



Helping you Achieve Highest Grades in IB

# IB Mathematics (Applications and Interpretations) Standard Level (SL)

## Mark Scheme

Fully in-lined with the First Assessment  
Examinations in 2021 & Beyond

### Paper: 1 (All Topics)

- Topic 1 - Number and Algebra
- Topic 2 - Functions
- Topic 3 - Geometry and Trigonometry
- Topic 4 - Statistics and Probability
- Topic 5 - Calculus

Marks: 187

Total Marks: / 187

Suitable for SL Students sitting the 2025 exams onwards  
However, HL students may also find these resources useful

## Markschemes

### EXM.1.SL.TZ0.1

a.

$$2400(1.04)^{10} = \$3552.59 \quad M1A1$$

**[2 marks]**

b. real interest rate =  $4 - 1.5 = 2.5\%$       A1     $2400(1.025)^{10} = \$3072.20 \quad M1A1$

**[3 marks]**

### EXM.1.SL.TZ0.2

a.

average equal ranks      M1

Competitor	A	B	C	D	E	F	G
Judge 1	1	2	3.5	3.5	5	6.5	6.5
Judge 2	2	3	1	4	5.5	5.5	7

A1A1

$$r_s = 0.817 \quad A2$$

**[5 marks]**

b. There is strong agreement between the two judges.

R1 [1 mark]

### EXM.1.SL.TZ0.4

Line from A to B will have the form  $y = \frac{1}{2}x + c \quad M1A1$

Through (4,6)  $\Rightarrow c = 4$  so line is  $y = \frac{1}{2}x + 4 \quad M1A1$

Intersection of  $y = -2x + 9$  and  $y = \frac{1}{2}x + 4$  is (2, 5)      M1A1

Let  $A = (p,q)$  then  $(2,5) = \left(\frac{p+4}{2}, \frac{q+6}{2}\right) \Rightarrow p = 0, q = 4 \quad M1A1A1$

$$A = (0,4)$$

**[9 marks]**

### 18M.1.SL.TZ2.T\_4

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

Set	Example of a number in the set
$\mathbb{N}$	<b>Any natural number</b>
$\mathbb{Z}$	<b>Any integer</b>
$\mathbb{Q}$	<b>Any rational number</b>
$\mathbb{R}$	<b>Any real number</b>

(A1)(A1)(A1)(A1)(C4)

[4 marks]

b. Incorrect (A1)

Natural numbers are positive integers. Integers can also be negative. (or equivalent)  
 (R1) (C2)

**Note:** Accept a correct justification. Do not award (RO)(A1).

[2 marks]

Accept: a statement with an example of an integer which is not natural.

## 16N.1.SL.TZ0.T\_1

$3.91 \times 10^{-2}$  (A1)(ft) (A1)(ft) (C2)

**Note:** Answer should be consistent with their answer to part (b)(ii). Award (A1)(ft) for 3.91, and (A1)(ft) for  $10^{-2}$ . Follow through from part (b)(ii).

[2 marks]

## 17N.1.SL.TZ0.S\_2

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

subtracting terms (M1)

eg  $5 - 8$ ,  $u_2 - u_1$

$d = -3$     A1    N2

[2 marks]

b. correct substitution into formula

$$\text{eg } u_{10} = 8 + (10 - 1)(-3), 8 - 27, -3(10) + 11 \quad u_{10} = -19$$

c. correct substitution into formula for sum

$$\text{eg } S_{10} = \frac{10}{2}(8 - 19), 5(2(8) + (10 - 1)(-3)) \quad S_{10} = -55$$

## 17M.1.SL.TZ1.S\_7

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct use  $\log x^n = n \log x$     A1

eg  $16 \ln x$

valid approach to find  $r$     (M1)

$$\text{eg } \frac{u_{n+1}}{u_n}, \frac{\ln x^8}{\ln x^{16}}, \frac{4 \ln x}{8 \ln x}, \ln x^4 = \ln x^{16} \times r^2$$

$$r = \frac{1}{2} \quad \text{A1} \quad \text{N2}$$

[3 marks]

## 18M.1.SL.TZ2.S\_7

**METHOD 1** (finding  $u_1$  and  $d$ )

recognizing  $\sum = S_{20}$  (seen anywhere)    (A1)

attempt to find  $u_1$  or  $d$  using  $\log_c c^k = k$     (M1)

eg  $\log_c c, 3 \log_c c$ , correct value of  $u_1$  or  $d$

$u_1 = 2, d = 3$  (seen anywhere)    (A1)(A1)

correct working    (A1)

$$\text{eg } S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3), S_{20} = \frac{20}{2}(2 + 59), 10(61)$$

$$\sum_{n=1}^{20} u_n = 610 \quad \text{A1 N2}$$

**METHOD 2** (expressing  $S$  in terms of  $c$ )

recognizing  $\sum = S_{20}$  (seen anywhere)    (A1)

correct expression for  $S$  in terms of  $c$  **(A1)**  
 eg  $10(2\log_c c^2 + 19\log_c c^3)$

