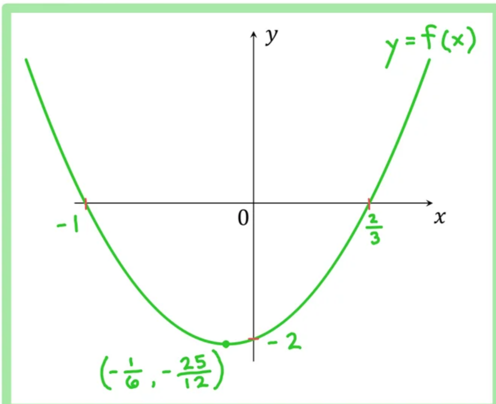
Question 1

Consider the functions $f(x) = 3x^2 + x - 2$ and $g(x) = -2x^2 + 3x + 5$.

(a) Sketch the graph of the function $f(x)$ on the axes provided, labelling the vertex as well as the x- and y-intercepts.



[3]

(b) Solve the inequality $f(x) < g(x)$.

[4]

Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$

a) $f(0) = 3(0)^2 + (0) - 2 = -2$

So y-intercept is $(0, -2)$

$3x^2 + x - 2 = 0$

$\Rightarrow (3x - 2)(x + 1) = 0$ Factorise

$\Rightarrow x = \frac{2}{3}$ or -1

So x-intercepts are $(-1, 0)$ and $(\frac{2}{3}, 0)$

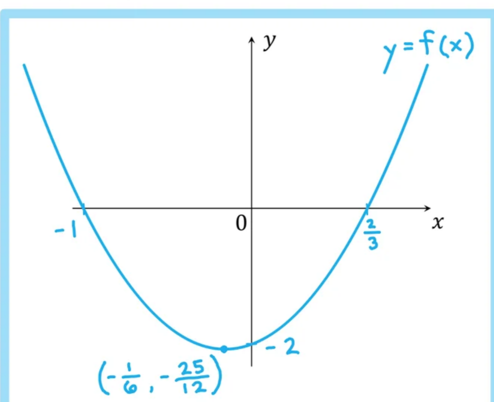
Axis of symmetry is $x = -\frac{1}{2(3)} = -\frac{1}{6}$

$f(-\frac{1}{6}) = 3(-\frac{1}{6})^2 + (-\frac{1}{6}) - 2 = \frac{1}{12} - \frac{1}{6} - 2 = -\frac{25}{12}$

So vertex is at $(-\frac{1}{6}, -\frac{25}{12})$

Consider the functions $f(x) = 3x^2 + x - 2$ and $g(x) = -2x^2 + 3x + 5$.

(a) Sketch the graph of the function $f(x)$ on the axes provided, labelling the vertex as well as the x- and y-intercepts.



[3]

(b) Solve the inequality $f(x) < g(x)$.

[4]

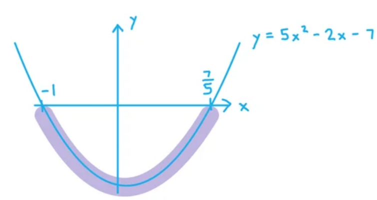
b) $f(x) < g(x) \Rightarrow 3x^2 + x - 2 < -2x^2 + 3x + 5$

$\Rightarrow 5x^2 - 2x - 7 < 0$ Rearrange to get zero on one side

$\Rightarrow \underbrace{(5x - 7)(x + 1)}< 0$ Factorise

This is equal to zero for $x = -1$ or $x = \frac{7}{5}$.

To see where it's less than zero, a sketch can help:



$$\boxed{-1 < x < \frac{7}{5}}$$

EXAM PAPERS PRACTICE

Question 2

Solve the inequality $5x^2 - 8x - 48 \geq 2x^2 + 4x - 12$.

[4]

$5x^2 - 8x - 48 \geq 2x^2 + 4x - 12$

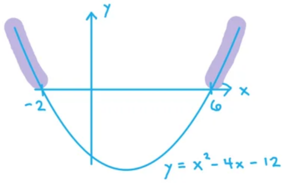
$\Longrightarrow 3x^2 - 12x - 36 \geq 0$ Rearrange to get zero on one side

$\Longrightarrow x^2 - 4x - 12 \geq 0$ Divide both sides by 3 to simplify

$\Longrightarrow \underbrace{(x-6)(x+2)}\geq 0$ Factorise

This is equal to zero for $x = -2$ or $x = 6$.

