EXAM PAPERS PRACTICE

Helping you Achieve Highest Grades in IB

IB Mathematics (Applications and Interpretations) Higher Level (HL)

Question Paper

Fully in-lined with the First Assessment
Examinations in 2021 & Beyond

Paper: 1 (All Topics)

- **Topic 1 - Number and Algebra**
- **Topic 2 - Functions**
- **Topic 3 - Geometry and Trigonometry**
- **Topic 4 - Statistics and Probability**
- **Topic 5 - Calculus**

Marks: 304

Total Marks: / 304

Suitable for HL Students sitting the 2026 exams and beyond
However, SL students may also find these resources useful

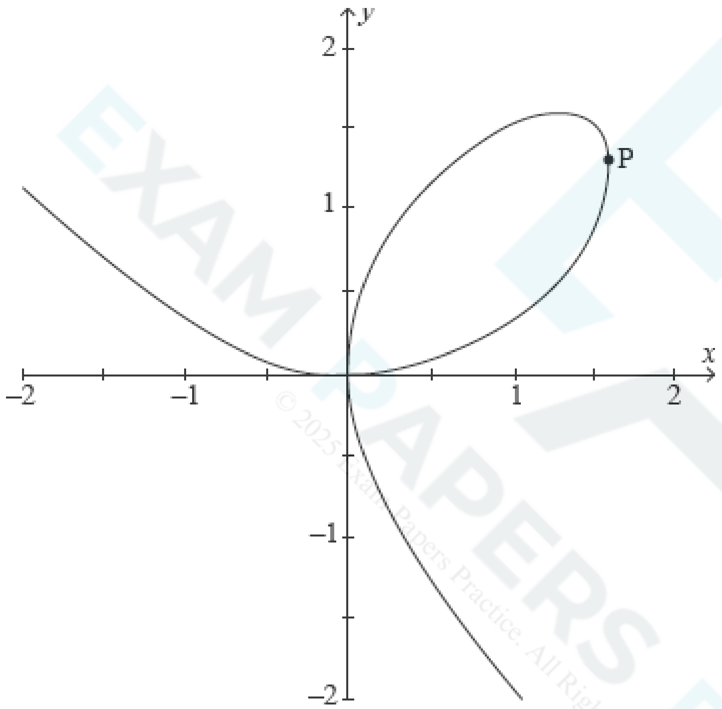
Questions

19M.1.AHL.TZ1.H_7

Find the coordinates of the points on the curve $y^3 + 3xy^2 - x^3 = 27$ at which $\frac{dy}{dx} = 0$.

17N.1.AHL.TZ0.H_7

The folium of Descartes is a curve defined by the equation $x^3 + y^3 - 3xy = 0$, shown in the following diagram.



Determine the exact coordinates of the point P on the curve where the tangent line is parallel to the y-axis.

18M.1.AHL.TZ1.H_4

Given that $\int_{-2}^{2} f(x)\,dx = 10$ and $\int_{0}^{2} f(x)\,dx = 12$, find

a. $\int_{-2}^{0} (f(x) + 2)\,dx$. [4]

b. $\int_{-2}^{0} f(x+2)\,dx$. [2]

17N.1.AHL.TZ0.H_5

A particle moves in a straight line such that at time t seconds ($t \geqslant 0$), its velocity v, in ms^{-1}, is given by $v = 10te^{-2t}$. Find the exact distance travelled by the particle in the first half-second.

17M.1.AHL.TZ2.H_4

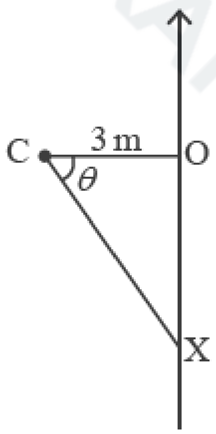
A particle moves along a straight line. Its displacement, s metres, at time t seconds is given by $s = t + \cos 2t,\ t \geqslant 0$. The first two times when the particle is at rest are denoted by t_1 and t_2, where $t_1 < t_2$.

a. Find t_1 and t_2. [5]

b. Find the displacement of the particle when $t = t_1$ [2]

19M.1.AHL.TZ1.H_5

A camera at point C is 3 m from the edge of a straight section of road as shown in the following diagram. The camera detects a car travelling along the road at $t = 0$. It then rotates, always pointing at the car, until the car passes O, the point on the edge of the road closest to the camera.



A car travels along the road at a speed of $24\ \text{ms}^{-1}$. Let the position of the car be X and let $\text{O}\hat{\text{C}}\text{X} = \theta$.

Find $\frac{\text{d}\theta}{\text{d}t}$, the rate of rotation of the camera, in radians per second, at the instant the car passes the point O .

16N.1.AHL.TZ0.H_9

A curve has equation $3x - 2y^2e^{x-1} = 2$.

a. Find an expression for $\frac{\text{d}y}{\text{d}x}$ in terms of x and y. [5]

b.

Find the equations of the tangents to this curve at the points where the curve intersects the line $x = 1$.

[4]

18M.1.AHL.TZ1.H_2

Let $y = \sin^2\theta, 0 \leqslant \theta \leqslant \pi$.

a. Find $\frac{dy}{d\theta}$ [2]

b. Hence find the values of θ for which $\frac{dy}{d\theta} = 2y$. [5]

21M.1.AHL.TZ1.12

A tank of water initially contains 400 litres. Water is leaking from the tank such that after 10 minutes there are 324 litres remaining in the tank.

