EXAM PAPERS PRACTICE

Helping you Achieve Highest Grades in IB

IB Mathematics (Analysis and Approaches) Standard Level (SL)

Mark Scheme

Fully in-lined with the First Assessment Examinations in 2021 & Beyond

Paper: 1 (All Topics)

- **Topic 1 - Number and Algebra**
- **Topic 2 - Functions**
- **Topic 3 - Geometry and Trigonometry**
- **Topic 4 - Statistics and Probability**
- **Topic 5 - Calculus**

Marks: 1566

Total Marks: / 1566

Suitable for SL Students sitting the 2025 exams onwards
However, HL students may also find these resources useful

Markschemes

SPM.1.SL.TZ0.8

a.

$f'(x) = x^2 + 2x - 15$ ***(M1)A1***

[2 marks]

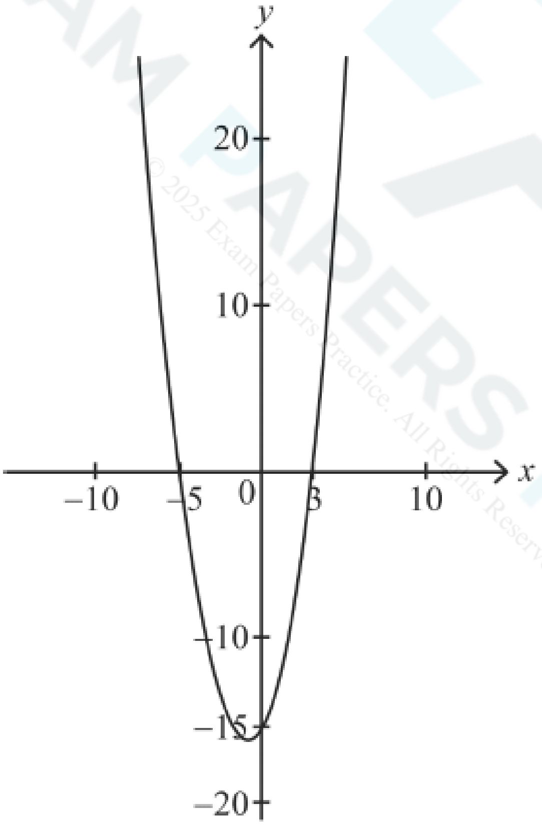
b. correct reasoning that $f'(x) = 0$ (seen anywhere) ***(M1)*** $x^2 + 2x - 15 = 0$

valid approach to solve quadratic ***M1*** $(x-3)(x+5)$, quadratic formula

correct values for x 3, −5 correct values for a and b a = −5 and b = 3 ***A1***

[3 marks]

c.i. ***[1 mark]***



A1

c.ii. first derivative changes from positive to negative at $x = a$ ***A1***

so local maximum at $x = a$ ***AG*** ***[1 mark]***

d.i. $f''(x) = 2x + 2$ ***A1*** substituting **their** b into **their** second derivative ***(M1)***

$f''(3) = 2 \times 3 + 2$ $f''(b) = 8$ ***(A1)*** ***[3 marks]***

d.ii. $f''(b)$ is positive so graph is concave up ***R1*** so local minimum at $x = b$ ***AG***

e. normal to f at $x = a$ is $x = -5$ (seen anywhere)

attempt to find y-coordinate at their value of b $f(3) = -10$

tangent at $x = b$ has equation $y = -10$ (seen anywhere)

intersection at $(-5, -10)$ $p = -5$ and $q = -10$

SPM.1.SL.TZ0.9

a.

attempt to use quotient rule ***(M1)***

correct substitution into quotient rule

$f'(x) = \frac{5kx\left(\frac{1}{5x}\right) - k\ln 5x}{(kx)^2}$ (or equivalent) ***A1***

$= \frac{k - k\ln 5x}{k^2x^2}, \ (k \in R^+)$ ***A1***

$= \frac{1 - \ln 5x}{kx^2}$ ***AG***

[3 marks]

b. $f'(x) = 0$ ***M1*** $\frac{1 - \ln 5x}{kx^2} = 0$ $\ln 5x = 1$ ***(A1)*** $x = \frac{e}{5}$ ***A1*** ***[3 marks]***

c. $f''(x) = 0$ ***M1*** $\frac{2\ln 5x - 3}{kx^3} = 0$ $\ln 5x = \frac{3}{2}$ ***A1*** $5x = e^{\frac{3}{2}}$ ***A1***

so the point of inflexion occurs at $x = \frac{1}{5}e^{\frac{3}{2}}$ ***AG*** ***[3 marks]***

d. attempt to integrate ***(M1)*** $u = \ln 5x \Rightarrow \frac{du}{dx} = \frac{1}{x}$ $\int \frac{\ln 5x}{kx}dx = \frac{1}{k}\int u\,du$ ***(A1)*** ***EITHER***

$= \frac{u^2}{2k}$ ***A1*** so $\frac{1}{k}\int_1^{\frac{3}{2}} u\,du = \left[\frac{u^2}{2k}\right]_1^{\frac{3}{2}}$ ***A1*** ***OR*** $= \frac{(\ln 5x)^2}{2k}$ ***A1***

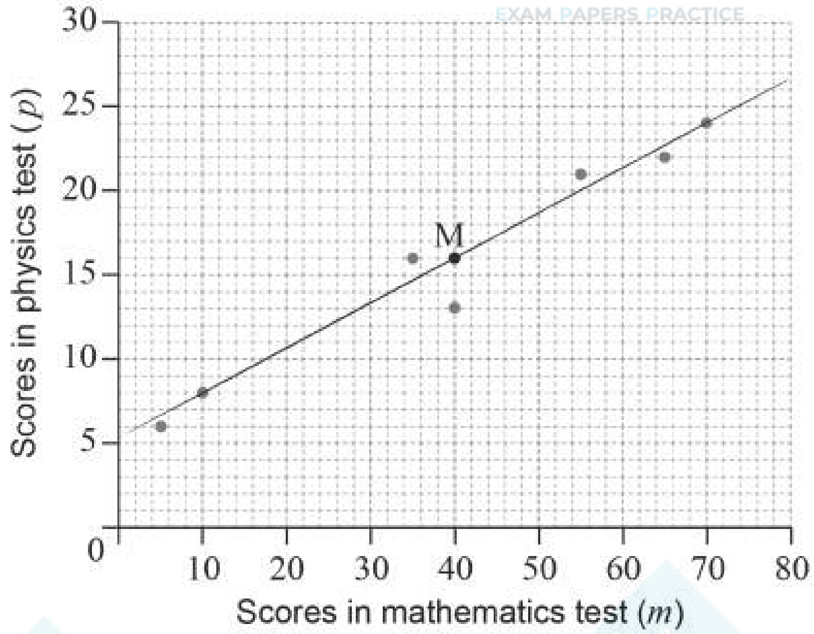
so $\int_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}} \frac{\ln 5x}{kx}dx = \left[\frac{(\ln 5x)^2}{2k}\right]_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}}$ ***A1*** ***THEN*** $= \frac{1}{2k}\left(\frac{9}{4} - 1\right)$ $= \frac{5}{8k}$ ***A1***

setting **their** expression for area equal to 3 ***M1*** $\frac{5}{8k} = 3$ $k = \frac{5}{24}$ ***A1*** ***[7 marks]***

18M.1.SL.TZ2.T_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1)(A1) (C2)

Note: Award ***(A1)*** for mean point plotted and ***(A1)*** for labelled M.

[2 marks]

b.

straight line through their mean point crossing the *p*-axis at 5±2 ***(A1)(ft)(A1)(ft) (C2)***

Note: Award ***(A1)*(ft)** for a straight line through their mean point. Award ***(A1)*(ft)** for a correct p-intercept if line is extended.

[2 marks]

c. point on line where *m* = 20 identified and an attempt to identify *y*-coordinate ***(M1)***

10.5 Follow through from their line in part (b).

19M.1.SL.TZ2.T_2

a.

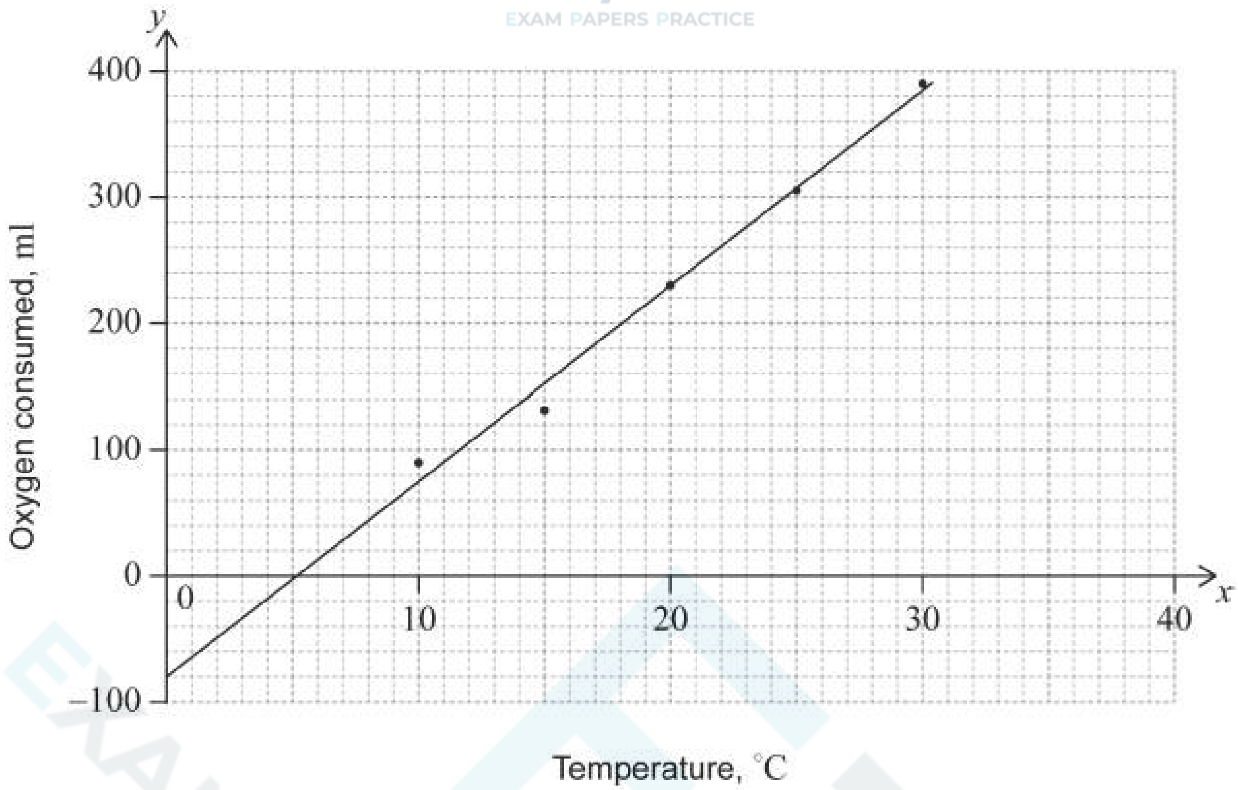
** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$y = 15.5x - 80$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for $15.5x$; ***(A1)*** for −80. Award at most ***(A1)(A0)*** if answer is not an equation. Award ***(A0)(A1)*(ft)** for $y = -80x + 15.5$.

[2 marks]

b.



Award for a straight line using a ruler passing through (20, 230); for correct y-intercept. If a ruler has not been used, award at most .

c. $a = 10$ $b = 30$ Accept [10, 30] or 10 ≤ x ≤ 30.

18M.1.SL.TZ1.T_15

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$[-2, \infty[$ or $[-2, \infty)$ **OR** $f(x) \geqslant -2$ or $y \geqslant -2$ **OR** $-2 \leqslant f(x) < \infty$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for −2 and ***(A1)*** for completely correct mathematical notation, including weak inequalities. Accept $f \geqslant -2$.

[2 marks]

b. −1 and 1.52 (1.51839...) ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for −1 and ***(A1)*** for 1.52 (1.51839). ***[2 marks]***

c. $x < -1, x > 1.52$ **OR** $(-\infty, -1) \cup (1.52, \infty)$. ***(A1)*(ft)*(A1)*(ft) *(C2)***

Note: Award ***(A1)*(ft)** for **both** critical values in inequality or range statements such as $x < -1$, $(-\infty, -1)$, $x > 1.52$ or $(1.52, \infty)$.

Award the second for correct strict inequality statements used with their critical values. If an incorrect use of strict and weak inequalities has already been penalized in (a), condone weak inequalities for this second mark and award .

17M.1.SL.TZ2.T_4

b.

$6 = -3(2) + c$ **OR** $(y - 6) = -3(x - 2)$ ***(M1)***

Note: Award ***(M1)*** for substitution of their gradient from part (a) into a correct equation with the coordinates $(2, 6)$ correctly substituted.

$y = -3x + 12$ ***(A1)(ft)*** ***(C2)***

Notes: Award ***(A1)(ft)*** for their correct equation. Follow through from part (a).

If no method seen, award ***(A1)(A0)*** for $y = -3x$.

Award ***(A1)(A0)*** for $-3x + 12$.

[2 marks]

c. $0 = -3x + 12$ ***(M1)***

Note: Award ***(M1)*** for substitution of $y = 0$ in their equation from part (b).

$(x =)\ 4$ ***(A1)(ft)*** ***(C2)***

Notes: Follow through from their equation from part (b). Do not follow through if no method seen. Do not award the final ***(A1)*** if the value of x is negative or zero.

[2 marks]

17N.1.SL.TZ0.T_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

Units are required in parts (a) and (c).

$\frac{\text{EB}}{\sin 53°} = \frac{1.2}{\sin 7°}$ ***(M1)(A1)***

Note: Award *(M1)* for substitution into sine formula, *(A1)* for correct substitution.

$(\text{EB} =)\ 7.86\text{ m}\textbf{OR}786\text{ cm}\ (7.86385\ \ldots\ \text{m}\textbf{OR}786.385\ \ldots\ \text{cm})$ ***(A1)*** ***(C3)***

[3 marks]

b. $34°$ ***(A1)*** ***(C1)*** ***[1 mark]***

c. **Units are required in parts (a) and (c).** $\sin 34° = \frac{\text{height}}{7.86385\ \ldots}$ ***(M1)***

Award for correct substitution into a trigonometric ratio.

$(\text{height} =)\ 4.40\text{ m}\quad 440\text{ cm}\ (4.39741\ \ldots\ \text{m}\quad 439.741\ \ldots\ \text{cm})$

Accept "BT" used for height. Follow through from parts (a) and (b). Use of 7.86 gives an answer of 4.39525....

19M.1.SL.TZ2.T_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\tan B = \frac{12}{9}$ ***(A1)(M1)***

Note: Award *(A1)* for 12 seen, *(M1)* for correct substitution into tan (or equivalent). Accept equivalent methods, such as Pythagoras, to find BC and correct substitution into other trig ratios. If $\tan^{-1}\left(\frac{16}{9}\right)$ seen award ***(A0)(M1)(A0)***.

53.1° (53.1301...°) ***(A1)*** ***(C3)***

Note: If radians are used the answer is 0.927295...; award at most ***(A1)(M1)(A0)***.

[3 marks]

b. $30\sin 70° + 4$ ***(M1)(M1)***

Note: Award *(M1)* for $\sin 70° = \frac{x}{30}$ (or equivalent) and *(M1)* for adding 4.