$\log_c c^2 = 2, \log_c c^3 = 3$  (seen anywhere) **(A1)(A1)**

correct working **(A1)**

eg  $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3), S_{20} = \frac{20}{2}(2 + 59), 10(61)$

$$\sum_{n=1}^{20} u_n = 610 \quad \mathbf{A1 N2}$$

### METHOD 3 (expressing $S$ in terms of $c$ )

recognizing  $\Sigma = S_{20}$  (seen anywhere) **(A1)**

correct expression for  $S$  in terms of  $c$

eg  $10(2\log_c c^2 + 19\log_c c^3)$

correct application of log law

eg  $2\log_c c^2 = \log_c c^4, 19\log_c c^3 = \log_c c^{57}, 10(\log_c(c^2)^2 + \log_c(c^3)^{19}), 10(\log_c c^4 + \log_c c^{57}), 10$

correct application of definition of log

eg  $\log_c c^{61} = 61, \log_c c^4 = 4, \log_c c^{57} = 57$

correct working

eg  $S_{20} = \frac{20}{2}(4 + 57), 10(61)$

$$\sum_{n=1}^{20} u_n = 610$$

### 18N.1.SL.TZ0.S\_3

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct working **(A1)**

eg  $-5 + (8 - 1)(3)$

$$u_8 = 16 \quad \mathbf{A1 N2}$$

**[2 marks]**

## 17N.1.SL.TZ0.S\_5

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to form composite (M1)

$$\text{eg } g(1 + e^{-x})$$

correct function A1 N2

$$\text{eg } (g \circ f)(x) = 2 + b + 2e^{-x}, 2(1 + e^{-x}) + b$$

**[2 marks]**

b. evidence of  $\lim_{x \rightarrow \infty} (2 + b + 2e^{-x}) = 2 + b + \lim_{x \rightarrow \infty} (2e^{-x})$  (M1)

eg  $2 + b + 2e^{-\infty}$ , graph with horizontal asymptote when  $x \rightarrow \infty$

**Note:** Award M0 if candidate clearly has incorrect limit, such as  $x \rightarrow 0$ ,  $e^\infty$ ,  $2e^0$ .

evidence that  $e^{-x} \rightarrow 0$  (seen anywhere) (A1)

eg  $\lim_{x \rightarrow \infty} (e^{-x}) = 0$ ,  $1 + e^{-x} \rightarrow 1$ ,  $2(1) + b = -3$ ,  $e^{\text{large negative number}} \rightarrow 0$ , graph of  $y = e^{-x}$  or

$y = 2e^{-x}$  with asymptote  $y = 0$ , graph of composite function with asymptote  $y = -3$

correct working (A1) eg  $2 + b = -3$   $b = -5$  A1 N2 [4 marks]

## 18M.1.SL.TZ2.S\_6

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

### METHOD 1

evidence of discriminant (M1)

$$\text{eg } b^2 - 4ac, \Delta$$

correct substitution into discriminant (A1)

$$\text{eg } q^2 - 4p(-4p)$$

correct discriminant A1

$$\text{eg } q^2 + 16p^2$$

$$16p^2 > 0 \text{ (accept } p^2 > 0 \text{)} \quad \text{A1}$$

$$q^2 \geq 0 \text{ (do not accept } q^2 > 0 \text{)} \quad \text{A1}$$

$$q^2 + 16p^2 > 0 \quad \text{A1}$$

## METHOD 2

$y$ -intercept =  $-4p$  (seen anywhere)

if  $p$  is positive, then the  $y$ -intercept will be negative

an upward-opening parabola with a negative  $y$ -intercept

eg sketch that must indicate  $p > 0$ .

if  $p$  is negative, then the  $y$ -intercept will be positive

a downward-opening parabola with a positive  $y$ -intercept

eg sketch that must indicate  $p < 0$ .

$f$  has 2 roots

## 16N.1.SL.TZ0.S\_1

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct approach **(A1)**

eg  $\frac{-(-4)}{2}, f'(x) = 2x - 4 = 0, (x^2 - 4x + 4) + 5 - 4$

$x = 2$  (must be an equation) **A1 N2**

**[2 marks]**

b. (i)  $h = 2$  **A1 N1** (ii) **METHOD 1** valid attempt to find  $k$  **(M1)** eg  $f(2) = 5$

correct substitution into **their** function **(A1)** eg  $(2)^2 - 4(2) + 5 = 1$  **A1 N2**

**METHOD 2** valid attempt to complete the square **(M1)** eg  $x^2 - 4x + 4$

correct working **(A1)** eg  $(x^2 - 4x + 4) - 4 + 5, (x - 2)^2 + 1 = 1$  **A1 N2**

**[4 marks]**

## 17N.1.SL.TZ0.T\_11

$$-\frac{b}{2a} = -2$$

$$a(-2)^2 - 2b + 5 = 3 \text{ or equivalent}$$

$$a(-4)^2 - 4b + 5 = 5 \text{ or equivalent}$$

$$2a(-2) + b = 0 \text{ or equivalent} \quad (\text{M1})$$

**Note:** Award **(M1)** for two of the above equations.

$$a = 0.5 \quad (\text{A1})(\text{ft})$$

$$b = 2 \quad (\text{A1})(\text{ft}) \quad (\text{C3})$$

**Note:** Award at most **(M1)(A1)(ft)(AO)** if the answers are reversed.

Follow through from parts (a) and (b).

