Helping you Achieve Highest Grades in IB

IB Mathematics (Applications and Interpretations) Higher Level (HL)

Mark Scheme

Fully in-lined with the First Assessment Examinations in 2021 & Beyond

Paper: 1 (All Topics)

- **Topic 1 - Number and Algebra**
- **Topic 2 - Functions**
- **Topic 3 - Geometry and Trigonometry**
- **Topic 4 - Statistics and Probability**
- **Topic 5 - Calculus**

Marks: 304

Total Marks: / 304

Suitable for HL Students sitting the 2026 exams and beyond
However, SL students may also find these resources useful

Markschemes

19M.1.AHL.TZ1.H_7

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempt at implicit differentiation ***M1***

$3y^2\frac{dy}{dx} + 3y^2 + 6xy\frac{dy}{dx} - 3x^2 = 0$ ***A1A1***

Note: Award ***A1*** for the second & third terms, ***A1*** for the first term, fourth term & RHS equal to zero.

substitution of $\frac{dy}{dx} = 0$ ***M1***

$3y^2 - 3x^2 = 0$

$\Rightarrow y = \pm x$ ***A1***

substitute either variable into original equation ***M1***

$y = x \Rightarrow x^3 = 9 \Rightarrow x = \sqrt[3]{9}$ (or $y^3 = 9 \Rightarrow y = \sqrt[3]{9}$) ***A1***

$y = -x \Rightarrow x^3 = 27 \Rightarrow x = 3$ (or $y^3 = -27 \Rightarrow y = -3$) ***A1***

$\left(\sqrt[3]{9}, \sqrt[3]{9}\right)$, (3, −3) ***A1***

[9 marks]

17N.1.AHL.TZ0.H_7

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$x^3 + y^3 - 3xy = 0$

$3x^2 + 3y^2\frac{dy}{dx} - 3x\frac{dy}{dx} - 3y = 0$ ***M1A1***

Note: Differentiation wrt y is also acceptable.

$\frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x}\left(= \frac{y - x^2}{y^2 - x}\right)$ ***(A1)***

Note: All following marks may be awarded if the denominator is correct, but the numerator incorrect.

$y^2 - x = 0$ ***M1***

EITHER

$x = y^2$

$y^6 + y^3 - 3y^3 = 0$ ***M1A1***

$y^6 - 2y^3 = 0$

$y^3(y^3 - 2) = 0$

$(y \neq 0) \therefore y = \sqrt[3]{2}$ ***A1***

$x = \left(\sqrt[3]{2}\right)^2 \left(= \sqrt[3]{4}\right)$ ***A1***

OR

$x^3 + xy - 3xy = 0$

$x(x^2 - 2y) = 0$

$x \neq 0 \Rightarrow y = \frac{x^2}{2}$

$y^2 = \frac{x^4}{4}$

$x = \frac{x^4}{4}$

$x(x^3 - 4) = 0$

$(x \neq 0) \therefore x = \sqrt[3]{4}$

$y = \frac{\left(\sqrt[3]{4}\right)^2}{2} = \sqrt[3]{2}$

18M.1.AHL.TZ1.H_4

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\int_{-2}^{0} f(x)\,dx = 10 - 12 = -2$ ***(M1)(A1)***

$\int_{-2}^{0} 2dx = [2x]_{-2}^{0} = 4$ ***A1***

$\int_{-2}^{0} (f(x) + 2)\,dx = 2$ ***A1***

[4 marks]

b. $\int_{-2}^{0} f(x+2)\,\mathrm{d}x = \int_{0}^{2} f(x)\,\mathrm{d}x$ $= 12$

17N.1.AHL.TZ0.H_5

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$s = \int_0^{\frac{1}{2}} 10t\mathrm{e}^{-2t}\mathrm{d}t$

attempt at integration by parts ***M1***

$= \left[-5t\mathrm{e}^{-2t}\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} -5\mathrm{e}^{-2t}\mathrm{d}t$ ***A1***

$= \left[-5t\mathrm{e}^{-2t} - \frac{5}{2}\mathrm{e}^{-2t}\right]_0^{\frac{1}{2}}$ ***(A1)***

Note: Condone absence of limits (or incorrect limits) and missing factor of 10 up to this point.

$s = \int_0^{\frac{1}{2}} 10t\mathrm{e}^{-2t}\mathrm{d}t$ ***(M1)***

$= -5\mathrm{e}^{-1} + \frac{5}{2} \left(= \frac{-5}{\mathrm{e}} + \frac{5}{2} \right) \left(= \frac{5\mathrm{e} - 10}{2\mathrm{e}} \right)$ ***A1***

[5 marks]

17M.1.AHL.TZ2.H_4

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$s = t + \cos 2t$

$\frac{\mathrm{d}s}{\mathrm{d}t} = 1 - 2\sin 2t$ ***M1A1***

$= 0$ ***M1***

$\Rightarrow \sin 2t = \frac{1}{2}$

$t_1 = \frac{\pi}{12}(s),\ t_2 = \frac{5\pi}{12}(s)$ ***A1A1***

Award if answers are given in degrees.

b. $s = \frac{\pi}{12} + \cos\frac{\pi}{6}\left(s = \frac{\pi}{12} + \frac{\sqrt{3}}{2}(m)\right)$

19M.1.AHL.TZ1.H_5

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

let OX = x

METHOD 1

$\frac{dx}{dt} = 24$ (or −24) ***(A1)***

$\frac{d\theta}{dt} = \frac{dx}{dt} \times \frac{d\theta}{dx}$ ***(M1)***

$3\tan\theta = x$ ***A1***

EITHER

$3\sec^2\theta = \frac{dx}{d\theta}$ ***A1***

$\frac{d\theta}{dt} = \frac{24}{3\sec^2\theta}$

attempt to substitute for $\theta = 0$ into their differential equation ***M1***

OR

$\theta = \arctan\left(\frac{x}{3}\right)$

$\frac{d\theta}{dx} = \frac{1}{3} \times \frac{1}{1 + \frac{x^2}{9}}$ ***A1***

$\frac{d\theta}{dt} = 24 \times \frac{1}{3\left(1 + \frac{x^2}{9}\right)}$

attempt to substitute for $x = 0$ into their differential equation ***M1***

THEN

$\frac{d\theta}{dt} = \frac{24}{3} = 8$ (rad s^{-1}) ***A1***

Note: Accept −8 rad s^{-1}.

METHOD 2

$\frac{dx}{dt} = 24$ (or -24) ***(A1)***

$3\tan\theta = x$ ***A1***

attempt to differentiate implicitly with respect to t ***M1***

$3\sec^2\theta \times \frac{d\theta}{dt} = \frac{dx}{dt}$ ***A1***

$\frac{d\theta}{dt} = \frac{24}{3\sec^2\theta}$

attempt to substitute for $\theta = 0$ into their differential equation ***M1***

$\frac{d\theta}{dt} = \frac{24}{3} = 8$ (rad s^{-1})

Accept -8 rad s^{-1}.

Can be done by consideration of CX, use of Pythagoras.

let the position of the car be at time t be $d - 24t$ from O

$\tan\theta = \frac{d-24t}{3}\left(= \frac{d}{3} - 8t\right)$

For $\tan\theta = \frac{24t}{3}$ award and follow through.

attempt to differentiate implicitly with respect to t

$\sec^2\theta \frac{d\theta}{dt} = -8$

attempt to substitute for $\theta = 0$ into their differential equation

$\theta = \arctan\left(\frac{d}{3} - 8t\right)$

$\frac{d\theta}{dt} = \frac{8}{1 + \left(\frac{d}{3} - 8t\right)^2}$

at O, $t = \frac{d}{24}$

$\frac{d\theta}{dt} = -8$

16N.1.AHL.TZ0.H_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempt to differentiate implicitly ***M1***

$3 - \left(4y\frac{dy}{dx} + 2y^2\right)e^{x-1} = 0$ ***A1A1A1***

Note: Award ***A1*** for correctly differentiating each term.

$\frac{dy}{dx} = \frac{3 \cdot e^{1-x} - 2y^2}{4y}$ ***A1***

Note: This final answer may be expressed in a number of different ways.

[5 marks]

b. $3 - 2y^2 = 2 \Rightarrow y^2 = \frac{1}{2} \Rightarrow y = \pm\sqrt{\frac{1}{2}}$ ***A1*** $\frac{dy}{dx} = \frac{3 - 2 \cdot \frac{1}{2}}{\pm 4\sqrt{\frac{1}{2}}} = \pm\frac{\sqrt{2}}{2}$ ***M1***

at $\left(1, \sqrt{\frac{1}{2}}\right)$ the tangent is $y - \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}(x - 1)$ and ***A1***

at $\left(1, -\sqrt{\frac{1}{2}}\right)$ the tangent is $y + \sqrt{\frac{1}{2}} = -\frac{\sqrt{2}}{2}(x - 1)$ ***A1***

Note: These equations simplify to $y = \pm\frac{\sqrt{2}}{2}x$.

Note: Award ***A0M1A1A0*** if just the positive value of y is considered and just one tangent is found.

[4 marks]

18M.1.AHL.TZ1.H_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempt at chain rule or product rule ***(M1)***

$\frac{dy}{d\theta} = 2\sin\theta\cos\theta$ ***A1***

[2 marks]

b. $2\sin\theta\cos\theta = 2\sin^2\theta$ $\sin\theta = 0$ $\theta = 0, \pi$

obtaining $\cos\theta = \sin\theta$ $\tan\theta = 1$ $\theta = \frac{\pi}{4}$

21M.1.AHL.TZ1.12

a.