32.2 (32.1907...) (m) ***(A1)*** ***(C3)***

Note: If radians are used the answer is 27.2167...; award at most ***(M1)(M1)(A0)***. ***[3 marks]***

EXN.1.SL.TZ0.1

** This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.*

correct working ***(A1)***

eg $-5 + (8 - 1)(3)$

$u_8 = 16$ ***A1 N2***

[2 marks]

METHOD 1

$f x = \int 3\sqrt{x} \ \ \mathrm{d}x$ **(A1)**

attempts to integrate **(M1)**

$f x = 2x^{\frac{3}{2}} + C \qquad = 2x\sqrt{x} + C$ **A1**

uses $f 1 = 3$ to obtain $3 = 21^{\frac{3}{2}} + C$ and so $C = 1$ **M1**

substitutes $x = 4$ into their expression for $f x$ **(M1)**

so $f 4 = 17$ **A1**

METHOD 2

$\int_1^4 f'x \ \ \mathrm{d}x = \int_1^4 3\sqrt{x} \ \ \mathrm{d}x$ **(A1)**

attempts to integrate both sides **(M1)**

$f x_1^4 = 2x^{\frac{3}{2}}{}_1^4$ **A1**

$f 4 - f 1 = 16 - 2$ **M1**

uses $f 1 = 3$ to find their value of $f 4$ **(M1)**

$f 4 - 3 = 16 - 2$

so $f 4 = 17$ **A1**

EXN.1.SL.TZ0.2

** This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.*

METHOD 1

$2 \ln x - \ln 9 = 4$

uses $m \ln x = \ln x^m$ **(M1)**

$\ln x^2 - \ln 9 = 4$

uses $\ln a - \ln b = \ln \frac{a}{b}$ **(M1)**

$\ln \frac{x^2}{9} = 4$

$\frac{x^2}{9} = e^4$ **A1**

$x^2 = 9e^4 \Rightarrow x = \sqrt{9e^4}$ $x > 0$ **A1**

$x = 3e^2$ $p = 3,$ $q = 2$ **A1**

METHOD 2

expresses 4 as $4 \ln e$ and uses $\ln x^m = m \ln x$ **(M1)**

$2 \ln x = 2 \ln 3 + 4 \ln e$ $\ln x = \ln 3 + 2 \ln e$ **A1**

uses $2 \ln e = \ln e^2$ and $\ln a + \ln b = \ln ab$ **(M1)**

$\ln x = \ln 3e^2$ **A1**

$x = 3e^2$ $p = 3,$ $q = 2$ **A1**

METHOD 3

expresses 4 as $4 \ln e$ and uses $m \ln x = \ln x^m$ **(M1)**

$\ln x^2 = \ln 3^2 + \ln e^4$ **A1**

uses $\ln a + \ln b = \ln ab$ **(M1)**

$\ln x^2 = \ln 3^2 e^4$

$x^2 = 3^2 e^4 \Rightarrow x = \sqrt{3^2 e^4}$ $x > 0$ **A1**

so $x = 3e^2$ $x > 0$ $p = 3,$ $q = 2$

EXN.1.SL.TZ0.3

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uses $\sum P(X = x) = 1$ **(M1)**

$k^2 + 7k + 2 + -2k + 3k^2 = 1$

$4k^2 + 5k + 1 = 0$ **A1**

EITHER

attempts to factorize their quadratic **M1**

$k + 14k + 1 = 0$

OR

attempts use of the quadratic formula on their equation **M1**

$k = \frac{-5 \pm \sqrt{5^2 - 441}}{8} = \frac{-5 \pm 3}{8}$

THEN

$k = -1, \ -\frac{1}{4}$ **A1**

rejects $k = -1$ as this value leads to invalid probabilities, for example, $PX = 2 = -5 < 0$ **R1**

so $k = -\frac{1}{4}$ **A1**

Note: Award **R0A1** if $k = -\frac{1}{4}$ is stated without a valid reason given for rejecting $k = -1$.

EXN.1.SL.TZ0.4

a.

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EITHER

uses $u_2 - u_1 = u_3 - u_2$ **(M1)**

$5u_1 - 8 - u_1 = 3u_1 + 8 - 5u_1 - 8$

$6u_1 = 24$ **A1**

OR

uses $u_2 = \frac{u_1 + u_3}{2}$ **(M1)**

$5u_1 - 8 = \frac{u_1 + 3u_1 + 8}{2}$

$3u_1 = 12$ **A1**

THEN

so $u_1 = 4$

b. $d = 8$ uses $S_n = \frac{n}{2}2u_1 + n - 1d$ $S_n = \frac{n}{2}8 + 8n - 1$ $= 4n^2$

$= 2n^2$

The final can be awarded for clearly explaining that $4n^2$ is a square number.

so sum of the first n terms is a square number

EXN.1.SL.TZ0.5

** This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.*

$f \circ gx = ax + b - 2$ **(M1)**

$f \circ g2 = -3 \Rightarrow 2a + b - 2 = -3 \quad 2a + b = -1$ **A1**

$g \circ fx = ax - 2 + b$ **(M1)**

$g \circ f1 = 5 \Rightarrow -a + b = 5$ **A1**

a valid attempt to solve their two linear equations for a and b **M1**

so $a = -2$ and $b = 3$ **A1**

[6 marks]

EXN.1.SL.TZ0.7

a.

** This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor nces in formatting compared to formal exam papers.*

$P = 2x + 2y$ **(A1)**

$= 2x + 24 - x^2$ **A1**

so $P = -2x^2 + 2x + 8$ **AG**

[2 marks]

b. **METHOD 1** **EITHER** uses the axis of symmetry of a quadratic **(M1)** $x = -\frac{2}{2 \cdot 2}$

OR forms $\frac{dP}{dx} = 0$ **(M1)** $-4x + 2 = 0$ **THEN** $x = \frac{1}{2}$ **A1**

substitutes their value of x into $y = 4 - x^2$ **(M1)** $y = 4 - \frac{1}{2}^2$ $y = \frac{15}{4}$ **A1**

so the dimensions of rectangle ORST of maximum perimeter are $\frac{1}{2}$ by $\frac{15}{4}$ **EITHER**

substitutes their value of x into $P = -2x^2 + 2x + 8$ **(M1)** $P = -2\frac{1}{2}^2 + 2\frac{1}{2} + 8$ **OR**

substitutes their values of x and y into $P = 2x + 2y$ **(M1)** $P = 2\frac{1}{2} + 2\frac{15}{4}$

$P = \frac{17}{2}$ **A1** so the maximum perimeter is $\frac{17}{2}$ **METHOD 2**

attempts to complete the square **M1** $P = -2x - \frac{1}{2}^2 + \frac{17}{2}$ **A1** $x = \frac{1}{2}$ **A1**

substitutes their value of x into $y = 4 - x^2$ **(M1)** $y = 4 - \frac{1}{2}^2$ $y = \frac{15}{4}$ **A1**

so the dimensions of rectangle ORST of maximum perimeter are $\frac{1}{2}$ by $\frac{15}{4}$ $P = \frac{17}{2}$ **A1**

so the maximum perimeter is $\frac{17}{2}$ **[6 marks]**

c. substitutes $y = 4 - x^2$ into $A = xy$ **(M1)** $A = x4 - x^2$ $= 4x - x^3$ **A1**

[2 marks]

d. $\frac{dA}{dx} = 4 - 3x^2$ **A1** attempts to solve their $\frac{dA}{dx} = 0$ for x **(M1)** $4 - 3x^2 = 0$

$\Rightarrow x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $x > 0$ **A1**

substitutes their (positive) value of x into $y = 4 - x^2$ **(M1)** $y = 4 - \frac{2}{\sqrt{3}}^2$ $y = \frac{8}{3}$ **A1**

[5 marks]

e. $A = \frac{16}{3\sqrt{3}}$ $= \frac{16\sqrt{3}}{9}$

EXN.1.SL.TZ0.8

a.

for example,

a reflection in the x-axis (in the line $y = 0$) **A1**

a horizontal translation (shift) 3 units to the left **A1**

a vertical translation (shift) down by 1 unit **A1**

Note: Award **A1** for each correct transformation applied in a correct position in the sequence. Do not accept use of the "move" for a translation.

Note: Award **A1A1A1** for a correct alternative sequence of transformations. For example,

a vertical translation (shift) down by 1 unit, followed by a horizontal translation (shift) 3 units to the left and then a reflection in the line $y = -1$.

[3 marks]

b. range is $fx \le -1$ **A1**

Note: Correct alternative notations include $]-\infty, -1]$, $(-\infty, -1]$ or $y \le -1$.

[1 mark]

c. $-1 - \sqrt{y+3} = x$ **M1**

Note: Award **M1** for interchanging x and y (can be done at a later stage).

$\sqrt{y+3} = -x - 1 = -x + 1$ **A1** $y + 3 = x + 1^2$ **A1**

so $f^{-1}x = x + 1^2 - 3$ $f^{-1}x = x^2 + 2x - 2$ **A1** domain is $x \le -1$ **A1**

Note: Correct alternative notations include $]-\infty, -1]$ or $(-\infty, -1]$. **[5 marks]**

d. the point of intersection lies on the line $y = x$ **EITHER** $x + 1^2 - 3 = x$ **M1**

attempts to solve their quadratic equation **M1**

for example, $x + 2x - 1 = 0$ or $x = \frac{-1 \pm \sqrt{1^2 - 41-2}}{2}$ $x = \frac{-1 \pm 3}{2}$ **OR** $-1 - \sqrt{x+3} = x$ **M1**

$-1 - \sqrt{x+3}^2 = x^2 \Rightarrow 2\sqrt{x+3} + x + 4 = x^2$

substitutes $2\sqrt{x+3} = -2x + 1$ to obtain $-2x + 1 + x + 4 = x^2$

attempts to solve their quadratic equation **M1**

for example, $x + 2x - 1 = 0$ or $x = \frac{-1 \pm \sqrt{1^2 - 41-2}}{2}$ $x = \frac{-1 \pm 3}{2}$ **THEN** $x = -2, 1$ **A1**

as $x \le -1$, the only solution is $x = -2$ **R1** so the coordinates of the point of intersection are $-2, -2$

Award if $-2, -2$ is stated without a valid reason given for rejecting $1, 1$.

EXN.1.SL.TZ0.9

a.

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attempts to find $h0$ **(M1)**

$h0 = 0.4 \cos 0 + 1.8 = 2.2$

2.2 (m) (above the ground) **A1**

[2 marks]

b. **EITHER** uses the minimum value of $\cos \pi t$ which is -1 **M1** $0.4 \text{-} 1 + 1.8$ (m)

OR the amplitude of motion is 0.4 (m) and the mean position is 1.8 (m) **M1** **OR**

finds $h't = -0.4\pi \sin \pi t$, attempts to solve $h't = 0$ for t and determines that the minimum height above the ground occurs at $t = 1, 3, \ldots$ **M1**

$0.4 \text{-} 1 + 1.8$ (m) **THEN** 1.4 (m) (above the ground) **A1** **[2 marks]**

c. **EITHER**

the ball is released from its maximum height and returns there a period later **R1**

the period is $\frac{2\pi}{\pi} = 2$ s **A1** **OR** attempts to solve $ht = 2.2$ for t **M1** $\cos \pi t = 1$

$t = 0, 2, \ldots$ **A1** **THEN**

so it takes 2 seconds for the ball to return to its initial position for the first time **AG**

[2 marks]

d. $0.4 \cos \pi t + 1.8 = 1.8 + 0.2\sqrt{2}$ **(M1)** $0.4 \cos \pi t = 0.2\sqrt{2}$ $\cos \pi t = \frac{\sqrt{2}}{2}$ **A1**

$\pi t = \frac{\pi}{4}, \frac{7\pi}{4}$ **(A1)** **Note:** Accept extra correct positive solutions for πt.

$t = \frac{1}{4}, \frac{7}{4}$ $0 \le t \le 2$ **A1**

Note: Do not award **A1** if solutions outside $0 \le t \le 2$ are also stated.

the ball is less than $1.8 + 0.2\sqrt{2}$ metres above the ground for $\frac{7}{4} - \frac{1}{4}$ (s) 1.5 (s) **A1**

e. attempts to find $h'(t)$

recognizes that $h'(t)$ is required $h'(t) = -0.4\pi \sin(\pi t)$

attempts to evaluate their $h'\left(\frac{1}{3}\right)$ $h'\left(\frac{1}{3}\right) = -0.4\pi \sin\left(\frac{\pi}{3}\right) = 0.2\pi\sqrt{3}$ ms^{-1}

Accept equivalent correct answer forms where $p \in \mathbb{Q}$. For example, $-\frac{1}{5}\pi\sqrt{3}$.

19N.1.SL.TZ0.S_1

a.

valid approach ***(M1)***

eg $11 - 5$, $11 = 5 + d$

$d = 6$ ***A1 N2***

[2 marks]

b. valid approach ***(M1)*** *eg* $u_2 - d$, $5 - 6$, $u_1 + (3-1)(6) = 11$

$u_1 = -1$ ***A1 N2*** ***[2 marks]***

c. correct substitution into sum formula

eg $\frac{20}{2}(2(-1) + 19(6))$, $\frac{20}{2}(-1 + 113)$ ***(A1)*** $S_{20} = 1120$ ***A1 N2*** ***[2 marks]***

21M.1.SL.TZ2.1

a.

minor arc AB has length r ***(A1)***

recognition that perimeter of shaded sector is $3r$ ***(A1)***

$3r = 12$

$r = 4$ ***A1***

[3 marks]

b. **EITHER**

$\theta = 2\pi - \text{A}\hat{\text{O}}\text{B} \quad = 2\pi - 1$ ***(M1)***

Area of non-shaded region $= \frac{1}{2}(2\pi - 1)4^2$ ***(A1)*** **OR** area of circle - area of shaded sector $16\pi - \frac{1}{2} \times 1 \times 4^2$

area $= 16\pi - 8 \quad = 82\pi - 1$

21M.1.SL.TZ2.2

attempt to subtract squares of integers ***(M1)***

$n + 1^2 - n^2$

EITHER

correct order of subtraction and correct expansion of $n + 1^2$, seen anywhere ***A1A1***

$= n^2 + 2n + 1 - n^2 \quad (= 2n + 1)$

OR

correct order of subtraction and correct factorization of difference of squares ***A1A1***

$= (n + 1 - n)(n + 1 + n)(= 2n + 1)$

THEN

$= n + n + 1 = \text{RHS}$ ***A1***

Note: Do not award final ***A1*** unless all previous working is correct.

which is the sum of n and $n + 1$ ***AG***

Note: If expansion and order of subtraction are correct, award full marks for candidates who find the sum of the integers as $2n + 1$ and then show that the difference of the squares (subtracted in the correct order) is $2n + 1$.

[4 marks]

19N.1.SL.TZ0.S_10

a.

$\text{B}(a, 0)$ (accept $\text{B}(q + 1, 0)$) ***A2 N2***

[2 marks]

b.

Note: There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may work with the equation of the line before finding a.

FINDING a valid attempt to find an expression for a in terms of q ***(M1)***

$g(0) = a, p^0 + q = a$ $a = q + 1$ ***(A1)*** **FINDING THE EQUATION OF** L_1

EITHER

attempt to substitute tangent gradient and coordinates into equation of straight line ***(M1)***

eg $y - 0 = f'(a)(x - a), y = f'(a)(x - (q + 1))$

correct equation in terms of a and p ***(A1)*** *eg* $y - 0 = \frac{1}{\ln(p)}(x - a)$

OR attempt to substitute tangent gradient and coordinates to find b

eg $0 = \frac{1}{\ln(p)}(a) + b$ $b = \frac{-a}{\ln(p)}$ ***(A1)***

THEN (must be in terms of **both** p and q)

$y = \frac{1}{\ln p}(x - q - 1), y = \frac{1}{\ln p}x - \frac{q+1}{\ln p}$ ***A1 N3***

Note: Award ***A0*** for final answers in the form $L_1 = \frac{1}{\ln p}(x - q - 1)$ ***[5 marks]***

c.

Note: There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may find q in terms of p before finding a value for p.

FINDING p valid approach to find the gradient of the tangent ***(M1)***

eg $m_1 m_2 = -1, -\frac{1}{\frac{1}{\ln\left(\frac{1}{3}\right)}}, -\ln\left(\frac{1}{3}\right), -\frac{1}{\ln p} = \frac{1}{\ln\left(\frac{1}{3}\right)}$

correct application of log rule (seen anywhere) ***(A1)***

eg $\ln\left(\frac{1}{3}\right)^{-1}, -(\ln(1) - \ln(3))$ correct equation (seen anywhere) ***A1***

eg $\ln p = \ln 3, p = 3$ **FINDING** q

correct substitution of $(-2, -2)$ into L_2 equation ***(A1)***

eg $-2 = (\ln p)(-2) + q + 1$ $q = 2\ln p - 3, q = 2\ln 3 - 3$ (seen anywhere) ***A1***

FINDING L_1 correct substitution of **their** p and q into **their** L_1 ***(A1)***

$y = \frac{1}{\ln 3}(x - (2\ln 3 - 3) - 1)$

$y = \frac{1}{\ln 3}(x - 2\ln 3 + 2), y = \frac{1}{\ln 3}x - \frac{2\ln 3 - 2}{\ln 3}$

Award for final answers in the form $L_1 = \frac{1}{\ln 3}(x - 2\ln 3 + 2)$.

19N.1.SL.TZ0.S_2

a.

$q = 5$ ***A1 N1***

[1 mark]

b. valid approach ***(M1)*** *eg* $(18 + 10 + 5) - 30,\ 28 - 25,\ 18 + 10 - n = 25$

$n = 3$ ***A1 N2 [2 marks]***

c. valid approach for finding m or p (may be seen in part (b)) ***(M1)***

eg $18 - 3,\ 3 + p = 10$ $m = 15,\ p = 7$ ***A1A1 N3 [3 marks]***

19N.1.SL.TZ0.S_3

a.

valid attempt to substitute coordinates ***(M1)***

eg $g(-1) = 8$

correct substitution ***(A1)***

eg $(-1)^2 + b(-1) + 11 = 8,\ 1 - b + 11 = 8$

$b = 4$ ***A1 N2***

[3 marks]

b. valid attempt to solve ***(M1)*** *eg* $(x^2 + 4x + 4) + 7,\ h = \frac{-4}{2},\ k = g(-2)$

correct working ***A1*** *eg* $(x + 2)^2 + 7,\ h = -2,\ k = 7$

translation or shift (do not accept move) of vector $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$ (accept left by 2 and up by 7
) ***A1A1 N2***

[4 marks]

21M.1.SL.TZ2.5

a.