**[3 marks]**

### 18M.1.SL.TZ2.T\_14

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$x^3 \quad (\text{A1}) \quad (\text{C1})$$

**Note:** Award **(AO)** for  $\frac{4x^3}{4}$  and not simplified to  $x^3$ .

**[1 mark]**

$$\text{b. } \left(-\frac{1}{2}\right)^3 \quad (\text{M1})$$

**Note:** Award **(M1)** for correct substitution of  $-\frac{1}{2}$  into their derivative.

$$-\frac{1}{8} (-0.125) \quad (\text{A1})(\text{ft}) \quad (\text{C2}) \quad \text{Note: Follow through from their part (a).} \quad \text{[2 marks]}$$

$$\text{c. } x^3 = 8 \quad (\text{A1})(\text{M1})$$

**Note:** Award **(A1)** for 8 seen maybe seen as part of an equation  $y = 8x + c$ , **(M1)** for equating their derivative to 8.

$$(x =) 2 \quad (\text{A1}) \quad (\text{C3}) \quad \text{Note: Do not accept (2, 4).} \quad \text{[3 marks]}$$

## SPM.1.SL.TZ0.11

$$\text{volume} = 240 \left( \pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664 \dots \right) \quad \mathbf{M1M1M1}$$

**Note:** Award **M1**  $240 \times \text{area}$ , award **M1** for correctly substituting area sector formula, award **M1** for subtraction of their area of the sector from area of circle.

$$= 45800 (= 45811.96071) \quad \mathbf{A1}$$

**[4 marks]**

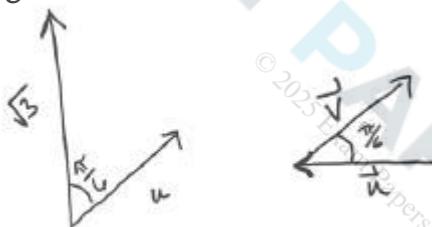
## 19M.1.SL.TZ1.S\_6

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**METHOD 1 (cosine rule)**

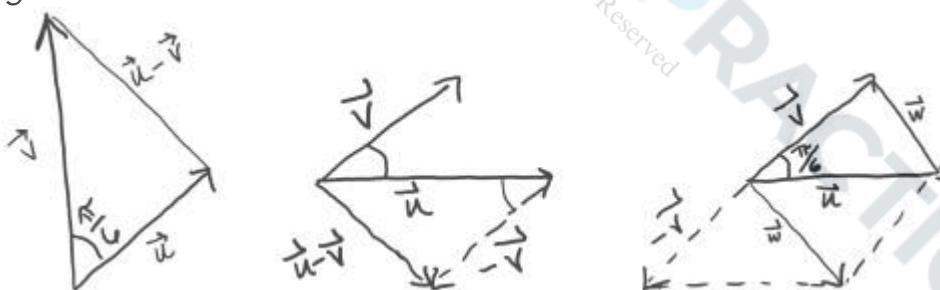
diagram including  $\mathbf{u}$ ,  $\mathbf{v}$  and included angle of  $\frac{\pi}{6}$  **(M1)**

eg



sketch of triangle with  $\mathbf{w}$  (does not need to be to scale) **(A1)**

eg



choosing cosine rule **(M1)**

$$\text{eg } a^2 + b^2 - 2ab\cos C$$

correct substitution **A1**

$$\text{eg } 4^2 + (\sqrt{3})^2 - 2(4)(\sqrt{3})\cos\frac{\pi}{6}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \text{ (seen anywhere)} \quad \mathbf{(A1)}$$

correct working **(A1)**

eg  $16 + 3 - 12$

$$| | = \sqrt{7}$$

valid approach, in terms of  $u$  and  $v$  (seen anywhere)

$$\text{eg } | |^2 = (\quad - \quad) \cdot (\quad - \quad), | |^2 = \bullet - 2 \bullet + \bullet, | |^2 = (u_1 - v_1)^2 + (u_2 - v_2)^2,$$

$$| | = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2}$$

correct value for  $\bullet$  (seen anywhere)

$$\text{eg } | |^2 = 16, \bullet = 16, u_1^2 + u_2^2 = 16$$

correct value for  $\bullet$  (seen anywhere)

$$\text{eg } | |^2 = 16, \bullet = 3, v_1^2 + v_2^2 + v_3^2 = 3$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \text{ (seen anywhere)}$$

$$\bullet = 4 \times \sqrt{3} \times \frac{\sqrt{3}}{2} (= 6) \text{ (seen anywhere)}$$

correct substitution into  $\bullet - 2 \bullet + \bullet$  or  $u_1^2 + u_2^2 + v_1^2 + v_2^2 - 2(u_1 v_1 + u_2 v_2)$  (2 or 3 dimensions)

$$\text{eg } 16 - 2(6) + 3 (= 7)$$

$$| | = \sqrt{7}$$

### 17N.1.SL.TZ0.T\_3

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$\frac{149600000}{300000 \times 60} \quad (\text{M1})(\text{M1})$$

**Note:** Award **(M1)** for dividing the **correct** numerator (which can be presented in a different form such as  $149.6 \times 10^6$  or  $1.496 \times 10^8$ ) by 300000 and **(M1)** for dividing by 60.