$\frac{dV}{dt} = -kV^{\frac{1}{2}}$

use of separation of variables ***(M1)***

$\Rightarrow \int V^{-\frac{1}{2}}\ dV = \int -k\ \ dt$ ***A1***

$2V^{\frac{1}{2}} = -kt\ \ +c$ ***A1***

considering initial conditions $40 = c$ ***A1***

$2\sqrt{324} = -10k + 40$

$\Rightarrow k = 0.4$ ***A1***

$2\sqrt{V} = -0.4t + 40$

$\Rightarrow \sqrt{V} = 20 - 0.2t$ ***A1***

Note: Award ***A1*** for any correct intermediate step that leads to the ***AG***.

$\Rightarrow V = 20 - \frac{t}{5}^2$ ***AG***

Note: Do not award the final ***A1*** if the ***AG*** line is not stated.

[6 marks]

b. $0 = 20 - \frac{t}{5}^2 \Rightarrow t = 100$ minutes ***(M1)A1*** ***[2 marks]***

21N.1.AHL.TZ0.17

a.

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attempt to use $V = \pi \int_a^b x^2 \quad \mathrm{d}\, y$ *(M1)*

$x = e^{\frac{y}{6}}$ or any reasonable attempt to find x in terms of y *(M1)*

$V = \pi \int_0^h e^{\frac{y}{3}} \quad \mathrm{d}\, y$ ***A1***

Note: Correct limits must be seen for the ***A1*** to be awarded.

$= \pi 3e^{\frac{y}{3}}{}_0^h$ *(A1)*

Note: Condone the absence of limits for this ***A1*** mark.

$= 3\pi e^{\frac{h}{3}} - e^0$ ***A1***

$= 3\pi e^{\frac{h}{3}} - 1$ ***AG***

Note: If the variable used in the integral is x instead of y (i.e. $V = \pi \int_0^h e^{\frac{x}{3}} \quad \mathrm{d}\, x$) and the candidate has not stated that they are interchanging x and y then award at most ***M1M1A0A1A1AG***.

[5 marks]

b. maximum volume when $h = 9 \text{ cm}$ *(M1)* max volume $= 180 \text{ cm}^3$ ***A1***

22M.1.AHL.TZ1.17

substitute coordinates of A

$f0 = pe^{q \cos 0} = 6.5$

$6.5 = pe^q$ *(A1)*

substitute coordinates of B

$f5.2 = pe^{q \cos 5.2r} = 0.2$

EITHER

$f't = -pqr \sin rte^{q \cos rt}$ *(M1)*

minimum occurs when $-pqr \sin 5.2re^{q \cos 5.2r} = 0$

$\sin rt = 0$

$r \times 5.2 = \pi$ ***(A1)***

OR

minimum value occurs when $\cos rt = -1$ ***(M1)***

$r \times 5.2 = \pi$ ***(A1)***

OR

period $= 2 \times 5.2 = 10.4$ ***(A1)***

$r = \frac{2\pi}{10.4}$ ***(M1)***

THEN

$r = \frac{\pi}{5.2} = 0.604152 \ldots \quad 0.604$

$0.2 = p \quad e^{-q}$

eliminate p or q

$e^{2q} = \frac{6.5}{0.2} \qquad 0.2 = \frac{p^2}{6.5}$

$q = 1.74 \quad 1.74062 \ldots$

$p = 1.14017 \ldots \quad 1.14$

EXM.1.AHL.TZ0.22

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

Attempting to multiply matrices ***(M1)***

$$\begin{pmatrix} 1 & x & -1 \\ 3 & 1 & 4 \end{pmatrix}\begin{pmatrix} 3 \\ x \\ 2 \end{pmatrix} = \begin{pmatrix} 3 + x^2 - 2 \\ 9 + x + 8 \end{pmatrix}\left(= \begin{pmatrix} 1 + x^2 \\ 17 + x \end{pmatrix}\right)$$ ***A1A1 N3***

[3 marks]

b. Setting up equation ***M1***

$$eg\ \ 2\begin{pmatrix} 1+x^2 \\ 17+x \end{pmatrix} = \begin{pmatrix} 20 \\ 28 \end{pmatrix},\ \begin{pmatrix} 2+2x^2 \\ 34+2x \end{pmatrix} = \begin{pmatrix} 20 \\ 28 \end{pmatrix},\ \begin{pmatrix} 1+x^2 \\ 17+x \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

$$\begin{matrix} 2+2x^2=20 \\ 34+2x=28 \end{matrix}\ \begin{pmatrix} 1+x^2=10 \\ 17+x=14 \end{pmatrix} \qquad x=-3$$

EXM.1.AHL.TZ0.11

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of addition ***(M1)***

eg at least two correct elements

$\boldsymbol{A}+\boldsymbol{B}=\begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}$ ***A1 N2***

[2 marks]

b. evidence of multiplication ***(M1)*** *eg* at least two correct elements

$-3\boldsymbol{A}=\begin{pmatrix} -3 & -6 \\ -9 & 3 \end{pmatrix}$ ***A1 N2*** ***[2 marks]***

c. evidence of matrix multiplication (in correct order) ***(M1)***

eg $\boldsymbol{AB}=\begin{pmatrix} 1(3)+2(-2) & 1(0)+2(1) \\ 3(3)+(-1)(-2) & 3(0)+(-1)(1) \end{pmatrix}$ $\boldsymbol{AB}=\begin{pmatrix} -1 & 2 \\ 11 & -1 \end{pmatrix}$ ***A2 N3***

[3 marks]

EXM.1.AHL.TZ0.14

$\ln v = n\ln w + \ln k$ ***M1A1***

gradient $=\frac{17.5-4.3}{7.1+1.7}\ (=1.5)$ ***M1***

$n=1.5$ ***A1***

y-intercept $=1.5\times1.7+4.3\ (=6.85)$ ***M1***

$k=e^{6.85}=944$ ***M1A1***

[7 marks]

EXM.1.AHL.TZ0.18

a.

$\begin{vmatrix} a-\lambda & 1-b \\ 1-a & b-\lambda \end{vmatrix} = 0 \Rightarrow (a-\lambda)(b-\lambda) - (1-b)(1-a) = 0$ ***M1A1***

$\Rightarrow \lambda^2 - (a+b)\lambda + a + b - 1 = 0 \Rightarrow (\lambda - 1)(\lambda + (1-a-b)) = 0$ ***A1***

$\Rightarrow \lambda = 1 \text{or} \lambda = a+b-1$ ***AGA1***

[4 marks]

b. $\begin{pmatrix} a & 1-b \\ 1-a & b \end{pmatrix}\begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix} \Rightarrow ap + 1 - b - p + bp = p$ ***M1A1***

$\Rightarrow 1-b = (2-a-b)p \Rightarrow p = \frac{1-b}{2-a-b}$ ***M1*** So vector is $\begin{pmatrix} \frac{1-b}{2-a-b} \\ \frac{1-a}{2-a-b} \end{pmatrix}$ ***A1A1***

[5 marks]

EXM.1.AHL.TZ0.25

a.

$A^2 = \begin{pmatrix} 2a & -2 \\ -a & 2a+1 \end{pmatrix}$ ***(M1)A1***

[2 marks]

b. **METHOD 1** det $A^2 = 4a^2 + 2a - 2a = 4a^2$ ***M1***

$a = \pm 2$ ***A1A1 N2*** **METHOD 2** det $A = -2a$ ***M1*** det $A = \pm 4$

$a = \pm 2$ ***A1A1 N2*** ***[3 marks]***

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EXM.1.AHL.TZ0.26

a.

$BA = \left(\begin{pmatrix} 1 & 3 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}\right) = \begin{pmatrix} 18 & -14 \\ -4 & 4 \end{pmatrix}$ ***A2***

Note: Award ***A1*** for one error, ***A0*** for two or more errors.

[2 marks]

b. det(BA) = (72 – 56) = 16 ***(M1)A1*** ***[2 marks]***

c. **EITHER** $A(A^{-1}B + 2A^{-1})A = BA + 2A$ ***(M1)A1*** $= \begin{pmatrix} 24 & -18 \\ 6 & -4 \end{pmatrix}$ ***A1***

$^{-1} = -\frac{1}{2}\begin{pmatrix} -4 & 2 \\ -5 & 3 \end{pmatrix}$ an attempt to evaluate

$^{-1} + 2\ ^{-1} = -\frac{1}{2}\begin{pmatrix} 0 & -16 \\ 1 & -21 \end{pmatrix} - \begin{pmatrix} -4 & 2 \\ -5 & 3 \end{pmatrix}$

$(\ ^{-1} + 2\ ^{-1}) = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}\begin{pmatrix} 4 & 6 \\ 4.5 & 7.5 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$

$= \begin{pmatrix} 3 & 3 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} 24 & -18 \\ 6 & -4 \end{pmatrix}$

EXM.1.AHL.TZ0.27

a.

$(\boldsymbol{A} + \boldsymbol{B})^2 = \boldsymbol{A}^2 + \boldsymbol{AB} + \boldsymbol{BA} + \boldsymbol{B}^2$ ***A2***

Note: Award ***A1*** in parts (a) to (c) if error is correctly identified, but not corrected.

[2 marks]

b. $(\boldsymbol{A} - k\boldsymbol{I})^3 = \boldsymbol{A}^3 - 3k\boldsymbol{A}^2 + 3k^2\boldsymbol{A} - k^3\boldsymbol{I}$ ***A2***

Note: Award ***A1*** in parts (a) to (c) if error is correctly identified, but not corrected.

[2 marks]

c. $\boldsymbol{CA} = \boldsymbol{B} \Rightarrow \boldsymbol{C} = \boldsymbol{BA}^{-1}$ ***A2***

Note: Award ***A1*** in parts (a) to (c) if error is correctly identified, but not corrected.

