IB Mathematics: Applications and Interpretation HL

Examination: May 2026 Paper 1 (2226-5220) — Worked Solutions & Mark Scheme

Time Allowed: 2 hours | **Maximum Mark:** 110 marks | **GDC Required:** Yes

Note: Unless otherwise stated, all numerical answers are given exactly or correct to three significant figures.

Section A & B: Complete Worked Solutions (Questions 1 to 15)

Question 1 [Maximum mark: 6] — Probability & Contingency Tables

(a) Total students surveyed = 200. Number of left-handed students preferring geometry = 16.
P(left-handed and geometry) = 16 / 200 = **0.08** (or 2/25) **[3 marks: M1A2]**

(b) Total left-handed students = 16 + 6 = 22.
P(geometry | left-handed) = 16 / 22 = **8 / 11** (or ≈ 0.727) **[1 mark: A1]**

(c) Total right-handed students = 67 + 111 = 178.
Using the complement rule for selection without replacement:
P(at least one left-handed) = 1 – P(both right-handed) = 1 – (178/200 × 177/199) = 1 – 31506 / 39800 = **4147 / 19900** (≈ 0.208) **[2 marks: M1A1]**

Question 2 [Maximum mark: 6] — Inverse Variation & Inequalities

(a) Since cost per person c varies inversely with n: $c = k / n$.
When $n = 4$, $c = 187.50 \Rightarrow k = 4 \times 187.50 = 750$.
Equation: $c = 750 / n$ **[2 marks: M1A1]**

(b) (i) We require $c < 100 \Rightarrow 750 / n < 100 \Rightarrow n > 7.5$.
Minimum number of people needed = **8** **[2 marks: M1A1]**

(ii) When $n = 8$: $c = 750 / 8 =$ **$93.75** **[2 marks: M1A1]**

Question 3 [Maximum mark: 7] — 3D Geometry, Units & Percentage Error

(a) Cuboid volume = height × length × width = 299 × 218 × 318 = 20,728,116 mm^3.
Volume to nearest cubic millimetre = **20,728,116 mm^3** **[2 marks: M1A1]**

(b) Actual volume = 20 litres = 20 × 1000 ml = 20,000 ml.
Since 1 mm^3 = 0.001 ml, 1 ml = 1,000 mm^3 ⇒ Actual volume = 20,000,000 mm^3.
Percentage error = |(20,728,116 – 20,000,000) / 20,000,000| × 100% = (728,116 / 20,000,000) × 100% = **3.64%** (3 s.f.) **[4 marks: M2A2]**

(c) **Real carboys have rounded corners, indented walls, or a narrower neck/handle opening**, which reduces the internal capacity compared to a perfect rectangular cuboid. **[1 mark: R1]**

Question 4 [Maximum mark: 10] — Quadratic Modelling (Volleyball Serve)

(a) Using coordinates (1, 2.8), (2, 2.86), and (3, 2.89) in $h(x) = ax^2 + bx + c$:
1) $\boldsymbol{a + b + c = 2.8}$ | 2) $\boldsymbol{4a + 2b + c = 2.86}$ | 3) $\boldsymbol{9a + 3b + c = 2.89}$ **[2 marks: A2]**

(b) Solving the simultaneous equations using GDC:
$a = -0.015$, $b = 0.105$, $c = 2.71 \Rightarrow$ $\boldsymbol{h(x) = -0.015x^2 + 0.105x + 2.71}$ **[2 marks: M1A1]**

(c) $c = 2.71$ m represents **the initial height above the ground at which Emily hits the ball** (at $x = 0$). **[1 mark: R1]**

(d) Checking clearance over the net ($x = 9$):
$h(9) = -0.015(9)^2 + 0.105(9) + 2.71 = -1.215 + 0.945 + 2.71 = 2.44$ m.
Since 2.44 m > 2.24 m, the ball successfully clears the net.
Checking landing point before the end line ($x = 18$):
$h(18) = -0.015(18)^2 + 0.105(18) + 2.71 = -4.86 + 1.89 + 2.71 = -0.26$ m.
Since $h(18) < 0$, the ball hits the court before reaching the 18 m end line (landing at $x \approx 17.3$ m).
Conclusion: **Yes, Emily's serve will be in play.** **[5 marks: M2A2R1]**

