EXAM PAPERS PRACTICE

Indices

Model Answers

Question 1

Find the value of

(a) $(\sqrt{5})^8$, [1]

To find the value of $(\sqrt{5})^8$, you can raise $\sqrt{5}$ to the power of 8 .

$(\sqrt{5})^8=\sqrt{5^8}=\sqrt{390625}=625$

So, $(\sqrt{5})^8$ is equal to 625 .

(b) $\left(\frac{1}{27}\right)^{-\frac{2}{3}}$. [1]

To simplify the expression $\left(\frac{1}{27}\right)^5$, remember that when you have a negative exponent, you can move the base to the other side of the fraction and change the sign of the exponent:

$\left(\frac{1}{27}\right)^{-\frac{2}{3}}=\frac{1}{\left(\frac{1}{27}\right)^{\frac{2}{3}}}$

Now, let's simplify the expression inside the parentheses:

$\left(\frac{1}{27}\right)^{\frac{2}{3}}=\left(\left(\frac{1}{3^3}\right)^{\frac{2}{3}}\right)=\left(\frac{1}{3^{3\times\frac{2}{3}}}\right)=\left(\frac{1}{3^2}\right)=\frac{1}{9}$

Now, substitute this back into the original expression:

$\frac{1}{\left(\frac{1}{27}\right)^{\frac{2}{3}}}=\frac{1}{\frac{1}{9}}=\frac{1}{1}\times\frac{9}{1}=9$

So, $\left(\frac{1}{27}\right)^{-\frac{2}{3}}$ simplifies to 9 .

Question 2

(a) Find the value of

(i) $\left(\frac{1}{4}\right)^{0.5}$, [1]

To evaluate $\left(\frac{1}{4}\right)^{0.5}$, you're essentially finding the square root of $\frac{1}{4}$. The square root of a number is a value that, when multiplied by itself, gives the original number.

$\left(\frac{1}{4}\right)^{0.5}=\sqrt{\frac{1}{4}}=\frac{\sqrt{1}}{\sqrt{4}}=\frac{1}{2}$

So, $\left(\frac{1}{4}\right)^{0.5}=\frac{1}{2}$.

$\mathbb{B}$

(ii) $(-8)^{\frac{2}{3}}$.

To evaluate $(-8)^{\frac{2}{3}}$, you can first find the cube root of -8 and then square the result.

1. Find the cube root of -8 :

$\sqrt[3]{-8}=-2$

2. Square the result: [1]

$(-2)^2=4$

Therefore, $(-8)^{\frac{2}{3}}=4$.

(b) Use a calculator to find the decimal value of $\frac{\sqrt{29-3\times 32^{0.4}}}{3}$. [1]

Let's break down the expression step by step:

$\frac{\sqrt{29-3\times 32^{10.4}}}{3}$

1. Calculate $32^{0.4}$:

$32^{0.4}=2$

2. Substitute this back into the expression:

$\sqrt{29-3\times 2}$

3. Simplify further:

$\sqrt{29-6}$

4. Subtract 6 from 29:

$\sqrt{23}$

5. Now, divide by 3 :

$\frac{\sqrt{23}}{3}$

This expression doesn't simplify further without using a calculator. To find the decimal value, you can evaluate it:

$\frac{\sqrt{23}}{3}\approx\frac{4.795}{3}\approx 1.598$

Therefore, the decimal value of the given expression is approximately 1.598 .

Question 3

Simplify the following.

(a) $(4pq^2)^3$

$\left(4pq^2\right)^3 = 4^3 \cdot p^3 \cdot \left(q^2\right)^3$ [2]

Now, calculate each term:

$4^3 = 64$

$p^3 = p \cdot p \cdot p$

$\left(q^2\right)^3 = q^{2\cdot 3} = q^6$

Combine the results:

$\left(4pq^2\right)^3 = 64p^3q^6$

So, $\left(4pq^2\right)^3$ simplifies to $64p^3q^6$.

(b) $(16x^8)^{-\frac{1}{4}}$

To simplify $\left(16x^8\right)^{‡‡}$, you can use the property that $\left(a^b\right)^c = a^{b\cdot c}$. Apply this property to each term inside the parentheses:

$\left(16x^8\right)^{-\frac{1}{4}} = 16^{\frac{1}{4}} \cdot \left(x^8\right)^{-\frac{1}{4}}$ [2]

Now, simplify each term:

1. $16^{\frac{1}{4}}$:

$16^{\frac{1}{4}} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$

2. $\left(x^8\right)^{-\frac{1}{4}}$:

Apply the power rule $\left(a^b\right)^c = a^{b-c}$:

$\left(x^8\right)^{-\frac{1}{3}} = x^{8\cdot\left(-\frac{1}{4}\right)} = x^{-2}$

Combine the results:

$\left(16x^8\right)^{-\frac{1}{4}} = \frac{1}{2}x^{-2} = \frac{1}{2x^2}$

So, $\left(16x^8\right)^{-\frac{1}{4}}$ simplifies to $\frac{1}{2x^2}$.

Question 4

$a \times 10^7 + b \times 10^6 = c \times 10^6$

Find c in terms of a and b.
Give your answer in its simplest form. [2]

To find c in terms of a and b in the equation $a \times 10^7 + b \times 10^6 = c \times 10^6$,you can set up the equation and then solve for c :

$a \times 10^7 + b \times 10^6 = c \times 10^6$

First, subtract $b \times 10^6$ from both sides:

$a \times 10^7 = c \times 10^6 - b \times 10^6$

Factor out 10^6 from the right side:

$a \times 10^7 = (c - b) \times 10^6$

Now, divide both sides by 10^6 to solve for $c - b$:

$c - b = \frac{a \times 10^7}{10^6} = a \times 10$

Finally, add b to both sides to solve for c :

$c = a \times 10 + b$

So, in terms of a and b, c is equal to $a \times 10 + b$.

Question 5

$3^x \times 9^4 = 3^n$

Find n in terms of x. [2]

To find n in terms of x in the equation $3^x \times 9^4 = 3^n$, we can use the fact that 9 can be expressed as 3^2 :

$3^x \times (3^2)^4 = 3^n$

Now, apply the exponent rules. When you have a power raised to another power, you multiply the exponents:

$3^x \times 3^{2\times4} = 3^n$

Simplify further:

$3^x \times 3^8 = 3^n$

Now, since the bases are the same (both are 3), you can combine the terms by adding the exponents:

$3^{x+8} = 3^n$

Now, set the exponents equal to each other:

$x + 8 = n$

So, in terms of x, n is equal to $x + 8$.

Question 6

Simplify $\frac{5}{8}x^{\frac{3}{2}} \div \frac{1}{2}x^{-\frac{5}{2}}$. [2]

1. Combine the fractions by multiplying the numerator and denominator of the first fraction by 2 :

$\frac{5}{8}x^{\frac{3}{2}} \div \frac{1}{2}x^{\frac{5}{2}} = \frac{5\times2}{8\times2}x^{\frac{3}{2}} = \frac{10}{16}x^{\frac{3}{2}}$

2. Combine the exponents with the same base by subtracting them:

$\frac{10}{16}x^{\frac{3}{2}} = \frac{5}{8}x^{\frac{3}{2}-\frac{5}{2}}$

3. Simplify the exponent:

$\frac{5}{8}x^{-\frac{2}{2}} = \frac{5}{8}x^{-1}$

So, $\frac{5}{8}x^{\frac{3}{2}} \div \frac{1}{2}x^{-\frac{5}{2}}$ simplifies to $\frac{5}{8}x^{-1}$.

