

# IB Maths: AI HL

## Integration

### Topic Questions

**These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Topic Questions**

Course	IB Maths
Section	5. Calculus
Topic	5.3 Integration
Difficulty	Medium

**Level: IB Maths**

**Subject: IB Maths AI HL**

**Board: IB Maths**

**Topic: Integration**

### Question 1

A curve  $y = f(x)$  passes through point  $A(4, 2)$  and has a gradient of  $f'(x) = 5x - 2$ .

(a) Find the gradient of the curve at point A.

[2 marks]

(b) Find the equation of the tangent to the curve at point A.

Give your answer in the form  $y = mx + c$ .

[2 marks]

(c) Determine the equation of the curve  $y = f(x)$ .

[3 marks]

### Question 2

A point  $P(3, 8)$  lies on the curve  $y = f(x)$  that has a gradient of  $f'(x) = -2x^2 + 11$ .

(a) Find the gradient of the curve at point P.

[2 marks]

(b) Find the equation of the tangent to the curve at point P.

Give your answer in the form  $y = mx + c$ .

[2 marks]

(c) Determine the equation of the curve  $y = f(x)$ .

[3 marks]

### Question 3

The following table shows the  $x$  and  $y$  coordinates of five points that lie on a curve  $y = f(x)$ .

$x$	0	0.25	0.5	0.75	1
$y = f(x)$	1	2.25	4	6.25	9

(a) Estimate the area under the curve over the interval  $0 \leq x \leq 1$ .

[2 marks]

The equation of the curve was found to be  $y = (2x + 1)^2$ .

(b) Find the exact value of the area under the curve over the interval  $0 \leq x \leq 1$ .

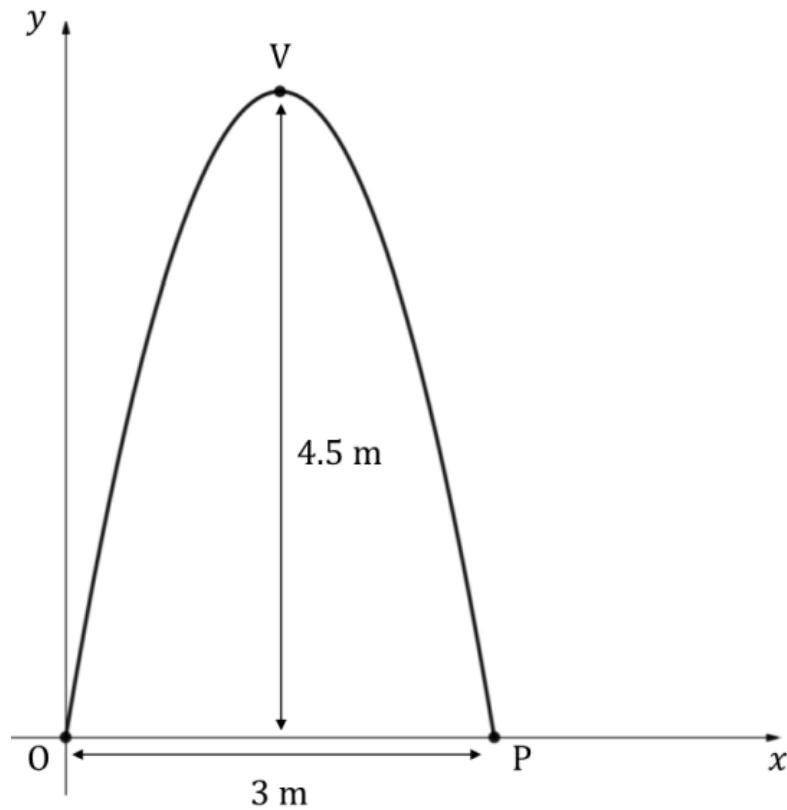
[2 marks]

(c) Find the percentage error between the estimation in part (a) and the exact value in part (b). Provide a reason for the difference.

[2 marks]

### Question 4

The following diagram shows an arch that is 4.5 m tall and 3 m wide. The arch crosses the  $x$ -axis at the origin,  $O$ , and at point  $P$ , and its vertex is at point  $V$ . The arch may be represented by a curve with an equation of the form  $y = x(ax + 6)$ , where all units are measured in metres.



(a) Find

- (i) the coordinates of  $P$
- (ii) the coordinates of  $V$
- (iii) the value of  $a$ .