$\ln x^2 - 16 = 0$ ***(M1)***

$e^0 = x^2 - 16 = 1$

$x^2 = 17$ OR $x = \pm\sqrt{17}$ ***(A1)***

$a = \sqrt{17}$ ***A1***

[3 marks]

b. attempt to differentiate (must include $2x$ and/or $\frac{1}{x^2 - 16}$) ***(M1)***

$f'x = \frac{2x}{x^2 - 16}$ ***A1*** setting their derivative $= \frac{1}{3}$ ***M1*** $\frac{2x}{x^2 - 16} = \frac{1}{3}$

$x^2 - 16 = 6x$ OR $x^2 - 6x - 16 = 0$ (or equivalent) ***A1***

valid attempt to solve their quadratic ***(M1)*** $x = 8$ ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $x = -2,\ 8$).

21M.1.SL.TZ2.6

METHOD 1

attempt to use the cosine rule to find the value of x ***(M1)***

$100 = x^2 + 4x^2 - 2x2x\frac{3}{4}$ ***A1***

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}$ $= 5\sqrt{2}$ ***A1***

attempt to find $\sin\ \hat{C}$ (seen anywhere) ***(M1)***

$\sin^2\ \hat{C} + \frac{3}{4}^2 = 1$ OR $x^2 + 3^2 = 4^2$ or right triangle with side 3 and hypotenuse 4

$\sin\ \hat{C} = \frac{\sqrt{7}}{4}$ ***(A1)***

Note: The marks for finding $\sin\ \hat{C}$ may be awarded independently of the first three marks for finding x.

correct substitution into the area formula using their value of x (or x^2) and their value of $\sin\ \hat{C}$ ***(M1)***

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or $A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

attempt to find the height, h, of the triangle in terms of x

$h^2 + \frac{3}{4}x^2 = x^2$ OR $h^2 + \frac{5}{4}x^2 = 10^2$ OR $h = \frac{\sqrt{7}}{4}x$

equating their expressions for either h^2 or h

$x^2 - \frac{3}{4}x^2 = 10^2 - \frac{5}{4}x^2$ OR $\sqrt{100 - \frac{25}{16}x^2} = \frac{\sqrt{7}}{4}x$ (or equivalent)

$x^2 = 50$ OR $x = \sqrt{50}$ $\quad = 5\sqrt{2}$

correct substitution into the area formula using their value of x (or x^2)

$A = \frac{1}{2} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}\sqrt{50}$ OR $A = \frac{1}{2}2 \times 5\sqrt{2}\frac{\sqrt{7}}{4}5\sqrt{2}$

$A = \frac{25\sqrt{7}}{2}$

21M.1.SL.TZ2.7

a.

evidence of median position ***(M1)***

40 students

median $= 14$ (hours) ***A1***

[2 marks]

b. recognizing there are 8 students in the top 10% ***(M1)***

72 students spent less than k hours ***(A1)*** $k = 18$ (hours) ***A1*** ***[3 marks]***

c. 15 hours is 60 students OR $p = 60 - 4$ ***(M1)*** $p = 56$ ***A1***

21 hours is 76 students OR $q = 80 - 76$ OR $q = 80 - 4 - 56 - 16$ ***(A1)***

$q = 4$ ***A1*** ***[4 marks]***

d. 20 of the 80 students OR $\frac{1}{4}$ spend more than 15 hours doing homework ***(A1)***

$\frac{20}{80} = \frac{x}{320}$ OR $\frac{1}{4} \times 320$ OR 4×20 ***(A1)*** 80 (students) ***A1*** ***[3 marks]***

e.i.

only year 12 students surveyed OR amount of homework might be different for different year levels

e.ii. stratified sampling OR survey students in all years

19N.1.SL.TZ0.S_4

a.

valid approach ***(M1)***

eg $11 - a = 9,\ \frac{11!}{9!(11-9)!}$

$a = 2$ ***A1 N2***

[2 marks]

b. valid approach for expansion using $n = 11$ ***(M1)***

eg $\binom{11}{r} x^{11-r} 3^r,\ a^{11}b^0 + \binom{11}{1} a^{10} b^1 + \binom{11}{2} a^9 b^2 + \ldots$

evidence of choosing correct term ***A1*** *eg* $\binom{11}{2} 3^2,\ \binom{11}{2} x^9 3^2,\ \binom{11}{9} 3^2$

correct working for binomial coefficient (seen anywhere, do not accept factorials) ***A1***

eg $55,\ \binom{11}{2} = 55,\ 55 \times 3^2,\ (55 \times 9)x^9,\ \frac{11 \times 10}{2} \times 9$ 495 ***A1 N2***

Note: If there is clear evidence of adding instead of multiplying, award ***A1*** for the correct working for binomial coefficient, but no other marks. For example, $55x^9 \times 3^2$ would earn ***M0A0A1A0***.

Do not award final ***A1*** for a final answer of $495x^9$, even if 495 is seen previously. If no working shown, award ***N1*** for $495x^9$.

[4 marks]

19N.1.SL.TZ0.S_5

a.

correct substitution into $b^2 - 4ac$ ***(A1)***

eg $(5k)^2 - 4(2)(3k^2 + 2),\ (5k)^2 - 8(3k^2 + 2)$

correct expansion of each term ***A1***

eg $25k^2 - 24k^2 - 16,\ 25k^2 - (24k^2 + 16)$

$k^2 - 16$ ***AG N0***

[2 marks]

b. valid approach ***M1*** *eg* $f'(x) > 0,\ f'(x) \geqslant 0$

recognizing discriminant < 0 or $\leqslant 0$ ***M1*** *eg* $D < 0,\ k^2 - 16 \leqslant 0,\ k^2 < 16$

two correct values for k/endpoints (even if inequalities are incorrect) ***(A1)***

eg $k = \pm 4,\ k < -4$ and $k > 4,\ |k| < 4$ correct interval ***A1 N2***

eg $-4 < k < 4,\ -4 \leqslant k \leqslant 4$

Note: Candidates may work with an equation, then write the intervals with inequalities at the end. If inequalities are not seen until the candidate's final correct answer, may be awarded.
If candidate is working with incorrect inequalitie(s) at the beginning, then gets the correct final answer, award or or in line with the markscheme.

21M.1.SL.TZ2.8

a.

$6 + 6 \cos x = 0$ (or setting their $f'x = 0$) ***(M1)***

$\cos x = -1$ (or $\sin x = 0$)

$x = \pi,\quad x = 3\pi$ ***A1A1***

[3 marks]

b. attempt to integrate $\int_{\pi}^{3\pi} 6 + 6 \cos x \, \mathrm{d}x$ ***(M1)***

$= 6x + 6 \sin x_{\pi}^{3\pi}$ ***A1A1***

substitute their limits into their integrated expression and subtract ***(M1)***

$= 18\pi + 6 \sin 3\pi - 6\pi + 6 \sin \pi$ $= 63\pi + 0 - 6\pi + 0$ $= 18\pi - 6\pi$ ***A1***

area $= 12\pi$ ***AG*** ***[5 marks]***

c. attempt to substitute into formula for surface area (including base) ***(M1)***

$\pi 2^2 + \pi 2l = 12\pi$ ***(A1)*** $4\pi + 2\pi l = 12\pi$ $2\pi l = 8\pi$ $l = 4$ ***A1***

[3 marks]

d. valid attempt to find the height of the cone ***(M1)*** e.g. $2^2 + h^2 = \text{their } l^2$

$h = \sqrt{12} \quad = 2\sqrt{3}$

attempt to use $V = \frac{1}{3}\pi r^2 h$ with their values substituted $\qquad \frac{1}{3}\pi 2^2 \sqrt{12}$

volume $= \frac{4\pi\sqrt{12}}{3} = \frac{8\pi\sqrt{3}}{3} = \frac{8\pi}{\sqrt{3}}$

19N.1.SL.TZ0.S_6

METHOD 1 – FINDING INTERVALS FOR *x*

$4\cos\left(\frac{x}{2}\right) + 1 > 2\sqrt{2} + 1$

correct working ***(A1)***

eg $\quad 4\cos\frac{x}{2} = 2\sqrt{2},\ \cos\frac{x}{2} > \frac{\sqrt{2}}{2}$

recognizing $\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ ***(A1)***

one additional correct value for $\frac{x}{2}$ (ignoring domain and equation/inequalities) ***(A1)***

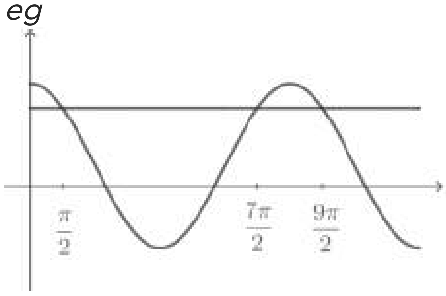
eg $\quad -\frac{\pi}{4}, \frac{7\pi}{4}, 315°, \frac{9\pi}{4}, -45°, \frac{15\pi}{4}$

three correct values for x ***A1A1***

eg $\quad \frac{\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$

valid approach to find intervals ***(M1)***

eg



correct intervals (must be in radians) ***A1A1 N2***

$0 \leqslant x < \frac{\pi}{2},\ \frac{7\pi}{2} < x < \frac{9\pi}{2}$

Note: If working shown, award ***A1A0*** if inclusion/exclusion of endpoints is incorrect. If no working shown award ***N1***.
If working shown, award ***A1A0*** if both correct intervals are given, **and** additional intervals are given. If no working shown award ***N1***.
Award ***A0A0*** if inclusion/exclusion of endpoints are incorrect **and** additional intervals are given.

METHOD 2 – FINDING INTERVALS FOR $\frac{x}{2}$

$4\cos\frac{x}{2} + 1 > 2\sqrt{2} + 1$

correct working ***(A1)***

eg $4\cos\frac{x}{2} = 2\sqrt{2},\ \cos\frac{x}{2} > \frac{\sqrt{2}}{2}$

recognizing $\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

one additional correct value for $\frac{x}{2}$ (ignoring domain and equation/inequalities)

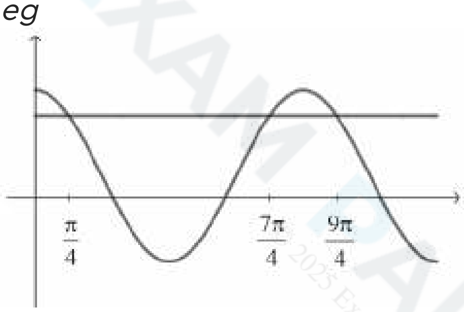
eg $-\frac{\pi}{4}, \frac{7\pi}{4}, 315°, \frac{9\pi}{4}, -45°, \frac{15\pi}{4}$

three correct values for $\frac{x}{2}$

eg $\frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$

valid approach to find intervals

eg



one correct interval for $\frac{x}{2}$

eg $0 \leqslant \frac{x}{2} < \frac{\pi}{4}, \frac{7\pi}{4} < \frac{x}{2} < \frac{9\pi}{4}$

correct intervals (must be in radians)

$0 \leqslant x < \frac{\pi}{2},\ \frac{7\pi}{2} < x < \frac{9\pi}{2}$

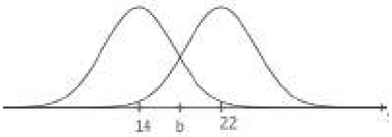
If working shown, award if inclusion/exclusion of endpoints is incorrect. If no working shown award .
If working shown, award if both correct intervals are given, additional intervals are given. If no working shown award .
Award if inclusion/exclusion of endpoints are incorrect additional intervals are given.

19N.1.SL.TZ0.S_7

a.

METHOD 1

cognizing that b is midway between the means of 14 and 22. ***(M1)*** *eg*



, $b = \frac{14+22}{2}$

$b = 18$ ***A1 N2***

METHOD 2

valid attempt to compare distributions ***(M1)***

eg $\frac{b-14}{a} = \frac{b-22}{a}$, $b - 14 = 22 - b$

$b = 18$ ***A1 N2***

[2 marks]

b. valid attempt to compare distributions (seen anywhere) ***(M1)***

eg Y is a horizontal translation of X of 8 units to the right,

$\text{P}(16 < Y < 28) = \text{P}(8 < X < 20), \text{P}(Y > 22 + 6) = \text{P}(X > 14 + 6)$

valid approach using symmetry ***(M1)***

eg $1 - 2\text{P}(X > 20), 1 - 2\text{P}(Y < 16), 2 \times \text{P}(14 < x < 20), \text{P}(X < 8) = \text{P}(X > 20)$

correct working *eg* $1 - 2(0.112), 2 \times (0.5 - 0.112), 2 \times 0.388, 0.888 - 0.112$

$\text{P}(16 < Y < 28) = 0.776$

21M.1.SL.TZ2.9

a.

setting $st = 0$ ***(M1)***

$8t$ - $t^2 = 0$

$t8$ - $t = 0$

$p = 8$ (accept $t = 8$, 8, 0) ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $p = 0$, 8).

[2 marks]

b.i.

recognition that when particle changes direction $v = 0$ OR local maximum on graph of s OR vertex of parabola ***(M1)***

$q = 4$ (accept $t = 4$) ***A1*** ***[2 marks]***

b.ii. substituting their value of q into st OR integrating vt from $t = 0$ to $t = 4$ ***(M1)***

$\text{displacement} = 16 \ (\text{m})$ ***A1*** ***[2 marks]***

c. $s10 = -20$ OR $\text{distance} = st$ OR integrating vt from $t = 0$ to $t = 10$ ***(M1)***

$\text{distance} = 20 \ (\text{m})$ ***A1*** ***[2 marks]***

d. 16 forward + 36 backward OR $16 + 16 + 20$ OR $\int_0^{10} vt \, \mathrm{d}\, t$ ***(M1)***

$d = 52 \ \text{m}$

e. graphical method with triangles on vt graph

$49 + \frac{x2x}{2}$ $49 + x^2 = 52, \quad x = \sqrt{3}$ $k = 7 + \sqrt{3}$

recognition that distance $= \int vt \, \mathrm{d}\, t$ $\int_0^7 14 - 2t \, \mathrm{d}\, t + \int_7^k 2t - 14 \, \mathrm{d}\, t$

$14t - t^2{}_0^7 + t^2 - 14t_7^k$ $147 - 7^2 + k^2 - 14k - 7^2 - 147 = 52$

$k = 7 + \sqrt{3}$

19N.1.SL.TZ0.S_8

a.

$y = 12 - 4x$ ***A1 N1***

[1 mark]

b. correct substitution into volume formula ***(A1)***

eg $3x \times x \times y, x \times 3x \times (12 - x - 3x), (12 - 4x)(x)(3x)$

$V = 3x^2(12 - 4x) \left(= 36x^2 - 12x^3 \right)$ ***A1 N2***

Note: Award ***A0*** for unfinished answers such as $3x^2(12 - x - 3x)$. ***[2 marks]***

c. $\frac{\mathrm{d}V}{\mathrm{d}x} = 72x - 36x^2$ ***A1A1 N2*** **Note:** Award ***A1*** for $72x$ and ***A1*** for $-36x^2$.

[2 marks]

d.i. valid approach to find maximum ***(M1)*** *eg* $V' = 0, 72x - 36x^2 = 0$

correct working *(A1)*

eg $x(72-36x), \frac{-72 \pm \sqrt{72^2-4\cdot(-36)\cdot 0}}{2(-36)}, 36x=72, 36x(2-x)=0$ $x=2$ ***A2 N2***

Note: Award ***A1*** for $x=2$ and $x=0$. ***[4 marks]***

d.ii. valid approach to explain that V is maximum when $x=2$

eg attempt to find V'', sign chart (must be labelled V') correct value/s

eg $V''(2)=72-72\times 2,\ V'(a)$ where $a<2$ $V'(b)$ where $b>2$

correct reasoning

eg $V''(2)<0,\ V'$ is positive for $x<2$ negative for $x>2$

Do not award unless has been awarded.