The volume of water, V litres, remaining in the tank after t minutes, can be modelled by the differential equation

$$\frac{dV}{dt} = -k\sqrt{V}, \text{ where } k \text{ is a constant.}$$

a. Show that $V = 20 - \frac{t}{5}^2$. [6]

b. Find the time taken for the tank to empty. [2]

21N.1.AHL.TZ0.17

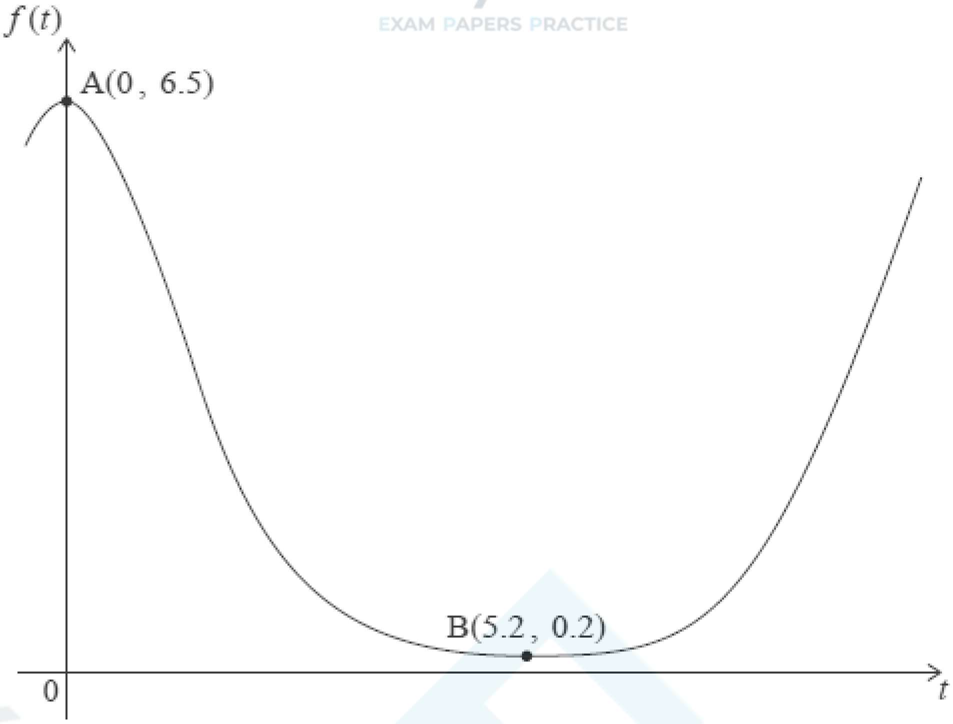
The sides of a bowl are formed by rotating the curve $y = 6 \ln x, \quad 0 \leq y \leq 9$, about the y-axis, where x and y are measured in centimetres. The bowl contains water to a height of h cm.

a. Show that the volume of water, V, in terms of h is $V = 3\pi e^{\frac{h}{3}} - 1$. [5]

b. Hence find the maximum capacity of the bowl in cm^3. [2]

22M.1.AHL.TZ1.17

A function f is of the form $ft = pe^{q \cos rt}, \quad p, \quad q, \quad r \in \mathbb{R}^+$. Part of the graph of f is shown.



The points A and B have coordinates $A(0, \ 6.5)$ and $B(5.2, \ 0.2)$, and lie on f.

The point A is a local maximum and the point B is a local minimum.

Find the value of p, of q and of r.

EXM.1.AHL.TZ0.22

Let $\boldsymbol{A} = \begin{pmatrix} 1 & x & -1 \\ 3 & 1 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 \\ x \\ 2 \end{pmatrix}$.

a. Find $\boldsymbol{AB}$. [3]

b. The matrix $\boldsymbol{C} = \begin{pmatrix} 20 \\ 28 \end{pmatrix}$ and $2\boldsymbol{AB} = \boldsymbol{C}$. Find the value of x. [3]

EXM.1.AHL.TZ0.11

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$.

a. Find $\boldsymbol{A} + \boldsymbol{B}$. [2]

b. Find $-3\boldsymbol{A}$. [2]

c. Find $\boldsymbol{AB}$. [3]

EXM.1.AHL.TZ0.14

It is believed that two variables, v and w are related by the equation $v = kw^n$, where $k,n \in R$. Experimental values of v and w are obtained. A graph of $\ln v$ against $\ln w$ shows a straight line passing through (−1.7, 4.3) and (7.1, 17.5).

Find the value of k and of n.

EXM.1.AHL.TZ0.18

A 2×2 transition matrix for a Markov chain will have the form
$M = \begin{pmatrix} a & 1-b \\ 1-a & b \end{pmatrix}, 0 < a < 1, 0 < b < 1.$

a.

Show that $\lambda = 1$ is always an eigenvalue for **M** and find the other eigenvalue in terms of a and b.

[4]

b. Find the steady state probability vector for **M** in terms of a and b. [5]

EXM.1.AHL.TZ0.25

Consider the matrix $\boldsymbol{A} = \begin{pmatrix} 0 & 2 \\ a & -1 \end{pmatrix}$.

a. [2]

Find the matrix $\boldsymbol{A}^2$.

b. [3]

If $\det \boldsymbol{A}^2 = 16$, determine the possible values of a.

EXM.1.AHL.TZ0.26

Consider the matrices

$$\boldsymbol{A} = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} 1 & 3 \\ 2 & -2 \end{pmatrix}.$$

a. [2]

Find $\boldsymbol{BA}$.

b. [2]

Calculate $\det(\boldsymbol{BA})$.

c. [3]

Find $\boldsymbol{A}(\boldsymbol{A}^{-1}\boldsymbol{B} + 2\boldsymbol{A}^{-1})\boldsymbol{A}$.