To see where it's greater than zero, a sketch can help:



$\boxed{x \leq -2 \text{ or } x \geq 6}$

Question 3

Consider the inequality $\frac{x^2-3x-10}{x-1} < 0$.

(a) Explain why you need to consider the cases $x < 1$, $x = 1$ and $x > 1$ separately when rearranging the inequality to find a solution.

[2]

(b) Solve the inequality.

[5]

a) When $x = 1$, $x - 1 = 0$ and $\frac{x^2-3x-10}{x-1}$ is undefined.

When $x > 1$, $x - 1 > 0$ and so we can multiply by $x - 1$ without affecting the inequality sign.

When $x < 1$, $x - 1 < 0$ and so multiplying by $x - 1$ will 'flip' the inequality sign from $<$ to $>$.

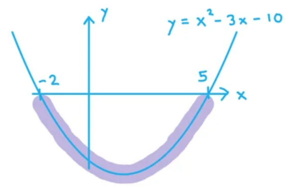
Consider the inequality $\frac{x^2-3x-10}{x-1} < 0$.

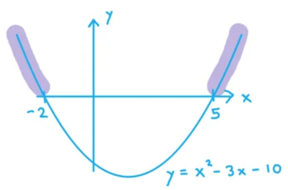
(a) Explain why you need to consider the cases $x<1$, $x=1$ and $x>1$ separately when rearranging the inequality to find a solution.

[2]

(b) Solve the inequality.

[5]





b) If $x > 1$

$\frac{x^2-3x-10}{x-1} < 0 \Rightarrow x^2-3x-10 < 0$ Multiply by $x-1$

$\Rightarrow (x+2)(x-5) < 0$ Factorise

$\Rightarrow -2 < x < 5$

$\Rightarrow 1 < x < 5$ Because $x>1$ must also be true here!

If $x < 1$

$x-1<0$, so inequality 'flips'

$\frac{x^2-3x-10}{x-1} < 0 \Rightarrow x^2-3x-10 > 0$ Multiply by $x-1$

$\Rightarrow (x+2)(x-5) > 0$ Factorise

$\Rightarrow x < -2$ or $x > 5$

$\Rightarrow x < -2$ Because $x<1$ must also be true here!

$\boxed{x < -2 \text{ or } 1 < x < 5}$

Question 4

The functions f and g are defined such that $f(x) = \frac{x+4}{2x-1}$ and $g(x) = 2x-4$.

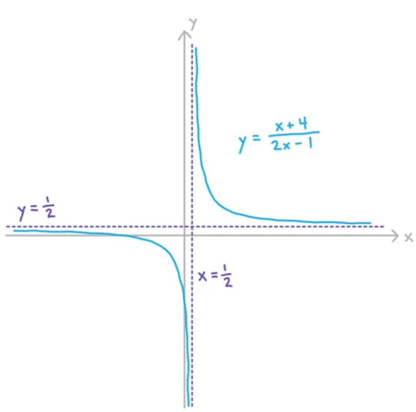
Given that f has the largest possible valid domain,

(a) state the domain and range of f.

[2]

(b) Solve the inequality $f(x) \leq g(x)$.

[4]



Because denominator cannot be zero!

a) $\boxed{\text{Domain}: x \in \mathbb{R},\ x \neq \tfrac{1}{2}}$

The graph of $y=f(x)$ has a vertical asymptote at $x=\frac{1}{2}$. Now consider what happens as $x \to \pm\infty$:

$$\lim_{x\to\pm\infty} \frac{x+4}{2x-1} = \lim_{x\to\pm\infty} \frac{1+4/x}{2-1/x} = \frac{1+0}{2-0} = \frac{1}{2}$$

Divide top and bottom by x

So there is also a horizontal asymptote at $y=\frac{1}{2}$.

$\boxed{\text{Range}: f(x) \in \mathbb{R},\ f(x) \neq \tfrac{1}{2}}$

The functions f and g are defined such that $f(x) = \frac{x+4}{2x-1}$ and $g(x) = 2x - 4$.

