

Boost your performance and confidence with these topic-based exam questions

Practice questions created by actual examiners and assessment experts

Detailed mark scheme

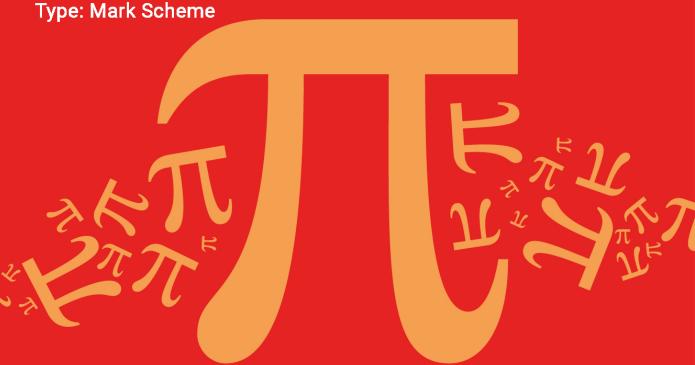
Suitable for all boards

Designed to test your ability and thoroughly prepare you

Level: IB Maths AA HL

Subject: Maths

Topic: AA HL Maths
Type: Mark Scheme



Maths AA HL IB
To be used for all exam preparation for 2025+

MATHS

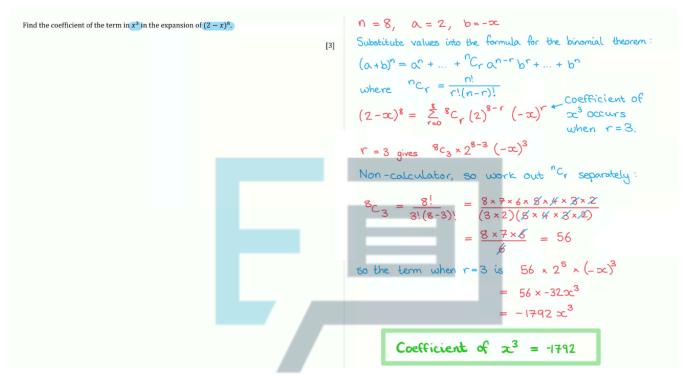
IB

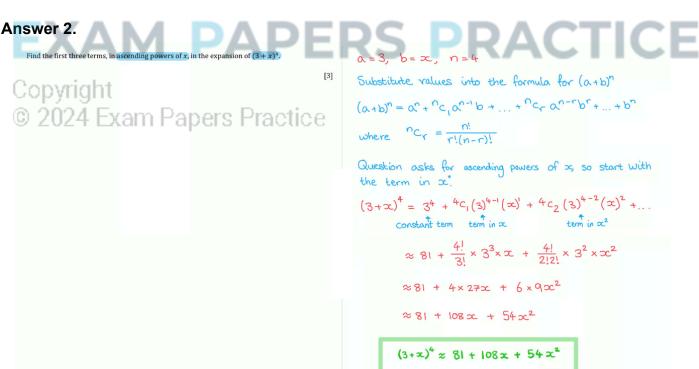
This to be used by all students studying AA HL IB Maths But students of other boards may find it useful



Mark Scheme

Answer 1.







Answer 3.

In the expansion of $(a-x)^4$, the coefficient of the x^2 term is 96. Given that a>0, find the value of a.

a=a, $b=-\infty$, n=4

Substitute values into the formula for (a+b)"

$$(a+b)_{u} = \sum_{v=0}^{c} C^{v} \sigma_{v-v} \rho_{v}$$

 $(\alpha - \alpha)^4 = \sum_{r=0}^4 4c_r \alpha^{4-r} (-\alpha)^r$ given the coefficient of the term in α^* , so evaluate the term when r=2.

Term in
$$x^2 = 4c_2(\alpha)^{4-2}(-x)^2 = 96x^2$$

$$6\alpha^{2}(-x)^{2} = 96x^{2}$$

$$6a^2 = 96$$

$$\alpha = \pm 4$$

It is given in the question that
$$a>0 \Rightarrow a=4$$

Answer 4.