EXM.1.AHL.TZ0.27

Let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ be non-singular 2×2 matrices, $\boldsymbol{I}$ the 2×2 identity matrix and k a scalar. The following statements are **incorrect**. For each statement, write down the correct version of the right hand side.

a. $(\boldsymbol{A}+\boldsymbol{B})^2 = \boldsymbol{A}^2 + 2\boldsymbol{AB} + \boldsymbol{B}^2$ [2]

b. $(\boldsymbol{A}-k\boldsymbol{I})^3 = \boldsymbol{A}^3 - 3k\boldsymbol{A}^2 + 3k^2\boldsymbol{A} - k^3$ [2]

c. $\boldsymbol{CA} = \boldsymbol{B} \quad \boldsymbol{C} = \frac{B}{A}$ [2]

EXM.1.AHL.TZ0.28

Consider the matrix $\boldsymbol{A} = \begin{pmatrix} e^x & e^{-x} \\ 2+e^x & 1 \end{pmatrix}$, where $x \in R$.

Find the value of x for which $\boldsymbol{A}$ is singular.

EXM.1.AHL.TZ0.31

Find the determinant of $\boldsymbol{A}$, where $\boldsymbol{A} = \begin{pmatrix} 3 & 1 & 2 \\ 9 & 5 & 8 \\ 7 & 4 & 6 \end{pmatrix}$.

EXM.1.AHL.TZ0.32

If $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}$ and $\boldsymbol{A}^2$ is a matrix whose entries are all 0, find k.

EXM.1.AHL.TZ0.33

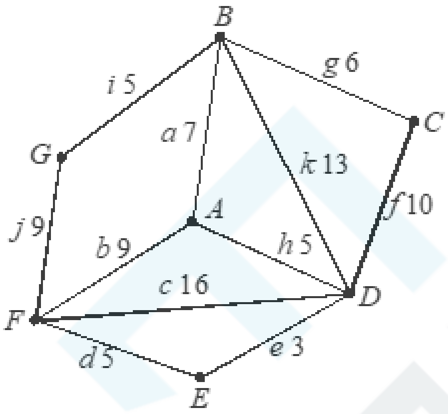
Given that $\boldsymbol{M} = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$ and that $\boldsymbol{M}^2 - 6\boldsymbol{M} + k\boldsymbol{I} = 0$ find k.

EXM.1.AHL.TZ0.34

The square matrix $\boldsymbol{X}$ is such that $\boldsymbol{X}^3 = 0$. Show that the inverse of the matrix $(\boldsymbol{I} - \boldsymbol{X})$ is $\boldsymbol{I} + \boldsymbol{X} + \boldsymbol{X}^2$.

EXM.1.AHL.TZ0.36

Apply Prim's algorithm to the weighted graph given below to obtain the minimal spanning tree starting with the vertex A.



Find the weight of the minimal spanning tree.

EXM.1.AHL.TZ0.37

In this part, marks will only be awarded if you show the correct application of the required algorithms, and show all your working.

In an offshore drilling site for a large oil company, the distances between the planned wells are given below in metres.

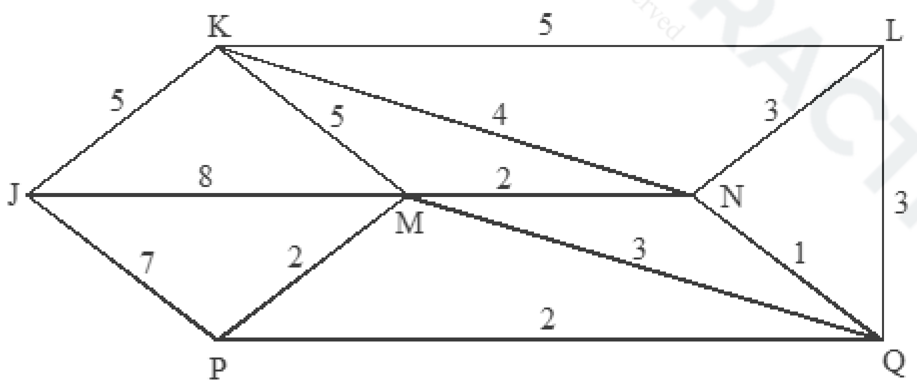
	1	2	3	4	5	6	7	8	9	10
2	30									
3	40	60								
4	90	190	130							
5	80	200	10	160						
6	70	40	20	40	130					
7	60	120	50	90	30	60				
8	50	140	90	70	140	70	40			
9	40	170	140	60	50	90	50	70		
10	200	80	150	110	90	30	190	90	100	
11	150	30	200	120	190	120	60	190	150	200

It is intended to construct a network of paths to connect the different wells in a way that minimises the sum of the distances between them.

Use Prim's algorithm, starting at vertex 3, to find a network of paths of minimum total length that can span the whole site.

EXM.1.AHL.TZ0.38

The diagram below shows a weighted graph.



Use Prim's algorithms to find a minimal spanning tree, starting at J. Draw the tree, and find its total weight.

17M.1.AHL.TZ1.H_7

An arithmetic sequence u_1, u_2, u_3 ... has $u_1 = 1$ and common difference $d \neq 0$. Given that u_2, u_3 and u_6 are the first three terms of a geometric sequence

Given that $u_N = -15$

a. find the value of d. [4]

b. determine the value of $\sum_{r=1}^{N} u_r$. [3]

17M.1.AHL.TZ2.H_3

The 1st, 4th and 8th terms of an arithmetic sequence, with common difference d, $d \neq 0$, are the first three terms of a geometric sequence, with common ratio r. Given that the 1st term of both sequences is 9 find

a. the value of d; [4]

b. the value of r; [1]

19M.1.AHL.TZ2.H_1

In an arithmetic sequence, the sum of the 3rd and 8th terms is 1.

