EXAM PAPERS PRACTICE

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Detailed mark scheme

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Level: IB Maths AA HL
Subject: Maths
Topic: AA HL Maths
Type: Mark Scheme

Maths AA HL IB
To be used for all exam preparation for 2025+

MATHS IB

This to be used by all students studying AA HL IB Maths
But students of other boards may find it useful

Mark Scheme

Answer 1.

Find the coefficient of the term in x^3 in the expansion of $(2-x)^8$.

[3]

$n = 8, \quad a = 2, \quad b = -x$

Substitute values into the formula for the binomial theorem:

$(a+b)^n = a^n + \ldots + {}^nC_r a^{n-r} b^r + \ldots + b^n$

where ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

$(2-x)^8 = \sum_{r=0}^{8} {}^8C_r (2)^{8-r} (-x)^r$ ← Coefficient of x^3 occurs when $r = 3$.

$r = 3$ gives ${}^8C_3 \times 2^{8-3} (-x)^3$

Non-calculator, so work out ${}^nC_r$ separately:

${}^8C_3 = \dfrac{8!}{3!(8-3)!} = \dfrac{8 \times 7 \times 6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}}{(3 \times 2)(\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2})}$

$= \dfrac{8 \times 7 \times \cancel{6}}{\cancel{6}} = 56$

so the term when $r = 3$ is $56 \times 2^5 \times (-x)^3$

$= 56 \times -32x^3$

$= -1792x^3$

$\boxed{\text{Coefficient of } x^3 = -1792}$

Answer 2.

Find the first three terms, in ascending powers of x, in the expansion of $(3+x)^4$.

[3]

$a = 3, \quad b = x, \quad n = 4$

Substitute values into the formula for $(a+b)^n$

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + \ldots + {}^nC_r a^{n-r} b^r + \ldots + b^n$

where ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

Question asks for ascending powers of x, so start with the term in x^0.

$(3+x)^4 = \underbrace{3^4}_{\text{constant term}} + \underbrace{{}^4C_1 (3)^{4-1} (x)^1}_{\text{term in } x} + \underbrace{{}^4C_2 (3)^{4-2} (x)^2}_{\text{term in } x^2} + \ldots$

$\approx 81 + \dfrac{4!}{3!} \times 3^3 \times x + \dfrac{4!}{2!2!} \times 3^2 \times x^2$

$\approx 81 + 4 \times 27x + 6 \times 9x^2$

$\approx 81 + 108x + 54x^2$

$\boxed{(3+x)^4 \approx 81 + 108x + 54x^2}$

Answer 3.

In the expansion of $(a-x)^4$, the coefficient of the x^2 term is 96.
Given that $a > 0$, find the value of a.

[4]

$a = a, \quad b = -x, \quad n = 4$

Substitute values into the formula for $(a+b)^n$

$$(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

$$(a-x)^4 = \sum_{r=0}^{4} {}^4C_r\, a^{4-r} (-x)^r$$

given the coefficient of the term in x^2, so evaluate the term when $r = 2$.

Term in x^2 $= {}^4C_2 (a)^{4-2} (-x)^2 = 96x^2$

$$6a^2(-x)^2 = 96x^2$$

coefficient of x^2

$$6a^2 = 96$$

$$a^2 = 16$$

$$a = \pm 4$$

It is given in the question that $a > 0$ $\Rightarrow a = 4$

$$\boxed{a = 4}$$

Answer 4.

Find the first three terms, in ascending powers of x, in the expansion of $(9-2x)^5$.

[3]

$a = 9 \quad b = -2x \quad n = 5$

Substitute values into the formula for $(a+b)^n$

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + \ldots + {}^nC_r a^{n-r} b^r + \ldots + b^n$$

Question asks for ascending powers of x, so start with the constant term, a^n.

$$(9-2x)^5 = 9^5 + {}^5C_1 (9)^{5-1}(-2x) + {}^5C_2 (9)^{5-2}(-2x)^2 + \ldots$$

$$\approx 59049 + 5 \times 6561 \times -2x + 10 \times 729 \times 4x^2$$

$$\approx 59049 - 65610x + 29160x^2$$

$$\boxed{(9-2x)^5 \approx 59049 - 65610x + 29160x^2}$$

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Answer 5.

In the expansion of $(a-2x)^5$, the coefficient of the x^2 term is equal to the coefficient of the x^3 term. Find the value of a.

[4]

$a = a,\quad b = -2x,\quad n = 5$

Substitute values into the formula for $(a+b)^n$

$$(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

$$(a-2x)^5 = \sum_{r=0}^{5} {}^5C_r\, a^{5-r}(-2x)^r$$

The coefficients of the terms in x^2 and x^3 are equal, so evaluate the terms when $r=2$ and $r=3$.