V is maximum when $x=2$

e. correct substitution into expression for volume

eg $3\times 2^2(12-4\times 2),\ 36\left(2^2\right)-12\left(2^3\right)$ $V=48$ (cm^3)

19N.1.SL.TZ0.S_9

a.i.

correct substitution into either $\overrightarrow{OA}\cdot\overrightarrow{OC}$ or into $\overrightarrow{OB}\cdot\overrightarrow{OC}$ (in (ii)) *(A1)*

eg $-2\times(-1)+4\times k,\ 6\times(-1)+8\times k$

correct expression ***A1 N1***

eg $2+4k,\ 4k+2$

[2 marks]

a.ii. correct expression ***A1 N1*** *eg* $8k-6,\ -6+8k$ ***[1 mark]***

b. finding magnitudes (seen anywhere) ***A1A1***

eg $\sqrt{(-2)^2+(4)^2+(-4)^2}(=6),\ \sqrt{(6)^2+(8)^2+0^2}(=10)$

correct substitution of their values into formula for angle AOC *(A1)*

eg $\cos\theta=\frac{2+4k}{\sqrt{(-2)^2+(4)^2+(-4)^2}\left|\overrightarrow{OC}\right|}$

correct substitution of their values into formula for angle BOC *(A1)*

eg $\cos\theta=\frac{8k-6}{\sqrt{(6)^2+(8)^2+0^2}\left|\overrightarrow{OC}\right|}$

recognizing that $\cos A\hat{O}C = \cos B\hat{O}C$ (seen anywhere) ***(M1)***

eg $\frac{2+4k}{\left|\overrightarrow{OC}\right|\sqrt{(-2)^2+(4)^2+(-4)^2}} = \frac{8k-6}{\left|\overrightarrow{OC}\right|\sqrt{6^2+(8)^2+0^2}}, \ \frac{2+4k}{6\sqrt{1+k^2}} = \frac{8k-6}{10\sqrt{1+k^2}}$

correct working (without radicals) ***(A2)***

eg $10(2+4k) = 6(8k-6), \ 11k^2 - 79k + 14 = 0$

correct working clearly leading to the required answer ***A1***

eg $20+36 = 48k - 40k, \ 56 = 8k, \ k = 7$ and $k = \frac{2}{11}, \ (k-7)(11k-2) = 0$

$k = 7$

c. finding magnitude of $\overrightarrow{OC}$ (seen anywhere) *eg* $\sqrt{(-1)^2+7^2+0^2}, \ \sqrt{50}$

valid attempt to find $\cos\theta$

eg $\cos\theta = \frac{2+28}{6\sqrt{(-1)^2+7^2+0^2}}, \ \cos\theta = \frac{56-6}{10\sqrt{(-1)^2+7^2+0^2}},$

$(\sqrt{26})^2 = 6^2 + \left(\sqrt{50}\right)^2 - 2(6)\sqrt{50}\cos\theta$

finding $\cos\theta$ *eg* $\cos\theta = \frac{5}{\sqrt{50}}\left(=\frac{1}{\sqrt{2}}\right)$

valid approach to find $\sin\theta$ (seen anywhere)

eg $\theta = \frac{\pi}{4}, \ \sin\theta = \cos\theta, \ \sin\theta = \sqrt{1-\frac{25}{50}}, \ \sin\theta = \sqrt{1-\cos^2\theta}, \ \sin\theta = \frac{\sqrt{2}}{2}$

correct substitution of values into $\frac{1}{2}ab\sin C$

eg $\frac{1}{2}\times 6\times\sqrt{50}\times\sqrt{1-\frac{25}{50}}, \ \frac{1}{2}\times 6\times\sqrt{50}\times\frac{5}{\sqrt{50}}$ area is 15

17N.1.SL.TZ0.S_8

valid approach ***(M1)***

eg $\int L - f, \ \int_{-1}^{1}(1-x^2)\mathrm{d}x$, splitting area into triangles and integrals

correct integration ***(A1)(A1)***

eg $\left[x - \frac{x^3}{3}\right]_{-1}^{1}, \ -\frac{x^3}{3} - \frac{x^2}{2} + \frac{x^2}{2} + x$

substituting **their** limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $1 - \frac{1}{3} - \left(-1 - \frac{-1}{3}\right)$

Award for substituting into original or differentiated function.

area $= \frac{4}{3}$

19M.1.SL.TZ1.S_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

recognizing relationship between v and s ***(M1)***

eg $\int v = s,\ s' = v$

$s(4) - s(2) = 9$ ***A1 N2***

[2 marks]

b.

correctly interpreting distance travelled in first 2 seconds (seen anywhere, including part (a) or the area of 15 indicated on diagram) ***(A1)***

eg $\int_0^2 v = 15,\ s(2) = 15$ valid approach to find total distance travelled ***(M1)***

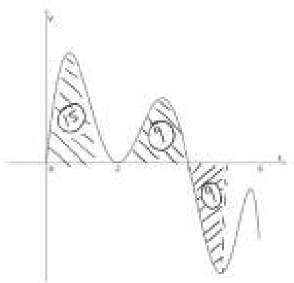
eg sum of 3 areas, $\int_0^4 v + \int_4^5 v,$ shaded areas in diagram between 0 and 5

Note: Award ***M0*** if only $\int_0^5 |v|$ is seen.

correct working towards finding distance travelled between 2 and 5 (seen anywhere including within total area expression or on diagram) ***(A1)***

eg $\int_2^4 v - \int_4^5 v,\ \int_2^4 v = \int_4^5 |v|,\ \int_4^5 v\mathrm{d}t = -9,\ s(4) - s(2) - [s(5) - s(4)],$

equal areas correct working using $s(5) = s(2)$ ***(A1)***



eg $15 + 9 - (-9),\ 15 + 2[s(4) - s(2)],\ 15 + 2(9),\ 2 \times s(4) - s(2),\ 48 - 15$

total distance travelled = 33 (m) ***A1 N2 [5 marks]***

19M.1.SL.TZ1.S_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct working ***(A1)***

eg $\sin\left(\frac{\pi}{4}x\right) = 1$, $\sqrt{x}\left(1 - \sin\left(\frac{\pi}{4}x\right)\right) = 0$

$\sin\left(\frac{\pi}{2}\right) = 1$ (seen anywhere) ***(A1)***

correct working (ignore additional values) ***(A1)***

eg $\frac{\pi}{4}x = \frac{\pi}{2}$, $\frac{\pi}{4}x = \frac{\pi}{2} + 2\pi$

x = 2, 10 ***A1A1 N1N1***

[5 marks]

c. valid approach ***(M1)*** *eg* first intersection at $x = 0$, $n = 20$

correct working ***A1*** *eg* $-6 + 8 \times 20$, $2 + (20 - 1) \times 8$, $u_{20} = 154$

P(154, $\sqrt{154}$) (accept $x = 154$ and $y = \sqrt{154}$) ***A1A1 N3 [4 marks]***

d. valid attempt to find upper boundary ***(M1)***

eg half way between u_{20} and u_{21}, $u_{20} + \frac{d}{2}$, 154 + 4, $-2 + 8n$, at least two values of new sequence {6, 14, ...}

upper boundary at $x = 158$ (seen anywhere) ***(A1)***

correct integral expression (accept missing $\mathrm{d}x$) ***A1A1 N4***

eg $\int_0^{158}\left(\sqrt{x}\sin\left(\frac{\pi}{4}x\right) + \sqrt{x}\right)\mathrm{d}x$, $\int_0^{158}(g + f)\ \mathrm{d}x)$, $\int_0^{158}\sqrt{x}\sin\left(\frac{\pi}{4}x\right)\mathrm{d}x - \int_0^{158} -\sqrt{x}\,\mathrm{d}x$

Note: Award ***A1*** for two correct limits and ***A1*** for correct integrand. The ***A1*** for correct integrand may be awarded independently of all the other marks.

[4 marks]

17M.1.SL.TZ1.S_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of summing to 1 ***(M1)***

eg $\sum p = 1$

correct equation ***A1***

$\cos\theta + 2\cos 2\theta = 1$

correct equation in $\cos\theta$ ***A1***

eg $\cos\theta + 2(2\cos^2\theta - 1) = 1,\ 4\cos^2\theta + \cos\theta - 3 = 0$

evidence of valid approach to solve quadratic ***(M1)***

eg factorizing equation set equal to 0, $\frac{-1 \pm \sqrt{1 - 4\times 4\times(-3)}}{8}$

correct working, clearly leading to required answer ***A1***

eg $(4\cos\theta - 3)(\cos\theta + 1),\ \frac{-1\pm 7}{8}$

correct reason for rejecting $\cos\theta \neq -1$ ***R1***

eg $\cos\theta$ is a probability (value must lie between 0 and 1), $\cos\theta > 0$

Note: Award ***R0*** for $\cos\theta \neq -1$ without a reason.

$\cos\theta = \frac{3}{4}$ ***AG N0***

b. valid approach ***(M1)***

eg sketch of right triangle with sides 3 and 4, $\sin^2 x + \cos^2 x = 1$ correct working ***(A1)***

eg missing side $= \sqrt{7},\ \frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$ $\tan\theta = \frac{\sqrt{7}}{3}$ ***A1 N2*** ***[3 marks]***

c. attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\pi\int_\theta^{\frac{\pi}{4}} f^2,\ \int\left(\frac{1}{\cos x}\right)^2$ correct substitution of both limits and function ***(A1)***

eg $\pi\int_\theta^{\frac{\pi}{4}}\left(\frac{1}{\cos x}\right)^2 \mathrm{d}x$ correct integration ***(A1)*** *eg* $\tan x$

substituting limits into integrated function and subtracting

eg $\tan\frac{\pi}{4} - \tan\theta$

Award if they substitute into original or differentiated function.

$\tan\frac{\pi}{4} = 1$ *eg* $1 - \tan\theta$ $V = \pi - \frac{\pi\sqrt{7}}{3}$

19M.1.SL.TZ2.S_10

b.

integrating by inspection from (a) or by substitution ***(M1)***

eg $\frac{2}{3}\int\frac{3}{2}\left(3x^2+1\right)\sqrt{x^3+x}\mathrm{d}x,\ u = x^3 + x,\ \frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2 + 1,\ \int u^{\frac{1}{2}},\ \frac{u^{\frac{3}{2}}}{1.5}$

correct integrated expression in terms of x ***A2 N3***

eg $\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}+C$, $\frac{\left(x^3+x\right)^{1.5}}{1.5}+C$

[3 marks]

c. integrating and subtracting functions (in any order) ***(M1)*** *eg* $\int g-f$, $\int f-\int g$

correct integral (including limits, accept absence of $\mathrm{d}x$) ***A1 N2***

eg $\int_0^1 (g-f)\,\mathrm{d}x$, $\int_0^1 6-3x^2\sqrt{x^3+x}-\sqrt{x^3+x}\,\mathrm{d}x$, $\int_0^1 g(x)-\int_0^1 f(x)$ ***[2 marks]***

d.

recognizing $\sqrt{x^3+x}$ is a common factor (seen anywhere, may be seen in part (c)) ***(M1)***

eg $\left(-3x^2-1\right)\sqrt{x^3+x}$, $\int 6-\left(3x^2+1\right)\sqrt{x^3+x}$, $\left(3x^2-1\right)\sqrt{x^3+x}$

correct integration ***(A1)(A1)*** eg $6x-\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}$

Note: Award ***A1*** for $6x$ and award ***A1*** for $-\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $6-\frac{2}{3}\left(1^3+1\right)^{\frac{3}{2}}$, $0-\left[6-\frac{2}{3}\left(1^3+1\right)^{\frac{3}{2}}\right]$ correct working ***(A1)***

eg $6-\frac{2}{3}\times 2\sqrt{2}$, $6-\frac{2}{3}\times\sqrt{4}\times\sqrt{2}$

area of $R=6-\frac{4\sqrt{2}}{3}\left(=6-\frac{2}{3}\sqrt{8},6-\frac{2}{3}\times 2^{\frac{3}{2}},\frac{18-4\sqrt{2}}{3}\right)$

18M.1.SL.TZ1.S_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct working ***(A1)***

eg $\int\frac{1}{2x-1}\mathrm{d}x$, $\int(2x-1)^{-1}$, $\frac{1}{2x-1}$, $\int\left(\frac{1}{\sqrt{u}}\right)^2\frac{\mathrm{d}u}{2}$

$\int(f(x))^2\mathrm{d}x=\frac{1}{2}\ln(2x-1)+c$ ***A2 N3***

Note: Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

[3 marks]

b.

attempt to substitute either limits or the function into formula involving f^2 (accept absence of π / dx) ***(M1)***

eg $\int_1^9 y^2 \mathrm{d}x, \pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2 \mathrm{d}x, \left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) ***(M1)***

eg $\frac{\pi}{2}(\ln(17)-\ln(1)), \pi\left(0-\frac{1}{2}\ln(2\times 9-1)\right)$

correct working involving calculating a log value or using log law

eg $\ln(1)=0, \ln\left(\frac{17}{1}\right)$ $\frac{\pi}{2}\ln 17\left(\text{accept } \pi\ln\sqrt{17}\right)$

Full may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two marks unless they involve logarithms.

18N.1.SL.TZ0.T_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\sqrt{\frac{4(350)^3}{243\pi}}$ ***OR*** $\sqrt{\frac{171500000}{763.407\ldots}}$ ***(M1)***

Note: Award **(M1)** for substitution of 350 into volume formula.

= 473.973... ***(A1)***

= 474 (cm^3) ***(A1)*(ft)** ***(C3)***

Note: The final **(A1)(ft)** is awarded for rounding **their** answer to 1 decimal place provided the unrounded answer is seen.

[3 marks]

b. 474 (cm^3) ***(A1)*(ft)** ***(C1)*** **Note:** Follow through from part (a). ***[1 mark]***

c. 4.74×10^2 (cm^3) ***(A1)*(ft)*(A1)*(ft)** ***(C2)*** **Note:** Follow through from **part (b) only**.

Award for answers of the type 0.474×10^3.

18M.1.SL.TZ1.T_14

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\sqrt{15^2 - 12^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras theorem.

OR

$\frac{\text{radius}}{21} = \frac{15}{35}$ ***(M1)***

Note: Award ***(M1)*** for a correct equation.

= 9 (cm) ***(A1) (C2)***

[2 marks]

b. $\pi \times 9 \times 15$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into curved surface area of a cone formula.

$= 424\text{cm}^2\left(135\pi, 424.115\ldots\text{cm}^2\right)$ ***(A1)*****(ft)** ***(C2)*** **Note**: Follow through from part (a).

[2 marks]

c. $\pi \times 21 \times 35 - 424.115\ldots$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into curved surface area of a cone formula and for subtracting their part (b).

$= 1880\text{cm}^2\left(600\pi, 1884.95\ldots\text{cm}^2\right)$ ***(A1)*****(ft)** ***(C2)*** **Note:** Follow through from part (b).

[2 marks]

18M.1.SL.TZ2.T_8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$(AC^2 =)\ 62^2 + 79^2 - 2 \times 62 \times 79 \times \cos(52°)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting in the cosine rule formula, ***(A1)*** for correct substitution.

63.7 (63.6708…) (km) ***(A1) (C3)***

b. $\frac{1}{2} \times 62 \times 79 \times \sin(52°)$

Award for substituting in the area of triangle formula, for correct substitution.

1930 km^2 (1929.83…km^2)

EXM.1.SL.TZ0.2

a.

$LQ = \frac{x_1 + x_2}{2}, UQ = \frac{x_4 + x_5}{2}, IQR = \frac{x_4 + x_5 - x_1 - x_2}{2}$ ***M1A1***

[2 marks]

b. $UQ + 1.5IQR = 1.25x_4 + 1.25x_5 - 0.75x_1 - 0.75x_2 \geqslant x_5$ ***M1A1***

Since $1.25x_4 + 0.25x_5 \geqslant 0.75x_1 + 0.75x_2$ due to the ascending order. ***R1***

Similarly $LQ - 1.5IQR = 1.25x_1 + 1.25x_2 - 0.75x_4 - 0.75x_5 \leqslant x_1$ ***M1A1***

Since $0.25x_1 + 1.25x_2 \leqslant 0.75x_3 + 0.75x_4$ due to the ascending order.

So there are no outliers for a data set of 5 numbers. ***AG*** ***[5 marks]***

c. For example 1, 2, 3, 4, 5, 6, 100 where $IQR = 4$ ***A1A1*** ***[2 marks]***

18M.1.SL.TZ2.T_15

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

(slant height^2 =) 10^2 + r^2 ***(M1)***

Note: For correct substitution of 10 and r into Pythagoras' Theorem.

$\sqrt{10^2 + r^2}$ ***(A1) (C2)***

[2 marks]

b. $\pi r^2 + 2\pi r \times 12 + \pi r\sqrt{100 + r^2} = 570$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in curved surface area of cylinder and area of the base, ***(M1)*** for their correct substitution in curved surface area of cone, ***(M1)*** for adding their 3 surface areas and equating to 570. Follow through their part (a).

= 4.58 (4.58358…) ***(A1)*(ft)** ***(C4)***

Note: Last line must be seen to award final ***(A1)***. Follow through from part (a). ***[4 marks]***

17N.1.SL.TZ0.T_3

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{149600000}{300000 \times 60}$ ***(M1)(M1)***

Note: Award ***(M1)*** for dividing the **correct** numerator (which can be presented in a different form such as 149.6×10^6 or 1.496×10^8) by 300000 and ***(M1)*** for dividing by 60.

$= 8.31$ (minutes) (8.31111 … , 8 minutes 19 seconds) ***(A1)*** ***(C3)***

[3 marks]

b. 323×9467280 ***(M1)***

Note: Award ***(M1)*** for multiplying 323 by 9467280, seen with **any** power of 10; therefore only penalizing incorrect power of 10 once.