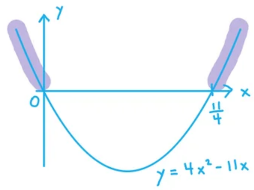
Given that f has the largest possible valid domain,

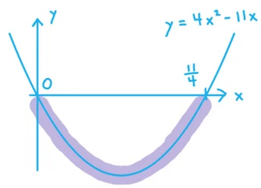
(a) state the domain and range of f.

[2]

(b) Solve the inequality $f(x) \leq g(x)$.

[4]





b) $f(x) \leq g(x) \implies \frac{x+4}{2x-1} \leq 2x-4$

If $x > \frac{1}{2}$

$\implies x+4 \leq (2x-1)(2x-4) = 4x^2 - 10x + 4$

$\implies 4x^2 - 11x \geq 0$

$\implies x(4x-11) \geq 0$ Factorise

$\implies x \leq 0$ or $x \geq \frac{11}{4}$

$\implies x \geq \frac{11}{4}$ Because $x > \frac{1}{2}$ must also be true here!

If $x < \frac{1}{2}$

$2x-1 < 0$, so inequality 'flips'

$\implies x+4 \geq (2x-1)(2x-4) = 4x^2 - 10x + 4$

$\implies 4x^2 - 11x = x(4x-11) \leq 0$

$\implies 0 \leq x \leq \frac{11}{4}$

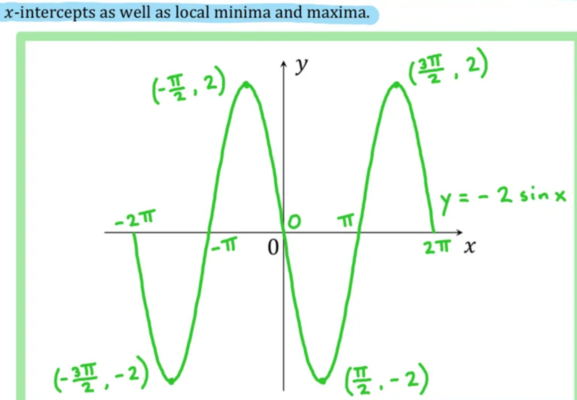
$\implies 0 \leq x < \frac{1}{2}$ Because $x < \frac{1}{2}$ must also be true here!

$\boxed{0 \leq x < \frac{1}{2} \quad \text{or} \quad x \geq \frac{11}{4}}$

Question 5

Consider the function $f(x) = -2\sin x$ in the interval $-2\pi \leq x \leq 2\pi$.

(a) Sketch a graph of the function over the given interval on the axes provided, labelling all x-intercepts as well as local minima and maxima.



[3]

a) You should be able to sketch this using your knowledge of the sine function and of graph transformations!

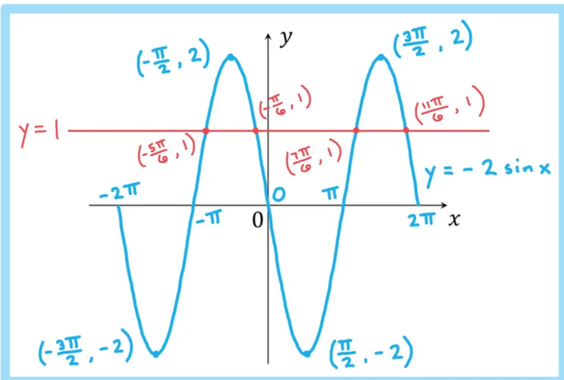
But your GDC can also help.

(b) Solve the inequality $f(x) > 1$.

[4]

Consider the function $f(x) = -2\sin x$ in the interval $-2\pi \le x \le 2\pi$.

(a) Sketch a graph of the function over the given interval on the axes provided, labelling all x-intercepts as well as local minima and maxima.



[3]

(b) Solve the inequality $f(x) > 1$.

[4]

b) Use your GDC to find intersections with the line $y = 1$.

The solutions to the inequality are the x-values in between those intersections, in the regions where $y = -2\sin x$ lies above $y = 1$.

$$\boxed{-\frac{5\pi}{6} < x < -\frac{\pi}{6} \quad \text{or} \quad \frac{7\pi}{6} < x < \frac{11\pi}{6}}$$

Note: These are the solutions in the region $-2\pi \le x \le 2\pi$. There are other solutions outside the region.

Question 6

Solve the inequality $\frac{3x-2}{5} + 3 > \frac{4x-4}{5}$.

