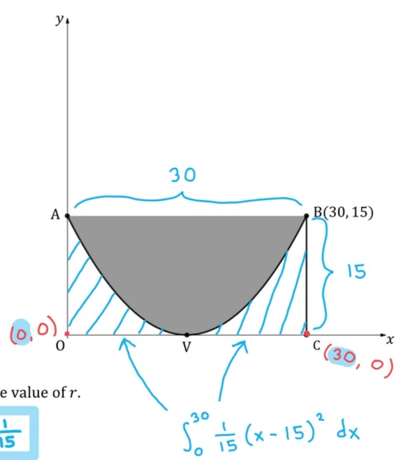
$$r(15)^2 = 15$$

$$225r = 15$$

$$r = \frac{15}{225}$$

$$\boxed{r = \frac{1}{15}}$$

A trough has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in centimetres. The curved bottom of the trough has an equation in the form $y = r(x - 15)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the trough, is situated on the x-axis.



(a) Determine the value of r.

$\boxed{r = \frac{1}{15}}$

[2]

(b) Find the cross-sectional area of the trough.

[4]

The length of the trough is 1.2 m.

(c) Find the volume of the trough. Give your answer in cm^3.

[2]

b) The shaded area is the area of OABC minus $\int_0^{30} \frac{1}{15}(x-15)^2\,dx$

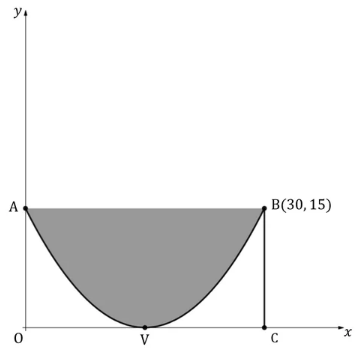
Area of OABC $= 30 \times 15 = 450$

$\int_0^{30} \frac{1}{15}(x-15)^2\,dx = 150$ from GDC

$450 - 150 = 300$

$\boxed{\text{The cross-sectional area is } 300 \text{ cm}^2}$

A trough has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in centimetres. The curved bottom of the trough has an equation in the form $y = r(x - 15)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the trough, is situated on the x-axis.



(a) Determine the value of r.

[2]

(b) Find the cross-sectional area of the trough.

The cross-sectional area is 300 cm^2

[4]

The length of the trough is 1.2 m.

(c) Find the volume of the trough. Give your answer in cm^3.

[2]

c) 1.2 m = 120 cm

Don't forget to convert units!

300 × 120 = 36000

$\boxed{\text{Volume} = 36000 \text{ cm}^3}$

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