[2 marks]

EXM.1.AHL.TZ0.28

finding det $\boldsymbol{A} = e^x - e^{-x}(2 + e^x)$ or equivalent ***A1***

$\boldsymbol{A}$ is singular $\Rightarrow$ det $\boldsymbol{A} = 0$ ***(R1)***

$e^x - e^{-x}(2 + e^x) = 0$

$e^{2x} - e^x - 2 = 0$ ***A1***

solving for e^x ***(M1)***

$e^x > 0$ (or equivalent explanation) ***(R1)***

$e^x = 2$

$x = \ln 2$ (only) ***A1 N0***

[6 marks]

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EXM.1.AHL.TZ0.31

det $\boldsymbol{A} = -2$ ***A2***

[2 marks]

EXM.1.AHL.TZ0.32

$\boldsymbol{A}^2 = \begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}$ ***M1***

$= \begin{pmatrix} 1+2k & 0 \\ 0 & 2k+1 \end{pmatrix}$ ***A2***

Note: Award ***A2*** for 4 correct, ***A1*** for 2 or 3 correct.

$1 + 2k = 0$ ***M1***

$k = -\frac{1}{2}$ ***A1***

[5 marks]

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EXM.1.AHL.TZ0.33

$\boldsymbol{M}^2 = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 7 & -6 \\ -18 & 19 \end{pmatrix}$ ***M1A1***

$\Rightarrow \begin{pmatrix} 7 & -6 \\ -18 & 19 \end{pmatrix} - \begin{pmatrix} 12 & -6 \\ -18 & 24 \end{pmatrix} + k\boldsymbol{I} = 0$ ***(M1)***

$\Rightarrow \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} + k\boldsymbol{I} = 0$ ***(A1)***

$\Rightarrow k = 5$ ***A1***

[5 marks]

EXM.1.AHL.TZ0.34

For multiplying $(\boldsymbol{I} - \boldsymbol{X})(\boldsymbol{I} + \boldsymbol{X} + \boldsymbol{X}^2)$ ***M1***

$= \boldsymbol{I}^2 + \boldsymbol{IX} + \boldsymbol{IX}^2 - \boldsymbol{XI} - \boldsymbol{X}^2 - \boldsymbol{X}^3 = \boldsymbol{I} + \boldsymbol{X} + \boldsymbol{X}^2 - \boldsymbol{X} - \boldsymbol{X}^2 - \boldsymbol{X}^3$ ***(A1)(A1)***

$= \boldsymbol{I} - \boldsymbol{X}^3$ ***A1***

$= I$ ***A1***

$AB = I \Rightarrow A^{-1} = B$ ***(R1)***

$(-)(+ + {}^2) = \Rightarrow (-)^{-1} = + + {}^2$

EXM.1.AHL.TZ0.36

We start with point A and write S as the set of vertices and T as the set of edges. The weights on each edge will be used in applying Prim's algorithm.
Initially, $S = \{A\}$, $T = \Phi$. In each subsequent stage, we shall update S and T.

Step 1: Add edge h: So $S = \{A, D\}$, $T = \{h\}$
Step 2: Add edge e: So $S = \{A, D, E\}$ $T = \{h, e\}$
Step 3: Add edge d: Then $S = \{A, D, E, F\}$ $T = \{h, e, d\}$
Step 4: Add edge a: Then $S = \{A, D, E, F, B\}$ $T = \{h, e, d, a\}$
Step 5: Add edge i: Then $S = \{A, D, E, F, B, G\}$ $T = \{h, e, d, a, i\}$
Step 6: Add edge g: Then $S = (A, D, E, F, B, G, C\}$ $T = \{h, e, d, a, I, g\}$ ***(M4)(A3)***

Notes: Award ***(M4)(A3)*** for all 6 correct,
(M4)(A2) for 5 correct;
(M3)(A2) for 4 correct,
(M3)(A1) for 3 correct;
(M1)(A1) for 2 correct,
(M1)(AO) for 1 correct

OR
(M2) for the correct definition of Prim's algorithm,
(M2) for the correct application of Prim's algorithm,
(A3) for the correct answers at the last three stages.

Now S has all the vertices and the minimal spanning tree is obtained.

The weight of the edges in T is $5 + 3 + 5 + 7 + 5 + 6$

$= 31$ ***(A1)***

[8 marks]

EXM.1.AHL.TZ0.37

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Vertices added to the Tree	Edge added	Weight
3	∅	0
5	3, 5	10
6	3, 6	20
7	5, 7	30
10	6, 10	30
1	3, 1	40
2	1, 2	30
11	2, 11	30
9	1, 9	40
4	6, 4	40
8	7, 8	40
		310

(R2)(A4)(M1)

(A1)

N t Award ***(R2)*** for correct algorithms, ***(R1)*** for 1 error, ***(R0)*** for 2 or more errors.
Award for correct calculations, for 1 error, for 2 errors, for 3 errors, for 4 or more errors.
Award for tree/table/method.
Award for minimum weight.

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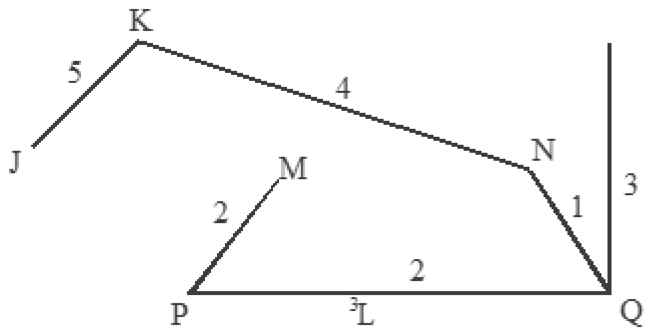
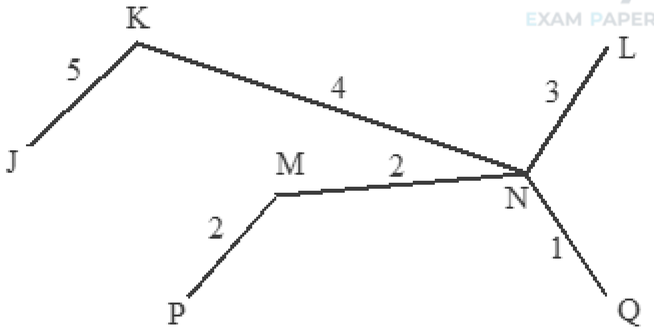
EXM.1.AHL.TZ0.38



(C4)

OR



Total weight = 17

There are other possible spanning trees.

17M.1.AHL.TZ1.H_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

use of $u_n = u_1 + (n-1)d$ ***M1***

$(1+2d)^2 = (1+d)(1+5d)$ (or equivalent) ***M1A1***

$d = -2$ ***A1***

[4 marks]

b. $1 + (N-1) \times -2 = -15$ $N = 9$ ***(A1)*** $\sum_{r=1}^{9} u_r = \frac{9}{2}(2 + 8 \times -2)$ ***(M1)***

$= -63$ ***A1*** ***[3 marks]***

17M.1.AHL.TZ2.H_3

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

EITHER

the first three terms of the geometric sequence are 9, $9r$ and $9r^2$ ***(M1)***

$9 + 3d = 9r(\Rightarrow 3 + d = 3r)$ and $9 + 7d = 9r^2$ ***(A1)***

attempt to solve simultaneously ***(M1)***

$9+7d=9\left(\frac{3+d}{3}\right)^2$

OR

the 1^{st}, 4^{th} and 8^{th} terms of the arithmetic sequence are

$9,\ 9+3d,\ 9+7d$ ***(M1)***

$\frac{9+7d}{9+3d}=\frac{9+3d}{9}$ ***(A1)***

attempt to solve ***(M1)***

THEN

$d=1$

b. $r=\frac{4}{3}$

Accept answers where a candidate obtains d by finding r first. The first two marks in either method for part (a) are awarded for the same ideas and the third mark is awarded for attempting to solve an equation in r.

19M.1.AHL.TZ2.H_1

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempting to form two equations involving u_1 and d ***M1***

$(u_1+2d)+(u_1+7d)=1$ and $\frac{7}{2}[2u_1+6d]=35$

$2u_1+9d=1$

$14u_1+42d=70\ (2u_1+6d=10)$ ***A1***

Note: Award ***A1*** for any two correct equations

attempting to solve their equations: ***M1***

$u_1=14,\ d=-3$ ***A1***

[4 marks]

18M.1.AHL.TZ1.H_5

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$(\ln x)^2 - (\ln 2)(\ln x) - 2(\ln 2)^2 (= 0)$

EITHER

$\ln x = \frac{\ln 2 \pm \sqrt{(\ln 2)^2 + 8(\ln 2)^2}}{2}$ ***M1***

$= \frac{\ln 2 \pm 3\ln 2}{2}$ ***A1***

OR

$(\ln x - 2\ln 2)(\ln x + 2\ln 2)(= 0)$ ***M1A1***

THEN

$\ln x = 2\ln 2$ or $-\ln 2$ ***A1***

$\Rightarrow x = 4$ or $x = \frac{1}{2}$ ***(M1)A1***

Note: ***(M1)*** is for an appropriate use of a log law in either case, dependent on the previous ***M1*** being awarded, **A1** for both correct answers.

solution is $\frac{1}{2} < x < 4$ ***A1***

16N.1.AHL.TZ0.H_7

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempt to form a quadratic in 2^x ***M1***

$(2^x)^2 + 4 \cdot 2^x - 3 = 0$ ***A1***

$2^x = \frac{-4 \pm \sqrt{16 + 12}}{2} \left(= -2 \pm \sqrt{7}\right)$ ***M1***

$2^x = -2 + \sqrt{7} \left(\text{as } -2 - \sqrt{7} < 0\right)$ ***R1***

$x = \log_2\left(-2 + \sqrt{7}\right) \left(x = \frac{\ln\left(-2 + \sqrt{7}\right)}{\ln 2}\right)$ ***A1***

Note: Award ***R0 A1*** if final answer is $x = \log_2\left(-2 + \sqrt{7}\right)$.