$$= 8.31 \text{ (minutes)} (8.31111 \dots, 8 \text{ minutes 19 seconds}) \quad (\text{A1}) \quad (\text{C3})$$

**[3 marks]**

### 16N.1.SL.TZ0.S\_5

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid interpretation (may be seen on a Venn diagram) **(M1)**

$$\text{eg } P(A \cap B) + P(A' \cap B), 0.2 + 0.6$$

$$P(B) = 0.8 \quad \mathbf{A1} \quad \mathbf{N2}$$

**[2 marks]**

b. valid attempt to find  $P(A)$  **(M1)**  $\text{eg } P(A \cap B) = P(A) \times P(B), 0.8 \times A = 0.2$

correct working for  $P(A)$  **(A1)**  $\text{eg } 0.25, \frac{0.2}{0.8}$  correct working for  $P(A \cup B)$  **(A1)**

$$\text{eg } 0.25 + 0.8 - 0.2, 0.6 + 0.2 + 0.05 \quad P(A \cup B) = 0.85 \quad \mathbf{A1} \quad \mathbf{N3} \quad \mathbf{[4 marks]}$$

## 19M.1.SL.TZ1.S\_2

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct equation **(A1)**

$$\text{eg } -3 + 6s = 15, 6s = 18$$

$$s = 3 \quad \mathbf{(A1)}$$

substitute their  $s$  value into  $z$  component **(M1)**

$$\text{eg } 10 + 3(2), 10 + 6$$

$$c = 16 \quad \mathbf{A1 N3}$$

**[4 marks]**

b.  $r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix} (= \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + t(6\mathbf{i} + 2\mathbf{k}) \quad \mathbf{A2 N2}$

**Note:** Accept any scalar multiple of  $\begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$  for the direction vector.

Award **A1** for  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$ , **A1** for  $L_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$ , **A0** for  $r = \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ .

**[2 marks]**

## 16N.1.SL.TZ0.S\_4

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid attempt to find direction vector (M1)

$$\text{eg } \overrightarrow{PQ}, \overrightarrow{QP}$$

correct direction vector (or multiple of) (A1)

$$\text{eg } 6\mathbf{i} + \mathbf{j} - 3\mathbf{k}$$

any correct equation in the form  $\mathbf{r} = \mathbf{a} + t\mathbf{b}$  (any parameter for  $t$ ) A2 N3

where  $\mathbf{a}$  is  $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  or  $7\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ , and  $\mathbf{b}$  is a scalar multiple of  $6\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

$$\text{egr } \mathbf{r} = 7\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + t(6\mathbf{i} + \mathbf{j} - 3\mathbf{k}), \mathbf{r} = \begin{pmatrix} 1+6s \\ 2+1s \\ -1-3s \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} -6 \\ -1 \\ 3 \end{pmatrix}$$

**Notes:** Award A1 for the form  $\mathbf{a} + t\mathbf{b}$ , A1 for the form  $\mathbf{L} = \mathbf{a} + t\mathbf{b}$ , A0 for the form  $\mathbf{r} = \mathbf{b} + t\mathbf{a}$ .

[4 marks]

b. correct expression for scalar product (A1)

$$\text{eg } 6 \times 2 + 1 \times 0 + (-3) \times n, -3n + 12$$

setting scalar product equal to zero (seen anywhere) (M1)

$$\text{eg } \mathbf{u} \cdot \mathbf{v} = 0, -3n + 12 = 0 \quad n = 4 \quad \mathbf{A1} \quad \mathbf{N2} \quad [3 \text{ marks}]$$

## 19M.1.SL.TZ2.S\_2

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach (M1)

$$\text{eg } \mathbf{b} = 2\mathbf{a}, \mathbf{a} = k\mathbf{b}, \cos \theta = 1, \mathbf{a} \cdot \mathbf{b} = -|\mathbf{a}||\mathbf{b}|, 2p = 18$$

$$p = 9 \quad \mathbf{A1} \mathbf{N2}$$

[2 marks]



b. evidence of scalar product *eg* (0)(0) + (3)(6) + p(18)

recognizing  $\bullet = \circ$  (seen anywhere)

correct working

$$\text{eg } 18 + 18p = 0, 18p = -18$$

$$p = -1$$

## 17M.1.SL.TZ2.S\_6

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

expressing  $h(1)$  as a product of  $f(1)$  and  $g(1)$  **(A1)**

$$\text{eg } f(1) \times g(1), 2(9)$$

$$h(1) = 18 \quad \mathbf{A1} \quad \mathbf{N2}$$

**[2 marks]**

b. attempt to use product rule (do **not** accept  $h' = f' \times g'$ ) **(M1)**

$$\text{eg } h' = fg' + gf', h'(8) = f'(8)g(8) + g'(8)f(8)$$

correct substitution of values into product rule **(A1)**

$$\text{eg } h'(8) = 4(5) + 2(-3), -6 + 20 \quad h'(8) = 14 \quad \mathbf{A1 N2} \quad \mathbf{[3 marks]}$$