Given that the sum of the first seven terms is 35, determine the first term and the common difference.

18M.1.AHL.TZ1.H_5

Solve $(\ln x)^2 - (\ln 2)(\ln x) < 2(\ln 2)^2$.

16N.1.AHL.TZ0.H_7

Solve the equation $4^x + 2^{x+2} = 3$.

17N.1.AHL.TZ0.H_1

Solve the equation $\log_2(x+3) + \log_2(x-3) = 4$.

17M.1.AHL.TZ1.H_1

Find the solution of $\log_2 x - \log_2 5 = 2 + \log_2 3$.

17M.1.AHL.TZ1.H_2

Consider the complex numbers $z_1 = 1 + \sqrt{3}\mathrm{i}$, $z_2 = 1 + \mathrm{i}$ and $w = \frac{z_1}{z_2}$.

a.i.

By expressing z_1 and z_2 in modulus-argument form write down the modulus of w;

[3]

a.ii.

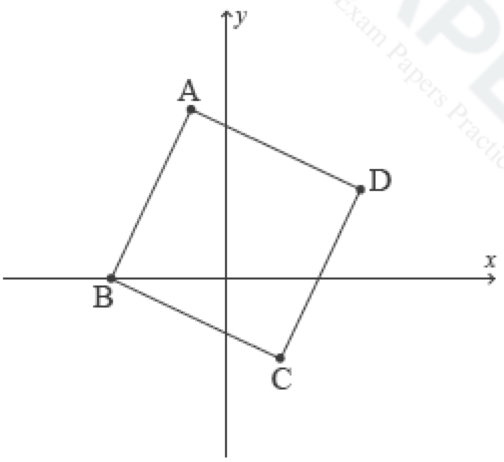
By expressing z_1 and z_2 in modulus-argument form write down the argument of w.

[1]

b. Find the smallest positive integer value of n, such that w^n is a real number. [2]

17M.1.AHL.TZ2.H_5

In the following Argand diagram the point A represents the complex number $-1 + 4\mathrm{i}$ and the point B represents the complex number $-3 + 0\mathrm{i}$. The shape of ABCD is a square. Determine the complex numbers represented by the points C and D.



17M.1.AHL.TZ2.H_2

The function f is defined by $f(x) = 2x^3 + 5,\ -2 \leqslant x \leqslant 2$.

a. Write down the range of f. [2]

b. Find an expression for $f^{-1}(x)$. [2]

c. Write down the domain and range of f^{-1}. [2]

18M.1.AHL.TZ1.H_3

Two unbiased tetrahedral (four-sided) dice with faces labelled 1, 2, 3, 4 are thrown and the scores recorded. Let the random variable T be the maximum of these two scores.

The probability distribution of T is given in the following table.

t	1	2	3	4
$P(T = t)$	$\frac{1}{16}$	a	b	$\frac{7}{16}$

a. Find the value of a and the value of b. [3]

b. Find the expected value of T. [2]

16N.1.AHL.TZ0.H_1

Find the coordinates of the point of intersection of the planes defined by the equations $x + y + z = 3$, $x - y + z = 5$ and $x + y + 2z = 6$.

16N.1.AHL.TZ0.H_2

The faces of a fair six-sided die are numbered 1, 2, 2, 4, 4, 6. Let X be the discrete random variable that models the score obtained when this die is rolled.

a. Complete the probability distribution table for X. [2]

x				
$P(X = x)$				

b. Find the expected value of X. [2]

19M.1.AHL.TZ1.H_6

Let X be a random variable which follows a normal distribution with mean μ. Given that $P(X < \mu - 5) = 0.2$, find

a. $P(X > \mu + 5)$. [2]

b. $P(X < \mu + 5 | X > \mu - 5)$. [5]

EXM.1.AHL.TZ0.40

The weights of the edges of a complete graph G are shown in the following table.

	A	B	C	D	E	F
A	–	5	4	7	6	2
B	5	–	6	3	5	4
C	4	6	–	8	1	6
D	7	3	8	–	7	3
E	6	5	1	7	–	3
F	2	4	6	3	3	–

Starting at *B*, use Prim's algorithm to find and draw a minimum spanning tree for *G*. Your solution should indicate the order in which the vertices are added. State the total weight of your tree.

EXM.1.AHL.TZ0.41

The weights of the edges in a simple graph G are given in the following table.

Vertices	A	B	C	D	E	F
A	–	4	6	16	15	17
B	4	–	5	17	9	16
C	6	5	–	15	8	14
D	16	17	15	–	15	7
E	15	9	8	15	–	18
F	17	16	14	7	18	–

Use Prim's Algorithm, starting with vertex F, to find and draw the minimum spanning tree for *G*. Your solution should indicate the order in which the edges are introduced.

EXM.1.AHL.TZ0.42

Given the matrix $\boldsymbol{A} = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$ find the values of the real number k for which $\det(A - kI) = 0$ where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

EXM.1.AHL.TZ0.44

Find the values of the real number k for which the determinant of the matrix $\begin{pmatrix} k-4 & 3 \\ -2 & k+1 \end{pmatrix}$ is equal to zero.