$= 3.06 \times 10^9$ $(= 3.05793 \ldots \times 10^9)$ ***(A1)(A1)*** ***(C3)*** **Note:** Award ***(A1)*** for 3.06.

Award ***(A1)*** for $\times 10^9$ Award ***(A0)(A0)*** for answers of the type: 30.6×10^8 ***[3 marks]***

19M.1.SL.TZ2.T_6

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

median ***(A1) (C1)***

[1 mark]

b.i. 18 − 12 ***(A1)*** **Note:** Award ***(M1)*** for correct quartiles seen. 6 (g) ***(A1) (C2)***

[2 marks]

b.ii. 125 ***(A1) (C1)*** ***[1 mark]***

c. Cafeteria 2 ***(A1) (C1)*** 75 % > 50 % (do not meet the requirement) ***(R1) (C1)***

OR 25 % < 50 % (meet the requirement) ***(R1) (C1)***

Note: Do not award ***(A1)(R0)***. Award the ***(R1)*** for a correct comparison of percentages for both cafeterias, which may be in words. The percentage values or fractions must be seen. It is possible to award ***(A0)(R1)***.

17M.1.SL.TZ1.T_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$100 = \frac{1}{3}\pi r^2(8)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into volume of cone formula.

$r = 3.45 \text{ (cm)} \ \left(3.45494 \ldots \text{ (cm)}\right)$ ***(A1)*** ***(C2)***

[2 marks]

b. $l^2 = 8^2 + (3.45494 \ldots)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras' theorem.

$l = 8.71 \text{ (cm)} \ \left(8.71416 \ldots \text{ (cm)}\right)$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a). ***[2 marks]***

c. $\pi \times 3.45494 \ldots \times 8.71416 \ldots$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions into curved surface area of a cone formula.

$= 94.6 \text{ cm}^2 \ (94.5836 \ldots \text{ cm}^2)$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from parts (a) and (b). Accept 94.4 cm^2 from use of 3 sf values.

[2 marks]

16N.1.SL.TZ0.T_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

(i) 32.5 ***(A1)***

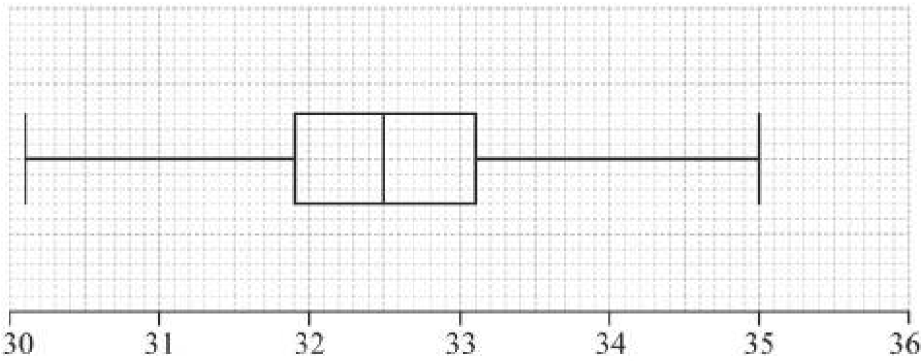
(ii) 31.9 ***(A1)***

(iii) 33.1 ***(A1)*** ***(C3)***

Note: Answers must be given correct to 1 decimal place.

[3 marks]

b.



Award for correct median, for correct quartiles and box, for correct end points of whiskers and straight whiskers.

Award at most if a horizontal line goes right through the box or if the whiskers are not well aligned with the midpoint of the box.

Follow through from part (a).

SPM.1.SL.TZ0.1

a.

valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

b. $\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)*** area $= 4\sqrt{11}$ ***A1*** ***[2 marks]***

18M.1.SL.TZ1.T_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{x+11}{2} = 10$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into median formula or for arranging all 9 values into ascending/descending order.

$(x =)\,9$ ***(A1) (C2)***

b.i. 2.69 (2.69072...) Follow through from part (a).

b.ii. 13 − 8 = 5
Award for 13 and 8 seen. Follow through from part (a).

SPM.1.SL.TZ0.2

attempt to substitute into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

Note: Accept use of Venn diagram or other valid method.

$0.6 = 0.5 + 0.4 - \mathrm{P}(A \cap B)$ ***(A1)***

$\mathrm{P}(A \cap B) = 0.3$ (seen anywhere) ***A1***

attempt to substitute into $\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ ***(M1)***

$= \frac{0.3}{0.4}$

$\mathrm{P}(A|B) = 0.75\left(= \frac{3}{4}\right)$ ***A1***

[5 marks]

SPM.1.SL.TZ0.4

attempt to integrate ***(M1)***

$u = 2x^2 + 1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4x$

$\int \frac{8x}{\sqrt{2x^2+1}}\mathrm{d}x = \int \frac{2}{\sqrt{u}}\mathrm{d}u$ ***(A1)***

EITHER

$= 4\sqrt{u}\,(+C)$ ***A1***

OR

$= 4\sqrt{2x^2+1}\,(+C)$ ***A1***

THEN

correct substitution into **their** integrated function (must have *C*) ***(M1)***

$5 = 4 + C \Rightarrow C = 1$

$f(x) = 4\sqrt{2x^2+1} + 1$ ***A1***

[5 marks]

SPM.1.SL.TZ0.5

a.

attempt to form composition ***M1***

correct substitution $g\left(\frac{x+3}{4}\right) = 8\left(\frac{x+3}{4}\right) + 5$ ***A1***

$(g \circ f)(x) = 2x + 11$ ***AG***

[2 marks]

b. attempt to substitute 4 (seen anywhere) ***(M1)***

correct equation $a = 2 \times 4 + 11$ ***(A1)*** $a = 19$ ***A1*** ***[3 marks]***

18N.1.SL.TZ0.T_9

a.i.

3 (cm) ***(A1) (C1)***

[1 mark]

a.ii. **units are required in part (a)(ii)** $\frac{1}{2} \times \frac{4\pi \times (3)^3}{3} + 3 \times (6)^2$ ***(M1)(M1)***

Note: Award ***(M1)*** for **their** correct substitution in volume of sphere formula divided by 2, ***(M1)*** for adding **their** correctly substituted volume of the cuboid.

= 165 cm^3 (164.548...) ***(A1)*(ft)** ***(C3)***

Note: The answer is 165 cm^3; the units are required. Follow through from part (a)(i).

[3 marks]

b. their 164.548... × 2.56 ***(M1)***

Note: Award ***(M1)*** for multiplying their part (a)(ii) by 2.56.

= 421 (g) (421.244...(g)) ***(A1)*(ft)** ***(C2)*** **Note:** Follow through from part (a)(ii).

[2 marks]

SPM.1.SL.TZ0.7

a.

evidence of median position ***(M1)***

80th employee

40 minutes ***A1***

[2 marks]

b. valid attempt to find interval (25–55) ***(M1)*** 18 (employees), 142 (employees) ***A1***

124 ***A1*** ***[3 marks]***

c. recognising that there are 16 employees in the top 10% ***(M1)***

144 employees travelled more than k minutes ***(A1)*** $k = 56$ ***A1*** ***[3 marks]***

d. $b = 70$ ***A1*** ***[1 mark]***

e.i. recognizing a is first quartile value ***(M1)*** 40 employees $a = 33$ ***A1***

[2 marks]

e.ii. 47 − 33 IQR = 14

f. attempt to find 1.5 × IQR 33 − 21 12

19M.1.SL.TZ1.T_1

evidence of 10 mm = 1 cm ***(A1)***

Note: Award ***(A1)*** for dividing their volume from part (a) or part (b) by 1000.

529 (cm^3) (528.612 (cm^3)) ***(A1)*(ft)** ***(C2)***

Note: Follow through from parts (a) or (b). Accept answers written in scientific notation.

[2 marks]

19M.1.SL.TZ2.T_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{3474000 \times \pi}{1000}$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct numerator and ***(M1)*** for dividing by 1000 **OR** equivalent, such as $\frac{3474000 \times 2 \times \pi}{2000}$ ie diameter.
Do not accept use of area formula ie πr^2.

10 913.89287... (km) ***(A1)*** ***(C3)***

[3 marks]

b. 10 900 (km) Follow through from part (a).

c. 1.09×10^4

Follow through from part (b) only. Award for 1.09, and $\times 10^4$. Award for answers of the type: 10.9×10^3.

19M.1.SL.TZ2.T_11

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

A = {3, 6, 9, 12} **AND** B = {2, 4, 6, 8, 10, 12, 14} ***(M1)***

Note: Award ***(M1)*** for listing all elements of sets A and B. May be seen in part (b). Condone the inclusion of 15 in set A when awarding the ***(M1)***.

6, 12 ***(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for each correct element. Award ***(A1)(A0)*** if one additional value seen. Award ***(A0)(A0)*** if two or more additional values are seen.

[3 marks]

b.i. 3, 9 ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Follow through from part (a) but only if their A and B are explicitly listed. Award ***(A1)*(ft)** for each correct element. Award ***(A1)(A0)*** if one additional value seen. Award ***(A0)(A0)*** if two or more additional values are seen.

[2 marks]

b.ii. 2 ***(A1)*(ft)** ***(C1)*** **Note:** Follow through from part (b)(i). ***[1 mark]***

18M.1.SL.TZ2.T_5

a.i.

$1000\left(1+\frac{3.5}{4\times 100}\right)^{4\times 5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitution.

OR

$N = 5$

$I = 3.5$

= 1000

$P/Y = 1$

$C/Y = 4$

Note: Award ***(A1)*** for $C/Y = 4$ seen, ***(M1)*** for other correct entries.

OR

$N = 5 \times 4$

$I = 3.5$

$PV = 1000$

$P/Y = 1$

$C/Y = 4$

Note: Award ***(A1)*** for $C/Y = 4$ seen, ***(M1)*** for other correct entries.

= 1190.34 (USD) ***(A1)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitution.

a.ii. 190.34 (USD)

Award for subtraction of 1000 from their part (a)(i). Follow through from (a)(i).

b. $\frac{170}{190.34}$ Award for division of 170 by their part (a)(ii).

= 0.89 Follow through from their part (a)(ii).

19M.1.SL.TZ1.T_12

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{1}{3} \times \frac{1}{3}$ **OR** $\left(\frac{1}{3}\right)^2$ ***(M1)***

Note: Award ***(M1)*** for multiplying correct probabilities.

$\frac{1}{9}$ (0.111, 0.111111..., 11.1%) ***(A1)*** ***(C2)***

[2 marks]

b. $\left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{6} \times \frac{1}{3}\right) + \frac{1}{3}$ ***(M1)(M1)***

Note: Award *(M1)* for $\left(\frac{1}{2}\times\frac{1}{3}\right)$ and $\left(\frac{1}{6}\times\frac{1}{3}\right)$ or equivalent, and *(M1)* for $\frac{1}{3}$ **and** adding only the three correct probabilities.

OR $1-\left(\frac{2}{3}\right)^2$ *(M1)(M1)*

Award for $\frac{2}{3}$ seen and for subtracting $\left(\frac{2}{3}\right)^2$ from 1. This may be shown in a tree diagram with "yellow" and "not yellow" branches.

$\frac{5}{9}$ (0.556, 0.555555..., 55.6%)

Follow through marks may be awarded if their answer to part (a) is used in a correct calculation.

c. $\frac{1}{3}$ (0.333, 0.333333..., 33.3%)

17M.1.SL.TZ2.T_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

0.93 (93%) *(A1)* *(C1)*

[1 mark]

b.i. 0.93×0.93 *(M1)* **Note:** Award *(M1)* for squaring their answer to part (a).

0.865 (0.8649; 86.5%) *(A1)*(ft) *(C2)* **Notes:** Follow through from part (a).

Accept 0.86 $\left(\text{unless it follows } \frac{93}{100}\times\frac{92}{99}\right)$. ***[2 marks]***

b.ii. $1-0.8649$ *(M1)* **Note:** Follow through from their answer to part (b)(i).

OR $0.07\times0.07+2\times(0.07\times0.93)$ *(M1)* **Note:** Follow through from part (a).

0.135 (0.1351; 13.5%) *(A1)*(ft) *(C2)* ***[2 marks]***

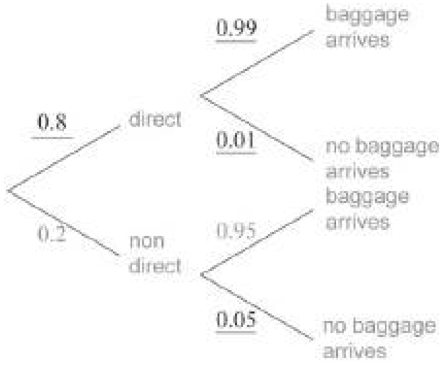
c. $1-a^3$ *(A1)* *(C1)*

Note: Accept $3a^2(1-a)+3a(1-a)^2+(1-a)^3$ or equivalent. ***[1 mark]***

17M.1.SL.TZ1.T_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1)(A1)(A1) ***(C3)***

Note: Award ***(A1)*** for each correct pair of probabilities.

[3 marks]

b. $0.8 \times 0.99 + 0.2 \times 0.95$ ***(A1)*****(ft)*****(M1)***

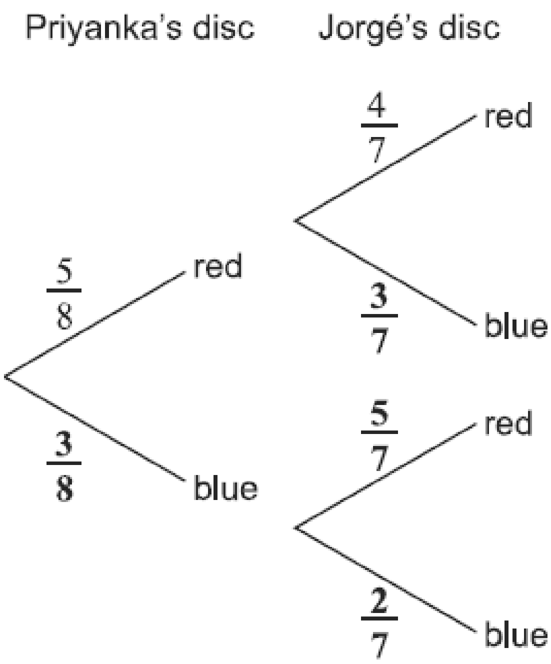
Note: Award ***(A1)*****(ft)** for two correct products of probabilities taken from their diagram, ***(M1)*** for the addition of their products.

$= 0.982 \left(98.2\%, \frac{491}{500}\right)$ Follow through from part (a).

18N.1.SL.TZ0.T_8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



Award for each correct pair of branches.

b. $\frac{5}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{5}{7}$

Award for two correct products from their tree diagram. Follow through from part (a), award for adding their two products. Award if additional products or terms are added.

$= \frac{5}{8}$ $\left(\frac{35}{56}, 0.625, 62.5\%\right)$

Follow through from their tree diagram, only if probabilities are [0,1].

19M.1.SL.TZ2.T_14

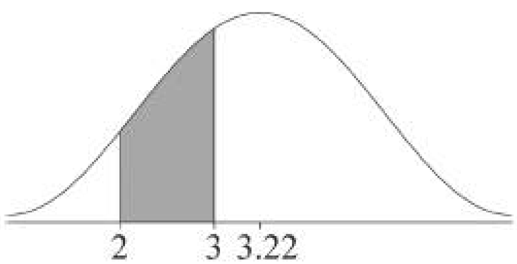
a.ii.

4.90 ***(A1) (C1)***

[1 mark]

b. 0.323 (0.323499…; 32.3 %) ***(A2) (C2)***

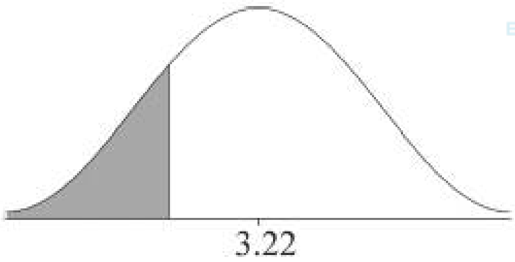
Note: If final answer is incorrect, ***(M1)(A0)*** may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement "P(2 ≤ X ≤ 3)" (accept other variables for X or "price" and strict inequalities).



[2 marks]

c. 2.51 (2.51303…) ***(A2) (C2)***

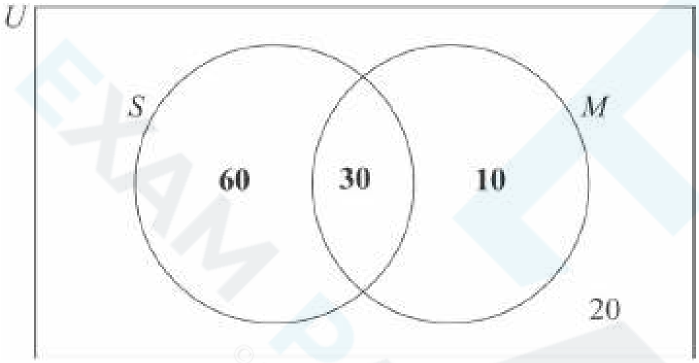
Note: If final answer is incorrect, ***(M1)(A0)*** may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement "P(X ≤ a) = 0.2" (accept other variables and strict inequalities).