[5 marks]

17N.1.AHL.TZ0.H_1

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* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\log_2(x+3) + \log_2(x-3) = 4$

$\log_2(x^2 - 9) = 4$ ***(M1)***

$x^2 - 9 = 2^4 \ (= 16)$ ***M1A1***

$x^2 = 25$

$x = \pm 5$ ***(A1)***

$x = 5$ ***A1***

[5 marks]

17M.1.AHL.TZ1.H_1

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\log_2 x - \log_2 5 = 2 + \log_2 3$

collecting at least two log terms ***(M1)***

eg $\log_2 \frac{x}{5} = 2 + \log_2 3$ or $\log_2 \frac{x}{15} = 2$

obtaining a correct equation without logs ***(M1)***

eg $\frac{x}{5} = 12$ **OR** $\frac{x}{15} = 2^2$ ***(A1)***

$x = 60$ ***A1***

[4 marks]

17M.1.AHL.TZ1.H_2

a.i.

$z_1 = 2\text{cis}\left(\frac{\pi}{3}\right)$ and $z_2 = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$ ***A1A1***

Note: Award ***A1A0*** for correct moduli and arguments found, but not written in mod-arg form.

$|w| = \sqrt{2}$ ***A1***

[3 marks]

a.ii. $z_1 = 2\text{cis}\left(\frac{\pi}{3}\right)$ and $z_2 = \sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$ ***A1A1***

Award for correct moduli and arguments found, but not written in mod-arg form.

$\arg w = \frac{\pi}{12}$

Allow from incorrect answers for z_1 and z_2 in modulus-argument form.

b. $\sin\left(\frac{\pi n}{12}\right) = 0$ $\arg(w^n) = \pi$ $\frac{n\pi}{12} = \pi$

$\therefore n = 12$

17M.1.AHL.TZ2.H_5

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

C represents the complex number $1 - 2\text{i}$ ***A2***

D represents the complex number $3 + 2\text{i}$ ***A2***

[4 marks]

17M.1.AHL.TZ2.H_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$-11 \leqslant f(x) \leqslant 21$ ***A1A1***

Note: ***A1*** for correct end points, ***A1*** for correct inequalities.

[2 marks]

b. $f^{-1}(x) = \sqrt[3]{\frac{x-5}{2}}$ ***(M1)A1*** ***[2 marks]***

c. $-11 \leqslant x \leqslant 21,\ -2 \leqslant f^{-1}(x) \leqslant 2$ ***A1A1*** ***[2 marks]***

18M.1.AHL.TZ1.H_3

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a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$a = \frac{3}{16}$ and $b = \frac{5}{16}$ ***(M1)A1A1***

[3 marks]

Note: Award ***M1*** for consideration of the possible outcomes when rolling the two dice.

b. $\mathrm{E}(T) = \frac{1 + 6 + 15 + 28}{16} = \frac{25}{8}(= 3.125)$ ***(M1)A1***

Note: Allow follow through from part (a) even if probabilities do not add up to 1.

[2 marks]

16N.1.AHL.TZ0.H_1

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1

for eliminating one variable from two equations ***(M1)***

eg, $\begin{cases} (x + y + z = 3) \\ 2x + 2z = 8 \\ 2x + 3z = 11 \end{cases}$ ***A1A1***

for finding correctly one coordinate

eg, $\begin{cases} (x + y + z = 3) \\ (2x + 2z = 8) \\ z = 3 \end{cases}$ ***A1***

for finding correctly the other two coordinates ***A1***

$\Rightarrow \begin{cases} x = 1 \\ y = -1 \\ z = 3 \end{cases}$

the intersection point has coordinates $(1, -1, 3)$

METHOD 2

for eliminating two variables from two equations or using row reduction ***(M1)***

eg, $\begin{cases} (x + y + z = 3) \\ -2 = 2 \\ z = 3 \end{cases}$ **or** $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$ ***A1A1***

for finding correctly the other coordinates ***A1A1***

$$\Rightarrow \begin{cases} x = 1 \\ y = -1 \\ (z = 3) \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

the intersection point has coordinates $(1, \ -1, \ 3)$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -2$$

attempt to use Cramer's rule

$$x = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 5 & -1 & 1 \\ 6 & 1 & 2 \end{vmatrix}}{-2} = \frac{-2}{-2} = 1$$

$$y = \frac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 5 & 1 \\ 1 & 6 & 2 \end{vmatrix}}{-2} = \frac{2}{-2} = -1$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 1 & -1 & 5 \\ 1 & 1 & 6 \end{vmatrix}}{-2} = \frac{-6}{-2} = 3$$

Award only if candidate attempts to determine at least one of the variables using this method.

16N.1.AHL.TZ0.H_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

x	1	2	4	6
$\mathrm{P}(X = x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

A1A1

Note: Award ***A1*** for each correct row.

[2 marks]

b. $\mathrm{E}(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{3} + 4 \times \frac{1}{3} + 6 \times \frac{1}{6} = \frac{19}{6} \left(= 3\frac{1}{6} \right)$

If the probabilities in (a) are not values between 0 and 1 or lead to $\mathrm{E}(X) > 6$ award to correct method using the incorrect probabilities; otherwise allow marks.

19M.1.AHL.TZ1.H_6

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

use of symmetry *eg* diagram ***(M1)***

$\mathrm{P}\left(X > \mu + 5\right) = 0.2$ ***A1***

[2 marks]

b. **EITHER** $\mathrm{P}\left(X < \mu + 5 | X > \mu - 5\right) = \frac{\mathrm{P}(X > \mu - 5 \cap X < \mu + 5)}{\mathrm{P}(X > \mu - 5)}$ ***(M1)***

$= \frac{\mathrm{P}(\mu - 5 < X < \mu + 5)}{\mathrm{P}(X > \mu - 5)}$ ***(A1)*** $= \frac{0.6}{0.8}$ ***A1A1***

Note: ***A1*** for denominator is independent of the previous ***A*** marks. **OR**

use of diagram ***(M1)***

Note: Only award ***(M1)*** if the region $\mu - 5 < X < \mu + 5$ is indicated and used.

$\mathrm{P}\left(X > \mu - 5\right) = 0.8$ $\mathrm{P}\left(\mu - 5 < X < \mu + 5\right) = 0.6$ ***(A1)***

Note: Probabilities can be shown on the diagram. $= \frac{0.6}{0.8}$ ***M1A1*** **THEN**

$= \frac{3}{4} = (0.75)$ ***A1*** ***[5 marks]***

EXM.1.AHL.TZ0.40

Different notations may be used but the edges should be added in the following order.

Using Prim's Algorithm, ***(M1)***

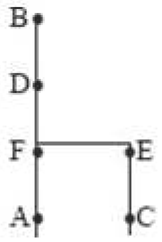
BD ***A1***

DF ***A1***

FA ***A1***

FE ***A1***

EC *A1*

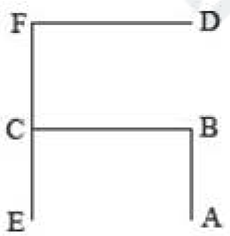


Total weight = 12

EXM.1.AHL.TZ0.41

The edges are introduced in the following order:

FD, FC, CB, BA, CE *A2A2A2A2A2*



A2

[12 marks]

EXM.1.AHL.TZ0.42

$\det(A-kI)=0$

$\Rightarrow \begin{vmatrix} 3-k & 2 \\ -1 & -k \end{vmatrix} = 0$ ***(M1)***

$\Rightarrow k^2-3k+2=0$ ***(M1)***

$\Rightarrow (k-2)(k-1)=0$

$\Rightarrow k=1,2$ ***(A2) (C4)***

[4 marks]

EXM.1.AHL.TZ0.44

$\begin{vmatrix} k-4 & 3 \\ -2 & k+1 \end{vmatrix} = 0$

$\Rightarrow (k-4)(k+1)+6=0$ ***(M1)***

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$\Rightarrow k^2 - 3k + 2 = 0$ ***(M1)***

$\Rightarrow (k-2)(k-1) = 0$

$\Rightarrow k = 2$ or $k = 1$ ***(A1) (C3)***

EXM.1.AHL.TZ0.2

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$2p^2 + 12p = 14$ ***(M1)(A1)***

$p^2 + 6p - 7 = 0$

$(p+7)(p-1) = 0$ ***(A1)***

$p = -7$ or $p = 1$ ***(A1) (C4)***

Note: Both answers are required for the final ***(A1)***.

[4 marks]

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EXM.1.AHL.TZ0.54

a.

Attempting to find det ***A*** ***(M1)***

det ***A*** $= k^2 + 2k - 1$ ***A1 N2***

[2 marks]

b. System has a unique solution provided det ***A*** ≠ 0 ***(R1)***

$k^2 + 2k - 1 \neq 0$ ***(A1)*** Solving $k^2 + 2k - 1 \neq 0$ or equivalent for k ***M1***

$k \in R \backslash \{-1 \pm \sqrt{2}\}$ (accept$k \neq -1 \pm \sqrt{2}, k \neq -2.41, 0.414$) ***A1 N3 [4 marks]***

EXM.1.AHL.TZ0.3

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$B = (BA)A^{-1}$ ***(M1)***

$= -\frac{1}{4}\begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ -2 & 5 \end{pmatrix}$ ***(M1)***

$$= -\frac{1}{4}\begin{pmatrix} -4 & -12 \\ -16 & -48 \end{pmatrix} \quad \textit{(A1)}$$

$$= \begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix} \quad \textit{(A1)}$$

OR

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix} \quad \textit{(M1)}$$

$$\Rightarrow \left.\begin{array}{c} 5a + 2b = 11 \\ 2a = 2 \end{array}\right\}$$

$\Rightarrow a = 1,\ b = 3$ ***(A1)***

$$\left.\begin{array}{c} 5c + 2d = 44 \\ 2c = 8 \end{array}\right\}$$

$\Rightarrow c = 4,\ d = 12$ ***(A1)***

$$B = \begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix}$$

Correct solution with inversion (ie *AB* instead of *BA*) earns FT marks, (maximum
).

