IB Mathematics: Analysis and Approaches HL

Paper 1 Mark Scheme — No Calculator

14 May 2026 · Zone A/B/C · Afternoon · 110 marks total

Q1	Quadratic Functions	
Question	**Answer**	**Explanation**
(a)	**q = 6**	The axis of symmetry of f(x)=3(x–2)(x–q) is x=(2+q)/2. Setting (2+q)/2=4 gives 2+q=8, so q=6.
(b)	**Vertex = (4, –12)**	Substitute x=4: f(4)=3(4–2)(4–6)=3(2)(–2)=–12. The vertex is the point (4,–12).
(c)	**f(x) ≥ –12**	Since the parabola opens upward (leading coefficient 3>0) and the vertex is (4,–12), the range is all values at or above the minimum, y ≥ –12.

Q2	Integration & Kinematics	
Question	**Answer**	**Explanation**
(a)	**ln|t+1| – e^(–t) + C**	Integrate term by term: ∫1/(t+1)dt = ln|t+1| and ∫e^(–t)dt = –e^(–t). Constant of integration required.
(b)	**s = ln(t+1) – e^(–t) + 1**	Integrate v to get s = ln(t+1) – e^(–t) + C. Apply initial condition s(0)=0: 0 = ln1 – e0 + C = –1+C, so C=1.

Q3	Geometric Sequences	
Question	**Answer**	**Explanation**
(a)	**r = 2**	r2 = u8/u6 = (6.4×105)/(1.6×105) = 4. Since all terms are positive, r=+2.
(b)	**u1 = 5×103**	Use u6=u1r5: 1.6×105 = u1×32, so u1 = 1.6×105/32 = 5×103. Here a=5, k=3.

Q4	Logarithms	
Question	**Answer**	**Explanation**
(a)	**log25(81x2) = log59 + log5x ✓**	Split using log product rule: log25(81)+log25(x2). Apply change of base (/2): log5(81)/2=log5(92)/2=log59. Similarly log25(x2)=log5x. Sum equals RHS.
(b)	**x = 8/27**	Using (a): (1/3)log5x = log59+log5x–log54. Rearranging: (–2/3)log5x=log5(9/4), so x^(–2/3)=9/4, giving x=(4/9)^(3/2)=8/27.

Q5	Probability	
Question	**Answer**	**Explanation**
(a)	**P(A) = (19–x)/24 ✓**	P(A) = P(A∩B′)+P(A∩B) = (5+x)/24+(7–x)/12. Converting to common denominator 24: (5+x+14–2x)/24 = (19–x)/24.
(b)	**x = 1 or x = 3**	P(B)=(9–x)/12. Independence requires P(A∩B)=P(A)P(B): (7–x)/12=[(19–x)/24]×[(9–x)/12]. This simplifies to x2–4x+3=0, giving x=1 or x=3. Both satisfy 0≤x≤7.

Q6	Circles & Sectors	
Question	**Answer**	**Explanation**
	r = √60	After removing the sector (angle π/3), remaining angle is 5π/3. Shaded area=½(36)(5π/3)=30π. Unshaded=½r2(5π/3)–30π. Setting ratio to 2/3: (5πr2/6–30π)/30π=2/3 gives r2=60, r=√60.

Q7	Conditional Probability	
Question	**Answer**	**Explanation**
(a)	**P(positive) = 0.48**	Total probability: P(+)=P(preg)P(+|preg)+P(not preg)P(+|not preg) = 0.4(0.9)+0.6(0.2) = 0.36+0.12 = 0.48.
(b)	**P(pregnant | +) = 3/4**	By Bayes: P(preg|+) = P(preg∩+)/P(+) = (0.4×0.9)/0.48 = 0.36/0.48 = 0.75.

Q8	Polynomials & Complex Numbers	
Question	**Answer**	**Explanation**
(a)(i)	**a = –1 ✓**	Conjugate pairs: roots are 1/3, a±i, 2±bi. Sum = 7/3 (Vieta's). So 1/3+2a+4=7/3, giving 2a=–2, a=–1.
(a)(ii)	**b = 3**	Product of roots = 26/3 (Vieta's). Product = (1/3)(a2+1)(4+b2) = (1/3)(2)(4+b2) = 26/3, so b2=9 and b=3 (since b>1).
(b)	**p(z) = (3z–1)(z2+2z+2)(z2–4z+13)**	Factor using all 5 roots: 1/3 → (3z–1); roots –1±i → z2+2z+2; roots 2±3i → z2–4z+13. Integers: A=3,B=–1,C=2,D=2,E=–4,F=13.