17N.1.SL.TZ0.T_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1)(A1) ***(C2)***

Note: Award ***(A1)*** for 30 in correct area, ***(A1)*** for 60 and 10 in the correct areas.

[2 marks]

b. $\frac{30}{90}\left(\frac{1}{3}, 0.333333\ldots, 33.3333\ldots\%\right)$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Award ***(A1)*(ft)** for correct numerator of 30, ***(A1)*(ft)** for correct denominator of 90. Follow through from their Venn diagram.

[2 marks]

c. $\mathrm{P}(S)\times\mathrm{P}(M)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$ ***(R1)*** **Note:** Award ***(R1)*** for multiplying their by $\frac{1}{3}$.

therefore the events are independent $\left(\text{as } \mathrm{P}(S\cap M)=\frac{1}{4}\right)$ ***(A1)*(ft)** ***(C2)***

Note: Award ***(R1)(A1)*(ft)** for an answer which is consistent with their Venn diagram.

Do not award ***(R0)(A1)*(ft)**.

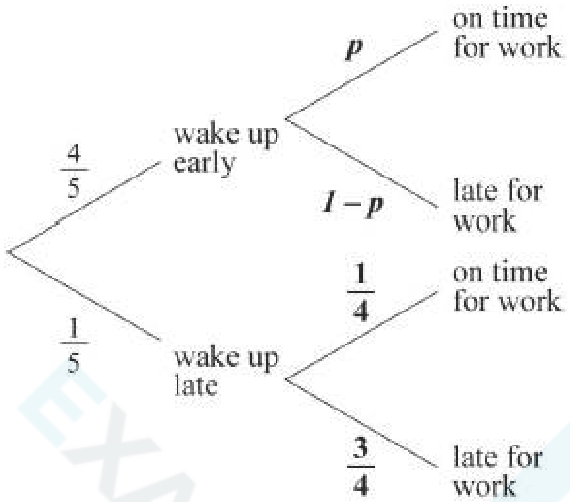
Do not award final ***(A1)*** if $\mathrm{P}(S)\times\mathrm{P}(M)$ is not calculated. Follow through from part (a).

[2 marks]

16N.1.SL.TZ0.T_12

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1)(A1) ***(C2)***

Note: Award ***(A1)*** for each correct pair of probabilities.

[2 marks]

b. $\frac{4}{5}p + \frac{1}{5} \times \frac{1}{4} = \frac{3}{5}$ ***(A1)*(ft)*(M1)(M1)***

Note: Award ***(A1)*(ft)** for two correct products from part (a), ***(M1)*** for adding their products, ***(M1)*** for equating the sum of any two probabilities to $\frac{3}{5}$.

$(p =)\ \frac{11}{16}\ (0.688,\ 0.6875)$ ***(A1)*(ft)** ***(C4)***

Note: Award the final ***(A1)*(ft)** only if $0 \leqslant p \leqslant 1$. Follow through from part (a).

[4 marks]

18M.1.SL.TZ1.T_10

$4 + 9 + 11 + 15 + x + (7 - x) + (11 - x) + (13 - x) = 60$ ***(M1)***

Note: Award ***(M1)*** for equating the sum of at least seven of the entries in their Venn diagram to 60.

$(x =)\ 5$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a), but only if answer is positive.

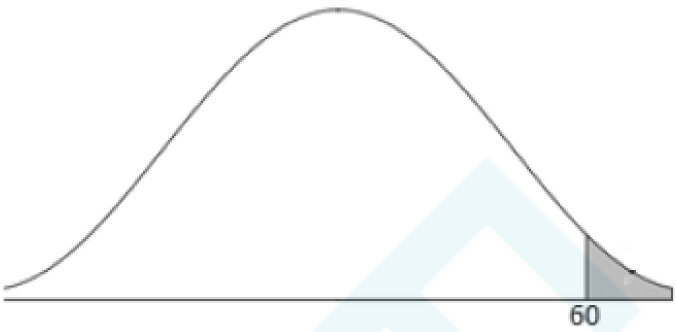
[2 marks]

18M.1.SL.TZ1.T_13

a.i.

0.0548 (0.054799…, 5.48%) ***(A2) (C2)***

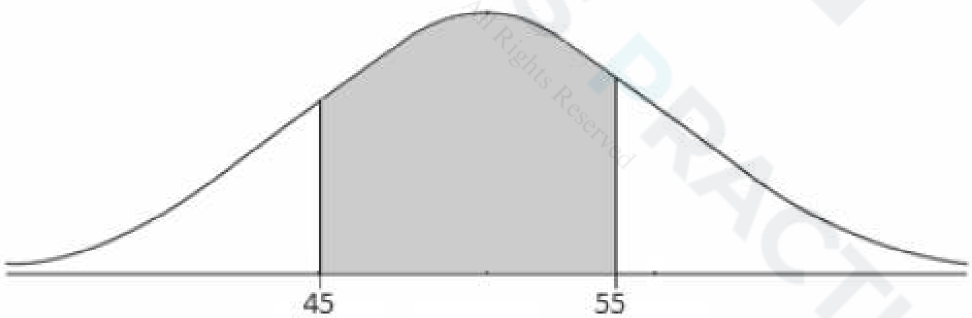
Note: Award ***(M1)(A0)*** for a correct probability statement, $\mathrm{P}(X > 60)$, or normal distribution graph with correctly shaded region, leading to incorrect or no answer.



[2 marks]

a.ii. 0.645 (0.6449900…, 64.5%) ***(A2) (C2)***

Note: Award ***(M1)(A0)*** for a correct probability statement, $\mathrm{P}(45 < X < 55)$, or normal distribution graph with correctly shaded region, leading to incorrect or no answer.



[2 marks]

b. $\frac{15}{0.0548}$ **(M1)** **Note:** Award ***(M1)*** for dividing 15 by their part (a)(i).

Accept an equation of the form $15 = x \times 0.0548$ for ***(M1)***. 274 (273.722…) ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a)(i). Accept 273. ***[2 marks]***

18M.1.SL.TZ2.T_9

a.i.

A' ***(A1)***

Note: Accept alternative set notation for complement such as $U - A$.

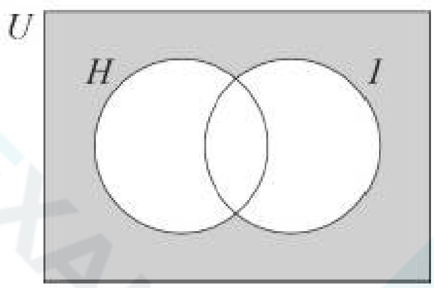
[1 mark]

a.ii. $C \cap D'$ **OR** $D' \cap C$ ***(A1)*** **Note:** Accept alternative set notation for complement.

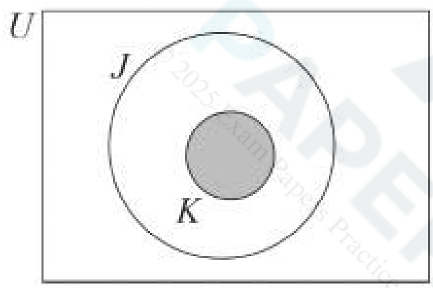
[1 mark]

a.iii. $(E \cap F) \cup G$ **OR** $G \cup (E \cap F)$ ***(A2) (C4)***

Note: Accept equivalent answers, for example $(E \cup G) \cap (F \cup G)$. ***[2 marks]***

b.i.  ***[1 mark]***

(A1)

b.ii. 

(A1) (C2)
[1 mark]

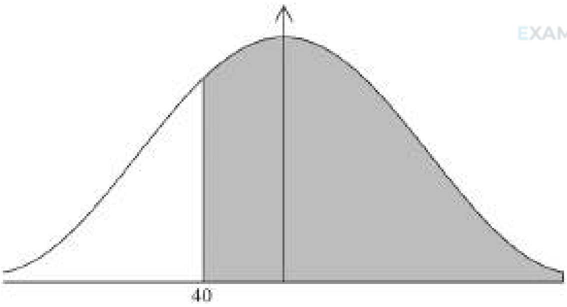
17N.1.SL.TZ0.T_13

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

0.787 (0.787433..., 78.7%) ***(M1)(A1)*** ***(C2)***

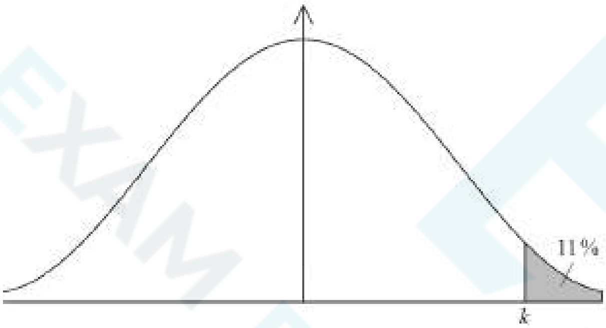
Note: Award ***(M1)*** for a correct probability statement, $\mathrm{P}(X > 40)$, or a correctly shaded normal distribution graph.



[2 marks]

b. 73.0 (minutes) (72.9924…) ***(M1)(A1)*** ***(C2)***

Note: Award ***(M1)*** for a correct probability statement, $\mathrm{P}(X > k) = 0.11$, or a correctly shaded normal distribution graph.



[2 marks]

c. $0.0423433\ \dots\ \times 400$

Award for multiplying a probability by 400. Do not award for 0.11×400.

Use of a lower bound less than zero gives a probability of 0.0429172….

$= 16$

Accept a final answer of 17. Do not accept a final answer of 18. Accept a non-integer final answer either 16.9 (16.9373…) from use of lower bound zero or 17.2 (17.1669…) from use of the default lower bound of -10^{99}.

17M.1.SL.TZ2.T_11

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$0.5\ \left(50\%,\ \frac{1}{2}\right)$ ***(A1)*** ***(C1)***

[1 mark]

b. $\mathrm{P}(X > a) = 0.25$ **OR** $\mathrm{P}(X < a) = 0.75$ ***(M1)***

Note: Award ***(M1)*** for a sketch of approximate normal curve with a vertical line drawn to the right of the mean with the area to the right of this line shaded.

$a = 434$ (g) $(433.724\ \ldots$ (g)$)$ ***(A1) (C2) [2 marks]***

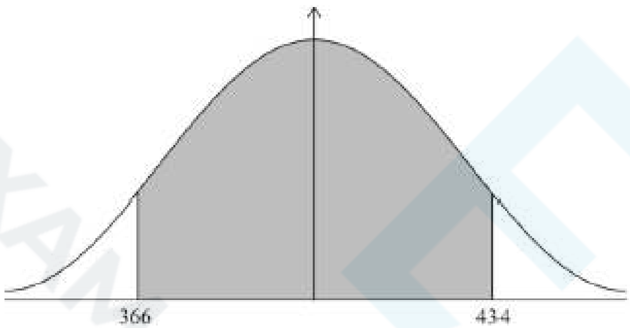
c. $33.7244\ \ldots \times 2$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for $33.7244\ \ldots$ (or $433.7244\ \ldots - 400$) seen, award ***(M1)*** for multiplying their 33.7244... by 2. Follow through from their answer to part (b).

OR $434 - 366.275\ \ldots$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their $366.275\ \ldots$ (366) seen, ***(M1)*** for difference between their answer to (b) and their 366.

OR



Award for their $366.275\ \ldots$ (366) seen. Award for correct symmetrical region indicated on labelled normal curve.

67.4 (g)

Accept an answer of 68 from use of rounded values Follow through from part (b).

17M.1.SL.TZ2.T_6

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

9 (cm) ***(A1) (C1)***

[1 mark]

b. 40 (leaves) ***(A1) (C1) [1 mark]***

c.i. $(200 \times 0.90 =)$ 180 or equivalent ***(M1)***

Note: Award ***(M1)*** for a horizontal line drawn through the cumulative frequency value of 180 and meeting the curve (or the corresponding vertical line from 10.5 cm).

$(k =)$ 10.5 (cm) ***(A1) (C2)*** **Note:** Accept an error of ±0.1. ***[2 marks]***

c.ii. $\left|\frac{9.5-10.5}{10.5}\right| \times 100\%$

Award for their correct substitution into the percentage error formula.

$9.52\ (\%)\ (9.52380\ldots\ (\%))$

Follow through from their answer to part (c)(i).

Award for an answer of -9.52 with or without working.

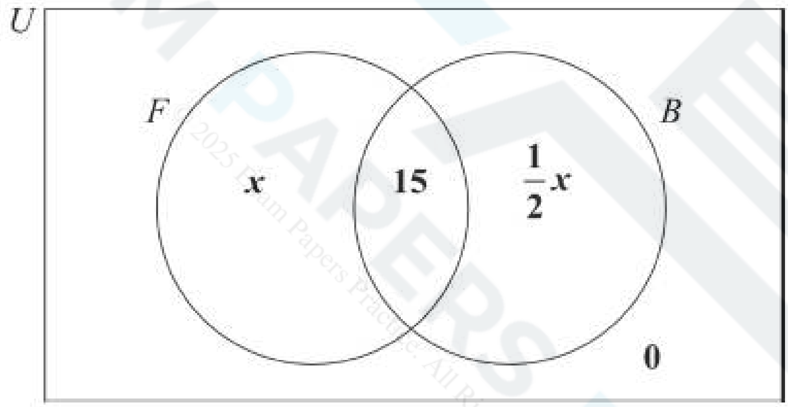
17M.1.SL.TZ2.T_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{1}{2}x$ ***(A1)*** ***(C1)***

[1 mark]

b. 

(A1)(A1)*(ft)** ***(C2)

Notes: Award ***(A1)*** for 15 placed in the correct position, award ***(A1)*(ft)** for x and their $\frac{1}{2}x$ placed in the correct positions of diagram. Do not penalize the absence of 0 inside the rectangle and award at most ***(A1)(A0)*** if any value other than 0 is seen outside the circles. Award at most ***(A1)(A0)*** if 35 and 70 are seen instead of x and their $\frac{1}{2}x$.

[2 marks]

c. $x+\frac{1}{2}x+15=120$ or equivalent ***(M1)***

Note: Award ***(M1)*** for adding the values in their Venn and equating to 120 (or equivalent).

$(x=)\ 70$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their Venn diagram, but only if the answer is a positive integer and x is seen in their Venn diagram.

[2 marks]

. 85 ***(A1)*(ft)** ***(C1)***

Follow through from their Venn diagram and their answer to part (c), but only if the answer is a positive integer and less than 120.

16N.1.SL.TZ0.T_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{\cos 36^\circ + \sin 18^\circ}{\sqrt{29^2 - 21.8}}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into formula.

$= 0.0390625$ ***(A1)*** ***(C2)***

Note: Accept $\frac{5}{128}$.

[2 marks]

b. (i) 0.04 ***(A1)*(ft)** (ii) 0.0391 ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a). ***[2 marks]***

c. 3.91×10^{-2} ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Answer should be consistent with their answer to part (b)(ii). Award ***(A1)*(ft)** for 3.91, and ***(A1)*(ft)** for 10^{-2}. Follow through from part (b)(ii).

[2 marks]

18M.1.SL.TZ2.T_3

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$4.8 \times 10^8 \times 2.5$ ***(M1)***

Note: Award ***(M1)*** for multiplying by 2.5.

1.2×10^9 (cm) ***(A1)*(ft)*(A1)*(ft)** ***(C3)***

Note: Award ***(A0)(A0)*** for answers of the type 12×10^8.

[3 marks]

b.i. 640 000 000 (cm) (6.4×10^8 (cm))

b.ii. $\frac{1.2 \times 10^9}{6.4 \times 10^8}$ Award for division by 640 000 000.

= 1.88 (1.875) Follow through from part (a) and part (b)(i).

19M.1.SL.TZ1.T_11

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

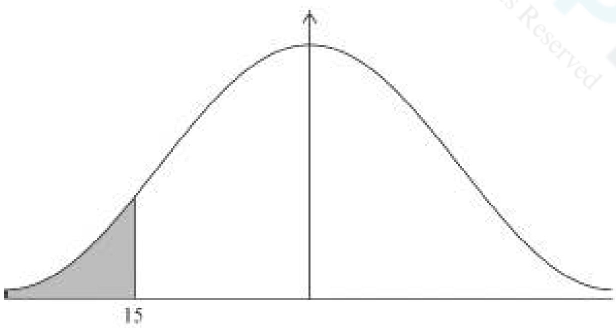
Mean and standard deviation	Graph
Mean $= -2$; standard deviation $= 0.707$	C
Mean $= 0$; standard deviation $= 0.447$	D

(A1)(A1) (C2)

Note: Award ***(A1)*** for each correct entry.