Question 7

Find the value of n in each of the following statements.

(a) $32^n = 1$ [1]

To find the value of n in the equation $32^n = 1$, you can use the fact that any non-zero number raised to the power of 0 is equal to 1 . Therefore, n must be 0 in this case.
So, the solution is $n = 0$, and the equation $32^n = 1$ is true when n is 0 .

(b) $32^n = 2$ [1]

To find the value of n in the equation $32^n = 2$, we need to determine the exponent that, when applied to 32 , results in 2.
$32^n = 2$
Since $32 = 2^5$, we can rewrite the equation using the base 2 :
$\left(2^5\right)^n = 2$
Now, apply the power of a power rule (multiply the exponents):
$2^{5n} = 2$
For the two sides of the equation to be equal, the exponents must be equal:
$5n = 1$
Now, solve for n :
$n = \frac{1}{5}$
So, the value of n is $\frac{1}{5}$.

(c) $32^n = 8$ [1]

To find the value of n in the equation $32^n = 8$, you can rewrite both sides of the equation using the same base. Since 8 can be expressed as 2^3, you can rewrite 32 as 2^5 :
$\left(2^5\right)^n = 2^3$
Now, apply the exponent rules by multiplying the exponents on the left side:
$2^{5n} = 2^3$
Now, since the bases are the same, you can set the exponents equal to each other:
$5n = 3$
Solve for n :
$n = \frac{3}{5}$
So, the value of n is $\frac{3}{5}$ in the equation $32^n = 8$.

Question 8

Simplify

(a) $\left(\frac{x^{27}}{27}\right)^{\frac{2}{3}}$, [2]

To simplify $\left(\frac{x^{27}}{27}\right)^{\frac{2}{3}}$, you can apply the exponent rules. When you raise a power to another power, you multiply the exponents. Here's how you can simplify it step by step:
$\left(\frac{x^{27}}{27}\right)^{\frac{2}{3}}$
1. Apply the power rule: $\left(a^b\right)^c = a^{b-c}$
$\frac{x^{27 \cdot \frac{2}{3}}}{27^2}$
1. Simplify the exponents:
$\frac{x^{18}}{\left(27^{\frac{1}{3}}\right)^2}$
1. Simplify the cube root of 27 :
$\frac{x^{18}}{3^2}$
1. Simplify further:
$\frac{x^{18}}{9}$
So, $\left(\frac{x^{27}}{27}\right)^{\frac{2}{3}}$ simplifies to $\frac{x^{15}}{9}$.

(b) $\left(\frac{x^{-2}}{4}\right)^{-\frac{1}{2}}$. [2]

To simplify $\left(\frac{x^{-2}}{4}\right)^{-\frac{1}{2}}$, you can apply the exponent rules. When you have a negative exponent, you can move the base to the other side of the fraction and change the sign of the exponent. Here's how you can simplify it step by step:
$\left(\frac{x^{-2}}{4}\right)^{-\frac{1}{2}}$
1. Move the base with the negative exponent to the denominator and change the sign of the exponent:
$\frac{1}{\left(\frac{x^{-2}}{4}\right)^{\frac{1}{2}}}$
1. Apply the power rule: $\left(a^b\right)^c = a^{b \cdot c}$
$\frac{1}{\frac{x^{-2\frac{1}{2}}}{4^{\frac{1}{2}}}}$
1. Simplify the exponents:
$\frac{1}{\frac{x^{-1}}{2}}$
1. Simplify further by multiplying the numerator and denominator by 2 :
$\frac{1}{\frac{1}{2}}$
1. Simplify by multiplying the numerator and denominator by x :
$\frac{x}{2}$
So, $\left(\frac{x^{-2}}{4}\right)^{-\frac{1}{2}}$ simplifies to $\frac{x}{2}$.

Question 9

Find the **exact** value of

(a) 3^{-2}, [1]

To find the exact value of 3^2, it means taking the reciprocal of 3^2.

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

So, the exact value of 3^{-2} is $\frac{1}{9}$.

(b) $\left(1\frac{7}{9}\right)^{\frac{1}{2}}$. [2]

To find the exact value of $(1\frac{7}{9})^{\frac{1}{2}}$, first convert the mixed number to an improper fraction.

$1\frac{7}{9} = \frac{9}{9} + \frac{7}{9} = \frac{16}{9}$

Now, take the square root of $\frac{16}{9}$:

$(\frac{16}{9})^{\frac{1}{2}} = \sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$

So, $(1\frac{7}{9})^{\frac{1}{2}}$ is equal to $\frac{4}{3}$.

Question 10

(a) Simplify $x^8 \div x^2$. [1]

To simplify $x^8 \div x^2$, you can use the rule $a^m \div a^n = a^{m-n}$, where a is a non-zero number. Apply this rule to your expression:
$x^8 \div x^2 = x^{8-2} = x^6$
So, $x^8 \div x^2$ simplifies to x^6.

(b) Simplify $\left(\frac{x^6}{27}\right)^{\frac{1}{3}}$. [2]

$$\left(\frac{x^6}{27}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{x^6}}{\sqrt[3]{27}}$$

Now, simplify the expression under the cube roots:

$$\frac{\sqrt[3]{x^6}}{\sqrt[3]{27}} = \frac{\sqrt[3]{(x^2)^3}}{\sqrt[3]{(3)^3}}$$

Cancel out the cubes:

$$\frac{x^2}{3}$$

So, $\left(\frac{x^6}{27}\right)^{\frac{1}{3}}$ simplifies to $\frac{x^2}{3}$.

Question 11

(a) $(2^{24})^{\frac{1}{2}} = p^4$ Find the value of p. [2]

To find the value of p in the equation $\left(2^{24}\right)^{\frac{1}{2}} = p^4$, you can simplify the left side of the equation:

$\left(2^{24}\right)^{\frac{1}{2}} = 2^{24\times\frac{1}{2}} = 2^{12}$

Now, set this equal to p^4 and solve for p :

$2^{12} = p^4$

Since both sides have the same base (2), you can equate the exponents:

$12 = 4 \times \log_2(p)$

Now, solve for p :

$4 \times \log_2(p) = 12$

$\log_2(p) = \frac{12}{4}$

$\log_2(p) = 3$

Now, raise 2 to the power of 3 to find p :

$p = 2^3 = 8$

So, the value of p is 8 .