Q9 Vectors (3D)

Question	Answer	Explanation
(a)	**v · a = 0 ✓**	v·a = (a·c)(b·a)–(a·b)(c·a). Since the dot product is commutative, both terms are equal and the expression equals zero.
(b)(i)	**b·(b×c) = 0 ✓**	The cross product b×c is perpendicular to b by definition, so b·(b×c) = |b||b×c|cos90° = 0.
(b)(ii)	**v·(b×c) = 0 ✓**	Expanding: v·(b×c) = (a·c)[b·(b×c)] – (a·b)[c·(b×c)]. Both bracket terms are zero (cross product perpendicular to each factor), so v·(b×c)=0.
(c)	**v is parallel to a×(b×c)**	v⊥a and v⊥(b×c) from parts (a) and (b). Since a×(b×c) is also perpendicular to both a and (b×c), and both are non-zero, v must be parallel to a×(b×c).

Q10 Cubic Functions & Calculus

Question	Answer	Explanation
(a)	**a = 3/2, b = 9 ✓**	f″(x)=6ax+2b. Inflection at x=–2: –12a+2b=0 → b=6a (eq.1). y-coord condition: –8a+4b=24 → –2a+b=6 (eq.2). Solving: a=3/2, b=9.
(b)(i)	**c = 27/2 ✓**	f′(x)=(9/2)x2+18x+c. Horizontal tangent at x=–3: f′(–3)=81/2–54+c=0, so c=27/2.
(b)(ii)	**k = –1**	f′(x)=0: (9/2)x2+18x+27/2=0 → x2+4x+3=0 → (x+1)(x+3)=0. So x=–1 or x=–3 (known). Thus k=–1.
(b)(iii)	**Local minimum at x=–1**	f″(–1)=9(–1)+18=9>0, so the graph is concave up at x=–1, confirming a local minimum.
(c)	**Tangent passes through P ✓**	P=(0,d) is the y-intercept. Computing f(–3)=(3/2)(–27)+81–(81/2)+d=d. The horizontal tangent y=d at x=–3 passes through (0,d)=P.

Q11 Trigonometric Identities & Equations

Question	Answer	Explanation
(a)	**cot2θ ≡ ½(cotθ–1/cotθ) ✓**	LHS = 1/tan2θ = (1–tan2θ)/(2tanθ) = ½(1/tanθ–tanθ) = ½(cotθ–tanθ) = ½(cotθ–1/cotθ) = RHS.
(b)	**θ = π/4, π/3, 2π/3, 3π/4**	Substitute (a): (t2–1)(3t2–1)=0 where t=cotθ, giving t=±1 or t=±1/√3. These yield tanθ=±1 or ±√3, so θ=π/4, π/3, 2π/3, 3π/4.
(c)	**4cosec22θ ≡ cosec2θ+sec2θ ✓**	Differentiate both sides of (a) w.r.t. θ: –2cosec22θ = ½(–cosec2θ–sec2θ). Multiply by –2 to get 4cosec22θ = cosec2θ+sec2θ.
(d)	**n = 2 (i.e. L:A = 2:1)**	PQ2=4cosec22θ–cosec2θ = sec2θ (using (c)), so PQ=secθ. Area A = ½cosecθ·secθ. Then L/A = 2cosec2θ/(½cosecθ·secθ) = 4sinθcosθ/sin2θ = 2.

Q12	Differential Equations — Rabbit Infection	
Question	**Answer**	**Explanation**
(a)	**½/(N–1) + ½/(3–N)**	Factorise: –3+4N–N2=(N–1)(3–N). Partial fractions give A=½ (set N=1) and B=½ (set N=3).
(b)	**ln|(N–1)/(3–N)| = 2kt–ln3 ✓**	Separate variables and integrate using (a): ½ln|N–1|–½ln|3–N|=kt+C. At t=0, N=1.5: C=½ln(1/3). Doubling gives the result with p=3.
(c)	**N = (3+3e^(–2kt))/(1+3e^(–2kt))**	Exponentiate (b): (N–1)/(3–N)=e^(2kt)/3. Solve for N: N=3(1+e^(2kt))/(3+e^(2kt)). Dividing top and bottom by e^(2kt) gives a=3,b=3,c=3.
(d)	**L = 3 (i.e. 3000 rabbits)**	As t→∞, e^(–2kt)→0 (since k>0). So N→(3+0)/(1+0)=3. The infected population approaches 3000 in the long term.