[2 marks]

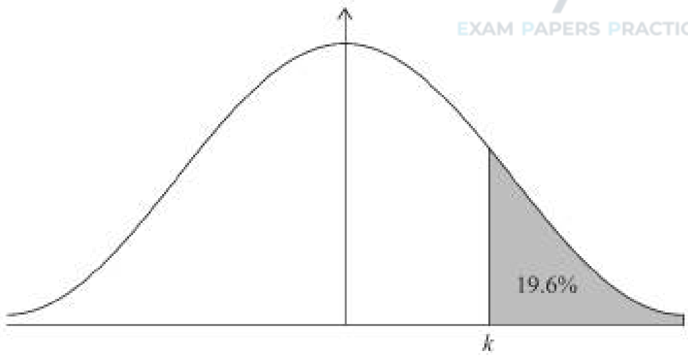
b.



(M1)

Note: Award ***(M1)*** for sketch with 15 labelled and left tail shaded **OR** for a correct probability statement, $P(X < 15)$.

0.0766 (0.0765637..., 7.66%) ***(A1) (C2) [2 marks]*** c.



Award for a sketch showing correctly shaded region to the right of the mean with 19.6% labelled (accept shading of the complement with 80.4% labelled) for a correct probability statement, $P(X > k) = 0.196$ or $P(X \leq k) = 0.804$.

23.0 (kg) (22.9959... (kg))

17M.1.SL.TZ1.T_2

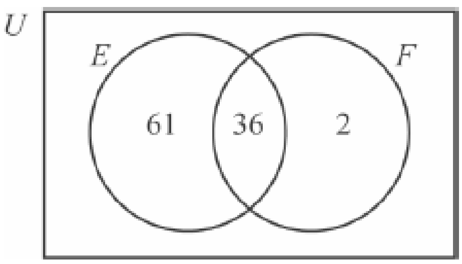
a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$97 - 36$ ***(M1)***

Note: Award ***(M1)*** for subtracting 36 from 97.

OR



(M1)

Note: Award ***(M1)*** for 61 **and** 36 seen in the correct places in the Venn diagram.

$= 61$ (%) ***(A1)*** ***(C2)***

Note: Accept 61.0 (%).

b. $\frac{36}{100} \times 985000$

Award for multiplying 0.36 (or equivalent) by 985000.

$= 355000\ (354600)$

c. $3.55 \times 10^5\ (3.546 \times 10^5)$

Award for 3.55 (3.546) match part (b), and $\times 10^5$.

Award for answers of the type: 35.5×10^4. Follow through from part (b).

18M.1.SL.TZ1.T_7

a.i.

$5d = 46 - 21$ **OR** $u_1 + 2d = 21$ and $u_1 + 7d = 46$ ***(M1)***

Note: Award ***(M1)*** for a correct equation in d or for two correct equations in u_1 and d.

$(d =)\ 5$ (kg) ***(A1) (C2)***

[2 marks]

a.ii. $u_1 + 2 \times 5 = 21$ ***(M1)*** ***OR*** $u_1 + 7 \times 5 = 46$ ***(M1)***

Note: Award ***(M1)*** for substitution of their d into either of the two equations.

$(u_1 =)\ 11$ (kg) ***(A1)*(ft)** ***(C2)*** **Note:** Follow through from part (a)(i). ***[2 marks]***

b. $\frac{12}{2}(2 \times 11 + (12 - 1) \times 5)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into arithmetic series formula.

$= 462$ (kg) ***(A1)*(ft)** ***(C2)*** **Note:** Follow through from parts (a) and (b). ***[2 marks]***

18M.1.SL.TZ1.T_4

a.i.

-0.974 $(-0.973745\ldots)$ ***(A2)***

Note: Award ***(A1)*** for an answer of 0.974 (minus sign omitted). Award ***(A1)*** for an answer of -0.973 (incorrect rounding).

[2 marks]

a.ii. $y = -0.365x + 17.9$ $(y = -0.365032\ldots x + 17.9418\ldots)$ ***(A1)(A1) (C4)***

Note: Award ***(A1)*** for $-0.365x$, ***(A1)*** for 17.9. Award at most ***(A1)(A0)*** if not an equation or if the values are reversed (eg $y = 17.9x - 0.365$).

b. $y = -0.365032... \times 18 + 17.9418...$

Award for correctly substituting 18 into their part (a)(ii).

= 11.4 (11.3712...) Follow through from part (a)(ii).

17M.1.SL.TZ2.T_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\sqrt[3]{\frac{2.78\times10^{11}}{3.12\times10^{-3}}}$ **OR** $\sqrt[3]{8.91025\ ...\ \times10^{13}}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into given expression.

44664.59503 ***(A1)*** ***(C2)***

Note: Award ***(A1)*** for a correct answer with at least 8 digits.

Accept 44664.5950301.

[2 marks]

b.i. 44664.60 ***(A1)*(ft)** ***(C1)***

Note: For a follow through mark, the answer to part (a) must be to at least 3 decimal places.

[1 mark]

b.ii. 44700 ***(A1)*(ft)** ***(C1)***

Notes: Answer to part (a) must be to at least 4 significant figures.

Accept any equivalent notation which is correct to 3 significant figures.

For example 447×10^2 or 44.7×10^3. Follow through from part (a). ***[1 mark]***

c. 4.47×10^4 ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Award for 4.47 and for 10^4.

Award for answers such as 44.7×10^3. Follow through from part (b)(ii) .

17M.1.SL.TZ2.T_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$4 + 3(n - 1) = 52$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the formula of the nth term of an arithmetic sequence, ***(A1)*** for correct substitution.

$n = 17$ ***(A1)*** ***(C3)***

[3 marks]

b. $\frac{24}{2}(2 \times 4 + 23 \times 3)$ **OR** $\frac{24}{2}(4 + 73)$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substitution into the sum of the first n terms of an arithmetic sequence formula, ***(A1)*(ft)** for their correct substitution, consistent with part (a).

924 ***(A1)*(ft)** ***(C3)*** **Note:** Follow through from part (a). ***[3 marks]***

16N.1.SL.TZ0.T_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$x = 3$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $x =$ constant, ***(A1)*** for the constant being 3.

The answer must be an equation.

[2 marks]

b. $\frac{-b}{2(-1)} = 3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into axis of symmetry formula. **OR**

$b - 2x = 0$

Award for correctly differentiating and equating to zero.

$c + b(-1) - (-1)^2 = 0$ (or equivalent) $c + b(3) - (3)^2 = 16$ (or equivalent)

Award for correct substitution of $(-1, 0)$ and $(3, 16)$ in the original quadratic function.

$(b =)\ 6$ Follow through from part (a).

c. $(-\infty, 16]$ $]-\infty, 16]$

Award for two correct interval endpoints, for left endpoint excluded right endpoint included.

17M.1.SL.TZ1.T_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$30 + (11 - 1) \times 27$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic sequence formula, ***(A1)*** for correct substitutions.

$= 300$ (cm) ***(A1)*** ***(C3)***

Note: Units are not required.

[3 marks]

b. $1050 \geqslant 30 + (n - 1) \times 27$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted arithmetic sequence formula $\leqslant 1050$, accept an equation, ***(A1)*** for correct substitutions.

$n = 38$ ***(A1)*****(ft)** ***(C3)***

Note: Follow through from their 27 in part (a). The answer must be an integer and rounded down.

17N.1.SL.TZ0.T_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$0.22(50) + 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 50 into equation of the regression line.

$(=)\ 26$ ***(A1)*** ***(C2)***

OR

$\frac{655}{25}$ ***(M1)***

Note: Award ***(M1)*** for correctly summing the h values of the points, and dividing by 25.

$(=)\ 26.2$ ***(A1)*** ***(C2)***

[2 marks]

b. line through $(50,\ 26 \pm 1)$ and $(0,\ 15)$ ***(A1)*(ft)*(A1)*** ***(C2)***

Note: Award ***(A1)*(ft)** for a straight line through (50, their $\bar{h}$), and ***(A1)*** for the line intercepting the y-axis at $(0,\ 15)$; this may need to be extrapolated. Follow through from part (a). Award at most ***(A0)(A1)*** if the line is not drawn with a ruler.

[2 marks]

c.

The correlation between h and a is positive.	✓
The correlation between h and a is negative.	
There is no correlation between h and a.	

(A1) (C1)

Note: Award ***(A0)*** if more than one tick (✓) is seen. ***[1 mark]***

d. 18 is less than the lowest age in the survey **OR** extrapolation. ***(A1)*** ***(C1)***

Note: Accept equivalent statements. ***[1 mark]***

16N.1.SL.TZ0.T_13

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$1064 + (5 - 1) \times 37$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic sequence formula, ***(A1)*** for correct substitution.

$= 1212$ ***(A1)*** ***(C3)***

[3 marks]

b. $2014 > 1064 + (n - 1) \times 37$ ***(M1)***

Note: Award ***(M1)*** for a correct substitution into arithmetic sequence formula.

Accept an equation. $(n <)\ 26.6756 \ldots$ ***(A1)*** 26 (times) ***(A1)*** ***(C3)***

Note: Award the final ***(A1)*** for the correct rounding **down** of their unrounded answer.

OR $2014 > 1064 + 37t$ ***(M1)***

Note: Award ***(M1)*** for a correct substitution into a linear model (where $t = n - 1$).

Accept an equation or weak inequality. Accept $\frac{2014 - 1064}{37}$ for ***(M1)***.

$(t <)\ 25.6756 \ldots$ ***(A1)*** 26 (times) ***(A1)*** ***(C3)***

Note: Award the final ***(A1)*** for adding 1 to the correct rounding down of their unrounded answer.

[3 marks]

17N.1.SL.TZ0.T_15

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

522 (kg) ***(A1)*** ***(C1)***

[1 mark]

b. $522(8 - 6.80)$ or equivalent ***(M1)***

Note: Award ***(M1)*** for multiplying their answer to part (a) by $(8 - 6.80)$.

626 (EUR) (626.40) ***(A1)(ft)*** ***(C2)*** **Note:** Follow through from part (a).

[2 marks]

c. $(W =)\ (882 - 45p)(p - 6.80)$ ***(A1)*** **OR**

$(W =) - 45p^2 + 1188p - 5997.6$ ***(A1)*** ***(C1)*** ***[1 mark]***

d. sketch of W with some indication of the maximum ***(M1)*** **OR**

$-90p + 1188 = 0$

Award for equating the correct derivative of their part (c) to zero.

$(p =)\ \frac{-1188}{2 \times (-45)}$

Award for correct substitution into the formula for axis of symmetry.

$(p =)\ 13.2$ (EUR)

Follow through from their part (c), if the value of p is such that $6.80 < p < 19.6$.

17M.1.SL.TZ2.T_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{1}{2}\ (0.5)$ ***(A1)*** ***(C1)***

[1 mark]

b. $18 \times \left(\frac{1}{2}\right)^4$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the geometric sequence formula. Accept a list of their five correct terms.

$1.125\ \left(1.13,\ \frac{9}{8}\right)$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their common ratio from part (a). ***[2 marks]***

c. $18 \times \left(\frac{1}{2}\right)^{n-1} < 10^{-3}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for their correct substitution into the geometric sequence formula with a variable in the exponent, ***(M1)*** for comparing their expression with $10^{-3}\ \left(\frac{1}{1000}\right)$.

Accept an equation. $n = 16$ ***(A1)*****(ft)** ***(C3)***

Note: Follow through from their common ratio from part (a). "n" must be a positive integer for the ***(A1)*** to be awarded.

[3 marks]

17M.1.SL.TZ1.T_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{162}{486}$ **OR** $\frac{54}{162}$ ***(M1)***

Note: Award ***(M1)*** for dividing any u_{n+1} by u_n.

$= \frac{1}{3}$ $(0.333,\ 0.333333\ \ldots)$ ***(A1)*** ***(C2)***

[2 marks]

b. $486\left(\frac{1}{3}\right)^{n-1} = 2$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into geometric sequence formula.

$n = 6$ ***(A1)*(ft)** ***(C2)*** **Note:** Follow through from part (a).

Award ***(A1)(A0)*** for $u_6 = 2$ or u_6 with or without working. ***[2 marks]***

c. $S_{30} = \frac{486\left(1 - \frac{1}{3}^{30}\right)}{1 - \frac{1}{3}}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into geometric series formula.

$= 729$ ***(A1)*(ft)** ***(C2)*** ***[2 marks]***

17M.1.SL.TZ2.T_13

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

3 ***(A1)*** ***(C1)***

Notes: Accept $y = 3$

[1 mark]

b. $3 = 0.5(1) + c$ **OR** $y - 3 = 0.5(x - 1)$ ***(A1)(A1)***

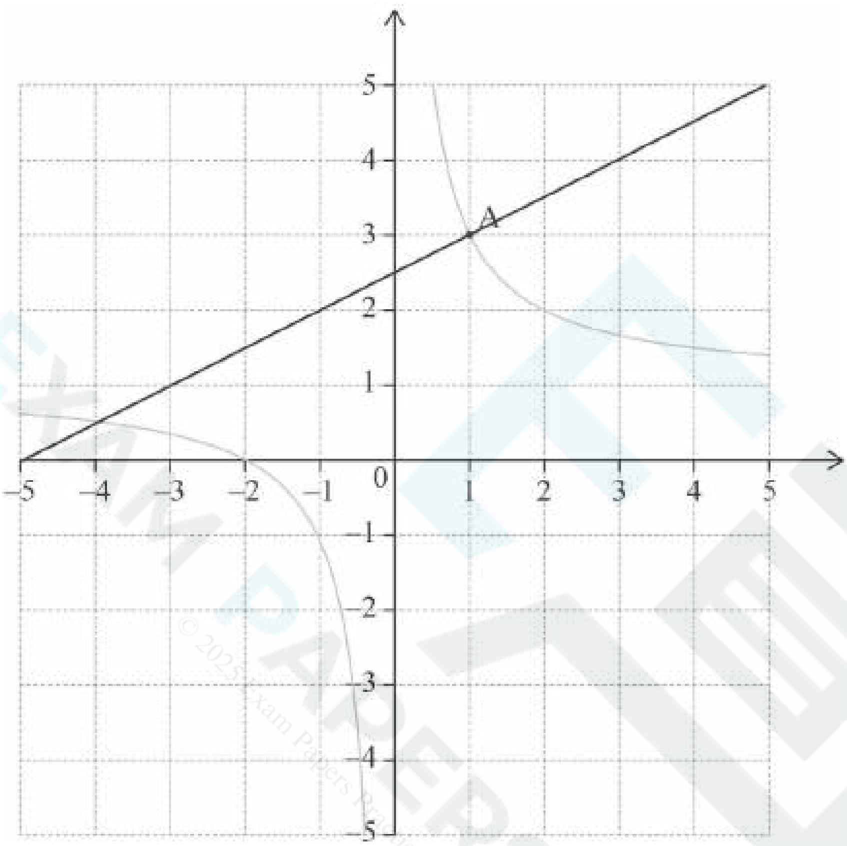
Note: Award ***(A1)*** for correct gradient, ***(A1)*** for correct substitution of $A(1, 3)$ in the equation of line.

$x - 2y + 5 = 0$ or any integer multiple ***(A1)*(ft)** ***(C3)***

Note: Award ***(A1)*(ft)** for their equation correctly rearranged in the indicated form.

The candidate's answer **must** be an equation for this mark. ***[3 marks]***

c.



Award for a straight line, with positive gradient, passing through $(1, 3)$, for line (or extension of their line) passing approximately through 2.5 or their intercept with the y-axis.

16N.1.SL.TZ0.T_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$48 = 64r$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into geometric sequence formula.

$0.75\left(\frac{3}{4}, \frac{48}{64}\right)$ ***(A1)*** ***(C2)***

[2 marks]

b. $64 \times (0.75)^7$ ***(M1)***

Award for correct substitution into geometric sequence formula or list of eight values using their r. Follow through from part (a), only if answer is positive.

$= 8.54 \text{ (cm) } (8.54296 \ldots \text{ cm})$

c. $\text{depth} = \frac{64\left(1-(0.75)^{10}\right)}{1-0.75}$

Award for correct substitution into geometric series formula. Follow through from part (a), only if answer is positive.

$= 242\text{(cm) } (241.583 \ldots)$

17N.1.SL.TZ0.T_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{560}{70} \times 100$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for dividing 560 by 0.7 or equivalent.

$= 800 \text{ (USD)}$ ***(A1)*** ***(C2)***

[2 marks]

b. $560\left(1+\frac{75}{12 \times 100}\right)^{12 \times \frac{1}{2}}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into interest formula, ***(A1)*** for their correct substitution.

OR $N = \frac{1}{2}$ $I\% = 75$ $PV = (\pm)560$ $P/Y = 1$ $C/Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 12$ seen, ***(M1)*** for all other entries correct. **OR** $N = 6$ $I\% = 75$ $PV = (\pm)560$ $P/Y = 12$ $C/Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 12$ seen, ***(M1)*** for all other entries correct.

$= 805.678 \ldots \text{ (USD)}$ ***(A1)***

Note: Award ***(A3)*** for 805.678... (806) seen without working.