In x^2 term, $r=2$:

$${}^5C_2\, a^{5-2}(-2x)^2$$
$$= 10a^3 \times 4x^2$$
$$= 40a^3x^2$$

coefficient of x^2 term

In x^3 term, $r=3$:

$${}^5C_3\, a^{5-3}(-2x)^3$$
$$= 10a^2 \times -8x^3$$
$$= -80a^2x^3$$

coefficient of x^3 term

Equating coefficients: $40a^3 = -80a^2$

$$a = \frac{-80}{40} = -2$$

$$\boxed{a = -2}$$

Answer 6.

In the expansion of $(3+px)^6$, the coefficient of the x^4 term is four times the coefficient of the x^2 term. Find the possible values of p.

[3]

$a = 3\quad b = px\quad n = 6$

$$(3+px)^6 = \sum_{r=0}^{6} {}^6C_r (3)^{6-r}(px)^r$$

evaluate the terms when $r=2$ and $r=4$.

In x^2 term, $r=2$:

$${}^6C_2 (3)^{6-2}(px)^2$$
$$= 15 \times 81 \times p^2x^2$$
$$= 1215p^2x^2$$

coefficient of x^2

In x^4 term, $r=4$:

$${}^6C_4 (3)^{6-4}(px)^4$$
$$= 15 \times 9 \times p^4x^4$$
$$= 135p^4x^4$$

coefficient of x^4

coefficient of x^4 = 4(coefficient of x^2)

$$135p^4 = 4(1215p^2)$$
$$135p^4 = 4860p^2$$
$$p^2 = \frac{4860}{135} = 36$$
$$\Rightarrow \quad p = \sqrt{36} = \pm 6$$

$$\boxed{p = 6 \text{ or } -6}$$

Answer 7.

Consider the expansion of $(4ax-3)^5$.

(a) Write down the number of terms in this expansion. [1]

(b) The coefficient of the term in x^4 is -61440.
Find the value of a where a is a positive constant. [4]

(a) A binomial expansion has $n+1$ terms.

$n=5 \Rightarrow n+1=6$

$\boxed{(4ax-3)^5 \text{ has } 6 \text{ terms}}$

Consider the expansion of $(4ax-3)^5$.

(a) Write down the number of terms in this expansion. [1]

(b) The coefficient of the term in x^4 is -61440.
Find the value of a where a is a positive constant. [4]

(b) $a=4ax$, $b=-3$, $n=5$

$(4ax-3)^5 = \sum_{r=0}^{5} {}^5C_r (4ax)^{5-r}(-3)^r$

we have been given the coefficient of the term in x^4, so evaluate the term when $r=1$.

Term in $x^4 = -61440x^4$

$\Rightarrow {}^5C_1(4ax)^{5-1}(-3)^1 = -61440x^4$

$5 \times -3 \times 4^4 \times a^4 \times x^4 = -61440x^4$

$-3840a^4 = -61440$

$a^4 = \frac{-61440}{-3840} = 16$

$a = \sqrt[4]{16} = \pm 2$

It is given in the question that a is a positive constant.

$\boxed{a = 2}$

Answer 8.

Consider the expansion of $(x^3 + \frac{4}{x})^4$.

(a) Write the first three terms in descending powers of x. [3]

(b) Find the value of the constant term. [3]

(a) $a = x^3, \quad b = \frac{4}{x}, \quad n = 4$

$$(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

$$(x^3 + \frac{4}{x})^4 = \sum_{r=0}^{4} {}^4C_r\, (x^3)^{4-r} \left(\frac{4}{x}\right)^r$$

The numerator has the greater power of x, so start with $r=0$ as the first term

$$= \sum_{r=0}^{4} {}^4C_r \left(\frac{4^r x^{3(4-r)}}{x^r}\right)$$

$$(x^3 + \frac{4}{x})^4 = {}^4C_0\left(\frac{4^0 x^{3(4-0)}}{x^0}\right) + {}^4C_1\left(\frac{4^1 x^{3(4-1)}}{x^1}\right) + {}^4C_2\left(\frac{4^2 x^{3(4-2)}}{x^2}\right) + \dots$$

$$\approx \frac{1 \times 1x^{12}}{1} + 4 \times \frac{4x^9}{x} + 6 \times \frac{16x^6}{x^2}$$

$$\approx x^{12} + 16x^8 + 96x^4$$

$$\boxed{(x^3 + \frac{4}{x})^4 \approx x^{12} + 16x^8 + 96x^4}$$

Consider the expansion of $(x^3 + \frac{4}{x})^4$.