Find the first three terms, in ascending powers of x, in the expansion of $(9-2x)^5$.

Copyright
© 2024 Exam Papers Practice

a = 9 b = -2x n = 5Substitute values into the formula $\log (a + b)^n$

Substitute values into the formula for $(a+b)^n$ $(a+b)^n = a^n + {n \choose 1} a^{n-1}b + ... + {n \choose r} a^{n-r}b^r + ... + b^n$

Question asks for ascending powers of ∞ so start with the constant term, a^n .

 $\left(9 - 2 \infty \right)^5 = 9^5 + 5 c_1 \left(9 \right)^{5-1} \left(-2 \infty \right) + 5 c_2 \left(9 \right)^{5-2} \left(-2 \infty \right)^2 + \dots$

 $\approx 59049 + 5 \times 6561 \times -2 \times + 10 \times 729 \times 4 \times^{2}$

 $\approx 59049 - 65610 \times + 29160 \times^{2}$

 $(9-2x)^5 \approx 59049 - 65610x + 29160x^2$



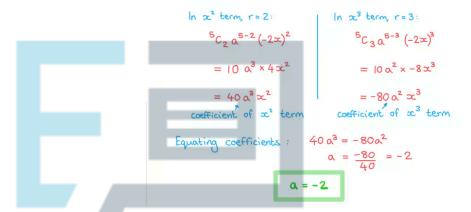
Answer 5.

In the expansion of $(a-2x)^5$, the coefficient of the x^2 term is equal to the coefficient of the x^3 term. Find the value of a.

a = a, $b = -2\infty$, n = 5Substitute values into the for

Substitute values into the formula for $(a+b)^n$ $(a+b)^n = \sum_{r=0}^n {}^n C_r a^{n-r} b^r$

 $(\alpha - 2x)^5 = \sum_{r=0}^{5} {}^5C_r \alpha^{5-r} (-2x)^r$ The coefficients of the terms in x^* and x^3 are equal, so evaluate the terms when r=2 and r=3



Answer 6.

In the expansion of $(3+px)^6$, the coefficient of the x^4 term is four times the coefficient of the x^2 term. Find the possible values of p.

Copyright

© 2024 Exam Papers Practice

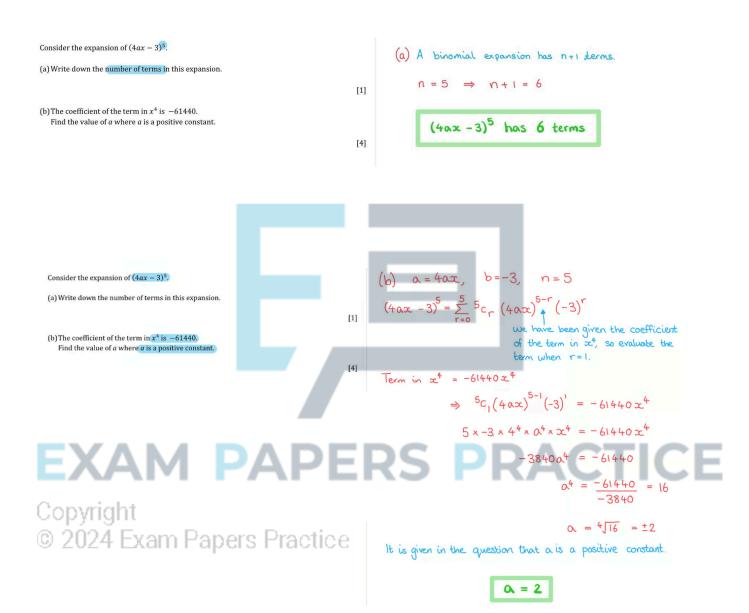
a = 3 b = px n = 6 $(3 + px)^6 = \sum_{r=0}^{6} {}^{6}C_{r}(3)^{6-r}(px)^{r} \text{ evaluate the terms when } r=2$ and r=4.