EXM.1.AHL.TZ0.2

If $A = \begin{pmatrix} 2p & 3 \\ -4p & p \end{pmatrix}$ and $\det A = 14$, find the possible values of p.

EXM.1.AHL.TZ0.54

Consider the system of equations $\boldsymbol{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ where $\boldsymbol{A} = \begin{pmatrix} k+1 & -k \\ 2 & k-1 \end{pmatrix}$ and $k \in R$.

a. Find $\det \boldsymbol{A}$. [2]

b. Find the set of values of k for which the system has a unique solution. [4]

EXM.1.AHL.TZ0.3

A and B are 2 × 2 matrices, where $A = \begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix}$ and $BA = \begin{bmatrix} 11 & 2 \\ 44 & 8 \end{bmatrix}$. Find B

EXM.1.AHL.TZ0.7

Let $\begin{pmatrix} b & 3 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 9 & 5 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ a & 15 \end{pmatrix}$.

a.i. Write down the value of a. [1]

a.ii. Find the value of b. [2]

b. Let $3\begin{pmatrix} -4 & 8 \\ 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 \\ q & -4 \end{pmatrix} = \begin{pmatrix} -22 & 24 \\ 9 & 23 \end{pmatrix}$. Find the value of q. [3]

EXM.1.AHL.TZ0.4

Consider the matrix $\boldsymbol{A} = \begin{pmatrix} 5 & -2 \\ 7 & 1 \end{pmatrix}$.

B, ***C*** and ***X*** are also 2 × 2 matrices.

a. Write down the inverse, $\boldsymbol{A}^{-1}$. [2]

b.i. Given that ***XA*** + ***B*** = ***C***, express ***X*** in terms of $\boldsymbol{A}^{-1}$, ***B*** and ***C***. [2]

b.ii. Given that $= \begin{pmatrix} 6 & 7 \\ 5 & -2 \end{pmatrix}$, and $= \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix}$, find . [2]

EXM.1.AHL.TZ0.8

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$, and $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Consider the equation ***AX*** = ***B***.

a. Write down the inverse matrix $\boldsymbol{A}^{-1}$. [2]

b.i. Express ***X*** in terms of $\boldsymbol{A}^{-1}$ and ***B***. [1]

b.ii. **Hence**, solve for ***X***. [3]

EXM.1.AHL.TZ0.46

If $\boldsymbol{A} = \begin{pmatrix} x & 4 \\ 4 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & y \\ 8 & 4 \end{pmatrix}$, find 2 values of x and y, given that ***AB*** = ***BA***.

EXM.1.AHL.TZ0.47

Given that $\boldsymbol{A} = \begin{pmatrix} 3 & -2 \\ -3 & 4 \end{pmatrix}$ and $\boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the values of λ for which $(\boldsymbol{A} - \lambda \boldsymbol{I})$ is a singular matrix.

EXM.1.AHL.TZ0.48

The matrices ***A***, ***B***, ***C*** and ***X*** are all non-singular 3 × 3 matrices.

Given that $\boldsymbol{A}^{-1}\boldsymbol{X}\boldsymbol{B} = \boldsymbol{C}$, express ***X*** in terms of the other matrices.

EXM.1.AHL.TZ0.49

The square matrix ***X*** is such that $\boldsymbol{X}^3 = 0$. Show that the inverse of the matrix $(\boldsymbol{I} - \boldsymbol{X})$ is $\boldsymbol{I} + \boldsymbol{X} + \boldsymbol{X}^2$.

EXM.1.AHL.TZ0.50

Given that $\boldsymbol{A} = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$, find $\boldsymbol{X}$ if $\boldsymbol{BX} = \boldsymbol{A} - \boldsymbol{AB}$.

EXM.1.AHL.TZ0.57

The number of cars passing a certain point in a road was recorded during 80 equal time intervals and summarized in the table below.

Number of cars	0	1	2	3	4	5
Frequency	4	18	19	20	11	8

Carry out a χ^2 goodness of fit test at the 5% significance level to decide if the above data can be modelled by a Poisson distribution.

EXM.1.AHL.TZ0.10

Let $\boldsymbol{A} = \begin{pmatrix} 3 & 2 \\ k & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find, in terms of k,

a. $2\boldsymbol{A} - \boldsymbol{B}$. [3]

b. $\det(2\boldsymbol{A} - \boldsymbol{B})$. [3]

18M.1.AHL.TZ2.H_4

Consider the curve $y = \frac{1}{1-x} + \frac{4}{x-4}$.

Find the x-coordinates of the points on the curve where the gradient is zero.

18M.1.AHL.TZ2.H_3

The discrete random variable X has the following probability distribution, where p is a constant.

x	0	1	2	3	4
$\mathrm{P}(X=x)$	p	$0.5-p$	0.25	0.125	p^3

a. Find the value of p. [2]

b.i. Find μ, the expected value of X. [2]

b.ii. Find $\mathrm{P}(X > \mu)$. [2]

18N.1.AHL.TZ0.H_5

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in R$.

a. Find and simplify an expression for $\boldsymbol{a} \bullet \boldsymbol{b}$ in terms of t. [2]

b.

Hence or otherwise, find the values of t for which the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtuse .

[4]

19M.1.AHL.TZ1.H_1

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$, $k \in R$.

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, find the possible values of k.

17M.1.AHL.TZ1.H_6

Consider the graphs of $y = |x|$ and $y = -|x| + b$, where $b \in Z^+$.

a. Sketch the graphs on the same set of axes. [2]

b.

Given that the graphs enclose a region of area 18 square units, find the value of b.