(b) Simplify $\dfrac{q^2 + q^2}{q^{\frac{1}{4}} \times q^{\frac{1}{4}}}$. [3]

To simplify the expression $\frac{q^2+q^2}{q^4\times q^4}$, combine like terms in the numerator and simplify the denominator. Here's the step-by-step process:

1. Combine like terms in the numerator:

$q^2 + q^2 = 2q^2$

2. Simplify the denominator by adding the exponents:

$q^{\frac{1}{4}} \times q^{\frac{1}{4}} = q^{\frac{1}{4}+\frac{1}{4}} = q^{\frac{1}{2}}$

Now, substitute these results back into the original expression:

$\frac{2q^2}{q^{\frac{1}{2}}}$

1. Subtract the exponents in the denominator from the exponent in the numerator:

$2q^2 \frac{1}{2} = 2q^{\frac{3}{2}}$

So, $\frac{q^2+q^2}{q^2\times q^{\frac{1}{4}}}$ simplifies to $2q^{\frac{3}{2}}$.

Question 12

1 Calculate $\dfrac{\sqrt[3]{16}}{1.3^2}$. [1]

To calculate $\frac{\sqrt[3]{16}}{1.3^2}$, let's break it down step by step:

1. Simplify the cube root of 16 :

$\sqrt[3]{16} = 2$

2. Square 1.3:

$1.3^2 = 1.69$

Now, substitute these values back into the original expression:

$\frac{\sqrt[3]{16}}{1.3^2} = \frac{2}{1.69}$

To get a decimal approximation, divide 2 by 1.69 :

$\frac{2}{1.69} \approx 1.1834$

So, $\frac{\sqrt[3]{16}}{1.3^2} \approx 1.1834$

Question 13

7 (a) Simplify $(3125t^{125})^{\frac{1}{5}}$

To simplify $(3125t^{125})^{\frac{1}{5}}$, apply the power rule for exponents, which states that $(a^b)^c = a^{b\cdot c}$. In this case, raise both the base and the exponent to the power of $\frac{1}{5}$:

[2]

$(3125t^{125})^{\frac{1}{5}} = 3125^{\frac{1}{5}} \cdot (t^{125})^{\frac{1}{5}}$

Now, simplify each term:

$3125^{\frac{1}{5}} = 5$, because $5^5 = 3125$.

$(t^{125})^{\frac{1}{5}} = t^{125\cdot\frac{1}{5}} = t^{25}$

Combine the results:

$(3125t^{125})^{\frac{1}{5}} = 5t^{25}$

So, $(3125t^{125})^{\frac{1}{5}}$ simplifies to $5t^{25}$.

(b) Find the value of p when $3^p = \frac{1}{9}$

To find the value of p when $3^p = \frac{1}{9}$, you can rewrite $\frac{1}{9}$ with a base of 3 .

$\frac{1}{9} = \frac{1}{3^2}$

Now, rewrite the equation with a common base:

[1]

$3^p = 3^{-2}$

Now, equate the exponents:

$p = -2$

So, the value of p is -2 when $3^p = \frac{1}{9}$.

(c) Find the value of w when $x^{72} \div x^w = x^8$.

[1]

To find the value of w when $x^{72} \div x^w = x^8$, apply the rule $a^m \div a^n = a^{m-n}$.

$x^{72} \div x^w = x^{72-w}$

Now, set this equal to x^8 and solve for w :

$x^{72-w} = x^8$

Since the bases are the same, you can equate the exponents:

$72 - w = 8$

Now, solve for w :

$w = 72 - 8 = 64$

So, the value of w is 64 when $x^{72} \div x^w = x^8$

Question 14

6 Simplify. $3x2y3 \times x4y$

To simplify the expression $3x^2y^3 \times x^4y$, you can combine the like terms by adding the exponents of x and y.

$3x^2y^3 \times x^4y = 3 \times x^{2+4} \times y^{3+1}$

Now, simplify the exponents:

$3 \times x^6 \times y^4$

So, $3x^2y^3 \times x^4y$ simplifies to $3x^6y^4$.

Question 15

(a) $3^x = \sqrt[4]{3^5}$

Find the value of x. [1]

To find the value of x in the equation $3^x = \sqrt[4]{3^5}$, first, simplify the right side:

$\sqrt[4]{3^5} = \sqrt[4]{243} = 3^{\frac{5}{4}}$

Now, set the exponents equal to each other:

$3^x = 3^{\frac{5}{4}}$

Since the bases are the same, the exponents must be equal:

$x = \frac{5}{4}$

So, the value of x is $\frac{5}{4}$ in the given equation.

(b) Simplify $(32y^{15})^{\frac{2}{5}}$. [2]

$(32y^{15})^{\frac{2}{5}} = 32^{\frac{2}{5}} \cdot (y^{15})^{\frac{2}{5}}$

1. Simplify the base $32^{\frac{2}{5}}$:

$32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^2 = 4$

2. Simplify the exponent of y^{15} by multiplying by $\frac{2}{5}$:

$(y^{15})^{\frac{2}{5}} = y^{\frac{15\times 2}{5}} = y^6$

Now, combine the results:

$(32y^{15})^{\frac{2}{5}} = 4y^6$

So, $(32y^{15})^{\frac{2}{5}}$ simplifies to $4y^6$.

Question 16

(a) Simplify $(64q^{-2})^{\frac{1}{2}}$. [2]

$\left(64q^{-2}\right)^{\frac{1}{2}} = 64^{\frac{1}{2}} \cdot \left(q^{-2}\right)^{\frac{1}{2}}$

1. Simplify the base $64^{\frac{1}{2}}$:

$64^{\frac{1}{2}} = \left(2^6\right)^{\frac{1}{2}} = 2^3 = 8$

2. Simplify the exponent of q^{-2} by multiplying by $\frac{1}{2}$:

$\left(q^{-2}\right)^{\frac{1}{2}} = q^{-1}$

Now, combine the results:

$\left(64q^{-2}\right)^{\frac{1}{2}} = 8q^{-1}$

So, $\left(64q^{-2}\right)^{\frac{1}{2}}$ simplifies to $8q^{-1}$.

(b) $5^7 \div 5^9 = p^2$

Find p. [2]

To find p in the equation $5^7 \div 5^y = p^2$, you can use the rule $a^m \div a^n = a^{m-n}$.

$5^7 \div 5^9 = 5^{7-9} = 5^{-2}$

Now, set this equal to p^2 and solve for p :

$5^{-2} = p^2$

Since the bases are the same, you can equate the exponents:

$p^2 = \frac{1}{5^2} = \frac{1}{25}$

So, the value of p is $\frac{1}{5}$ in the given equation.

Question 17

Write $(27x^{12})^{\frac{1}{3}}$ in its simplest form.

[2]

To simplify $\left(27x^{12}\right)^{\frac{1}{3}}$, you can use the rule $\left(a^b\right)^c = a^{b \cdot c}$. In this case, raise both the base and the exponent to the power of $\frac{1}{3}$:

$\left(27x^{12}\right)^{\frac{1}{3}} = 27^{\frac{1}{3}} \cdot \left(x^{12}\right)^{\frac{1}{3}}$

Now, simplify each term:

$27^{\frac{1}{3}} = 3$, because $3^3 = 27$.

$\left(x^{12}\right)^{\frac{1}{3}} = x^{12 \cdot \frac{1}{3}} = x^4$

Combine the results:

$\left(27x^{12}\right)^{\frac{1}{3}} = 3x^4$

So, $\left(27x^{12}\right)^{\frac{1}{3}}$ simplifies to $3x^4$.