## 17N.1.SL.TZ0.S\_7

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

### METHOD 1 – using discriminant

correct equation without logs **(A1)**

$$\text{eg } 6x - 3x^2 = k^2$$

valid approach **(M1)**

$$\text{eg } -3x^2 + 6x - k^2 = 0, 3x^2 - 6x + k^2 = 0$$

recognizing discriminant must be zero (seen anywhere) **M1**

$$\text{eg } \Delta = 0$$

correct discriminant **(A1)**

$$\text{eg } 6^2 - 4(-3)(-k^2), 36 - 12k^2 = 0$$

correct working **(A1)**

eg  $12k^2 = 36$ ,  $k^2 = 3$

$k = \sqrt{3}$     A2    N2

### METHOD 2 – completing the square

correct equation without logs    (A1)

eg  $6x - 3x^2 = k^2$

valid approach to complete the square    (M1)

eg  $3(x^2 - 2x + 1) = -k^2 + 3$ ,  $x^2 - 2x + 1 - 1 + \frac{k^2}{3} = 0$

correct working

eg  $3(x - 1)^2 = -k^2 + 3$ ,  $(x - 1)^2 - 1 + \frac{k^2}{3} = 0$

recognizing conditions for one solution

eg  $(x - 1)^2 = 0$ ,  $-1 + \frac{k^2}{3} = 0$

correct working

eg  $\frac{k^2}{3} = 1$ ,  $k^2 = 3$

$k = \sqrt{3}$

### 18M.1.SL.TZ2.T\_11

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$x = \textcircled{1}$     (A1)(A1) (C2)

**Note:** Award (A1) for  $x = \text{a constant}$  (A1) for = o. Award (AO)(AO) for an answer of "o".

**[2 marks]**

b.  $f(x) = -2$     ( $y = -2$ )    (A1)(A1) (C2)

**Note:** Award (A1) for  $y = \text{a constant}$  (A1) for = -2. Award (AO)(AO) for an answer of "-2".

**[2 marks]**

c.  $\frac{3}{x} - 2 = 0$     (M1)    **Note:** Award (M1) for equating  $f(x)$  to 0.

$$(x = ) \frac{3}{2} (1.5)$$

## 17N.1.SL.TZ0.T\_14

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$12x^2 - \frac{6}{x^3} \text{ or equivalent } \quad (\text{A1})(\text{A1})(\text{A1}) \quad (\text{C3})$$

**Note:** Award **(A1)** for  $12x^2$ , **(A1)** for  $-6$  and **(A1)** for  $\frac{1}{x^3}$  or  $x^{-3}$ . Award at most **(A1)(A1)(AO)** if additional terms seen.

**[3 marks]**

b.  $12x^2 - \frac{6}{x^3} = 6 \quad (\text{M1}) \quad \text{Note: Award } (\text{M1}) \text{ for equating their derivative to 6.}$

$(1, 4) \text{ OR } x = 1, y = 4 \quad (\text{A1})(\text{ft})(\text{A1})(\text{ft}) \quad (\text{C3})$

**Note:** A frequent wrong answer seen in scripts is  $(1, 6)$  for this answer with correct working award **(M1)(AO)(A1)** and if there is no working award **(C1)**.

**[3 marks]**

## 17M.1.SL.TZ2.S\_5

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach **(M1)**

$$\text{eg } \int f' dx, \int \frac{3x^2}{(x^3 + 1)^5} dx$$

correct integration by substitution/inspection **A2**

$$\text{egf}(x) = -\frac{1}{4}(x^3 + 1)^{-4} + c, \frac{-1}{4(x^3 + 1)^4}$$

correct substitution into **their** integrated function (must include  $c$ ) **M1**

$$\text{eg1} = \frac{-1}{4(0^3 + 1)^4} + c, -\frac{1}{4} + c = 1$$

**Note:** Award **MO** if candidates substitute into  $f'$  or  $f''$ .

$$c = \frac{5}{4}$$

$$f(x) = -\frac{1}{4}(x^3 + 1)^{-4} + \frac{5}{4} \left( = \frac{-1}{4(x^3 + 1)^4} + \frac{5}{4}, \frac{5(x^3 + 1)^4 - 1}{4(x^3 + 1)^4} \right)$$

## 16N.1.SL.TZ0.S\_6

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of integration **(M1)**

$$\text{eg } \int f'(x) dx$$

correct integration (accept missing C) **(A2)**

$$\text{eg } \frac{1}{2} \times \frac{\sin^4(2x)}{4}, \frac{1}{8}\sin^4(2x) + C$$

substituting initial condition into their integrated expression (must have +C) **M1**

$$\text{eg } 1 = \frac{1}{8}\sin^4\left(\frac{\pi}{2}\right) + C$$

**Note:** Award **M0** if they substitute into the original or differentiated function.

recognizing  $\sin\left(\frac{\pi}{2}\right) = 1$  **(A1)**

$$\text{eg } 1 = \frac{1}{8}(1)^4 + C$$

$$C = \frac{7}{8} \quad \textbf{(A1)}$$

$$f(x) = \frac{1}{8}\sin^4(2x) + \frac{7}{8} \quad \textbf{A1} \quad \textbf{N5}$$