[3]

18N.1.AHL.TZ0.H_1

Consider two events, A and B, such that $\mathrm{P}(A) = \mathrm{P}\left(A' \cap B\right) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$.

a. By drawing a Venn diagram, or otherwise, find $\mathrm{P}(A \cup B)$. [3]

b. Show that the events A and B are not independent. [3]

16N.1.AHL.TZ0.H_10

Consider two events A and A defined in the same sample space.

Given that $\mathrm{P}(A \cup B) = \frac{4}{9}$, $\mathrm{P}(B \mid A) = \frac{1}{3}$ and $\mathrm{P}(B \mid A') = \frac{1}{6}$,

a. Show that $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(A' \cap B)$. [3]

b. (i) show that $\mathrm{P}(A) = \frac{1}{3}$; (ii) hence find $\mathrm{P}(B)$. [6]

19M.1.AHL.TZ2.H_3

Consider the function $f(x) = x^4 - 6x^2 - 2x + 4$, $x \in R$.

The graph of f is translated two units to the left to form the function $g(x)$.

Express $g(x)$ in the form $ax^4 + bx^3 + cx^2 + dx + e$ where $a, b, c, d, e \in Z$.

SPM.1.AHL.TZ0.10

In a coffee shop, the time it takes to serve a customer can be modelled by a normal distribution with a mean of 1.5 minutes and a standard deviation of 0.4 minutes.

Two customers enter the shop together. They are served one at a time.

Find the probability that the total time taken to serve both customers will be less than 4 minutes.

Clearly state any assumptions you have made.

SPM.1.AHL.TZ0.12

Product research leads a company to believe that the revenue (R) made by selling its goods at a price (p) can be modelled by the equation.

$$R(p) = cpe^{dp}, \quad c, d \in R$$

There are two competing models, A and B with different values for the parameters c and d.

Model A has $c = 3$, $d = -0.5$ and model B has $c = 2.5$, $d = -0.6$.

The company experiments by selling the goods at three different prices in three similar areas and the results are shown in the following table.

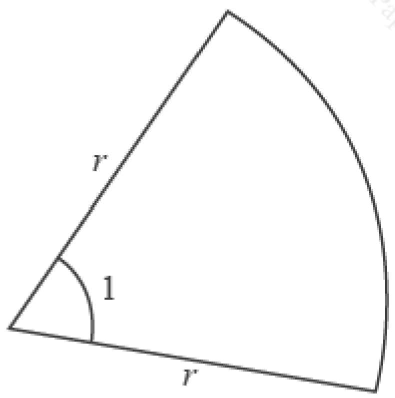
Area	Price (p)	Revenue (R)
1	1	1.5
2	2	1.8
3	3	1.5

The company will choose the model with the smallest value for the sum of square residuals.

Determine which model the company chose.

19M.1.AHL.TZ1.H_3

A sector of a circle with radius r cm , where $r > 0$, is shown on the following diagram. The sector has an angle of 1 radian at the centre.



Let the area of the sector be A cm^2 and the perimeter be P cm. Given that $A = P$, find the value of r.

17M.1.AHL.TZ1.H_3

Solve the equation $\sec^2 x + 2\tan x = 0,\ 0 \leqslant x \leqslant 2\pi$.

18M.1.AHL.TZ0.F_2

Let $\boldsymbol{A}^2 = 2\boldsymbol{A} + \boldsymbol{I}$ where $\boldsymbol{A}$ is a 2 × 2 matrix.

a. Show that $\boldsymbol{A}^4 = 12\boldsymbol{A} + 5\boldsymbol{I}$. [3]

b. Let $\boldsymbol{B} = \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix}$. Given that $\boldsymbol{B}^2 - \boldsymbol{B} - 4\boldsymbol{I} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, find the value of k. [3]

19M.1.AHL.TZ0.F_13

The function $f: \boldsymbol{M} \to \boldsymbol{M}$ where $\boldsymbol{M}$ is the set of 2 × 2 matrices, is given by $f(\boldsymbol{X}) = \boldsymbol{AX}$ where $\boldsymbol{A}$ is a 2 × 2 matrix.

a. Given that $\boldsymbol{A}$ is non-singular, prove that f is a bijection. [7]

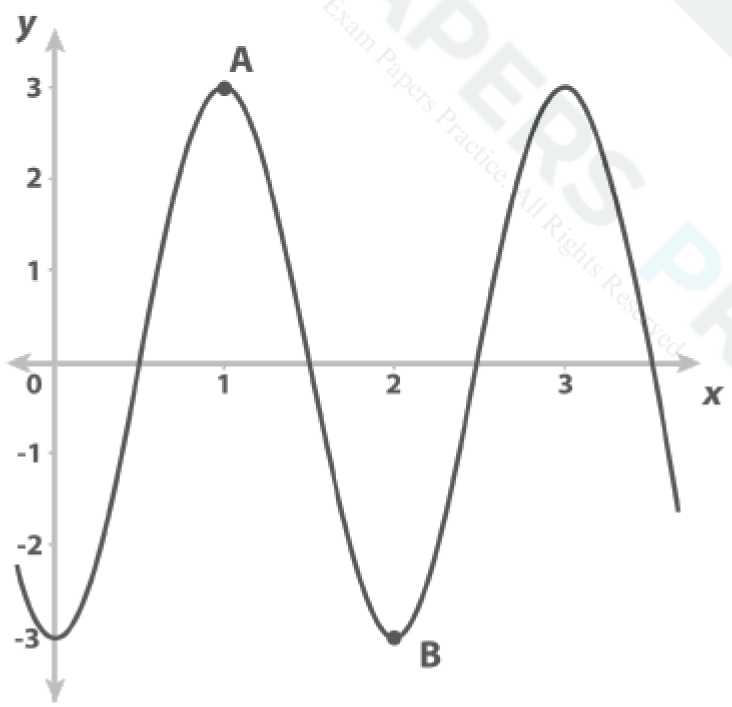
b. It is now given that $\boldsymbol{A}$ is singular.