EXM.1.AHL.TZ0.7

a.i.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$a = 5$ ***A1 N1***

[1 mark]

a.ii. $b + 9 = 4$ ***(M1)*** $b = -5$ ***A1 N2*** ***[2 marks]***

b. Comparing elements $3(2) - 5(q) = -9$ ***M1*** $q = 3$ ***A2 N2*** ***[3 marks]***

EXM.1.AHL.TZ0.4

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

det $\boldsymbol{A} = 5(1) - 7(-2) = 19$

$$A^{-1} = \frac{1}{19}\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix}$$ ***(A2)***

Note: Award ***(A1)*** for $\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix}$, ***(A1)*** for dividing by 19.

OR

$$A^{-1} = \begin{pmatrix} 0.0526 & 0.105 \\ -0.368 & 0.263 \end{pmatrix}$$ ***(G2)***

[2 marks]

b.i. $XA + B = C \Rightarrow XA = C - B$ ***(M1)*** $X = (C - B)A^{-1}$ ***(A1)*** **OR**

$X = (C - B)A^{-1}$ ***(A2)*** ***[2 marks]***

b.ii.

$$(C - B)A^{-1} = \begin{pmatrix} -11 & -7 \\ -13 & 9 \end{pmatrix}\begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix}$$ ***(A1)***

$$\Rightarrow X = \begin{pmatrix} \frac{38}{19} & \frac{-57}{19} \\ \frac{-76}{19} & \frac{19}{19} \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$$ ***(A1)*** **OR** $X = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$ ***(G2)***

Note: If premultiplication by $\mathbf{A}^{-1}$ is used, award ***(M1)(M0)*** in part (i) but award ***(A2)*** for $\begin{pmatrix} \frac{-37}{19} & \frac{11}{19} \\ \frac{12}{19} & \frac{94}{19} \end{pmatrix}$ in part (ii).

EXM.1.AHL.TZ0.8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$$A^{-1} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{5}{3} & -\frac{7}{3} \\ \frac{2}{3} & -\frac{4}{3} & \frac{5}{3} \end{pmatrix} \text{ or } \begin{pmatrix} -0.333 & 0.667 & -0.333 \\ -0.333 & 1.67 & -2.33 \\ 0.667 & -1.33 & 1.67 \end{pmatrix}$$ ***A2 N2***

[2 marks]

b.i. $X = A^{-1}B$ ***A1 N1*** ***[1 mark]*** b.ii. $= \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$

EXM.1.AHL.TZ0.46

$\boldsymbol{AB} = \begin{pmatrix} x & 4 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 2 & y \\ 8 & 4 \end{pmatrix} = \begin{pmatrix} 2x+32 & xy+16 \\ 24 & 4y+8 \end{pmatrix}$ ***(A1)***

$\boldsymbol{BA} = \begin{pmatrix} 2 & y \\ 8 & 4 \end{pmatrix}\begin{pmatrix} x & 4 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 2x+4y & 2y+8 \\ 8x+16 & 40 \end{pmatrix}$ ***(A1)***

$\boldsymbol{AB} = \boldsymbol{BA} \Rightarrow 8x + 16 = 24$ and $4y + 8 = 40$

This gives $x = 1$ and $y = 8$. ***(A1) (C3)***

[3 marks]

EXM.1.AHL.TZ0.47

singular matrix ⇒ det = 0 ***(R1)***

$\begin{vmatrix} 3-\lambda & -2 \\ -3 & 4-\lambda \end{vmatrix}$ ***(A1)***

$(3-\lambda)(4-\lambda) - 6 = 0$ ***(M1)***

$\Rightarrow \lambda^2 - 7\lambda + 6 = 0$ ***(A1)***

$\lambda = 1$ or 6 ***(A1)(A1) (C6)***

Note: Award ***(C2)*** for one correct answer with no working.

[6 marks]

EXM.1.AHL.TZ0.48

$\boldsymbol{AA}^{-1}\boldsymbol{XB} = \boldsymbol{AC}$ ***(M1)(A1)***

$\boldsymbol{IXBB}^{-1} = \boldsymbol{ACB}^{-1}$ ***(M1)(A1)***

$\boldsymbol{X} = \boldsymbol{ACB}^{-1}$ ***(M1)(A1) (C6)***

[6 marks]

EXM.1.AHL.TZ0.49

For multiplying $(\boldsymbol{I} - \boldsymbol{X})(\boldsymbol{I} + \boldsymbol{X} + \boldsymbol{X}^2)$ ***M1***

$= I^2 + IX + IX^2 - XI - X^2 - X^3 = I + X + X^2 - X - X^2 - X^3$ ***(A1)(A1)***

$= I - X^3$ ***A1***

$= I$ ***A1***

$AB = I \Rightarrow A^{-1} = B$ ***(R1)***

$(I - X)(I + X + X^2) = I \Rightarrow (I - X)^{-1} = I + X + X^2$ ***AG N0***

[6 ma ks]

EXM.1.AHL.TZ0.50

METHOD 1

$A - AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 4 & -9 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} -2 & 12 \\ -1 & -8 \end{pmatrix}$ ***(M1)(A1)***

$X = B^{-1}(A - AB) = B^{-1}\begin{pmatrix} -2 & 12 \\ -1 & -8 \end{pmatrix}$ ***(M1)***

$= -\frac{1}{6}\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -2 & 12 \\ -1 & -8 \end{pmatrix}$ ***(A1)***

$= \begin{pmatrix} -1 & 6 \\ \frac{1}{3} & \frac{8}{3} \end{pmatrix}$ ***(A2) (C6)***

METHOD 2

Attempting to set up a matrix equation ***(M2)***

$X = B^{-1}(A - AB)$ ***(A2)***

$= \begin{pmatrix} -1 & 6 \\ \frac{1}{3} & \frac{8}{3} \end{pmatrix}$ (from GDC) ***(A2) (C6)***

[6 marks]

EXM.1.AHL.TZ0.57

H_0 : The data can be modeled by a Poisson distribution.

H_1 : The data cannot be modeled by a Poisson distribution.

$\sum f = 80, \frac{\sum fx}{\sum f} = \frac{0 \times 4 + 1 \times 18 + 2 \times 19 + \ \dots\ + 5 \times 8}{80} = \frac{200}{80} = 2.5$ ***A1***

Theoretical frequencies are

$f(0) = 8.0e^{-2.5} = 6.5668$ ***(M1)(A1)***

$f(1) = \frac{2.5}{1} \times 6.5668 = 16.4170$ ***A1***

$f(2) = \frac{2.5}{2} \times 16.4170 = 20.5212$

$f(3) = \frac{2.5}{3} \times 20.5212 = 17.1010$

$f(4) = \frac{2.5}{4} \times 17.1010 = 10.6882$ ***A1***

Note: Award A1 for $f(2)$, $f(4)$, $f(4)$.

f(5 or more) $= 80 - (6.5668 + 16.4170 + 20.5212 + 17.1010 + 10.6882)$ ***A1***

$= 8.7058$

Number of cars	0	1	2	3	4	5 or more
O	4	18	19	20	11	8
E	6.5668	16.4170	20.5212	17.1010	10.6882	8.7058

$$\chi^2 = \frac{(4-6.5668)^2}{6.5668} + \frac{(18-16.4170)^2}{16.4170} + \frac{(19-20.5212)^2}{20.5212} + \frac{(20-17.1010)^2}{17.1010} + \frac{(11-10.6882)^2}{10.6882} + \frac{(8-8.7058)^2}{8.7058}$$

$= 1.83$ (accept 1.82) ***(M1)(A1)***

$v = 4$ (six frequencies and two restrictions) ***(A1)***

$\chi^2(4) = 9.488$ at the 5% level. ***A1***

Since 1.83 < 9.488 we accept H_0 and conclude that the distribution can be modeled by a Poisson distribution.

EXM.1.AHL.TZ0.10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$2\boldsymbol{A} = \begin{pmatrix} 6 & 4 \\ 2k & 8 \end{pmatrix}$ ***(A1)***

$2\boldsymbol{A} - \boldsymbol{B} = \begin{pmatrix} 4 & 2 \\ 2k-1 & 5 \end{pmatrix}$ ***A2 N3***

[3 marks]

b. Evidence of using the definition of determinant

Correct substitution *eg* $4(5) - 2(2k - 1)$, $20 - 2(2k - 1)$, $20 - 4k + 2$

det (2 −) = $22 - 4k$

18M.1.AHL.TZ2.H_4

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid attempt to find $\frac{dy}{dx}$ ***M1***

$\frac{dy}{dx} = \frac{1}{(1-x)^2} - \frac{4}{(x-4)^2}$ ***A1A1***

attempt to solve $\frac{dy}{dx} = 0$ ***M1***

$x = 2, x = -2$ ***A1A1***

[6 marks]

18M.1.AHL.TZ2.H_3

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

equating sum of probabilities to 1 ($p + 0.5 - p + 0.25 + 0.125 + p^3 = 1$) ***M1***

$p^3 = 0.125 = \frac{1}{8}$

$p = 0.5$ ***A1***

[2 marks]

b.i. $\mu = 0 \times 0.5 + 1 \times 0 + 2 \times 0.25 + 3 \times 0.125 + 4 \times 0.125$ ***M1***

$= 1.375 \left(= \frac{11}{8}\right)$ ***A1*** ***[2 marks]***

b.ii. $P(X > \mu) = P(X = 2) + P(X = 3) + P(X = 4)$ ***(M1)*** $= 0.5$ ***A1***

Note: Do not award follow through ***A*** marks in (b)(i) from an incorrect value of *p*.