(a) Write the first three terms in descending powers of x. [3]

(b) Find the value of the constant term. [3]

(b) $(x^3 + \frac{4}{x})^4 = \sum_{r=0}^{4} {}^4C_r\, (x^3)^{4-r} \left(\frac{4}{x}\right)^r$

$$= \sum_{r=0}^{4} {}^4C_r \left(\frac{4^r x^{3(4-r)}}{x^r}\right)$$

The constant term is the term in x^0, so we need r such that $3(4-r) - r = 0$

$$3(4-r) - r = 0$$
$$12 - 4r = 0$$
$$r = 3$$

$r = 3$ gives ${}^4C_3\left(\frac{4^3 x^{3(4-3)}}{x^3}\right) = 4 \times 4^3 \times 1 = 256$

$$\boxed{\text{constant term} = 256}$$

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Answer 9.

The coefficient of x^7 in the expansion of $x^3(ax+3)^5$ is 1215.
Find the possible values of a.

[4]

First expand $(ax+3)^5$, rearranging this to $(3+ax)^5$ makes it easier to spot the correct term to use.

$$(3+ax)^5 = \sum_{r=0}^{5} {}^5C_r \, (3)^{5-r} (ax)^r$$

The question gives the term in x^7, so here we are looking for the term in x^4. Evaluate for $r=4$.

The term when $r=4$:

$$(3+ax)^5 = \ldots + {}^5C_4 (3)^{5-4} (ax)^4 + \ldots$$

$$= \ldots + 15a^4x^4 + \ldots$$

The coefficient of x^7 in the expansion $x^3(ax+3)^5 = 1215$, therefore:

$$x^3(15a^4)x^4 = 1215x^7$$

$$15a^4 = 1215$$

$$a^4 = 81$$

$$a = \pm\sqrt[4]{81}$$

$$= \pm 3$$

$$\boxed{a = 3 \text{ or } -3}$$

Answer 10.

Consider the binomial expansion of $\frac{1}{1+x}$.

(a) Write down the first four terms.

[4]

(b) Find the values of x such that the complete expansion converges.

[2]

(c) Use the terms found in part (a) to estimate $\frac{1}{1.1}$.

[2]

(a) By the binomial theorem, for any value of n:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

Rewrite $\frac{1}{1+x}$ in the form $(1+x)^n$

$$\frac{1}{1+x} = (1+x)^{-1} \leftarrow n$$

Substitute values into the formula for $(1+x)^n$

$$(1+x)^{-1} = 1 + (-1)(x) + \frac{(-1)(-2)}{2!}(x)^2 + \frac{(-1)(-2)(-3)}{3!}(x)^3 + \ldots$$

$$= 1 - x + \frac{2}{2}x^2 + \frac{(-6)}{6}x^3 + \ldots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$$

$$\boxed{\text{First four terms: } 1 - x + x^2 - x^3}$$

Consider the binomial expansion of $\frac{1}{1+x}$.

(a) Write down the first four terms. [4]

First four terms : $1 - x + x^2 - x^3$

(b) Find the values of x such that the complete expansion converges. [2]

(c) Use the terms found in part (a) to estimate $\frac{1}{1.1}$. [2]

(c) Find the value of x for which $\frac{1}{1+x} = \frac{1}{1.1}$

$$\frac{1}{1+x} = \frac{1}{1.1}$$

$$1 + x = 1.1$$

$$x = 0.1$$

Substitute $x = 0.1$ into the expansion for $(1+x)^{-1}$

$$(1+(0.1))^{-1} = 1 - 0.1 + (0.1)^2 - (0.1)^3 + \ldots$$

$$= 1 - 0.1 + 0.01 - 0.001 + \ldots$$

$$\boxed{\frac{1}{1.1} \approx 0.909}$$

Answer 11.

Consider the binomial expansion of $\sqrt[3]{4(2+x)}$.

(a) Write down the first three terms. [4]

(b) State the interval of convergence for the complete expansion. [2]

(c) Use the terms found in part (a) to estimate $\sqrt[3]{12}$. Give your answer as a fraction. [2]

By the binomial theorem, for any value of n:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

(a) Rewrite $\sqrt[3]{4(2+x)}$ in the form $k(1 + \frac{x}{a})^n$

$$\sqrt[3]{4(2+x)} = 4^{\frac{1}{3}}(2+x)^{\frac{1}{3}}$$

$$= 4^{\frac{1}{3}}\left[2^{\frac{1}{3}}\left(1 + \frac{x}{2}\right)^{\frac{1}{3}}\right]$$