In x^2 term, r=2: ${}^{6}C_{2}(3)^{6-2}(px)^2$ $= 15 \times 81 \times p^2x^2$ In x^4 term, r=4: ${}^{6}C_{4}(3)^{6-4}(px)^4$ $= 15 \times 9 \times p^4x^4$

= $|215 p^2 \propto^2$ = $|35 p^4 \propto^4$ coefficient of x^2 coefficient of x^4 = $4(\text{coefficient of } x^2)$ $|35 p^4 = 4(1215 p^2)$ $|35 p^4 = 4860 p^2$ $|36 p^4 = 4860 p^2$ $|36 p^4 = 4860 p^2$



Answer 7.





[3]

[3]

Answer 8.

Consider the expansion of $(x^3 + \frac{4}{x})^4$.

(a) Write the first three terms in descending powers of x.

(b) Find the value of the constant term.

(a)
$$a = x^3$$
, $b = \frac{4}{x}$, $n = 4$
 $(a + b)^n = \sum_{r=0}^{n} {n \choose r} a^{n-r} b^r$

$$(x^3 + \frac{4}{x})^4 = \sum_{r=0}^{4} {}^{4}C_r (x^3)^{4-r} (\frac{4}{2c})^r$$
 The numerator has the greater power of x , so start with
$$= \sum_{r=0}^{4} {}^{4}C_r (\frac{4^r x^{3(4-r)}}{x^r})$$
 $r=0$ as the first term

$$(x^{3} + \frac{4}{x})^{4} \approx {}^{4}c_{0}\left(\frac{4^{0}x^{3(4-0)}}{x^{0}}\right) + {}^{4}c_{1}\left(\frac{4^{1}x^{3(4-1)}}{x^{1}}\right) + {}^{4}c_{2}\left(\frac{4^{2}x^{3(4-2)}}{x^{2}}\right) + \dots$$

$$\approx \frac{1 \times 1x^{12}}{1} + 4 \times \frac{4x^{9}}{x} + 6 \times \frac{16x^{6}}{x^{2}}$$

$$\approx x^{12} + 16x^{8} + 96x^{4}$$

$$(x^{3} + \frac{4}{x})^{4} \approx x^{12} + 16x^{8} + 96x^{4}$$

Consider the expansion of $(x^3 + \frac{4}{x})^4$. $(b) \qquad (x^3 + \frac{4}{x})^4 = \sum_{r=0}^4 C_r (x^3)^{4r-r} (\frac{4}{3c})^r$

(a) Write the first three terms in descending powers of x

(b) Find the value of the constant term.

C _ Z _ Z A _ Exam Papers Practice [3]

$$=\sum_{r=0}^{4} {}^{4}C_{r} \left(\frac{4^{r}x^{3(4-r)}}{x^{r}}\right)$$

The constant term is the term in ∞° , so we need r such that 3(4-r)-r=0

$$3(4-r)-r = 0$$
 $12-4r = 0$

$$r = 8$$
 gives ${}^{4}C_{3}\left(\frac{4^{3} \times 3^{(4-3)}}{x^{3}}\right) = 4 \times 4^{3} \times 1$

constant term = 266



Answer 9.

The coefficient of x^7 in the expansion of $x^3(ax + 3)^5$ is 1215.

Find the possible values of a.

First expand $(ax+3)^5$, rearranging this to $(3+ax)^5$ makes it easier to spot the correct term to use. $(3 + \alpha x)^5 = \sum_{r=0}^{5} {}^5C_r (3)^{5-r} (\alpha x)^r$ The question gives the term in x^3 , so here we are booking for the term The term when r=4:

 $(3+ax)^5 = ... + 5_{C_4}(3)^{5-4}(ax)^4 + ...$ = ... + 1504 x4 +...

The coefficient of x^2 in the expansion $x^3(ax+3)^5 = 1215$,

$$x^{3} (16\alpha^{4}) x^{4} = 1216x^{7}$$

$$15\alpha^{4} = 1215$$

$$\alpha^{4} = 81$$

$$\alpha = \pm \sqrt{81}$$

$$= \pm 3$$

$$\alpha = 3 \text{ or } -3$$

Answer 10.