Question 5 [Maximum mark: 9] — Differentiation, Normals & Transformations

(a) $f(x) = 0.5x^3 - 6.75x^2 + 25.5x - 13$.
$f'(x) = 1.5x^2 - 13.5x + 25.5 \Rightarrow$ At point A(6, 5): $f'(6) = 1.5(36) - 13.5(6) + 25.5 = -1.5$.
Gradient of the normal $m_N = -1 / f'(6) = -1 / (-1.5) =$ **2 / 3** (or 0.667) **[2 marks: M1A1]**

(b) Equation of normal: $y - 5 = (2/3)(x - 6) \Rightarrow$ $\boldsymbol{y = (2/3)x + 1}$ (or $2x - 3y + 3 = 0$) **[1 mark: A1]**

(c) A straight line drawn on the axes passing through (0, 1), (6, 5), and (9, 7). **[2 marks: A2]**

(d) Intersecting $f(x) = (2/3)x + 1$ using GDC:
$0.5x^3 - 6.75x^2 + 24.8333x - 14 = 0$. Roots are $x = 0.685$, $x = 6$ (point A), and $x = 6.82$.
Other intersection coordinates: **(0.685, 1.46) and (6.82, 5.54)** **[2 marks: A2]**

(e) For $g(x) = -f(3x) + 4$:
Horizontal scale factor 1/3 $\Rightarrow$ x-coordinate becomes 6 / 3 = 2.
Vertical reflection and shift +4 $\Rightarrow$ y-coordinate becomes $-5 + 4 = -1$.
Corresponding point on $g(x)$: **(2, –1)** **[2 marks: M1A1]**

Question 6 [Maximum mark: 8] — Bearings, Sine/Cosine Rules & Speed

(a) Magnetic north is 6° clockwise from true north. Bearing = **006°** (or 6°) **[1 mark: A1]**

(b) In triangle ABC, AB = 5 km (due True North) and AC = 5 km (on bearing 006°).
Since AB = AC = 5 km, triangle ABC is isosceles with vertex angle ∠BAC = 6°.
Angle $A\hat{C}B = (180° − 6°) / 2 =$ **87°** **[2 marks: M1A1]**

(c) Using the cosine rule (or splitting into right-angled triangles):
$BC^2 = 5^2 + 5^2 − 2(5)(5)\cos(6°) = 50 − 49.7261 = 0.2739 \Rightarrow BC =$ **0.523 km** (or 523 m) **[3 marks: M2A1]**

(d) Total distance hiked = AC + CB = 5 + 0.52336 = 5.52336 km.
Total time = 2 hours 15 minutes = 2.25 hours.
Average speed = 5.52336 / 2.25 = **2.45 km h^{-1}** **[2 marks: M1A1]**

Question 7 [Maximum mark: 7] — Transition Matrices & Steady State

(a) Let $C_0 = (x, y)^T$ be the previous month's vector such that $T C_0 = C$.
$0.8x + 0.3(1 − x) = 0.65 \Rightarrow 0.5x + 0.3 = 0.65 \Rightarrow 0.5x = 0.35 \Rightarrow x = 0.70$.
Proportion in previous month: **Regular = 0.70 (70%), Decaf = 0.30 (30%)** **[3 marks: M1A2]**

(b) At steady state, $T s = s$ where $s = (p, q)^T$ and $p + q = 1$.
$0.8p + 0.3q = p \Rightarrow 0.3q = 0.2p \Rightarrow 3q = 2p \Rightarrow p = 1.5q$.
Substituting into $p + q = 1 \Rightarrow 1.5q + q = 1 \Rightarrow 2.5q = 1 \Rightarrow q = 0.4$.
Then $p = 1 − 0.4 = 0.6$. Hence, the unique steady state vector is **$(0.6, 0.4)^T$**. **[4 marks: M2A2]**