[4]

$$\frac{3x-2}{5} + 3 > \frac{4x-4}{5}$$

$\Longrightarrow \quad 3x - 2 + 15 > 4x - 4$ Multiply both sides by 5

$\Longrightarrow \quad 4x - 4 < 3x + 13$

$\Longrightarrow \quad \boxed{x < 17}$

Question 7

Consider the functions $f(x) = x^2 - 9 + \frac{4}{x}$ and $g(x) = -x + 5$.

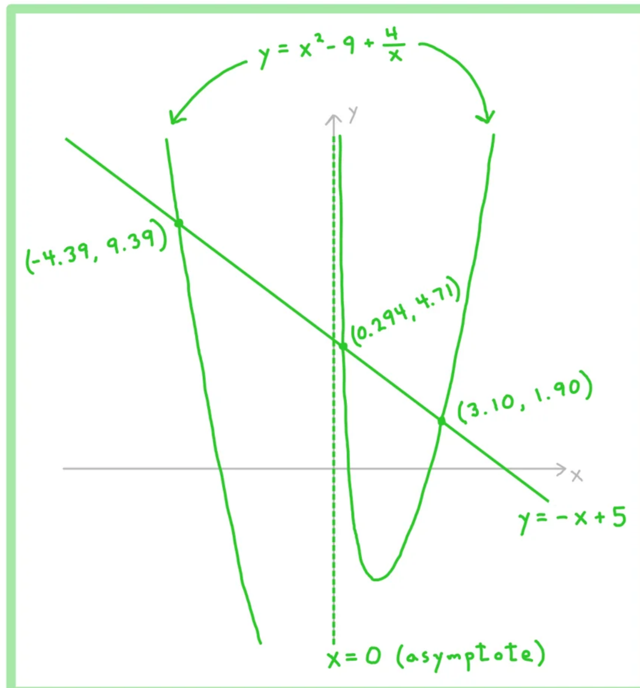
(a) Sketch the graphs of $f(x)$ and $g(x)$, clearly labelling any points of intersection or asymptotes.

[4]

(b) Determine the values of x such that $f(x) \geq g(x)$.

[3]

a)



Consider the functions $f(x) = x^2 - 9 + \frac{4}{x}$ and $g(x) = -x + 5$.

(a) Sketch the graphs of $f(x)$ and $g(x)$, clearly labelling any points of intersection or asymptotes.

[4]

(b) Determine the values of x such that $f(x) \geq g(x)$.

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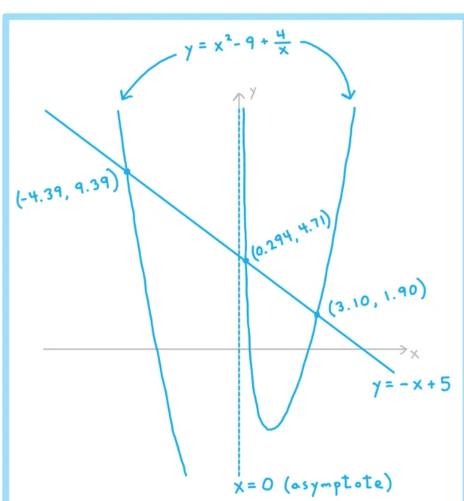


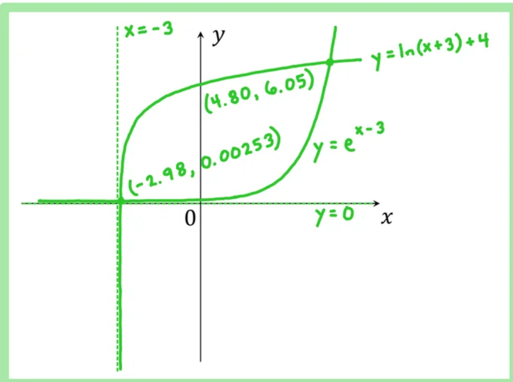
b) You can read the relevant regions off your graph.

$x \leq -4.39$, $0 < x \leq 0.294$, or $x \geq 3.10$ (all to 3 s.f.)

Question 8

Consider two functions, $f(x) = \ln(x+3) + 4$ and $g(x) = e^{x-3}$.

(a) Sketch both functions on the axes below, clearly labelling the asymptotes and points of intersection.