EXAM PAPERS PRACTICE

Question 18

(a) $\left(\frac{3}{8}\right)^{\frac{3}{8}} \times \left(\frac{3}{8}\right)^{\frac{1}{8}} = p^q$

Find the value of p and the value of q.

2]

1. $\left(\frac{3}{8}\right)^{\frac{3}{8}}$:

$\left(\frac{3}{8}\right)^{\frac{3}{8}} = \left(\frac{3^3}{8^3}\right)^{\frac{1}{8}} = \left(\frac{27}{512}\right)^{\frac{1}{8}}$

2. $\left(\frac{3}{8}\right)^{\frac{1}{8}}$:

$\left(\frac{3}{8}\right)^{\frac{1}{8}} = \left(\frac{3}{8}\right)^{\frac{1}{8}} = \frac{3}{8}$

Now, multiply the simplified terms:

$\left(\frac{27}{512}\right)^{\frac{1}{8}} \times \frac{3}{8} = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

So, $p = \frac{9}{64}$ and $q = 1$.

(b) $5^{-3} + 5^{-4} = k \times 5^{-4}$

[2]

Find the value of k.

To find the value of k in the equation $5^{-3} + 5^{-4} = k \times 5^{-4}$, let's simplify the left side of the equation first.

$5^{-3} + 5^{-4} = \frac{1}{5^3} + \frac{1}{5^4}$

Now, find a common denominator, which is 5^4 :

$\frac{5^4}{5^4} \times \frac{1}{5^3} + \frac{1}{5^4} = \frac{5^4+1}{5^4}$

So, the left side becomes:

$5^{-3} + 5^{-4} = \frac{5^4+1}{5^4}$

Now, set this equal to $k \times 5^{-4}$ and solve for k :

$\frac{5^4+1}{5^4} = k \times 5^{-4}$

Now, multiply both sides by 5^4 to isolate k :

$5^4 + 1 = k$

$625 + 1 = k$

$k = 626$

So, the value of k is 626 .

Question 19

Simplify $\left(256w^{256}\right)^{\frac{1}{4}}$.

[2]

To simplify $\left(256w^{256}\right)^{\frac{1}{4}}$, you can use the property $\left(a^b\right)^c = a^{b \cdot c}$. Here, you will apply this property to both the base and the exponent:

$\left(256w^{256}\right)^{\frac{1}{4}} = 256^{\frac{1}{4}} \cdot \left(w^{256}\right)^{\frac{1}{4}}$

Now, simplify each part:

$256^{\frac{1}{4}}$: This is the fourth root of 256 . Since $4^4 = 256$, $256^{\frac{1}{4}} = 4$.

$\left(w^{256}\right)^{\frac{1}{4}}$: This is the fourth root of w^{256}. Since $\left(w^4\right)^{64} = w^{256}$, $\left(w^{256}\right)^{\frac{1}{4}} = w^{64}$.

Now, combine the results:

$\left(256w^{256}\right)^{\frac{1}{4}} = 4w^{64}$

So, $\left(256w^{256}\right)^{\frac{1}{4}}$ simplifies to $4w^{64}$.

Question 20

Find the values of m and n.

(a) $2^m = 0.125$ [2]

To find the value of m in the equation $2^m = 0.125$, you can rewrite 0.125 as a power of 2 . Since 0.125 is equivalent to 2^{-3}, the equation becomes:
$2^m = 2^{-3}$
Now, set the exponents equal to each other:
$m = -3$
So, the value of m in the equation $2^m = 0.125$ is -3 .

(b) $2^{4n} \times 2^{2n} = 512$ [2]

To find the value of n in the equation $2^{4n} \times 2^{2n} = 512$, you can use the properties of exponents.
First, simplify the left side of the equation by combining the exponents:
$2^{4n} \times 2^{2n} = 2^{4n+2n} = 2^{6n}$
Now, set this equal to 512 , since $512 = 2^9$:
$2^{6n} = 2^9$
Since the bases are the same, the exponents must be equal:
$6n = 9$
Now, solve for n :
$n = \frac{9}{6} = \frac{3}{2}$
So, the value of n in the equation $2^{4n} \times 2^{2n} = 512$ is $\frac{3}{2}$.

Question 21

Find the value of $\left(\frac{27}{8}\right)^{-\frac{4}{3}}$.

Give your answer as an exact fraction. [2]

To find the value of $\left(\frac{27}{8}\right)^{-\frac{4}{3}}$, you can apply the reciprocal of the exponent. The reciprocal of $-\frac{4}{3}$ is $\frac{3}{4}$.

So,

$\left(\frac{27}{8}\right)^{-\frac{4}{3}} = \left(\frac{27}{8}\right)^{\frac{3}{4}}$

Now, take the cube root of the numerator and the fourth root of the denominator:

$\left(\frac{27}{8}\right)^{\frac{3}{4}} = \frac{\sqrt[4]{27^3}}{\sqrt[4]{8}}$

Simplify the expression under the radicals:

$\frac{\sqrt[4]{27^3}}{\sqrt[4]{8}} = \frac{\sqrt[4]{19683}}{\sqrt[4]{8}}$

Now, express both numbers with the same index:

$\frac{\sqrt[4]{19683}}{\sqrt[4]{8}} = \frac{\sqrt[4]{3^9}}{\sqrt[4]{2^3}}$

Combine the radicals:

$\frac{\sqrt[4]{3^9}}{\sqrt[4]{2^3}} = \frac{3^{\frac{9}{4}}}{2^{\frac{3}{4}}}$

So, $\left(\frac{27}{8}\right)^{-\frac{4}{3}}$ is equal to $\frac{3^{\frac{9}{4}}}{2^{\frac{3}{4}}}$ as an exact fraction.

Question 22

(a) Find m when $4^m \times 4^2 = 4^{12}$. [1]

To find the value of m in the equation $4^m \times 4^2 = 4^{12}$, you can use the properties of exponents.

First, simplify the left side of the equation by combining the exponents:

$4^m \times 4^2 = 4^{m+2}$

Now, set this equal to 4^{12} :

$4^{m+2} = 4^{12}$

Since the bases are the same, the exponents must be equal:

$m + 2 = 12$

Now, solve for m :

$m = 12 - 2 = 10$

So, the value of m in the equation $4^m \times 4^2 = 4^{12}$ is 10 .

(b) Find p when $6^p \div 6^5 = \sqrt{6}$. [1]

To find the value of p in the equation $6^p \div 6^5 = \sqrt{6}$, let's first simplify both sides of the equation.

$6^p \div 6^5 = \sqrt{6}$

Now, simplify the left side using the rule $a^m \div a^n = a^{m-n}$:

$6^p \div 6^5 = 6^{p-5}$

So, the equation becomes:

$6^{p-5} = \sqrt{6}$

To solve for p, set the exponents equal to each other:

$p - 5 = \frac{1}{2}$

Now, solve for p :

$p = \frac{1}{2} + 5 = \frac{11}{2}$

So, the value of p in the equation $6^p \div 6^5 = \sqrt{6}$ is $\frac{11}{2}$.

Question 23

Simplify

(a) $32x^8 \div 8x^{32}$, [2]

To simplify the expression $\frac{32x^n}{8x^{32}}$, you can apply the rule $\frac{a^m}{a^n} = a^{m-n}$ for non-zero a.