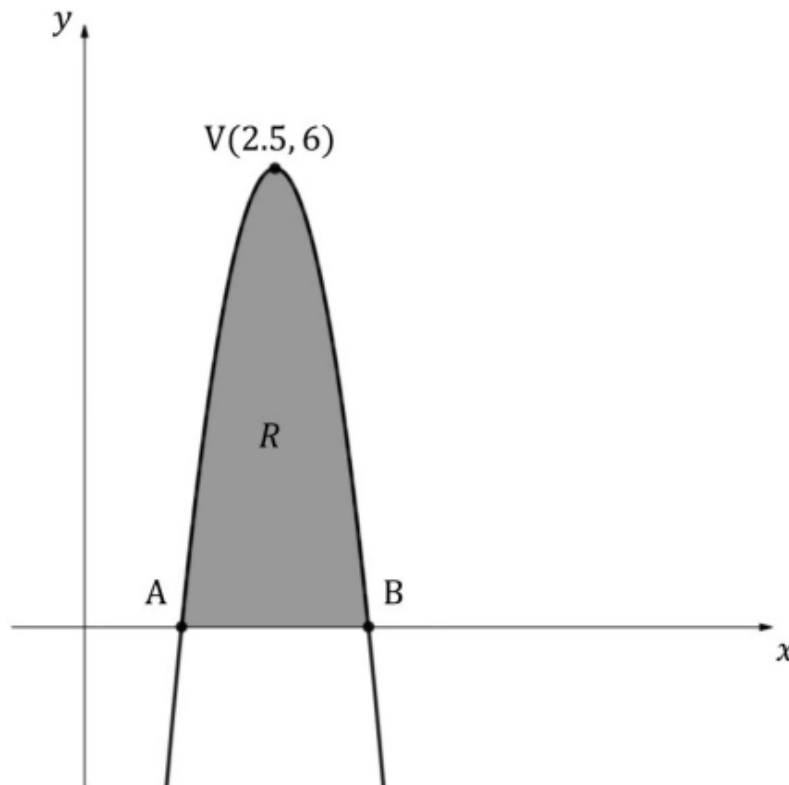
[4 marks]

(b) Find the cross-sectional area under the arch.

[2 marks]

### Question 5

The diagram below shows a part of the curve  $y = -4x^2 + px + q$ . Points A and B represent the  $x$ -intercepts, point  $V(2.5, 6)$  represents the vertex of the curve, and the shaded region  $R$  represents the area between the curve and the  $x$ -axis.



(a) Find the values of  $p$  and  $q$ .

[2 marks]

(b) Find the coordinates of points A and B.

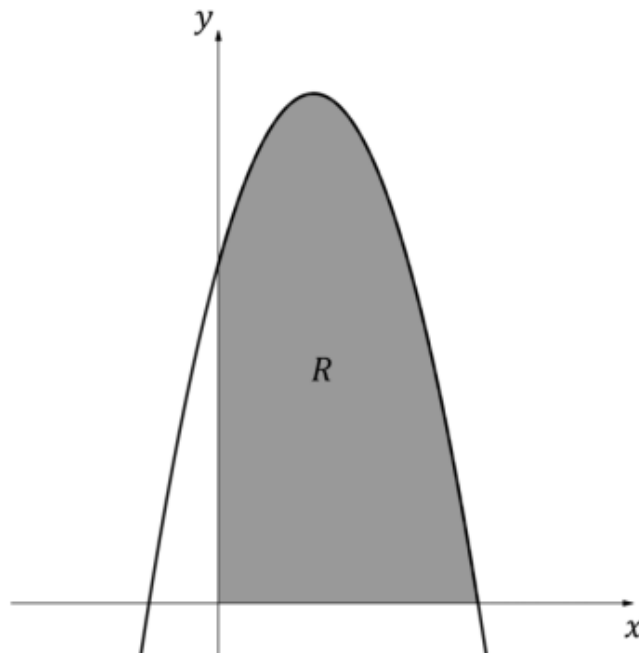
[4 marks]

(c) Find the area of region  $R$ .

[2 marks]

### Question 6

The following diagram shows part of the graph of  $f(x) = (5 - 2x)(2 + 3x)$ ,  $x \in \mathbb{R}$ . The shaded region  $R$  is bounded by the  $x$ -axis, the  $y$ -axis and the graph of  $f$ .