Note: Award ***M*** marks in both (b)(i) and (b)(ii) provided no negative probabilities, and provided a numerical value for μ has been found.

[2 marks]

18N.1.AHL.TZ0.H_5

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a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\boldsymbol{a} \bullet \boldsymbol{b} = (1 \times 0) + (1 \times -t) + (t \times 4t)$ ***(M1)***

$= -t + 4t^2$ ***A1***

[2 marks]

b. recognition that $\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$ ***(M1)***

$\boldsymbol{a} \bullet \boldsymbol{b} < 0$ or $-t + 4t^2 < 0$ or $\cos\theta < 0$ ***R1*** **Note:** Allow ≤ for ***R1***.

attempt to solve using sketch or sign diagram ***(M1)*** $0 < t < \frac{1}{4}$ ***A1*** ***[4 marks]***

19M.1.AHL.TZ1.H_1

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\boldsymbol{a} \bullet \boldsymbol{b} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$

$= -6 + k(k+2) - k$ ***A1***

$\boldsymbol{a} \bullet \boldsymbol{b} = 0$ ***(M1)***

$k^2 + k - 6 = 0$

attempt at solving their quadratic equation ***(M1)***

$(k+3)(k-2) = 0$

$k = -3, 2$ ***A1***

Note: Attempt at solving using $|\boldsymbol{a}||\boldsymbol{b}| = |\boldsymbol{a} \times \boldsymbol{b}|$ will be ***M1A0A0A0*** if neither answer found ***M1(A1)A1A0***
for one correct answer and ***M1(A1)A1A1*** for two correct answers.

[4 marks]

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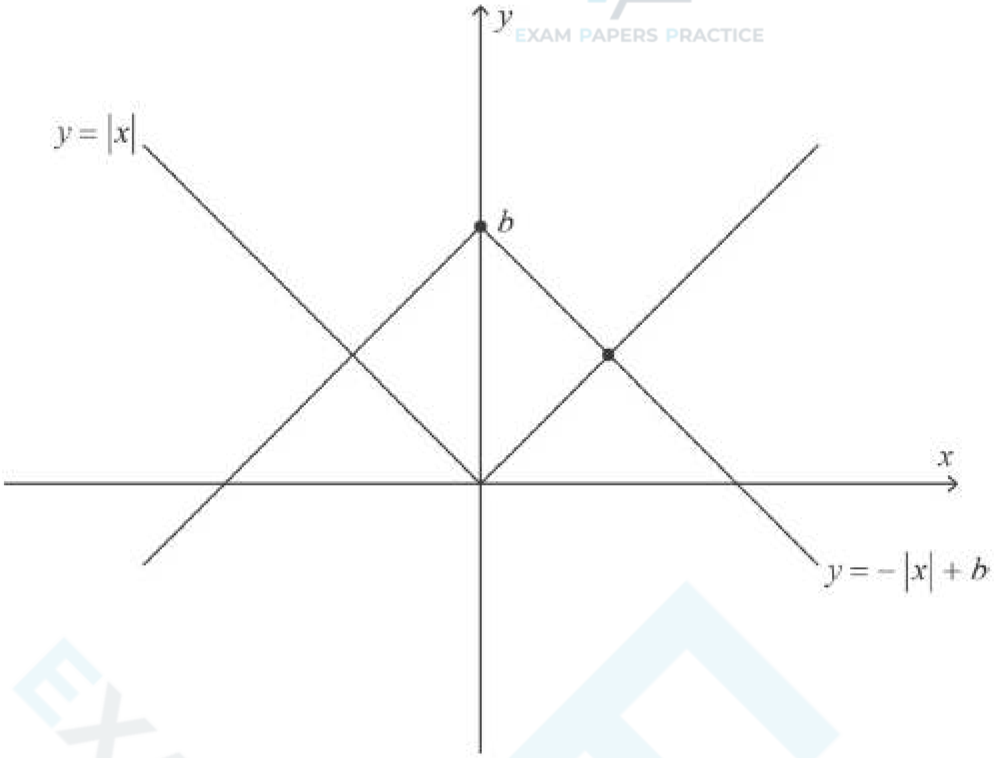
17M.1.AHL.TZ1.H_6

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



graphs sketched correctly (condone missing *b*) ***A1A1***

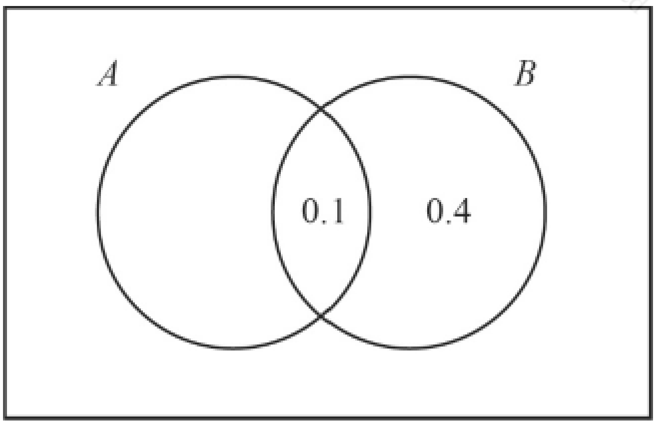
[2 marks]

b. $\frac{b^2}{2} = 18$ $\quad b = 6$

18N.1.AHL.TZ0.H_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(M1)

Note: Award ***M1*** for a Venn diagram with at least one probability in the correct region.

EITHER

$\mathrm{P}\left(A \cap B'\right) = 0.3$ ***(A1)***

$\mathrm{P}(A \cup B) = 0.3 + 0.4 + 0.1 = 0.8$ ***A1***

$\mathrm{P}(B) = 0.5$

$\mathrm{P}(A \cup B) = 0.5 + 0.4 - 0.1 = 0.8$

b. $\mathrm{P}(A)\mathrm{P}(B) = 0.4 \times 0.5$ = 0.2

statement that their $\mathrm{P}(A)\mathrm{P}(B) \neq \mathrm{P}(A \cap B)$

Award for correct reasoning from their value. $\Rightarrow A, B$ not independent

$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \frac{0.1}{0.5}$ = 0.2

statement that their $\mathrm{P}(A|B) \neq \mathrm{P}(A)$

Award for correct reasoning from their value. $\Rightarrow A, B$ not independent

Accept equivalent argument using $\mathrm{P}(B|A) = 0.25$.

16N.1.AHL.TZ0.H_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***M1***

$= \mathrm{P}(A) + \mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B) - \mathrm{P}(A \cap B)$ ***M1A1***

$= \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ ***AG***

METHOD 2

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***M1***

$= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \mid B) \times \mathrm{P}(B)$ ***M1***

$= \mathrm{P}(A) + \left(1 - \mathrm{P}(A \mid B)\right) \times \mathrm{P}(B)$

$= \mathrm{P}(A) + \mathrm{P}(A' \mid B) \times \mathrm{P}(B)$ ***A1***

$= \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ ***AG***

b. (i) use $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ and $\mathrm{P}(A' \cap B) = \mathrm{P}(B \mid A')\mathrm{P}(A')$

$\frac{4}{9} = \mathrm{P}(A) + \frac{1}{6}\left(1 - \mathrm{P}(A)\right)$ $\quad 8 = 18\mathrm{P}(A) + 3\left(1 - \mathrm{P}(A)\right)$ $\quad \mathrm{P}(A) = \frac{1}{3}$

(ii) $\mathrm{P}(B) = \mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B)$

$= \mathrm{P}(B \mid A)\mathrm{P}(A) + \mathrm{P}(B \mid A')\mathrm{P}(A')$ $\quad = \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{2}{3} = \frac{2}{9}$

$\mathrm{P}(A \cap B) = \mathrm{P}(B \mid A)\mathrm{P}(A) \Rightarrow \mathrm{P}(A \cap B) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\mathrm{P}(B) = \mathrm{P}(A \cup B) + \mathrm{P}(A \cap B) - \mathrm{P}(A)$ $\quad \mathrm{P}(B) = \frac{4}{9} + \frac{1}{9} - \frac{1}{3} = \frac{2}{9}$

19M.1.AHL.TZ2.H_3

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$g(x) = f(x+2)\left(= (x+2)^4 - 6(x+2)^2 - 2(x+2) + 4\right)$ ***M1***

attempt to expand $(x+2)^4$ ***M1***

$(x+2)^4 = x^4 + 4(2x^3) + 6\left(2^2x^2\right) + 4\left(2^3x\right) + 2^4$ ***(A1)***

$= x^4 + 8x^3 + 24x^2 + 32x + 16$ ***A1***

$g(x) = x^4 + 8x^3 + 24x^2 + 32x + 16 - 6\left(x^2 + 4x + 4\right) - 2x - 4 + 4$

$= x^4 + 8x^3 + 18x^2 + 6x - 8$ ***A1***

Note: For correct expansion of $f(x-2) = x^4 - 8x^3 + 18x^2 - 10x$ award max ***M0M1(A1)A0A1***.