$\frac{32x^8}{8x^{32}} = \frac{32}{8} \times \frac{x^8}{x^{32}}$

Simplify the numerical part:

$4 \times \frac{x^8}{x^{32}}$

Now, apply the rule for subtracting exponents:

$4 \times x^{8-32} = 4 \times x^{-24}$

To make the expression more readable, you can rewrite the negative exponent in the denominator:

$\frac{4}{x^{24}}$

So, $\frac{32x^8}{8x^{32}}$ simplifies to $\frac{4}{x^{24}}$.

(b) $\left(\frac{x^3}{64}\right)^{\frac{2}{3}}$. [2]

To simplify $\left(\frac{x^3}{64}\right)^{\frac{2}{3}}$, you can use the rule $\left(a^b\right)^c = a^{b \cdot c}$. In this case, you raise both the numerator and denominator to the power of $\frac{2}{3}$:

$\left(\frac{x^3}{64}\right)^{\frac{2}{3}} = \frac{x^{3 \cdot \frac{2}{3}}}{64^{\frac{2}{3}}}$

Now, simplify the exponents:

1. Simplify the numerator exponent:

$x^{3 \cdot \frac{2}{3}} = x^2$

2. Simplify the denominator exponent:

$64^{\frac{2}{3}} = \left(2^6\right)^{\frac{2}{3}} = 2^4 = 16$

Now, combine the results:

$\left(\frac{x^3}{64}\right)^{\frac{2}{3}} = \frac{x^2}{16}$

So, $\left(\frac{x^3}{64}\right)^{\frac{2}{3}}$ simplifies to $\frac{x^2}{16}$.

Question 24

Simplify the following.

(a) $(3x^3)^3$ [2]

$\left(3x^3\right)^3 = 3^3 \cdot \left(x^3\right)^3$

Now, simplify each part:

1. $3^3 = 27$
2. $\left(x^3\right)^3 = x^{3\cdot 3} = x^9$

Combine the results:

$\left(3x^3\right)^3 = 27x^9$

So, $\left(3x^3\right)^3$ simplifies to $27x^9$.

(b) $(125x^6)^{\frac{2}{3}}$ [2]

To simplify $\left(125x^6\right)^{\frac{2}{3}}$, you can use the rule $\left(a^b\right)^c = a^{b\cdot c}$. In this case, raise both the base and the exponent to the power of $\frac{2}{3}$:

$\left(125x^6\right)^{\frac{2}{3}} = 125^{\frac{2}{3}} \cdot \left(x^6\right)^{\frac{2}{3}}$

Now, simplify each part:

$125^{\frac{2}{3}} = \left(5^3\right)^{\frac{2}{3}} = 5^2 = 25$

$\left(x^6\right)^{\frac{2}{3}} = x^{6\cdot\frac{2}{3}} = x^4$

Combine the results:

$\left(125x^6\right)^{\frac{2}{3}} = 25x^4$

So, $\left(125x^6\right)^{\frac{2}{3}}$ simplifies to $25x^4$.

Question 25

Find the value of n in the following equations.

(a) $2^n = 1024$ [1]

To find n in the equation $2^n = 1024$, you can rewrite 1024 as a power of 2 .
$2^{10} = 1024$
Now, set the exponents equal to each other:
$n = 10$
So, the value of n in the equation $2^n = 1024$ is 10 .

(b) $4^{2n-3} = 16$ [2]

To find n in the equation $4^{2n-3} = 16$, you can start by expressing both sides of the equation with the same base.
$4^{2n-3} = 4^2$
Now, set the exponents equal to each other:
$2n - 3 = 2$
Solve for n :
$2n = 5$
$n = \frac{5}{2}$
So, the value of n in the equation $4^{2n-3} = 16$ is $\frac{5}{2}$.

Question 26

Simplify

(a) $\left(\frac{16}{81}x^{16}\right)^{\frac{1}{2}}$, [2]

To simplify $\left(\frac{16}{81}x^{16}\right)^{\frac{1}{2}}$, you can use the rule $\left(a^b\right)^c = a^{b\cdot c}$. In this case, raise both the numerator and denominator to the power of $\frac{1}{2}$:

$\left(\frac{16}{81}x^{16}\right)^{\frac{1}{2}} = \left(\frac{16^{\frac{1}{2}}}{81^{\frac{1}{2}}} \cdot \left(x^{16}\right)^{\frac{1}{2}}\right)$

Now, simplify each part:

$16^{\frac{1}{2}} = \sqrt{16} = 4$

$81^{\frac{1}{2}} = \sqrt{81} = 9$

$\left(x^{16}\right)^{\frac{1}{2}} = x^8$

Combine the results:

$\left(\frac{16}{81}x^{16}\right)^{\frac{1}{2}} = \frac{4x^8}{9}$

So, $\left(\frac{16}{81}x^{16}\right)^{\frac{1}{2}}$ simplifies to $\frac{4x^8}{9}$.

(b) $\frac{16y^{10} \times 4y^{-4}}{32y^7}$. [2]

To simplify the expression $\frac{16y^{10}\times 4y^{-4}}{32y^7}$, you can perform the following steps:

1. Combine the numerical coefficients in the numerator:

$\frac{16\times 4}{32} = \frac{64}{32} = 2$

2. Combine the y terms in the numerator by adding the exponents:

$y^{10} \times y^{-4} = y^{10-4} = y^6$

Now, rewrite the simplified expression:

$\frac{16y^{10}\times 4y^{-4}}{32y^7} = \frac{2y^6}{y^7}$

To divide with the same base, subtract the exponents:

$\frac{2y^6}{y^7} = 2y^{6-7} = \frac{2}{y}$

So, $\frac{16y^{10}\times 4y^{-4}}{32y^7}$ simplifies to $\frac{2}{y}$.

Question 27

Simplify

(a) $\left(\frac{p^4}{16}\right)^{0.75}$, [2]

$\left(\frac{p^4}{16}\right)^{0.75} = \frac{p^{4 \cdot 0.75}}{16^{0.75}}$

Now, simplify each part:

1. $p^{4 \cdot 0.75} = p^3$
2. $16^{0.75} = (2^4)^{0.75} = 2^3$

Combine the results:

$\left(\frac{p^4}{16}\right)^{0.75} = \frac{p^3}{2^3}$

Now, simplify the fraction:

$\frac{p^3}{2^3} = \frac{p^3}{8}$

So, $\left(\frac{p^4}{16}\right)^{0.75}$ simplifies to $\frac{p^3}{8}$.

(b) $3^2 q^{-3} \div 2^3 q^{-2}$. [2]

To simplify $3^2 \cdot q^{-3} \div 2^3 \cdot q^{-2}$, you can apply the rules of exponents and perform the necessary operations step by step.

1. Combine the numerical coefficients:

$3^2 \div 2^3 = \frac{9}{8}$

2. Combine the q terms by subtracting exponents:

$q^{-3} \div q^{-2} = q^{-3-(-2)} = q^{-1}$

Now, combine the results:

$3^2 \cdot q^{-3} \div 2^3 \cdot q^{-2} = \frac{9}{8} \cdot q^{-1}$

So, the simplified expression is $\frac{9}{8q}$.