[4]

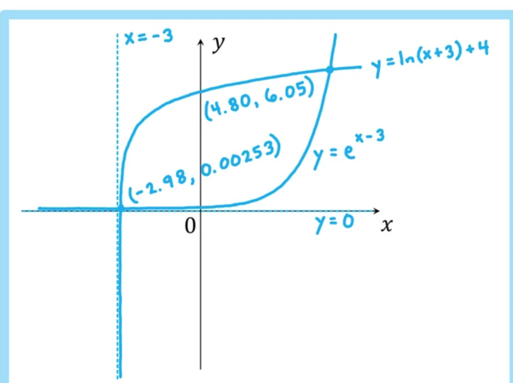
(b) Hence or otherwise, solve the inequality $f(x) \geq g(x)$.

[2]

a) Use your GDC to help you sketch the functions and find the intercepts.

Consider two functions, $f(x) = \ln(x+3) + 4$ and $g(x) = e^{x-3}$.

(a) Sketch both functions on the axes below, clearly labelling the asymptotes and points of intersection.



[4]

(b) Hence or otherwise, solve the inequality $f(x) \geq g(x)$.

[2]

b) You can read the relevant regions off your graph.

$$\boxed{-2.98 \leq x \leq 4.80 \quad \text{(all to 3 s.f.)}}$$

Question 9

Consider the polynomial $q(x) = x^3 - 8x^2 + 19x - 12$.

(a) Given that $(x-4)$ is a factor of $q(x)$, determine the x-intercepts of $q(x)$.

[4]

(b) Hence or otherwise, solve the inequality $x^3 + 19x \le 8x^2 + 12$.

[3]

a) $q(x) = (x-4)(x^2-4x+3)$ Factorise by synthetic division or by inspection

$q(x) = (x-4)(x-3)(x-1)$ Factorise the quadratic factor

The x-intercepts are

$(1,0)$, $(3,0)$, and $(4,0)$

Consider the polynomial $q(x) = x^3 - 8x^2 + 19x - 12$.

(a) Given that $(x-4)$ is a factor of $q(x)$, determine the x-intercepts of $q(x)$.

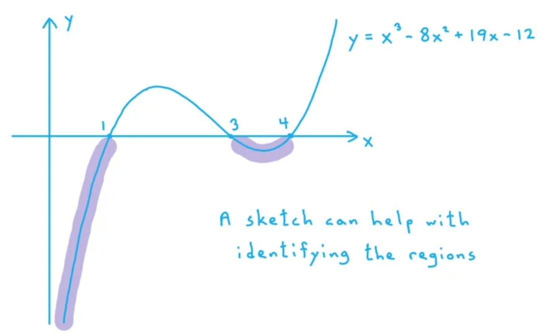
[4]

(b) Hence or otherwise, solve the inequality $x^3 + 19x \le 8x^2 + 12$.

[3]

b) $x^3 + 19x \le 8x^2 + 12$

$\Longrightarrow x^3 - 8x^2 + 19x - 12 \le 0$



$x \le 1$ or $3 \le x \le 4$

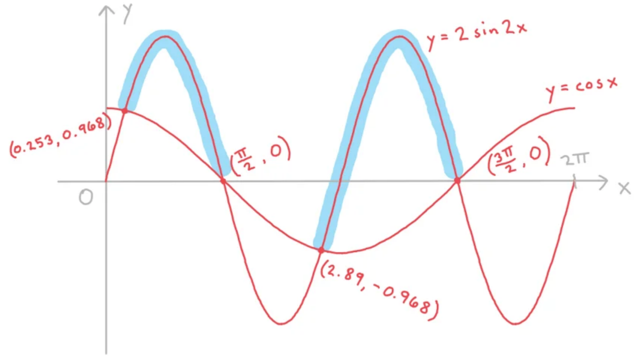
Question 10

Consider the two functions $f(x) = 2\sin 2x$ and $g(x) = \cos x$, both having the domain $0 \le x \le 2\pi$.

Solve the inequality $f(x) \ge g(x)$.

[3]

Use GDC to graph functions and find the points of intersection:



Then you can read the relevant regions off your graph.

$0.253 \le x \le \frac{\pi}{2}$ (3 s.f.) or $2.89 \le x \le \frac{3\pi}{2}$ (3 s.f.)