By considering appropriate determinants, prove that f is not a bijection. [4]

EXN.1.AHL.TZ0.8

Let $f x = a \ \cos \ \ bx - c, \quad a, b, c \in \mathbb{R}^+$.

Part of the graph of $y = f x$ is shown below. Point A is a local maximum and has coordinates $1, \ \ 3$ and point B is a local minimum with coordinates $2, \ \ -3$.



Write down a sequence of transformations that will transform the graph of $y = \cos \ \ x$ onto the graph of $y = f x$.

EXN.1.AHL.TZ0.16

The cars for a fairground ride hold four people. They arrive at the platform for loading and unloading every 30 seconds.

During the hour from 9 am the arrival of people at the ride in any interval of t minutes can be modelled by a Poisson distribution with a mean of $9t \quad 0 < t < 60$.

When the 9 am car leaves there is no one in the queue to get on the ride.

Shunsuke arrives at 9.01 am.

a. Find the probability that more than 7 people arrive at the ride before Shunsuke. [2]

b. Find the probability there will be space for him on the 9.01 car. [6]

EXN.1.AHL.TZ0.11

Nymphenburg Palace in Munich has extensive grounds with 9 points of interest (stations) within them.

These nine points, along with the palace, are shown as the vertices in the graph below. The weights on the edges are the walking times in minutes between each of the stations and the total of all the weights is 105 minutes.

A	Nymphenburg Palace
B	Fountain
C	Magdalenenklause
D	Pagodenburg
E	Large Cascade
F	Pan with Spring
G	Badenburg
H	Amalienburg
I	Summerhouse
J	Bridge

Anders decides he would like to walk along all the paths shown beginning and ending at the Palace (vertex A).

Use the Chinese Postman algorithm, clearly showing all the stages, to find the shortest time to walk along all the paths.

EXN.1.AHL.TZ0.13

Consider the second order differential equation

$\ddot{x} + 4\dot{x}^2 - 2t = 0$

where x is the displacement of a particle for $t \geq 0$.

a.

Write the differential equation as a system of coupled first order differential equations.

[2]

b. When $t = 0$, $x = \dot{x} = 0$

Use Euler's method with a step length of 0.1 to find an estimate for the value of the displacement and velocity of the particle when $t = 1$.

[4]

21N.1.AHL.TZ0.6

An infinite geometric sequence, with terms u_n, is such that $u_1 = 2$ and $\sum_{k=1}^{\infty} u_k = 10$.

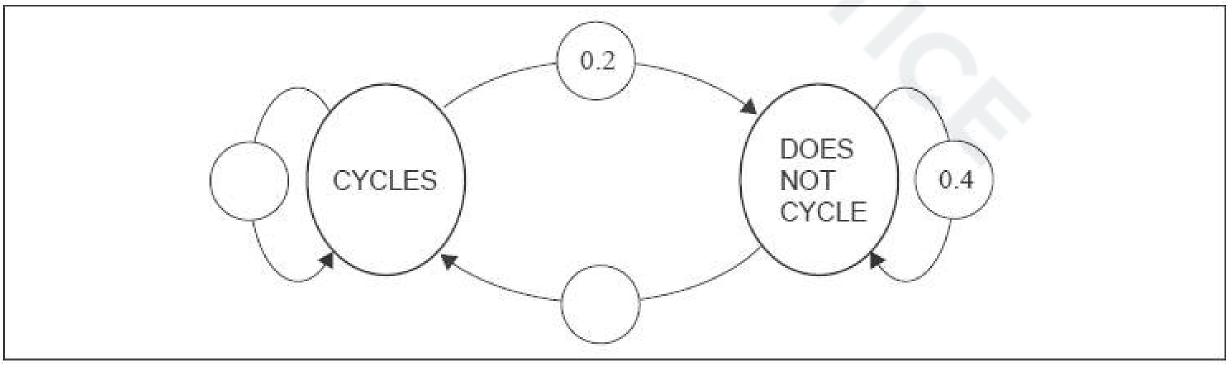
a. Find the common ratio, r, for the sequence. [2]

b. Find the least value of n such that $u_n < \frac{1}{2}$. [3]

21N.1.AHL.TZ0.9

Katie likes to cycle to work as much as possible. If Katie cycles to work one day then she has a probability of 0.2 of not cycling to work on the next work day. If she does not cycle to work one day then she has a probability of 0.4 of not cycling to work on the next work day.

a. Complete the following transition diagram to represent this information.



[2]

b. Katie works for 180 days in a year.