Question 28

Write $2^8 \times 8^2 \times 4^{-2}$ in the form 2^n. [2]

1. 2^8 remains as it is.
2. 8^2 can be written as $(2^3)^2 = 2^6$.
3. 4^{-2} is the reciprocal of 4^2 and can be written as $\frac{1}{4^2} = \frac{1}{2^4} = 2^{-4}$.

Now, combine the terms:

$2^8 \times 2^6 \times 2^{-4}$

To combine the terms with the same base (2), add the exponents:

$2^{8+6-4} = 2^{10}$

So, $2^8 \times 8^2 \times 4^{-2}$ can be written in the form 2^{10}.

Question 29

Simplify $(27x^3)^{\frac{2}{3}}$. [2]

$(27x^3)^{\frac{2}{3}} = 27^{\frac{2}{3}} \cdot (x^3)^{\frac{2}{3}}$

Now, simplify each part:

1. $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^2 = 9$
2. $(x^3)^{\frac{2}{3}} = x^{3 \cdot \frac{2}{3}} = x^2$

Combine the results:

$(27x^3)^{\frac{2}{3}} = 9x^2$

So, $(27x^3)^{\frac{2}{3}}$ simplifies to $9x^2$.

Question 30

(a) Simplify $(27x^6)^{\frac{1}{3}}$. [2]

We have $(27x^6)^{\frac{1}{3}} = (3^3x^6)^{\frac{1}{3}} = 3^{\frac{3}{3}}x^{6\cdot\frac{1}{3}} = 3^1x^2 = 3x^2 = 9x^2$.

(b) $(512)^{-\frac{2}{3}} = 2^p$. Find p. [2]

We have $(512)^{-\frac{2}{3}} = 2^p$. Note that $512 = 2^9$, and $2^{-2/3} = 2^{-2\cdot\frac{1}{3}} = 2^{-6/3} = 2^{-2}$. Hence, $2^p = 2^{-2}$ and $p = -2$.

Question 31

(a) $\sqrt{32} = 2^p$. Find the value of p.

[2]

To find the value of p, we need to solve the equation $\sqrt{32} = 2^p$.
Since the square root of 32 is equal to 2^5, we can substitute this into the equation to get:
$2^5 = 2^p$
Comparing the exponents, we see that $p = 5$.

(b) $\sqrt[3]{8} = 2^q$. Find the value of q.

[2]

First, we can re-write $\sqrt[3]{8}$ as $\sqrt[3]{2 \cdot 4 \cdot 2} = \sqrt[3]{2}\sqrt[3]{4}\sqrt[3]{2} = \sqrt[3]{2} \cdot \sqrt[3]{4} \cdot 2 = \sqrt[3]{2} \cdot \sqrt[3]{2^2} \cdot 2 = \sqrt[3]{2} \cdot 2 \cdot 2 = 4\sqrt[3]{2}$.
(Note that $\sqrt[3]{4} = \sqrt[3]{2^2} = 2$.)

Question 32

Simplify $\frac{2}{3}p^{12} \times \frac{3}{4}p^{8}$.

[2]

$\frac{2}{3}p^{12} \times \frac{3}{4}p^8$
We can simplify the expression by multiplying the numerators and the denominators.
Steps to solve:
1. Multiply the numerators and the denominators:
$\frac{2}{3}p^{12} \times \frac{3}{4}p^8 = \frac{2\times3}{3\times4} \times p^{12\times8}$
2. Simplify the fraction:
$\frac{2\times3}{3\times4} = \frac{1}{2}$
3. Combine the exponents:
$p^{12\times8} = p^{20}$
Answer:
$\frac{p^{20}}{2}$

Question 33

Simplify.

(a) $81^{\frac{3}{4}}$ [1]

To simplify $81^{\frac{3}{4}}$, we can first write 81 as 3^4. This gives us:

$81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}}$

Using the power of a power rule, $(a^m)^n = a^{m \cdot n}$, we get:

$(3^4)^{\frac{3}{4}} = 3^{4 \cdot \frac{3}{4}}$

Simplifying the exponent, we have:

$3^{4 \cdot \frac{3}{4}} = 3^3$

Evaluating the exponent, we get the simplified form:

$3^3 = 3 \cdot 3 \cdot 3 = 27$

[1]

(b) $x^{\frac{2}{3}} \div x^{-\frac{4}{3}}$

Since $x^a \div x^b = x^{a-b}$, we can simplify $x^{\frac{2}{3}} \div x^{-\frac{4}{3}}$ by subtracting the exponents:

$x^{\frac{2}{3}} \div x^{-\frac{4}{3}} = x^{\frac{2}{3}-\left(-\frac{4}{3}\right)} = x^{\frac{2}{3}+\frac{4}{3}} = x^{\frac{6}{3}} = x^2.$

Therefore, the simplified form of $x^{\frac{2}{3}} \div x^{-\frac{4}{3}}$ is x^2.

(c) $\left(\frac{8}{y^6}\right)^{-\frac{1}{3}}$ [2]

To simplify $\left(\frac{8}{y^6}\right)^{-\frac{1}{3}}$, we can use the following properties of exponents:

- $(x^a)^b = x^{ab}$
- $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

First, we can use the second property to distribute the exponent to the numerator and denominator:

$\left(\frac{8}{y^6}\right)^{-\frac{1}{3}} = \frac{8^{-\frac{1}{3}}}{y^{6 \cdot \left(-\frac{1}{3}\right)}}$

Next, we can use the first property to simplify the exponents:

$\frac{8^{-\frac{1}{3}}}{y^{6 \cdot \left(-\frac{1}{3}\right)}} = \frac{\left(\frac{1}{8}\right)^{\frac{1}{3}}}{y^{-2}}$

Finally, we can simplify the fraction:

$\frac{\left(\frac{1}{8}\right)^{\frac{1}{3}}}{y^{-2}} = \frac{\sqrt[3]{\frac{1}{8}}}{y^2}$

Therefore, the simplified form of $\left(\frac{8}{y^5}\right)^{-\frac{1}{3}}$ is $\frac{\sqrt[3]{\frac{1}{8}}}{y^2}$.

Question 34

(a) $2^r = \frac{1}{16}$

Find the value of r. [1]

Since $16 = 2^4$, then $\frac{1}{16} = \frac{1}{2^4} = 2^{-4}$. Therefore, $r = -4$.

(b) $3^t = \sqrt[5]{3}$

Find the value of t. [1]

Since $\sqrt[5]{3} = 3^{1/5}$, we can substitute this into the equation to get:
$3^t = 3^{1/5}$
Comparing the exponents, we see that $t = \frac{1}{5}$.

Question 35

Work out.

(a) $125^{\frac{2}{3}}$ [1]

Since $125 = 5^3$, then $125^{\frac{2}{3}} = \left(5^3\right)^{\frac{2}{3}} = 5^{3\times\frac{2}{3}} = 5^2 = 25$.