Question 8 [Maximum mark: 6] — Trapezoidal Rule & Numerical Integration

(a) Using 4 intervals of width $h = (8 − 0) / 4 = 2$ with points (0, 3.08), (2, 1.63), (4, 1.48), (6, 3.07), (8, 3.90):
$x(8) \approx (2 / 2) \times [3.08 + 2(1.63 + 1.48 + 3.07) + 3.90] = 1 \times [3.08 + 12.36 + 3.90] =$ **19.3 m** **[3 marks: M2A1]**

(b) Using GDC numerical integration for $x(8) = \int_0^8 \sqrt{[0.5(t − 3)^2 + 2\cos(t) + 3]}\, dt$:
$x(8) =$ **19.2 m** (or 19.2486...) **[3 marks: M1A2]**

Question 9 [Maximum mark: 7] — Slope Fields & Separation of Variables

(a) Sketching on the slope field: A smooth bell-shaped curve starting at (0, 1.5), following tangent arrows upwards to a peak at $x \approx 1.57$ (π/2) with height $y \approx 4.1$, then decreasing symmetrically towards $x = 3.14$ (π).
[2 marks: A2]

(b) Separating variables for $dy/dx = y \cos x$:
$\int (1/y)\, dy = \int \cos x\, dx \Rightarrow \ln|y| = \sin x + C \Rightarrow y = A e^{\sin x}$.
Using initial condition (0, 1.5): $1.5 = A e^{\sin 0} = A(1) \Rightarrow A = 1.5$.
Solution equation: **$y = 1.5 e^{\sin x}$** **[5 marks: M2A2A1]**

Question 10 [Maximum mark: 5] — Non-linear Modelling & Equation Solving

(a) At $t = 0$: $C(0) = 150 / (1 + 0) =$ **150 mg L^{-1}** **[1 mark: A1]**

(b) We need to find $C \geq 0$ such that $\ln(C + 1.5) - 1/(C + 1) = 2$.
Using GDC equation solver or graphical intersection of $y_1 = \ln(x + 1.5) - 1/(x + 1)$ and $y_2 = 2$:
$C =$ **6.89 mg L^{-1}** (3 s.f.) **[4 marks: M2A2]**

Question 11 [Maximum mark: 6] — Linear & Exponential Regression

(a) Applying least squares linear regression on $Y = \ln(N(t))$ against $X = t$ using GDC:
Slope $a = 0.318$, Intercept $b = -0.722 \Rightarrow$ **$\ln(N(t)) = 0.318t - 0.722$** **[3 marks: M1A2]**

(b) For exponential model $N(t) = C\,e^{dt}$, taking logarithms gives $\ln(N(t)) = \ln C + dt$.
Comparing coefficients with part (a): $d = a =$ **0.318**.
$\ln C = b = -0.722 \Rightarrow C = e^{-0.722} =$ **0.486** (or 0.485 depending on rounding). **[3 marks: M1A2]**

Question 12 [Maximum mark: 8] — Calculus: Chain Rule & Volume of Revolution

(a) For $g(x) = a \sin(x^n) + b$, differentiating by chain rule gives $g'(x) = a\,n\,x^{n-1} \cos(x^n)$.
Comparing with $g'(x) = 6x^2 \cos(x^3)$: $n = 3$ and $a(3) = 6 \Rightarrow a = 2$.
Using y-intercept $(0, 1)$: $g(0) = 2 \sin(0) + b = 1 \Rightarrow b = 1$.
Equation: **$g(x) = 2 \sin(x^3) + 1$** **[3 marks: A3]**

(b) (i) Finding upper limit of region bounded by $x = 0$ and $y = 3$:
$2 \sin(x^3) + 1 = 3 \Rightarrow \sin(x^3) = 1 \Rightarrow x^3 = \pi/2 \Rightarrow x = (\pi/2)^{1/3} \approx 1.1624$.
Volume of revolution about x-axis (washer method):
$V = \pi \int_0^{(\pi/2)^{1/3}} [3^2 - (2 \sin(x^3) + 1)^2]\,dx$ **[2 marks: M1A1]**

(ii) Evaluating the integral using GDC numerical integration:
$V =$ **20.1** (or 20.1139...) **[3 marks: M1A2]**