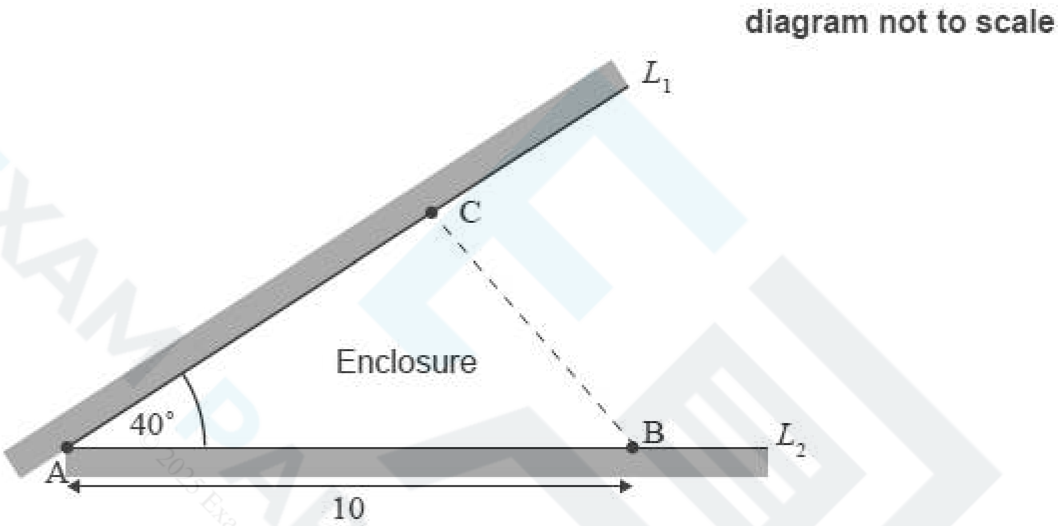
Find the probability that Katie cycles to work on her final working day of the year.

[3]

21N.1.AHL.TZ0.11

The following diagram shows a corner of a field bounded by two walls defined by lines L_1 and L_2. The walls meet at a point A, making an angle of $40°$.

Farmer Nate has $7\ \text{m}$ of fencing to make a triangular enclosure for his sheep. One end of the fence is positioned at a point B on L_2, $10\ \text{m}$ from A. The other end of the fence will be positioned at some point C on L_1, as shown on the diagram.



He wants the enclosure to take up as little of the current field as possible.

Find the minimum possible area of the triangular enclosure ABC.

21M.1.AHL.TZ1.1

George goes fishing. From experience he knows that the mean number of fish he catches per hour is 1.1. It is assumed that the number of fish he catches can be modelled by a Poisson distribution.

On a day in which George spends 8 hours fishing, find the probability that he will catch more than 9 fish.

21M.1.AHL.TZ1.11

A factory, producing plastic gifts for a fast food restaurant's Jolly meals, claims that just 1% of the toys produced are faulty.

A restaurant manager wants to test this claim. A box of 200 toys is delivered to the restaurant. The manager checks all the toys in this box and four toys are found to be faulty.

The restaurant manager performs a one-tailed hypothesis test, at the 10% significance level, to determine whether the factory's claim is reasonable. It is known that faults in the toys occur independently.

a. Identify the type of sampling used by the restaurant manager. [1]

b. Write down the null and alternative hypotheses. [2]

c. Find the p-value for the test. [2]

d. State the conclusion of the test. Give a reason for your answer. [2]

21M.1.AHL.TZ1.17

The graph of the function $f(x) = \ln x$ is translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ so that it then passes through the points $(0, 1)$ and $(e^3, 1 + \ln 2)$.

Find the value of a and the value of b.

21M.1.AHL.TZ2.10

A manufacturer of chocolates produces them in individual packets, claiming to have an average of 85 chocolates per packet.

Talha bought 30 of these packets in order to check the manufacturer's claim.

Given that the number of individual chocolates is x, Talha found that, from his 30 packets, $\Sigma x = 2506$ and $\Sigma x^2 = 209\ 738$.

a. Find an unbiased estimate for the mean number (μ) of chocolates per packet.

[1]

b.

Use the formula $s_{n-1}^2 = \frac{\Sigma x^2 - \frac{\Sigma x^2}{n}}{n-1}$ to determine an unbiased estimate for the variance of the number of chocolates per packet.

[2]

c.

Find a 95% confidence interval for μ. You may assume that all conditions for a confidence interval have been met.

[2]

d. Suggest, with justification, a valid conclusion that Talha could make. [1]

21N.1.AHL.TZ0.14

On Paul's farm, potatoes are packed in sacks labelled 50 kg. The weights of the sacks of potatoes can be modelled by a normal distribution with mean weight 49.8 kg and standard deviation 0.9 kg.

a. Find the probability that a sack is under its labelled weight. [2]

b. Find the lower quartile of the weights of the sacks of potatoes. [2]

c.

The sacks of potatoes are transported in crates. There are 10 sacks in each crate and the weights of the sacks of potatoes are independent of each other.

Find the probability that the total weight of the sacks of potatoes in a crate exceeds 500 kg.

[3]

22M.1.AHL.TZ2.10

The function $f x = \ln \frac{1}{x - 2}$ is defined for $x > 2, \quad x \in \mathbb{R}$.

a. Find an expression for $f^{-1}\left(x\right)$. You are not required to state a domain. [3]

b. Solve $f x = f^{-1}\left(x\right)$. [2]

22M.1.AHL.TZ2.14

The shape of a vase is formed by rotating a curve about the y-axis.

The vase is $10\ \text{cm}$ high. The internal radius of the vase is measured at $2\ \text{cm}$ intervals along the height:

Height (cm)	Radius (cm)
0	4
2	6
4	8
6	7
8	3
10	5

Use the trapezoidal rule to estimate the volume of water that the vase can hold.

22M.1.AHL.TZ2.16

The position vector of a particle, P, relative to a fixed origin O at time t is given by

$$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} \sin\ t^2 \\ \cos\ t^2 \end{pmatrix}.$$

a. Find the velocity vector of P. [2]

b.

Show that the acceleration vector of P is never parallel to the position vector of P.

[5]

22M.1.AHL.TZ2.7

The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

Find the third term. Justify your answer.