**[7 marks]**

## 19M.1.SL.TZ1.S\_5

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognizing to integrate **(M1)**

$$\text{eg } \int f', \int 2e^{-3x} dx, du = -3$$



correct integral (do not penalize for missing +C) **(A2)**

$$\text{eg } -\frac{2}{3}e^{-3x} + C$$

substituting  $\left(\frac{1}{3}, 5\right)$  (in any order) into **their** integrated expression (must have +C) **M1**

$$\text{eg } -\frac{2}{3}e^{-3(1/3)} + C = 5$$

Award if they substitute into original or differentiated function.

$$f(x) = -\frac{2}{3}e^{-3x} + 5 + \frac{2}{3}e^{-1} \text{ (or any equivalent form, eg } -\frac{2}{3}e^{-3x} + 5 - \frac{2}{-3e})$$

## 16N.1.SL.TZ0.T\_14

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$2x^3 - 3x \quad (\text{A1})(\text{A1}) \quad (\text{C2})$$

**Note:** Award **(A1)** for  $2x^3$ , award **(A1)** for  $-3x$ .

Award at most **(A1)(AO)** if there are any extra terms.

**[2 marks]**

$$\text{b. } 2x^3 - 3x = -10 \quad (\text{M1})$$

**Note:** Award **(M1)** for equating their answer to part (a) to  $-10$ .

$$x = -2 \quad (\text{A1})(\text{ft})$$

**Note:** Follow through from part (a). Award **(M0)(AO)** for  $-2$  seen without working.

$$y = \frac{1}{2}(-2)^4 - \frac{3}{2}(-2)^2 + 7 \quad (\text{M1})$$

**Note:** Award **(M1)** substituting their  $-2$  into the original function.

$$y = 9 \quad (\text{A1})(\text{ft}) \quad (\text{C4}) \quad \text{Note: Accept } (-2, 9). \quad \text{[4 marks]}$$

## 16N.1.SL.TZ0.T\_7

a.



\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

EXAM PAPERS PRACTICE

**Units are required in parts (a) and (b).**

$$\frac{4}{3}\pi \times 6^3 \quad (\text{M1})$$

**Note:** Award **(M1)** for correct substitution into volume of sphere formula.

$$= 905 \text{ cm}^3 (288\pi \text{ cm}^3, 904.778 \dots \text{ cm}^3) \quad (\text{A1}) \quad (\text{C2})$$

**Note:** Answers derived from the use of approximations of  $\pi$  (3.14; 22/7) are awarded **(AO).**

**[2 marks]**

b. **Units are required in parts (a) and (b).**

$$\frac{140}{100} \times 904.778 \dots = \frac{4}{3}\pi r^3 \text{ OR } \frac{140}{100} \times 288\pi = \frac{4}{3}\pi r^3 \text{ OR } 1266.69 \dots = \frac{4}{3}\pi r^3 \quad (\text{M1})(\text{M1})$$

**Note:** Award **(M1)** for multiplying their part (a) by 1.4 or equivalent, **(M1)** for equating to the volume of a sphere formula.

$$r^3 = \frac{3 \times 1266.69 \dots}{4\pi} \text{ OR } r = \sqrt[3]{\frac{3 \times 1266.69 \dots}{4\pi}} \text{ OR } r = \sqrt[3]{(1.4) \times 6^3} \text{ OR } r^3 = 302.4 \quad (\text{M1})$$

**Note:** Award **(M1)** for isolating  $r$ .  $(r = ) 6.71 \text{ cm} (6.71213 \dots ) \quad (\text{A1})(\text{ft}) \quad (\text{C4})$

Follow through from part (a).

**20N.1.SL.TZ0.T\_13**

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$2x + \frac{k}{x^2} \quad (\text{A1})(\text{A1})(\text{A1}) \quad (\text{C3})$$

**Note:** Award **(A1)** for  $2x$ , **(A1)** for  $+k$ , and **(A1)** for  $x^{-2}$  or  $\frac{1}{x^2}$ .  
 Award at most **(A1)(A1)(AO)** if additional terms are seen.

**[3 marks]**

b.  $-2.5 - \frac{5}{2} \quad (\text{A1}) \quad (\text{C1}) \quad [\text{1 mark}]$

c.  $-2.5 = 2 \times -2 + \frac{k}{-2^2} \quad (\text{M1})$

**Note:** Award **(M1)** for equating their gradient from part (b) to their substituted derivative from part (a).

$k = 6 \quad (\text{A1})(\text{ft}) \quad (\text{C2}) \quad \text{Note: Follow through from parts (a) and (b).} \quad [\text{2 marks}]$

## 20N.1.SL.TZ0.T\_15

a.

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$u_1 r = 30 \text{ and } u_1 r^4 = 240, \quad (\text{M1})$$

**Note:** Award **(M1)** for both the given terms expressed in the formula for  $u_n$ .

**OR**

$$30r^3 = 240 \quad r^3 = 8 \quad (\text{M1})$$

**Note:** Award **(M1)** for a correct equation seen.

$$r = 2 \quad (\text{A1}) \quad (\text{C2})$$

**[2 marks]**

b.  $u_1 \times 2 = 30 \text{ OR } u_1 \times 2^4 = 240 \quad (\text{M1})$

Award for their correct substitution in geometric sequence formula.

$$u_1 = 15$$

Follow through from part (a).

$$\text{c. } \frac{152^n - 1}{2 - 1} = 61425$$

Award for correctly substituted geometric series formula equated to 61425

$$n = 12 \text{ (slices)}$$

Follow through from parts (a) and (b).