(b) $\left(\frac{1}{3}\right)^{-2}$ [1]

Steps to solve:
1. Evaluate the exponent:
$\left(\frac{1}{3}\right)^{-2} = 3^2$
Answer:

Question 36

(a) Simplify. $(16x^{16})^{\frac{3}{4}}$ [2]

$\left(16x^{16}\right)^{\frac{3}{4}}$

We can simplify the expression using the following rules of exponents:

1. $(a^m)^n = a^{mn}$
2. $(a^m) \cdot (a^n) = a^{m+n}$

First, we can rewrite 16 as 2^4 using the rule $(a^m) \cdot (a^n) = a^{m+n}$:

$\left(16x^{16}\right)^{\frac{3}{4}} = \left(2^4x^{16}\right)^{\frac{3}{4}}$

Then, we can apply the rule $(a^m)^n = a^{mn}$ to simplify the expression:

$\left(2^4x^{16}\right)^{\frac{3}{4}} = 2^{4\cdot\frac{3}{4}} \cdot x^{16\cdot\frac{3}{4}}$

Simplifying the exponents, we get:

$2^3 \cdot x^{12}$

Therefore, the simplified form of the expression is $8x^{12}$.

(b) $2p^{\frac{3}{2}} = 54$ Find the value of p.

Dividing both sides by 2 , we get

$p^{\frac{3}{2}} = 27$

Cubing both sides, we get

$p^3 = 27^3 = 729$

Finally, taking the cube root of both sides, we get

$p = 9$

Question 37

Simplify. $\left(\frac{8}{a^{12}}\right)^{\frac{1}{3}}$

To simplify $\left(\frac{8}{a^{12}}\right)^{\frac{1}{3}}$, first we simplify the numerator. We can rewrite 8 as 2^3, giving us:

\$$\left(\frac{2^3}{a^{12}}\right)^{\frac{1}{3}}$\$ Next, we can apply the rule $(a^m)^n = a^{mn}$ to simplify the expression:

$\left(\frac{2^3}{a^{12}}\right)^{\frac{1}{3}} = 2^{3\cdot\frac{1}{3}}a^{-12\cdot\frac{1}{3}}$

Simplifying the exponents, we get:

$2 \cdot a^{-4}$

Therefore, the simplified form of the expression is $\frac{2}{a^4}$.

Question 38

Work out.

(a) $t^{24} \div t^4$ [1]

Steps to solve:

1. Cancel terms that are in both the numerator and denominator:

$\frac{t^{24}}{t^4} = \frac{t^{20}}{1}$

2. Divide by 1 :

$\frac{t^{20}}{1} = t^{20}$

Answer:

t^{20}

(b) $(x^5)^2$ [1]

We can simplify the expression by using the following rule of exponents:

$(a^m)^n = a^{m \cdot n}$

Steps to solve:

1. Apply the exponent rule:

$(x^5)^2 = x^{5 \cdot 2}$

2. Simplify:

$x^{5 \cdot 2} = x^{10}$

Answer:

x^{10}

(c) $(81m^8)^{\frac{3}{4}}$

[2]

To simplify $(81m^8)^{\frac{3}{4}}$, we can start by simplifying 81 . Since 81 is the cube of 3 , we can write it as$81 = 3^3$. This gives us:

$(3^3m^8)^{\frac{3}{4}}$

Using the power of a power rule, $(a^m)^n = a^{m \cdot n}$, we can simplify the expression to:

$(3^3)^{\frac{3}{4}} \cdot (m^8)^{\frac{3}{4}}$

Simplifying the exponents, we get:

$3^{\frac{9}{4}} \cdot m^{24}$

Finally, evaluating the exponents, we get the simplified form:

$27 \cdot m^{24}$

Therefore, the simplified form of $(81\ \mathrm{m}^8)^{\frac{3}{4}}$ is $27\ \mathrm{m}^{24}$.

Question 39

Simplify. $(36x^{16})^{\frac{1}{2}}$ [2]

We can simplify the expression using the following property of exponents: $(x^a)^b = x^{ab}$

$(36x^{16})^{\frac{1}{2}} = (6^2x^{16})^{\frac{1}{2}}$

Now we can apply the property above to simplify the expression:

$(6^2x^{16})^{\frac{1}{2}} = 6^{2 \cdot \frac{1}{2}} \cdot x^{16 \cdot \frac{1}{2}}$

Simplifying the exponents, we get:

$6 \cdot x^8$

Therefore, the simplified form of the expression is $6x^8$.

Question 40

Simplify. [2]

$$\left(\frac{1}{2}x^{\frac{2}{3}}\right)^3$$

Steps to solve:

1. Combine multiplied terms into a single fraction:

$$\left(\frac{1x^{\frac{2}{3}}}{2}\right)^3$$

2. Multiply by 1 :

$$\left(\frac{x^{\frac{2}{3}}}{2}\right)^3$$

3. Distribute exponent:

$$\frac{\left(x^{\frac{2}{3}}\right)^3}{2^3}$$

4. Power of a power:

$$\frac{x^{\frac{2}{3}\cdot 3}}{2^3}$$

5. Multiply the numbers:

$$\frac{x^2}{2^3}$$

6. Evaluate the exponent:

$$\frac{x^2}{8}$$

Answer:

$$\frac{x^2}{8}$$

Question 41

Simplify. $(32x^{10})^{\frac{3}{5}}$

To simplify the expression $(32x^{10})^{\frac{3}{5}}$, we can first break down the expression into its prime factorization and then apply the properties of exponents.

First, we factorize 32 as 2^5. This gives us:

$(2^5x^{10})^{\frac{3}{5}}$

Next, we apply the power of a power rule, $(a^m)^n = a^{mn}$, to simplify the expression:

$(2^5x^{10})^{\frac{3}{5}} = 2^{5\cdot\frac{3}{5}} \cdot x^{10\cdot\frac{3}{5}}$

Simplifying the exponents, we get:

$2^3 \cdot x^6$

Finally, evaluating the exponents, we get the simplified form:

$8x^6$

Therefore, the simplified form of the expression $(32x^{10})^{\frac{3}{5}}$ is $8x^6$.

Question 42

Work out.

$2^{-4} \times 2^5$

[1]

Steps to solve:
1. Evaluate the exponent:
$\frac{1}{16} \cdot 2^5$
2. Evaluate the exponent:
$\frac{1}{16} \cdot 32$
3. Multiply the numbers:
2
Answer:
2

Question 43

Simplify.

(a) $(m^5)^2$ [1]

We can simplify the expression using the power of a power rule: $(a^m)^n = a^{m \cdot n}$.
Steps to solve:
1. Apply the power of a power rule:
$m^{5 \cdot 2}$
2. Multiply the numbers:
m^{10}
Answer:
m^{10}

(b) $4x^3y \times 5x^2y$ [2]

We can simplify the expression by multiplying the numbers, combining exponents, and combining exponents again.
Steps to solve:
1. Multiply the numbers:
$20x^3yx^2y$
2. Combine exponents:
$20x^5yy$
3. Combine exponents:
$20x^5y^2$
Answer:
$20x^5y^2$

Question 44

Simplify. $(x^2)^5$

[1]

We can simplify the expression using the following rule of exponents:

$(x^m)^n = x^{m \cdot n}$

Steps to solve:

1. Apply the exponent rule:

$(x^2)^5 = x^{2 \cdot 5}$

2. Simplify:

$x^{2 \cdot 5} = x^{10}$

Answer:

x^{10}

Question 45

Simplify.