Consider the binomial expansion of $\frac{1}{1+x}$ (a) Write down the first four terms. [4] (b) Find the values of x such that the complete expansion converges.

© 2024 Exam Papers Practice 🛭

(c) Use the terms found in part (a) to estimate $\frac{1}{11}$.

Rewrite $\frac{1}{1+\infty}$ in the form $(1+\infty)^n$ $\frac{1}{1+\infty} = (1+\infty)^{-1}$

Substitute values into the formula for (1+x)"

 $(1+x)^{-1} = 1 + (-1)(x) + \frac{(-1)(-2)}{2!}(x)^2 + \frac{(-1)(-2)(-3)}{3!}(x)^3 + ...$ $= 1 - x + \frac{2}{2}x^2 + \frac{(-6)}{6}x^3 + \dots$

 $\frac{1}{1+\infty} = 1 - \infty + \infty^2 - \infty^3 + \dots$

First four terms: $1-x+x^2-x^3$

[2]



Consider the binomial expansion of $\frac{1}{1+x}$.

(a) Write down the first four terms.

First four terms:
$$1-x+x^2-x^3$$

(b) Find the values of x such that the complete expansion converges.

(c) Use the terms found in part (a) to estimate $\frac{1}{1.1}$.

(c) find the value of
$$\infty$$
 for which $\frac{1}{1+\infty} = \frac{1}{1.1}$

$$\frac{1}{1+\infty} = \frac{1}{1.1}$$

$$1+\infty = 1.1$$

$$\infty = 0.1$$

Substitute x = 0.1 into the expansion for $(1+x)^{-1}$

[2]
$$(1 + (0.1))^{-1} = 1 - 0.1 + (0.1)^{2} - (0.1)^{3} + \dots$$
$$= 1 - 0.1 + 0.01 - 0.001 + \dots$$

[2]

Answer 11.

Consider the binomial expansion of $\sqrt[3]{4(2+x)}$.

(a) Write down the first three terms.

[4]

(b) State the interval of convergence for the complete expansion.

(c) Use the terms found in part (a) to estimate $\sqrt[3]{12}$. Give your answer as a fraction.

© 2024 Exam Papers Practice 🛚

By the binomial theorem, for any value of n:

$$(1+x)^{n} = 1 + nx + \frac{n(n-1)}{2!}x^{2} + \frac{n(n-1)(n-2)}{3!}x^{3} + \dots$$

(a) Rewrite $\sqrt[3]{4(2+\infty)}$ in the form $k(1+\frac{\infty}{a})^n$

$$\frac{3\sqrt{4(2+\alpha)}}{4(2+\alpha)} = 4^{\frac{1}{3}}(2+\alpha)^{\frac{1}{3}}$$
Spot that $4^{\frac{1}{3}} = (2^{\frac{1}{3}})^{\frac{1}{3}} = 2^{\frac{2}{3}}$

$$= 2^{\frac{2}{3}}\left[2^{\frac{1}{3}}(1+\frac{\alpha}{2})^{\frac{1}{3}}\right]$$

$$= 2(1+\frac{\alpha}{2})^{\frac{1}{3}}$$

Substitute values into the formula for $(1+x)^2$

$$2(1 + \frac{x}{2})^{\frac{1}{3}} = 2\left[1 + \left(\frac{1}{3}\right)\left(\frac{x}{2}\right) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(\frac{x}{2}\right)^{2} + \dots\right]$$

$$= 2\left(1 + \frac{x}{6} + \frac{-\frac{2}{9}}{2}\left(\frac{x^{2}}{4}\right) + \dots\right)$$

$$= 2 + \frac{x}{3} - \frac{2}{9}\left(\frac{x^{2}}{4}\right) + \dots$$

$$= 2 + \frac{x}{3} - \frac{x^{2}}{18} + \dots$$

$$\sqrt[3]{4(2+\infty)} \approx 2 + \frac{\infty}{3} - \frac{\infty^2}{18}$$



[4]

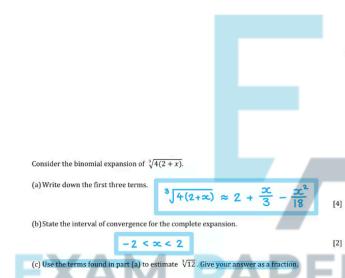
[2]

[2]

Consider the binomial expansion of $\sqrt[3]{4(2+x)}$. (a) Write down the first three terms. (b) State the interval of convergence for the complete expansion. (c) Use the terms found in part (a) to estimate $\sqrt[3]{12}$. Give your answer as a fraction.