[5 marks]

SPM.1.AHL.TZ0.10

let T be the time to serve both customers and T_i the time to serve the ith customer

assuming independence of T_1 and T_2 ***R1***

T is normally distributed and $T = T_1 + T_2$ ***(M1)***

$E(T) = 1.5 + 1.5 = 3$ ***A1***

$\mathrm{Var}(T) = 0.4^2 + 0.4^2 = 0.32$ ***M1A1***

$P(T < 4) = 0.961$ ***A1***

[6 marks]

EXAM PAPERS PRACTICE

SPM.1.AHL.TZ0.12

(Model A)

$R = 3pe^{-0.5p}$ ***M1***

predicted values

p	R
1	1.8196
2	2.2073
3	2.0082

(A1)

$SS_{res} = (1.8196 - 1.5)^2 + (2.2073 - 1.8)^2 + (2.0082 - 1.5)^2$ ***(M1)***

= 0.5263… ***A1***

(Model B)

$R = 2.5pe^{-0.6p}$

predicted values

p	R
1	1.372
2	1.506
3	1.2397

(A1)

$SS_{res} =$ 0.170576… ***A1***

chose model B ***A1***

Note: Method marks can be awarded if seen for either model A or model B. Award final ***A1*** if it is a correct deduction from their calculated values for A and B.

[7 marks]

19M.1.AHL.TZ1.H_3

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* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$A = P$

use of the correct formula for area and arc length ***(M1)***

perimeter is $r\theta + 2r$ ***(A1)***

Note: ***A1*** independent of previous ***M1***.

$\frac{1}{2}r^2(1) = r(1) + 2r$ ***A1***

$r^2 - 6r = 0$

$r = 6$ (as $r > 0$) ***A1***

Note: Do not award final ***A1*** if $r = 0$ is included.

[4 marks]

17M.1.AHL.TZ1.H_3

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

use of $\sec^2 x = \tan^2 x + 1$ ***M1***

$\tan^2 x + 2\tan x + 1 = 0$

$(\tan x + 1)^2 = 0$ ***(M1)***

$\tan x = -1$ ***A1***

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ ***A1A1***

METHOD 2

$\frac{1}{\cos^2 x} + \frac{2\sin x}{\cos x} = 0$ ***M1***

$1 + 2\sin x\cos x = 0$

$\sin 2x = -1$ ***M1A1***

$2x = \frac{3\pi}{2}, \frac{7\pi}{2}$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ ***A1A1***

Note: Award ***A1A0*** if extra solutions given or if solutions given in degrees (or both).

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18M.1.AHL.TZ0.F_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1

$\boldsymbol{A}^4 = 4\boldsymbol{A}^2 + 4\boldsymbol{A}\boldsymbol{I} + \boldsymbol{I}^2$ or equivalent ***M1A1***

$= 4(2\boldsymbol{A} + \boldsymbol{I}) + 4\boldsymbol{A} + \boldsymbol{I}$ ***A1***

$= 8\boldsymbol{A} + 4\boldsymbol{I} + 4\boldsymbol{A} + \boldsymbol{I}$

$= 12\boldsymbol{A} + 5\boldsymbol{I}$ ***AG***

[3 marks]

METHOD 2

$\boldsymbol{A}^3 = \boldsymbol{A}(2\boldsymbol{A} + \boldsymbol{I}) = 2\boldsymbol{A}^2 + \boldsymbol{A}\boldsymbol{I} = 2(2\boldsymbol{A} + \boldsymbol{I}) + \boldsymbol{A}(= 5\boldsymbol{A} + 2\boldsymbol{I})$ ***M1A1***

$\boldsymbol{A}^4 = \boldsymbol{A}(5\boldsymbol{A} + 2\boldsymbol{I})$ ***A1***

$= 5\boldsymbol{A}^2 + 2\boldsymbol{A} = 5(2\boldsymbol{A} + \boldsymbol{I}) + 2\boldsymbol{A}$

$= 12\boldsymbol{A} + 5\boldsymbol{I}$ ***AG***

[3 marks]

b. $\boldsymbol{B}^2 = \begin{bmatrix} 18 & 2 \\ 1 & 11 \end{bmatrix}$ ***(A1)*** $\begin{bmatrix} 18 & 2 \\ 1 & 11 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ ***(A1)***

$\Rightarrow k = 10$ ***A1*** ***[3 marks]***

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19M.1.AHL.TZ0.F_13

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

suppose $f(\boldsymbol{X}) = f(\boldsymbol{Y})$, ie $\boldsymbol{AX} = \boldsymbol{AY}$ ***(M1)***

then $\boldsymbol{A}^{-1}\boldsymbol{AX} = \boldsymbol{A}^{-1}\boldsymbol{AY}$ ***A1***

$\boldsymbol{X} = \boldsymbol{Y}$ ***A1***

since $f(\boldsymbol{X}) = f(\boldsymbol{Y}) \Rightarrow \boldsymbol{X} = \boldsymbol{Y}$, f is an injection ***R1***

now suppose $\boldsymbol{C} \in \boldsymbol{M}$ and consider $f(\boldsymbol{D}) = \boldsymbol{C}$, ie $\boldsymbol{AD} = \boldsymbol{C}$ ***M1***

then $\boldsymbol{D} = \boldsymbol{A}^{-1}\boldsymbol{C}$ ($\boldsymbol{A}^{-1}$ exists since $\boldsymbol{A}$ is non- singular) ***A1***

since given $\boldsymbol{C} \in \boldsymbol{M}$, there exists $\boldsymbol{D} \in \boldsymbol{M}$ such that $f(\boldsymbol{D}) = \boldsymbol{C}$, f is a surjection ***R1***

therefore f is a bijection ***AG***

b. suppose $f($ $) =$, ie = then det( )det( ) = det( )

since det( ) = 0, it follows that det( ) = 0

it follows that f is not surjective since the function cannot reach non-singular matrices

therefore f is not a bijection

EXN.1.AHL.TZ0.8

Vertical stretch, scale factor 3 **A1**

Horizontal stretch, scale factor $\frac{1}{\pi} \approx 0.318$ **A1**

Horizontal translation of 1 unit to the right **A1**

Note: The vertical stretch can be at any position in the order of transformations. If the order of the final two transformations are reversed the horizontal translation is π units to the right.

[3 marks]

EXN.1.AHL.TZ0.16

a.

** This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.*

Let X be the number of people who arrive between 9.00 am and 9.01 am

$X \sim \text{Po}9$

$\text{P}X > 7 = \text{P}X \geq 8$ **(M1)**

$0.676 \quad 0.67610 \ldots$ **A1**

[2 marks]

b. Mean number of people arriving each 30 seconds is 4.5 **(M1)**

Let X_1 be the number who arrive in the first 30 seconds and X_2 the number who arrive in the second 30 seconds.

P(Shunsuke will be able to get on the ride)

$= PX_1 \leq 4 \times PX_2 \leq 3 + PX_1 = 5 \times PX_2 \leq 2 + PX_1 = 6 \times PX_2 \leq 1 + PX_1 = 7 \times PX_2 = 0$

for first term, for any of the other terms. null

for one correct value, for four correct values.

$= 0.221 \quad 0.220531 \ldots$

EXN.1.AHL.TZ0.11

** This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.*

Odd vertices are B, F, H and I **(M1)A1**

Pairing the vertices **M1**

BF and HI $9 + 3 = 12$
BH and FI $4 + 11 = 15$
BI and FH $3 + 8 = 11$ **A2**

Note: award **A1** for two correct totals.

Shortest time is $105 + 11 = 116$ (minutes) **M1A1**

[7 marks]

EXN.1.AHL.TZ0.13

a.

** This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.*

$\dot{x} = y$ **M1**

$\dot{y} = 2t - 4y^2$ **A1**

[2 marks]

b. $t_{n+1} = t_n + 0.1 \quad x_{n+1} = x_n + 0.1y_n \quad y_{n+1} = y_n + 0.12t_n - 4y_n^2$ **(M1)(A1)**

Award for a correct attempt to substitute the functions in part (a) into the formula for Euler's method for coupled systems.

When $t = 1$ $x = 0.202$ $0.20201 \ldots$ $\dot{x} = 0.598$ $0.59822 \ldots$

Accept $y = 0.598$.

21N.1.AHL.TZ0.6

a.

$10 = \frac{2}{1 - r}$ ***(M1)***

$r = 0.8$ ***A1***

[2 marks]

b. $2 \times 0.8^{n-1} < 0.5$ **OR** $2 \times 0.8^{n-1} = 0.5$ ***(M1)*** $n >$ $7.212 \ldots$ ***(A1)***

$n = 8$ ***A1***

Note: If $n = 7$ is seen, with or without seeing the value $7.212 \ldots$ then award ***M1A1A0***.

[3 marks]

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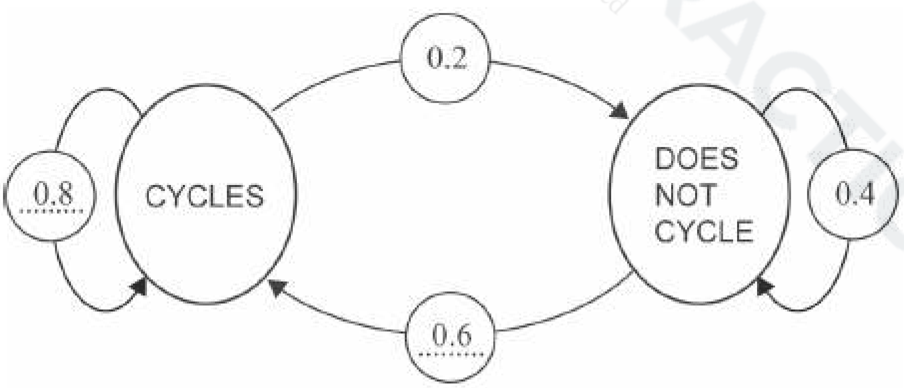
21N.1.AHL.TZ0.9

a.