Spot that $4^{\frac{1}{3}} = (2^2)^{\frac{1}{3}} = 2^{\frac{2}{3}}$

$$= 2^{\frac{2}{3}}\left[2^{\frac{1}{3}}\left(1 + \frac{x}{2}\right)^{\frac{1}{3}}\right]$$

$$= 2\left(1 + \frac{x}{2}\right)^{\frac{1}{3}}$$

Substitute values into the formula for $(1+x)^n$

$$2\left(1 + \frac{x}{2}\right)^{\frac{1}{3}} = 2\left[1 + \left(\frac{1}{3}\right)\left(\frac{x}{2}\right) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(\frac{x}{2}\right)^2 + \ldots\right]$$

$$= 2\left(1 + \frac{x}{6} + \frac{-\frac{2}{9}}{2}\left(\frac{x^2}{4}\right) + \ldots\right)$$

$$= 2 + \frac{x}{3} - \frac{2}{9}\left(\frac{x^2}{4}\right) + \ldots$$

$$= 2 + \frac{x}{3} - \frac{x^2}{18} + \ldots$$

$$\boxed{\sqrt[3]{4(2+x)} \approx 2 + \frac{x}{3} - \frac{x^2}{18}}$$

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Consider the binomial expansion of $\sqrt[3]{4(2+x)}$.

(a) Write down the first three terms.

[4]

(b) State the interval of convergence for the complete expansion.

[2]

(c) Use the terms found in part (a) to estimate $\sqrt[3]{12}$. Give your answer as a fraction.

[2]

(b) $n \geq 0$ and $n \notin \mathbb{N}$, so the series converges when $|x| < 1$

$$2\left(1+\frac{x}{2}\right)^{\frac{1}{3}}$$

x term

$$\left|\frac{x}{2}\right| < 1$$

$$|x| < 2$$

$$-2 < x < 2$$

$$\boxed{\text{Converges for } -2 < x < 2}$$

Consider the binomial expansion of $\sqrt[3]{4(2+x)}$.

(a) Write down the first three terms.

$$\boxed{\sqrt[3]{4(2+x)} \approx 2 + \frac{x}{3} - \frac{x^2}{18}}$$

[4]

(b) State the interval of convergence for the complete expansion.

$$\boxed{-2 < x < 2}$$

[2]

(c) Use the terms found in part (a) to estimate $\sqrt[3]{12}$. Give your answer as a fraction.

[2]

(c) Find the value of x for which $\sqrt[3]{4(2+x)} = \sqrt[3]{12}$

$$\sqrt[3]{4(2+x)} = \sqrt[3]{12}$$

$$4(2+x) = 12$$

$$2+x = 3$$

$$x = 1$$

$-2 < x < 2$ so can use the expansion

Substitute $x = 1$ into the expansion for $\sqrt[3]{4(2+x)}$

$$\sqrt[3]{4(2+x)} \approx 2 + \frac{x}{3} - \frac{x^2}{18}$$

$$\sqrt[3]{4(2+1)} \approx 2 + \frac{1}{3} - \frac{1^2}{18}$$

$$= 2\tfrac{1}{3} - \tfrac{1}{18}$$

$$= 2\tfrac{6}{18} - \tfrac{1}{18}$$

$$\boxed{\sqrt[3]{12} \approx 2\tfrac{5}{18}}$$

Answer 12.

Consider the binomial expansion of $\frac{1}{\sqrt{4+x}}$.

(a) Write down the first four terms. [4]

(b) State the interval of convergence for the complete expansion. [2]

(a) Rewrite $\frac{1}{\sqrt{4+x}}$ in the form $k(1+\frac{x}{a})^n$

$$\frac{1}{\sqrt{4+x}} = \frac{1}{(4+x)^{\frac{1}{2}}} = (4+x)^{-\frac{1}{2}}$$

$$4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \frac{1}{2} \rightarrow = 4^{-\frac{1}{2}}(1+\frac{x}{4})^{-\frac{1}{2}}$$

$$= \frac{1}{2}(1+\frac{x}{4})^{-\frac{1}{2}}$$

Substitute values into the formula for $(1+x)^n$

By the binomial theorem, for any value of n:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

$$\frac{1}{2}(1+\frac{x}{4})^{-\frac{1}{2}} \approx \frac{1}{2}\left[1 + (-\frac{1}{2})(\frac{x}{4}) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(\frac{x}{4})^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(\frac{x}{4})^3\right]$$

$$= \frac{1}{2}\left[1 + (-\frac{x}{8}) + \frac{(\frac{3}{4})}{2}(\frac{x^2}{16}) + \frac{(-\frac{15}{8})}{6}(\frac{x^3}{64})\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{8}x + \frac{3}{128}x^2 - \frac{5}{1024}x^3\right]$$

$$\frac{1}{2}(1+\frac{x}{4})^{-\frac{1}{2}} = \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \ldots$$

$$\boxed{\frac{1}{\sqrt{4+x}} \approx \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3}$$

Consider the binomial expansion of $\frac{1}{\sqrt{4+x}}$.

(a) Write down the first four terms. [4]

(b) State the interval of convergence for the complete expansion. [2]

(b)

$$\left|\frac{x}{4}\right| < 1$$

$$|x| < 4$$

$$\boxed{-4 < x < 4}$$