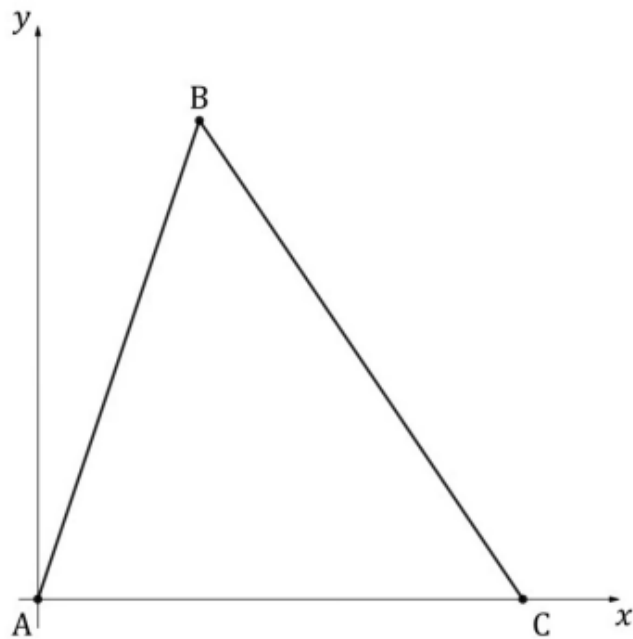
(a) Write down an integral for the area of region  $R$ .

[2 marks]

(b) Find the area of region  $R$ .

[1 mark]

The three points  $A(0, 0)$ ,  $B(4, h)$  and  $C(9, 0)$  define the vertices of a triangle.



(c) Find the value of  $h$ , the  $y$ -coordinate of  $B$ , given that the area of the triangle is equal to the area of region  $R$ .

[2 marks]

### Question 7

A rice farm sells  $x$  kg of rice every week.

It is known that  $\frac{dP}{dx} = -0.02x + 6$ ,  $x \geq 0$ , where  $P$  is the weekly profit, in dollars (\$), from the sale of  $x$  kg of rice.

(a) Find the amount of rice, in kg, that should be sold each week to maximise the profit.

[3 marks]

The profit from selling 250 kg of rice is \$480.

(b) Find  $P(x)$ .

[5 marks]

### Question 8

A paint company sells  $x$  hundred of litres of paint every week.

It is known that  $\frac{dP}{dx} = -1.9x + 145$ ,  $x \geq 0$ , where  $P$  is the weekly profit, in euros (€), from the sale of  $x$  hundred litres of paint.

(a) Find the number of litres that should be sold each week to maximise the profit.

[3 marks]

The profit from selling 7000 litres of paint is €5000.

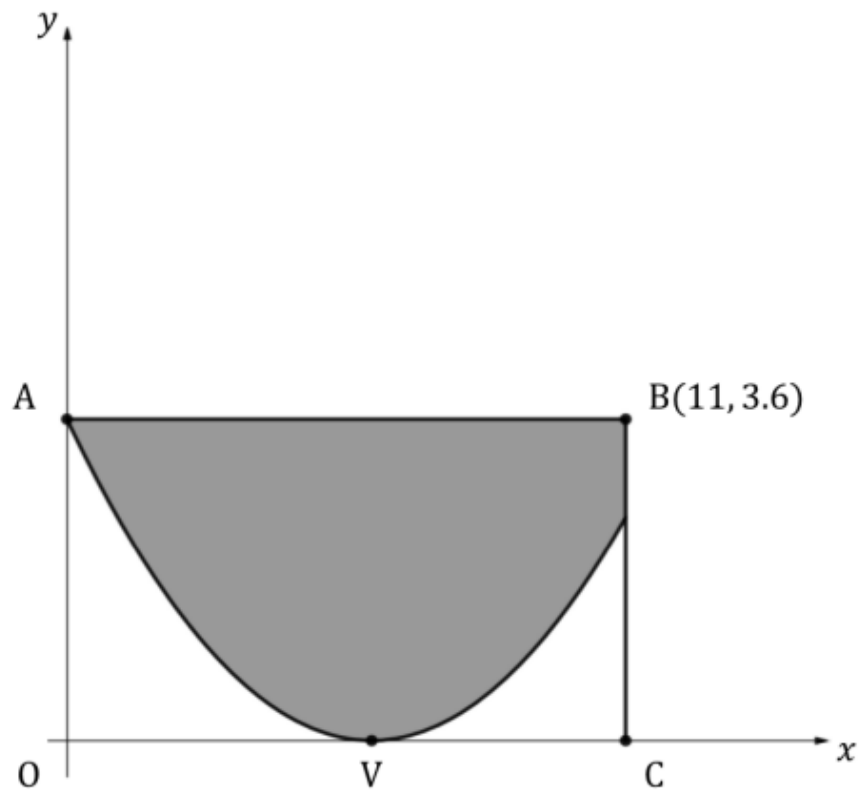
(b) Find  $P(x)$ .