A1A1

[2 marks]

b. $A = \begin{matrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{matrix}$ ***(A1)*** $A^{180} = \begin{matrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{matrix}$ ***(M1)*** 0.75 ***A1***

EXAM PAPERS PRACTICE

21N.1.AHL.TZ0.11

METHOD 1

attempt to find AC using cosine rule ***M1***

$7^2 = 10^2 + AC^2 - 2 \times 10 \times AC \times \cos\ 40°$ ***(A1)***

attempt to solve a quadratic equation ***(M1)***

$AC = 4.888\ldots$ **AND** $10.432\ldots$ ***(A1)***

Note: At least $AC = 4.888\ldots$ must be seen, or implied by subsequent working.

minimum area $= \frac{1}{2} \times 10 \times 4.888\ldots \times \sin 40°$ ***M1***

Note: Do not award ***M1*** if incorrect value for minimizing the area has been chosen.

$= 15.7\ \ m^2$ ***A1***

METHOD 2

attempt to find $A\hat{C}B$ using the sine Rule ***M1***

$\frac{\sin\ C}{10} = \frac{\sin\ 40}{7}$ ***(A1)***

$C = 66.674\ldots°$ **OR** $113.325\ldots°$ ***(A1)***

EITHER

$B = 180 - 40 - 113.325\ldots$

$B = 26.675\ldots°$ ***(A1)***

area $= \frac{1}{2} \times 10 \times 7 \times \sin 26.675\ldots°$ ***M1***

OR

sine rule or cosine rule to find $AC = 4.888\ldots$ ***(A1)***

minimum area $= \frac{1}{2} \times 10 \times 4.888\ldots \times \sin 40°$ ***M1***

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EXAM PAPERS PRACTICE

$= 15.7 \ \ \mathrm{m}^2$

Award if the wrong length AC or the wrong angle B selected but used correctly finding a value of $33.5 \ \ \mathrm{m}^2$ for the area.

21M.1.AHL.TZ1.1

$X \sim \mathrm{Po}8.8$ ***(M1)***

Note: Award ***(M1)*** for calculating the mean, 8.8, of the distribution

$\mathrm{P}X > 9 = \mathrm{P}X \geq 10$ **OR** $\mathrm{P}X > 9 = 1 - \mathrm{P}X \leq 9$ ***(M1)***

$\mathrm{P}X > 9 = 0.386 \quad (0.386260 \ldots)$ ***(M1)A1***

Note: Award ***(M1)(M0)(M1)A0*** for finding $\mathrm{P}X \geq 9 = 0.518 \quad (0.517719 \ldots)$ **OR** $\mathrm{P}X \leq 9 = 0.614 \quad (0.613740 \ldots)$.

[4 marks]

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21M.1.AHL.TZ1.11

a.

Convenience ***A1***

[1 mark]

b. H_0: 1% of the toys produced are faulty ***A1***

[2 marks]

H_1: More than 1% are faulty ***A1***

c. $X \sim \mathrm{B}200, \ 0.01$ ***(M1)*** $\mathrm{P}X \geq 4 = 0.142$ ***A1***

Note: Any attempt using Normal approximation to find p-value is awarded ***M0A0***.

d. $14\% > 10\%$

so there is insufficient evidence to reject H_0.

Do not award . Accept "fail to reject H_0" or "accept H_0".

21M.1.AHL.TZ1.17

new function is $f x - a + b = \ln x - a + b$ ***(M1)***

$f 0 = \ln\text{-}a + b = 1$ ***A1***

$f e^3 = \ln e^3 - a + b = 1 + \ln\ 2$ ***A1***

$\ln\text{-}a = \ln e^3 - a - \ln\ 2$ ***(M1)***

$\ln\text{-}a = \ln\frac{e^3 - a}{2}$

$-a = \frac{e^3 - a}{2}$

$-2a = e^3 - a$

$a = -e^3 = -20.0855 \ldots$ ***A1***

$b = 1 - \ln\ e^3 = 1 - 3 = -2$ ***(M1)A1***

[7 marks]

21M.1.AHL.TZ2.10

a.

$\bar{x} = \frac{\Sigma x}{n} = \frac{2506}{30} = 83.5 \quad 83.5333 \ldots$ ***A1***

[1 mark]

b. $s_{n-1}^2 = \frac{\Sigma x^2 - \frac{\Sigma x^2}{n}}{n - 1} = \frac{209738 - \frac{2506^2}{30}}{29}$ ***(M1)*** $= 13.9 \quad 13.9126 \ldots$ ***A1***

[2 marks]

c. $82.1, \ 84.9 \quad 82.1405 \ldots, \ 84.9261 \ldots$ ***A2***

[2 marks]

d.

85 is outside the confidence interval and therefore Talha would suggest that the manufacturer's claim is incorrect

The conclusion must refer back to the original claim.

Allow use of a two sided t-test giving a p-value rounding to $0.04 < 0.05$ and therefore Talha would suggest that the manufacturer's claims in incorrect.

21N.1.AHL.TZ0.14

a.

let X be the random variable "the weight of a sack of potatoes"

$\text{P}X < 50$ ***(M1)***

$= 0.588 \text{ kg} \quad 0.587929 \ldots$ ***A1***

[2 marks]

b. $\text{P}X < l = 0.25$ ***(M1)*** $49.2 \text{ kg} \quad 49.1929 \ldots$ ***A1***

[2 marks]

c. attempt to sum 10 independent random variables ***(M1)***

$Y = \sum_{i=1}^{10} X_i \sim \text{N}498, \quad 10 \times 0.9^2$ ***(A1)*** $\text{P}Y > 500 = 0.241$ ***A1***

[3 marks]

22M.1.AHL.TZ2.10

a.

$y = \ln \frac{1}{x - 2}$

an attempt to isolate x (or y if switched) ***(M1)***

$e^y = \frac{1}{x - 2}$

$x - 2 = e^{-y}$

$x = e^{-y} + 2$

switching x and y (seen anywhere) ***M1***

$f^{-1}(x) = e^{-x} + 2$

b. sketch of fx and $f^{-1}(x)$ $x = 2.12$ $2.12002 \ldots$

22M.1.AHL.TZ2.14

$V = \pi \int_0^{10} y^2 \ \mathrm{d}x$ **OR** $\pi \int_0^{10} x^2 \ \mathrm{d}y$ ***(M1)***

$h = 2$

$\approx \pi \times \frac{1}{2} \times 2 \times 4^2 + 5^2 + 2 \times 6^2 + 8^2 + 7^2 + 3^2$ ***M1A1***

$= 1120 \ \text{cm}^3$ $1121.548 \ldots$ ***A1***

Note: Do not award the second ***M1*** If the terms are not squared.

[4 marks]

22M.1.AHL.TZ2.16

a.

attempt at chain rule ***(M1)***

$v = \frac{\mathrm{d}\,OP}{\mathrm{d}\,t} = \begin{matrix} 2t \ \cos \ t^2 \\ -2t \ \sin \ t^2 \end{matrix}$ ***A1***

[2 marks]

b. attempt at product rule ***(M1)*** $a = \begin{matrix} 2 \ \cos \ t^2 - 4t^2 \ \sin \ t^2 \\ -2 \ \sin \ t^2 - 4t^2 \ \cos \ t^2 \end{matrix}$ ***A1***

let $S = \sin \ t^2$ and $C = \cos \ t^2$ finding $\cos \ \theta$ using

METHOD 1

$a \cdot \overrightarrow{\mathrm{OP}} = 2SC - 4t^2S^2 - 2SC - 4t^2C^2 = -4t^2$ ***M1*** $\overrightarrow{\mathrm{OP}} = 1$ $a = \sqrt{2C - 4t^2S^2 + -2S - 4t^2C^2}$

$= \sqrt{4 + 16t^4} > 4t^2$ if θ is the angle between them, then $\cos \ \theta = -\frac{4t^2}{\sqrt{4 + 16t^4}}$ ***A1***

so $-1 < \cos \ \theta < 0$ therefore the vectors are never parallel ***R1*** **METHOD 2** solve $\begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \end{pmatrix} = k \begin{pmatrix} \sin t^2 \\ \cos t^2 \end{pmatrix}$ ***M1*** then

$$k = \frac{2\cos t^2 - 4t^2 \sin t^2}{\sin t^2} = \frac{-2\sin t^2 - 4t^2 \cos t^2}{\cos t^2}$$

N t Condone candidates not excluding the division by zero case here. Some might go straight to the next line.

$2\cos^2 t^2 - 4t^2 \cos t^2 \sin t^2 = -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2$

$2\cos^2 t^2 + 2\sin^2 t^2 = 0 \quad 2 = 0$

this is never true so the two vectors are never parallel

embedding vectors in a 3d space and taking the cross product:

$$\begin{pmatrix} \sin t^2 \\ \cos t^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2 - 2\cos^2 t^2 + 4t^2 \text{ cc} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

since the cross product is never zero, the two vectors are never parallel

22M.1.AHL.TZ2.7

METHOD 1

$\frac{u_1}{1 - r} = 9$ ***A1***

therefore $u_1 = 9 - 9r$

$u_1 = 4 + u_1 r$ ***A1***

substitute or solve graphically: ***M1***

$9 - 9r = 4 + 9 - 9rr$ **OR** $\frac{4}{1 - r^2} = 9$

$9r^2 - 18r + 5 = 0$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

then $u_1 = 9 - 9 \times \frac{1}{3} = 6$

$u_3 = 6 \times \frac{1}{3}^2 = \frac{2}{3}$ ***A1***

METHOD 2

$\frac{u_1}{1 - r} = 9$ ***A1***

$r = \frac{u_1 - 4}{u_1}$ ***A1***

attempt to solve ***M1***

$\frac{u_1}{1 - \frac{u_1 - 4}{u_1}} = 9$

$\frac{u_1}{\frac{4}{u_1}} = 9$

$u_1{}^2 = 36$

$u_1 = \pm 6$

attempting to solve both possible sequences

6, 2, ... or -6, -10 ...

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists

$u_3 = 6 \times \frac{1}{3}^2 = \frac{2}{3}$

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