(a) $6w^0$

[1]

Steps to solve:

1. Any nonzero number raised to the 0 power is 1 :$6 \cdot 1$

2. Multiply the numbers:

6

Answer:

6

(b) $5x^3 - 3x^3$

[1]

Steps to solve:

1. Combine like terms:

$5x^3 - 3x^3 = (5-3)x^3 = 2x^3$

Answer:

$2x^3$

(c) $3y^6 \times 5y^{-2}$

[2]

Steps to solve:

1. Multiply the numbers:

$15y^6y^{-2}$

2. Combine the exponents:

$15y^{6-2}$

3. Simplify:

$15y^4$

Answer:

$15y^4$

Question 46

(a) Write 5^{-3} as a fraction. [1]

To write 5^{-3} as a fraction, we simply take the reciprocal of 5^3. Since $5^3 = 125$, we have: $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$.

(b) Write 0.004 56 in standard form. [1]

$$0.00456 = 4.56 \times 10^{-3}$$

Question 47

Simplify.

$36y^5 \div 4y^2$ [2]

Steps to solve:

1. Cancel the common factor 9y^{3} in the numerator and denominator:

$\frac{36y^5}{4y^2} = \frac{9y^3 \cdot 4y^2}{9y^3 \cdot \frac{4}{y^2}} = 9y^3 \cdot \frac{y^2}{\frac{4}{y^2}} = 9y^3 \cdot y^2 = 9y^{3+2} = 9y^5$

Answer:

$9y^5$

Question 48

Simplify $(16p^{16})^{\frac{1}{4}}$. [2]

To simplify $(16p^{16})^{\frac{1}{4}}$, we can start by simplifying 16 as 2^4. This gives us:

$(2^4p^{16})^{\frac{1}{4}}$

Using the power of a power rule, $(a^m)^n = a^{m \cdot n}$, we can simplify the expression to:

$2^{4 \cdot \frac{1}{4}} \cdot p^{16 \cdot \frac{1}{4}}$

Simplifying the exponents, we get:

$2 \cdot p^4$

Question 49

Simplify.

(a) $x^3y^4 \times x^5y^3$ [2]

Steps to solve:

1. Combine exponents:

$x^{3+5}y^{4+3} = x^8y^7$

Answer:

x^8y^7

(b) $(3p^2m^5)^3$ [2]

To simplify the expression $(3p^2m^5)^3$, you can use the property $(a^b)^c = a^{bc}$. Applying this property to the given expression:

$(3p^2m^5)^3 = 3^3 \cdot (p^2)^3 \cdot (m^5)^3$.

Now, simplify each part separately:

$3^3 = 27$

$(p^2)^3 = p^{2 \cdot 3} = p^6$

$(m^5)^3 = m^{5 \cdot 3} = m^{15}$

Now combine these results:

$27 \cdot p^6 \cdot m^{15}$

So, the simplified expression is $27p^6m^{15}$.

Question 50

Simplify.

$$\left(\frac{x^{64}}{16y^{16}}\right)^{\frac{1}{4}}$$

[3]

To simplify $\left(\frac{x^{64}}{16y^{16}}\right)^{\frac{1}{4}}$, we can first simplify the fraction in the parentheses. We can factor out a 4^2 from the numerator and a 4^2 from the denominator, which gives us:

$\left(\frac{4^2x^{64}}{4^2y^{16}}\right)^{\frac{1}{4}}$

Simplifying further, we get $\left(\frac{x^{64}}{y^{16}}\right)^{\frac{1}{4}}$.

Now, we can apply the power of a power rule, $(a^m)^n = a^{m \cdot n}$, to simplify the expression:

$(x^{64})^{\frac{1}{4}}(y^{16})^{-\frac{1}{4}}$

Simplifying the exponents, we get $x^{16}y^{-4}$.

Finally, we can simplify the expression by writing y^{-4} as $\frac{1}{y^4}$. This gives us:

$x^{16} \cdot \frac{1}{y^4} = \frac{x^{16}}{y^4}$

Therefore, the simplified form of the expression is $\frac{x^{16}}{y^4}$.

Question 51

Simplify.

$6uw^{-3} \times 4uw^{6}$

[2]

Steps to solve:

1. Multiply the numbers:

$24uw^{-3}uw^{6}$

2. Combine exponents:

$24u^2w^{-3}w^{6}$

3. Combine exponents:

$24u^2w^3$

Answer:

$24u^2w^3$

Question 52

$81^x = 3$

Find the value of x. [1]

Steps to solve:

1. Take the logarithm of both sides:

$x = \log_{81}(3)$

2. Apply the change of base rule:

$x = \frac{\log_3(3)}{\log_3(81)}$

3. Compute the logarithm of two numbers:

$x = \frac{1}{\log_3(81)}$

4. Compute the logarithm of two numbers:

$x = \frac{1}{4}$

Answer:

$x = \frac{1}{4}$

Question 53

Simplify.

(a) $12x^{12} \div 3x^3$ [2]

Steps to solve:

1. Cancel the common factor 4x^{9} :

$\frac{12x^{12}}{3x^3} = \frac{4 \cdot 3x^9}{3 \cdot x^3} = 4x^9$

Answer:

$4x^9$

(b) $(256y^{256})^{\frac{1}{8}}$ [2]

To simplify $(256y^{256})^{\frac{1}{8}}$, we can first simplify 256 to 2^8. This gives us:

$(2^8 y^{256})^{\frac{1}{8}}$

Now, we can apply the power of a power rule, $(a^m)^n = a^{m \cdot n}$, to simplify the expression:

$(2^8)^{\frac{1}{8}} (y^{256})^{\frac{1}{8}}$

Simplifying the exponents, we get $2 \cdot y^{32}$.

Therefore, the simplified form of the expression is $2y^{32}$.

Question 54

(a) Simplify

(i) x^0,

Steps to solve:

1. Any nonzero number raised to the 0 power is 1 :

$x^0 = 1$

Answer:

1

[1]

(ii) $m^4 \times m^3$,

Steps to solve:

1. Combine exponents:

$m^4 \times m^3 = m^{4+3} = m^7$

Answer:

m^7

(iii) $(8p^6)^{\frac{1}{3}}$

Steps to solve:

1. Simplify the expression inside the parentheses:

$8p^6 = 2^3 \cdot (p^3)^2 = 2^3 \cdot p^6$

2. Apply the power of a power rule:

$(2^3 \cdot p^6)^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} \cdot (p^6)^{\frac{1}{3}}$

3. Simplify the exponents:

$2 \cdot p^2$

Answer:

$2p^2$

[2]

(b) $243^x = 3^2$ Find the value of x.

Steps to solve:

1. Evaluate the exponent:

$243^x = 9$

2. Take logarithm of both sides:

$x = \log_{243}(9)$

3. Apply the logarithm change of base rule:

$x = \frac{\log_3(9)}{\log_3(243)}$

4. Compute the logarithm of two numbers:

$x = \frac{2}{\log_3(243)}$

5. Compute the logarithm of two numbers:

$x = \frac{2}{5}$

Answer:

$x = \frac{2}{5}$

[2]