Question 13 [Maximum mark: 7] — Complex Numbers & Argand Transformations

(a) For $z = -4 + (\sqrt{6} - \sqrt{2})i$:
$r = \sqrt{[(-4)^2 + (\sqrt{6} - \sqrt{2})^2]} = \sqrt{[16 + (6 - 2\sqrt{12} + 2)]} = \sqrt{[24 - 4\sqrt{3}]}$.
In the required form: **$r = \sqrt{(24 - 4\sqrt{3})}$** (where $a = 24$, $b = -4$) **[2 marks: M1A1]**

(b) Reference angle $\alpha = \arctan[(\sqrt{6} - \sqrt{2}) / 4] \approx 0.2532$ rad (14.5°).
Since $\mathrm{Re}(z) < 0$ and $\mathrm{Im}(z) > 0$, z is in Quadrant 2:
$\theta = \pi - 0.2532 =$ **2.89 rad** (or 165.5°) **[3 marks: M1A2]**

(c) For $w = z^3 = r^3 \text{ cis}(3\theta)$:

The vector length is cubed (stretched by a factor of $r^2 \approx 17.1$ to length $r^3 \approx 70.5$), and its direction is rotated anticlockwise about the origin by an angle of $2\theta \approx 5.78$ rad (or 331°), resulting in an argument of $3\theta \approx 2.38$ rad in Quadrant 2. **[2 marks: A2]**

Question 14 [Maximum mark: 10] — Truncated Cone Geometry & Related Rates

(a) (i) From the removed bottom cone (height 4 cm, base radius 2 cm): $\tan\theta = 2 / 4 =$ **$1 / 2$** **[1 mark: A1]**

(ii) For the full cone up to water surface (radius r, total height $4 + x$):

$\tan\theta = r / (4 + x) = 1 / 2 \Rightarrow 4 + x = 2r \Rightarrow$ **$x = 2r - 4$** **[2 marks: M1A1]**

(b) Water volume V = Total cone volume − Removed bottom cone volume:

$V = (1/3)\pi r^2(2r) - (1/3)\pi(2^2)(4) = (2/3)\pi r^3 - (16/3)\pi$.

Comparing with $V = (2/3)\pi r^3 - a\pi \Rightarrow$ **$a = 16 / 3$** (or ≈ 5.33) **[4 marks: M2A2]**

(c) Given $dV/dt = 2\pi r^2\,(dr/dt) = 5 \text{ cm}^3 \text{ s}^{-1}$ and $dr/dt = 0.06 \text{ cm s}^{-1}$:

$2\pi r^2(0.06) = 5 \Rightarrow r^2 = 5 / (0.12\pi) \approx 13.2629 \Rightarrow r = 3.6418$ cm.

Height $x = 2(3.6418) - 4 =$ **3.28 cm** **[3 marks: M1A2]**

Question 15 [Maximum mark: 8] — Systems of ODEs & Phase Portraits

(a) Coefficient matrix $A = [[2, -5], [1, -1]]$. Characteristic equation $\det(A - \lambda I) = 0$:

$(2 - \lambda)(-1 - \lambda) - (-5)(1) = 0 \Rightarrow \lambda^2 - \lambda + 3 = 0$.

Using quadratic formula: $\lambda = (1 \pm \sqrt{[1 - 12]}) / 2 =$ **$0.5 \pm 1.66i$** (or $1/2 \pm i\sqrt{11} / 2$) **[4 marks: M2A2]**

(b) Trajectory selection: **Anticlockwise spiral outwards from the origin** **[1 mark: A1]**

Justification:

1) The complex conjugate eigenvalues $\lambda = 0.5 \pm i\sqrt{11}/2$ have a **positive real part** ($\text{Re}(\lambda) = 0.5 > 0$), which causes the trajectory to grow exponentially in magnitude, creating an **outward spiral** away from the origin (unstable focus/source).

2) Checking the velocity vector at the starting point (1, 0): $dx/dt = 2(1) - 5(0) = 2 > 0$ and $dy/dt = 1(1) - 0 = 1 > 0$. Since the trajectory moves upwards into the first quadrant $(+x, +y)$ from the positive x-axis, the direction of rotation is **anticlockwise**. **[3 marks: R3]**