(b) n≥0 and n € IN, so the series converges when loc/<1 $2(1+\frac{x}{2})^{\frac{1}{3}}$ x term 2 < 1 |x| < 2 $-2 < \infty < 2$

Converges for -2 < x < 2



Copyright

(c) Find the value of ∞ for which $\sqrt[3]{4(2+\infty)} = \sqrt[3]{12}$ $3\sqrt{4(2+\infty)} = 3\sqrt{12}$ 4(2+x) = 12

© 2024 Exam Papers Practice

 $\sqrt[3]{4(2+\infty)} \approx 2 + \frac{\infty}{3} - \frac{\infty^2}{18}$ $\sqrt[3]{4(2+1)} \approx 2 + \frac{1}{3} - \frac{1^2}{18}$ $= 2\frac{1}{3} - \frac{1}{18}$ $=2\frac{6}{18}-\frac{1}{18}$ $\sqrt[3]{12} \approx 2\frac{5}{18}$



[4]

[2]

Answer 12.

Consider the binomial expansion of $\frac{1}{\sqrt{4+x}}$.

(a) Write down the first four terms.

(b) State the interval of convergence for the complete expansion.

(a) Rewrite $\frac{1}{\sqrt{4+x}}$ in the form $k(1+\frac{x}{a})^n$ $\frac{1}{\sqrt{4+x}} = \frac{1}{(4+x)^{\frac{1}{2}}} = (4+x)^{-\frac{1}{2}}$ $4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$ $= \frac{1}{2}(1+\frac{x}{4})^{-\frac{1}{2}}$ Substitute values into the formula for $(1+x)^n$

By the binomial theorem, for any value of n: $(1+x)^{n} = 1 + nx + \frac{n(n-1)}{2!}x^{2} + \frac{n(n-1)(n-2)}{3!}x^{3} + ...$ $\frac{1}{2} \left(1 + \frac{\infty}{4}\right)^{-\frac{1}{2}} \approx \frac{1}{2} \left[1 + \left(-\frac{1}{2}\right) \left(\frac{\infty}{4}\right) + \frac{\left(-\frac{1}{2}\right) \left(\frac{3}{2}\right)}{2!} \left(\frac{\infty}{4}\right)^2 + \frac{\left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{\infty}{4}\right)^3}{3!} \right]$ $= \frac{1}{2} \left[1 + \left(-\frac{\infty}{8}\right) + \frac{\left(\frac{3}{4}\right)}{2} \left(\frac{\infty^2}{16}\right) + \frac{\left(-\frac{15}{8}\right)}{6} \left(\frac{\infty^3}{64}\right)\right]$ $= \frac{1}{2} \left[1 - \frac{1}{8} \infty + \frac{3}{128} \infty^2 - \frac{5}{1024} \infty^3\right]$ $\frac{1}{2} \left(1 + \frac{\infty}{4}\right)^{-\frac{1}{2}} = \frac{1}{2} - \frac{1}{16} \infty + \frac{3}{256} \infty^2 - \frac{5}{2048} \infty^3 + \dots$

EXAM PAPERS PRACTICE

Copyright

© 2024 Exam Papers Practice

Consider the binomial expansion of $\frac{1}{\sqrt{4+x}}$.

(a) Write down the first four terms.

(b) State the interval of convergence for the complete expansion.

(P)

[4]

[2]

 $\left|\frac{x}{4}\right| < 1$ $\left|x\right| < 4$

-4< x < 4