## 20N.1.SL.TZ0.S\_2

a.

*\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

### METHOD 1 – (sine rule)

evidence of choosing sine rule (M1)

$$\text{eg } \frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b}$$

correct substitution (A1)

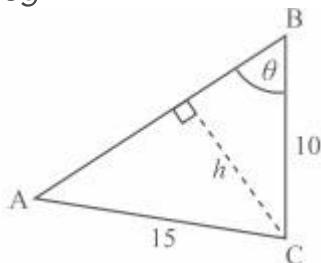
$$\text{eg } \frac{\sqrt{3}}{10} = \frac{\sin \theta}{15}, \quad \frac{\sqrt{3}}{30} = \frac{\sin \theta}{15}, \quad \frac{\sqrt{3}}{30} = \frac{\sin B}{15}$$

$$\sin \theta = \frac{\sqrt{3}}{2} \quad \text{A1 N2}$$

### METHOD 2 – (perpendicular from vertex C)

valid approach to find perpendicular length (may be seen on diagram) (M1)

eg



$$\frac{h}{15} = \frac{\sqrt{3}}{3}$$

correct perpendicular length

$$eg \quad \frac{15\sqrt{3}}{3}, \quad 5\sqrt{3}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

Do not award the final mark if candidate goes on to state  $\sin \theta = \frac{\pi}{3}$ , as this demonstrates a lack of understanding.

b. attempt to substitute into double-angle formula for cosine

$$1 - 2\left(\frac{\sqrt{3}}{3}\right)^2, \quad 2\left(\frac{\sqrt{6}}{3}\right)^2 - 1, \quad \frac{\sqrt{6}}{3}^2 - \frac{\sqrt{3}}{3}^2, \quad \cos 2\theta = 1 - 2\left(\frac{\sqrt{3}}{2}\right)^2, \quad 1 - 2\sin^2\frac{\sqrt{3}}{3}$$

$$\text{correct working} \quad eg \quad 1 - 2 \times \frac{3}{9}, \quad 2 \times \frac{6}{9} - 1, \quad \frac{6}{9} - \frac{3}{9}$$

$$\cos 2 \times \hat{CAB} = \frac{3}{9} = \frac{1}{3}$$

## 19N.1.SL.TZ0.T\_7

a.

$$9 = \left(\frac{8}{3}\right)r^3 \quad (M1)$$

**Note:** Award (M1) for correctly substituted geometric sequence formula equated to 9.

$$(r = ) 1.5 \left(\frac{3}{2}\right) \quad (A1) (C2)$$

**[2 marks]**

b. 4      (A1)(ft) (C1)    **Note:** Follow through from part (a).    [1 mark]

$$c. \quad 2500 < \frac{\left(\frac{8}{3}\right)\left((1.5)^k - 1\right)}{1.5 - 1} \quad (M1)$$

**Note:** Award (M1) for their correctly substituted geometric series formula compared to 2500.

$$k = 15.2 (15.17319 \dots) \quad (A1)(ft) \quad (k = ) 16 \quad (A1)(ft) \quad (C3)$$

**Note:** Answer must be an integer for the final (A1)(ft) to be awarded.    [3 marks]  
Follow through from part (a).

**17N.1.SL.TZ0.T\_2**

$$y = \frac{4}{3}x - \frac{5}{2} \quad (y = 1.33 \dots x - 2.5) \quad (\text{A1})(\text{ft}) \quad (\text{C1})$$

**Note:** Follow through from parts (c)(i) and (a). Award **(AO)** if final answer is not written in the form  $y = mx + c$ .

**[1 mark]**

**EXN.1.SL.TZ0.2**

a.

\* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$$C = kr^3 \quad (\text{M1})$$

$$375 = k \times 0.8^3 \Rightarrow k = 732 \quad 732.421 \dots \quad (\text{M1})$$

$$C = 732r^3$$

A1

**[3 marks]**

$$\text{b. } C = 732.42 \dots \times 1.1^3 \quad (\text{M1}) \quad C = \$ 975 \quad 974.853 \dots \quad \text{A1}$$

**Note:** accept \$ 974 from use of  $C = 732r^3$ . **[2 marks]**

**EXN.1.SL.TZ0.7**

a.

\* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$$\frac{dy}{dx} = 2x - 4 \quad \text{A1}$$

**[1 mark]**

$$\text{b. Gradient at } x = 1 \text{ is } -2 \quad \text{M1} \quad \text{Gradient of normal is } \frac{1}{2} \quad \text{A1}$$

$$\text{When } x = 1 \quad y = 1 - 4 + 2 = -1 \quad (\text{M1})\text{A1} \quad \text{EITHER} \quad y + 1 = \frac{1}{2}x - 1 \quad \text{M1}$$

$$2y + 2 = x - 1 \text{ or } y + 1 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x - \frac{3}{2}$$

$$2y - x + 3 = 0$$

$$1 = \frac{1}{2} \times 1 + c$$

## EXN.1.SL.TZ0.9

a.

\* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

Number of time periods  $12 \times 5 = 60$  (A1)

$N = 60$

$I\% = 3.1$

$PV = 0$

$PMT = 200$

$P/Y = 12$

$C/Y = 12$

Value \$ 12,961.91 (M1)A1

[3 marks]

b. METHOD 1 Real interest rate  $= 3.1 - 2.0 = 1.1\%$

(M1)A1

METHOD 2

$$\frac{1 + 0.031}{1 + 0.02} = 1.01078 \dots \quad (\text{M1}) \quad 1.08\% \text{ (accept } 1.1\%)$$

A1

[2 marks]

c.  $N = 60$

$I\% = 1.1$

$PV = 0$

$PMT = 200$

$P/Y = 12$

$C/Y = 12$

\$ 12,300 (12,330.33 ... ) (M1)A1

Note: Award A1 for \$ 12,300 only.

[2 marks]

## 21M.1.SL.TZ1.2

a.

14

**A1****[1 mark]**

b.  $\frac{14 + 15 + \dots}{10}$  **(M1)** = 13.1 **A1** **[2 marks]**

c. 2.21 (2.21133 ...) **A1** **[1 mark]**

**21M.1.SL.TZ1.3**

a.

$$\frac{1}{2} \times 4 \times \pi \times 6^2 + \pi \times 6^2 \text{ OR } 3 \times \pi \times 6^2$$

**(M1)(A1)(M1)**

**Note:** Award **M1** for use of surface area of a sphere formula (or curved surface area of a hemisphere), **A1** for substituting correct values into hemisphere formula, **M1** for adding the area of the circle.

$$= 339 \text{ mm}^2 \quad 108\pi, 339.292 \dots$$

**A1****[4 marks]**

b.  $\frac{339.292 \dots}{240}$  **(M1)** = 1.41 g  $\frac{9\pi}{20}, 0.45\pi, 1.41371 \dots$  **A1**

**[2 marks]****EXN.1.SL.TZ0.11**

a.

\* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$$\text{Area} = \frac{1}{2} \times 110 \times 85 \times \sin 55^\circ$$

**(M1)(A1)**

$$= 3830 \quad 3829.53 \dots \text{ m}^2$$

**A1**

**Note:** units must be given for the final **A1** to be awarded.

**[3 marks]**

b.  $BC^2 = 110^2 + 85^2 - 2 \times 110 \times 85 \times \cos 55^\circ$  (M1) A1

$BC = 92.7$  (92.7314 ...) (m) A1 **METHOD 1**

Because the height and area of each triangle are equal they must have the same length base R1

D must be placed half-way along BC  $BD = \frac{92.731 \dots}{2} \approx 46.4$  m

the final two marks are dependent on the being awarded.

Let  $\hat{CBA} = \theta^\circ$   $\frac{\sin \theta}{110} = \frac{\sin 55^\circ}{92.731 \dots}$   $\Rightarrow \theta = 76.3^\circ$  76.3354 ... Use of area formula

$\frac{1}{2} \times 85 \times BD \times \sin 76.33 \dots^\circ = \frac{3829.53 \dots}{2}$  BD = 46.4 (46.365 ...) (m)

### 21M.1.SL.TZ2.7

a.

$f^{-7} = 8$  and  $f^7 = 1$  (A1)

range is  $fx \leq 1$ ,  $fx \geq 8$  A1A1

**Note:** Award at most A1A1A0 if strict inequalities are used.

**[3 marks]**

b. EITHER sketch of  $f$  and  $y = 0$  or sketch of  $f^{-1}$  and  $x = 0$  (M1) OR

finding the correct expression of  $f^{-1}x = \frac{-2 - 5x}{x - 2}$  (M1) OR

$f^{-1}0 = \frac{-2 - 50}{0 - 2}$  (M1)  $fx = 0$  (M1)

**[2 marks]**

OR

THEN

$f^{-1}0 = 1$  A1

### 22M.1.SL.TZ1.3

a.

1.2 metres **A1**
**[1 mark]**

b.  $-4.8t^2 + 21t + 1.2 = 0$  **(M1)**  $t =$  4.43 s 4.431415 ... s **A1**

**Note:** If both values for  $t$  are seen do not award the **A1** mark unless the negative is explicitly excluded.

**[2 marks]**

c.  $0 \leq t \leq 4.43$  **OR** 0, 4.43 **A1A1**

**Note:** Award **A1** for correct endpoints and **A1** for expressing answer with correct notation. Award at most **A1A0** for use of  $x$  instead of  $t$ .

**[2 marks]**

### 22M.1.SL.TZ1.5

a.

$x + y + z = 600$  **A1**

$15x + 10y + 12z = 7816$  **A1**

$x = 2y$  **A1**

**Note:** Condone other labelling if clear, e.g.  $a$  (adult),  $c$  (child) and  $s$  (student). Accept equivalent, distinct equations e.g.  $2y + y + z = 600$ .

**[3 marks]**

b.  $x = 308$ ,  $y = 154$ ,  $z = 138$  **A1A1**

**Note:** Award **A1** for all three correct values seen, **A1** for correctly labelled as  $x$ ,  $y$  or  $z$ . Accept answers written in words: e.g. 308 adult tickets.

**[2 marks]**