[5 marks]



### Question 9

A river has a cross-sectional area shown by the shaded region of the diagram below, where the  $x$  and  $y$  values are in metres. The riverbed (the curved part of the region shown) has an equation of the form  $y = q(x - 6)^2$ . Point  $O$  is the origin, and points  $O, A, B$  and  $C$  are the vertices of a rectangle. Point  $V$ , the deepest point of the riverbed, is situated on the  $x$ -axis.



(a) Find

- (i) the coordinates of  $V$
- (ii) the area of the rectangle  $OABC$ .

[3 marks]

(b) Determine the value of  $q$ .

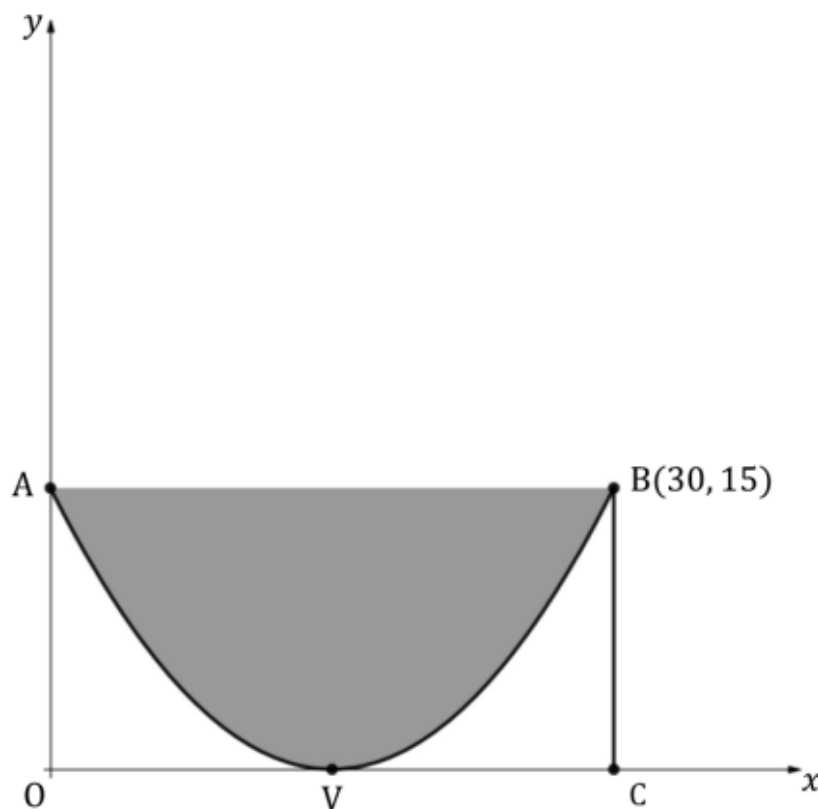
[2 marks]

(c) Find the cross-sectional area of the riverbed.

[3 marks]

### Question 10

A trough has a cross-sectional area shown by the shaded region of the diagram below, where the  $x$  and  $y$  values are in centimetres. The curved bottom of the trough has an equation in the form  $y = r(x - 15)^2$ . Point  $O$  is the origin, and points  $O$ ,  $A$ ,  $B$  and  $C$  are the vertices of a rectangle. Point  $V$ , the deepest point of the trough, is situated on the  $x$ -axis.



(a) Determine the value of  $r$ .

[2 marks]

(b) Find the cross-sectional area of the trough.

[4 marks]

The length of the trough is 1.2 m.

(c) Find the volume of the trough. Give your answer in  $\text{cm}^3$ .

[2 marks]