(Juan spends) 5.68 (USD) (5.67828... USD) (more than the original price) ***(A1)*(ft)** ***(C4)***

17M.1.SL.TZ1.T_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

Length of trout	Frequency
20 cm < trout length ≤ 30 cm	0
30 cm < trout length ≤ 40 cm	4
40 cm < trout length ≤ 50 cm	2
50 cm < trout length ≤ 60 cm	7
60 cm < trout length ≤ 70 cm	8
70 cm < trout length ≤ 80 cm	1

(A2) ***(C2)***

Note: Award ***(A2)*** for all correct entries, ***(A1)*** for 3 correct entries.

[2 marks]

b. continuous ***(A1)*** ***(C1)*** ***[1 mark]***

c. $60 \text{ (cm)} < \text{trout length} \leqslant 70 \text{ (cm)}$ ***(A1)*** ***(C1)***

Note: Accept equivalent notation such as $(60, 70]$ or $]60, 70]$.

Award ***(A0)*** for "60-70" (incorrect notation). ***[1 mark]***

d. $\frac{4}{22} \times 100$ ***(M1)*** **Note:** Award ***(M1)*** for their 4 divided by their 22.

$= 18.2 \ (18.1818 \ldots)$ ***(A1)(ft)*** ***(C2)***

Note: Follow through from their part (a). Do not accept 0.181818.... ***[2 marks]***

18M.1.SL.TZ1.T_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

(1, −2) ***(A1)(A1) (C2)***
Note: Award ***(A1)*** for 1 and ***(A1)*** for −2, seen as a coordinate pair.

Accept $x = 1$, $y = -2$. Award ***(A1)(A0)*** if x and y coordinates are reversed.

[2 marks]

b. $\frac{1-(-2)}{-3-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution, of their part (a), into gradient formula.

$= -\frac{3}{4}(-0.75)$ ***(A1)*(ft)** ***(C2)*** **Note:** Follow through from part (a). ***[2 marks]***

c. $y - 1 = -\frac{3}{4}(x+3)$ ***OR*** $y + 2 = -\frac{3}{4}(x-1)$ ***OR*** $y = -\frac{3}{4}x - \frac{5}{4}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their part (b) and a given point. ***OR***

$1 = -\frac{3}{4} \times -3 + c$ ***OR*** $-2 = -\frac{3}{4} \times 1 + c$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their part (b) and a given point.

$3x + 4y + 5 = 0$ (accept any integer multiple, including negative multiples) ***(A1)*(ft)** ***(C2)***

Follow through from parts (a) and (b). Where the gradient in part (b) is found to be $\frac{5}{0}$, award at most for either $x = -3$ or $x + 3 = 0$.

20N.1.SL.TZ0.S_10

a.i.

$f'x = -kx^{-2}$ ***(A1)***

$f'p = -kp^{-2} = -\frac{k}{p^2}$ ***A1*** ***N2***

[2 marks]

a.ii. attempt to use point and gradient to find equation of L_1 ***M1***

eg $y - \frac{k}{p} = -kp^{-2}x - p$, $\frac{k}{p} = -\frac{k}{p^2}p + b$ correct working leading to answer ***A1***

eg $p^2y - kp = -kx + kp$, $y - \frac{k}{p} = -\frac{k}{p^2}x + \frac{k}{p}$, $y = -\frac{k}{p^2}x + \frac{2k}{p}$

$kx + p^2y - 2pk = 0$ ***AG*** ***N0*** ***[2 marks]***

b. **METHOD 1 – area of a triangle** recognizing $x = 0$ at B ***(M1)***

correct working to find y-coordinate of null ***(A1)*** *eg* $p^2y - 2pk = 0$

y-coordinate of null at $y = \frac{2k}{p}$ (may be seen in area formula) ***A1***

correct substitution to find area of triangle ***(A1)*** *eg* $\frac{1}{2}2p\frac{2k}{p}$, $p \times \frac{2k}{p}$

area of triangle $\text{AOB} = 2k$ ***A1*** ***N3*** **METHOD 2 – integration**

recognizing to integrate L_1 between 0 and $2p$ ***(M1)*** *eg* $\int_0^{2p} L_1 \, \mathrm{d}x$, $\int_0^{2p} -\frac{k}{p^2}x + \frac{2k}{p}$

correct integration of **both** terms ***A1***

eg $-\frac{kx^2}{2p^2} + \frac{2kx}{p}$, $-\frac{k}{2p^2}x^2 + \frac{2k}{p}x + c$, $-\frac{k}{2p^2}x^2 + \frac{2k}{p}x \Big|_0^{2p}$

substituting limits into **their** integrated function and subtracting (in either order) ***(M1)***

eg $-\frac{k2p^2}{2p^2} + \frac{2k2p}{p} - 0$, $-\frac{4kp^2}{2p^2} + \frac{4kp}{p}$ correct working ***(A1)*** *eg* $-2k + 4k$

area of triangle $\text{AOB} = 2k$ ***A1*** ***N3*** ***[5 marks]***

c.

Note: In this question, the second ***M*** mark may be awarded independently of the other marks, so it is possible to award ***(M0)(A0)M1(A0)(A0)A0***.

recognizing use of transformation ***(M1)***

eg area of triangle AOB = area of triangle DEF, $gx = \frac{k}{x-4} + 3$, gradient of $L_2 =$ gradient of L_1, D4, 3, 2p+4, one correct shift

correct working

eg area of triangle $\text{DEF} = 2k$, $\text{CD} = 3$, $\text{DF} = 2p$, $\text{CG} = 2p$, E4, $\frac{2k}{p} + 3$, F$2p + 4$, 3, Q$p + 4$, $\frac{k}{p} + 3$,

gradient of $L_2 = -\frac{k}{p^2}$, $g'x = -\frac{k}{x-4^2}$, area of rectangle $\text{CDFG} = 2k$ valid approach

eg $\frac{\text{ED} \times \text{DF}}{2} = \text{CD} \times \text{DF}$, $2p \cdot 3 = 2k$, $\text{ED} = 2\text{CD}$, $\int_4^{2p+4} L_2 \, \mathrm{dx} = 4k$

correct working

eg $\text{ED} = 6$, E4, 9, $k = 3p$, gradient $= \frac{3 - \frac{2k}{p} + 3}{2p + 4 - 4}$, $\frac{-6}{\frac{2k}{3}}$, $-\frac{9}{k}$

correct expression for gradient (in terms of p)

eg $\frac{-6}{2p}$, $\frac{9-3}{4-2p+4}$, $-\frac{3p}{p^2}$, $\frac{3 - \frac{23p}{p} + 3}{2p + 4 - 4}$, $-\frac{9}{3p}$ gradient of L_2 is $-\frac{3}{p} = -3p^{-1}$

20N.1.SL.TZ0.S_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1 – (sine rule)

evidence of choosing sine rule ***(M1)***

eg $\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b}$

correct substitution ***(A1)***

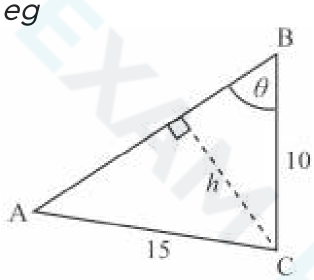
eg $\frac{\frac{\sqrt{3}}{3}}{10} = \frac{\sin \theta}{15}$, $\frac{\sqrt{3}}{30} = \frac{\sin \theta}{15}$, $\frac{\sqrt{3}}{30} = \frac{\sin \mathrm{B}}{15}$

$\sin \theta = \frac{\sqrt{3}}{2}$ ***A1 N2***

METHOD 2 – (perpendicular from vertex C)

valid approach to find perpendicular length (may be seen on diagram) ***(M1)***

eg



, $\frac{h}{15} = \frac{\sqrt{3}}{3}$

correct perpendicular length ***(A1)***

eg $\frac{15\sqrt{3}}{3}$, $5\sqrt{3}$

$\sin \theta = \frac{\sqrt{3}}{2}$ ***A1 N2***

Note: Do not award the final ***A*** mark if candidate goes on to state $\sin \theta = \frac{\pi}{3}$, as this demonstrates a lack of understanding.

[3 marks]

b. attempt to substitute into double-angle formula for cosine ***(M1)***

$1 - 2\frac{\sqrt{3}}{3}^{2}$, $2\frac{\sqrt{6}}{3}^{2} - 1$, $\frac{\sqrt{6}}{3}^{2} - \frac{\sqrt{3}}{3}^{2}$, $\cos 2\theta = 1 - 2\frac{\sqrt{3}}{2}^{2}$, $1 - 2 \sin^{2}\frac{\sqrt{3}}{3}$

correct working *eg* $1 - 2 \times \frac{3}{9}$, $2 \times \frac{6}{9} - 1$, $\frac{6}{9} - \frac{3}{9}$

$\cos 2 \times \mathrm{C\hat{A}B} = \frac{3}{9}$ $= \frac{1}{3}$

20N.1.SL.TZ0.S_3

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognize $fx = 0$ ***(M1)***

eg $\sqrt{12 - 2x} = 0,\quad 2x = 12$

$a = 6$ (accept $x = 6$, $6,\quad 0)$ ***A1 N2***

[2 marks]

b.

attempt to substitute either **their** limits or the function into volume formula (must involve $f^{\ 2}$) ***(M1)***

eg $\int_0^6 f^{\ 2}\ \mathrm{d}\,x$, $\pi\int\sqrt{12 - 2x}^2$, $\pi\int_0^6 12 - 2x\quad \mathrm{d}\,x$

correct integration of each term ***A1 A1*** *eg* $12x - x^2$, $12x - x^2 + c$, $12x - x^2{}_0^6$

substituting limits into **their integrated** function and subtracting (in any order) ***(M1)***

eg $\pi 126 - 6^2 - \pi 0$, $72\pi - 36\pi$, $126 - 6^2 - 0$

Note: Award ***M0*** if candidate has substituted into f, $f^{\ 2}$ or f'.

volume $= 36\pi$ ***A1 N2*** ***[5 marks]***

20N.1.SL.TZ0.S_5

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1 – (discriminant)

correct expression for g ***(A1)***

eg $--x^2 + 4x + 5 + k$, $x^2 - 4x - 5 + k = 0$

evidence of discriminant ***(M1)***

eg $b^2 - 4ac,\quad \Delta$

correct substitution into discriminant of g ***(A1)***

eg $-4^2 - 41\text{-}5 + k$, $16 - 4k - 5$

recognizing discriminant is negative ***(M1)***

eg $\Delta < 0$, $-4^2 - 41\text{-}5 + k < 0$, $16 < 4k - 5$, $16 - 4\text{-}15 < 0$

correct working (must be correct inequality) ***(A1)***

eg $-4k < -36$, $k - 5 > 4$, $16 + 20 - 4k < 0$

$k > 9$ ***A1 N3***

METHOD 2 – (transformation of vertex of f)

valid approach for finding $f x$ vertex ***(M1)***

eg $-\frac{b}{2a} = 2$, $f'x = 0$

correct vertex of $f x$ ***(A1)***

eg $2,\ 9$

correct vertex of $-f x$ ***(A1)***

eg $2,\ -9$

correct vertex of $g x$ ***(A1)***

eg $\begin{matrix} 2 \\ -9 \end{matrix} + \begin{matrix} 0 \\ k \end{matrix}$, $2,\ -9 + k$

recognizing when vertex is above x-axis ***(M1)***

eg $-9 + k > 0$, sketch

$k > 9$ ***A1 N3***

METHOD 3 – (transformation of f)

recognizing vertical reflection of $f x$ ***(M1)***

eg $-f x$, $x^2 - 4x - 5$, sketch

correct expression for $g x$

eg $x^2 - 4x - 5 + k$

valid approach for finding vertex of $g x$

eg $-\frac{b}{2a} = 2$, $g'x = 0$

correct y coordinate of vertex of $g x$

eg $y = -9 + k$, $2,\ -9 + k$

recognizing when vertex is above x-axis

eg $-9 + k > 0$, sketch

$k > 9$

20N.1.SL.TZ0.S_4

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempt to substitute coordinates (in any order) into f ***(M1)***

eg $a \log_3 13 - 4 = 7$, $a \log_3 7 - 4 = 13$, $a \log\ 9 = 7$

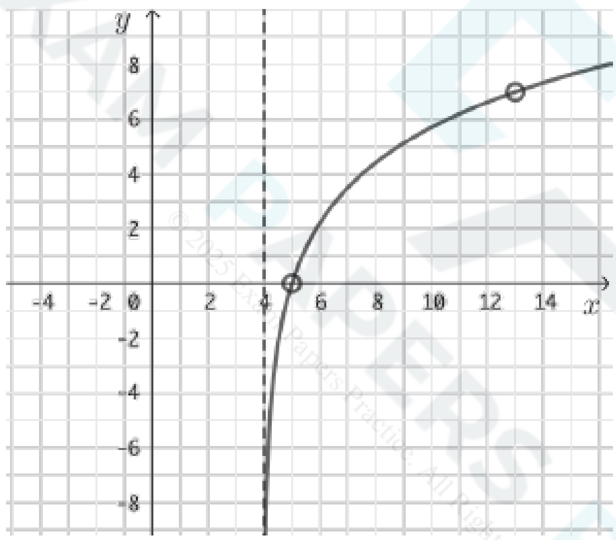
finding $\log_3 9 = 2$ (seen anywhere) ***(A1)***

eg $\log_3 9 = 2$, $2a = 7$

$a = \frac{7}{2}$ ***A1 N2***

[3 marks]

b.



A1A1A1 N3

Note: Award ***A1*** for correct shape of logarithmic function (must be increasing and concave down).
Only if the shape is correct, award the following:
A1 for being asymptotic to $x = 4$
A1 for curve including both points in circles.

[3 marks]

20N.1.SL.TZ0.S_8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach ***(M1)***

eg $Q_3 - Q_1$, $Q_3 - 1.1$, $4.5 - a = 1.1$

$a = 3.4$ ***A1 N2***

[2 marks]

b. $\frac{32}{5} = 6.4$ (km) ***A1 N1 [1 mark]***

c. **METHOD 1 (standard deviation first)** valid approach ***(M1)***

eg standard deviation $= \sqrt{\text{variance}}$, $\sqrt{\frac{16}{9}}$ standard deviation $= \frac{4}{3}$ (km) ***(A1)***

valid approach to convert **their** standard deviation ***(M1)*** *eg* $\frac{4}{3} \times \frac{5}{8}$, $\sqrt{\frac{16}{9}} = \frac{8}{5}M$

$\frac{20}{24}$ (miles) $= \frac{5}{6}$ ***A1 N3***

Note: If no working shown, award ***M1A1M0A0*** for the value $\frac{4}{3}$.
If working shown, and candidate's final answer is $\frac{4}{3}$, award ***M1A1M0A0***.

METHOD 2 (variance first) valid approach to convert variance ***(M1)***

eg $\frac{5}{8}^2$, $\frac{64}{25}$, $\frac{16}{9} \times \frac{5}{8}^2$ variance $= \frac{25}{36}$ ***(A1)*** valid approach ***(M1)***

eg standard deviation $= \sqrt{\text{variance}}$, $\sqrt{\frac{25}{36}}$, $\sqrt{\frac{16}{9} \times \frac{5}{8}^2}$ $\frac{20}{24}$ (miles) $= \frac{5}{6}$ ***A1 N3***

[4 marks]

d. correct frequency for 22 minutes ***(A1)*** *eg* 20

adding **their** frequency (do not accept 22 + 400) ***(M1)***

eg 20 + 400 , 420 athletes $m = 30$ (minutes) ***A1 N3 [3 marks]***

e. 27 (minutes) ***(A1)*** correct working ***(A1)***

eg 130 athletes between 22 and 27 minutes, $\text{P}22 < t < 27 = \frac{150 - 20}{600}$, $\frac{13}{60}$

evidence of conditional probability or reduced sample space ***(M1)***

eg $\text{P}A \quad B$, $\text{P}t < 27 \quad 22 < t < 30$, $\frac{\text{P}22 < t < 27}{\text{P}22 < t < m}$, $\frac{150}{400}$ correct working ***(A1)***

eg $\frac{\frac{130}{600}}{\frac{400}{600}}$, $\frac{150 - 20}{400}$ $\frac{130}{400}$ $\frac{13}{40} = \frac{78000}{240000} = \frac{390}{1200} = 0.325$

If no other working is shown, award for answer of $\frac{150}{400}$.
Award for answer of $\frac{3}{8}$ with no other working shown.

19M.1.SL.TZ1.T_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

continuous ***(A1) (C1)***

[1 mark]

b. 75.5 (km h^{-1}) ***(A1) (C1)*** **Note:** Answer must be exact. ***[1 mark]***

c. 294 (km h^{-1}) ***(A1) (C1)*** ***[1 mark]***

d.i. $\frac{300+97+80+80+71+64+21+6}{8}$ **OR** $\frac{719}{8}$ ***(M1)***

Note: Award ***(M1)*** for correct sum divided by 8. 89.9 (89.875)(km h^{-1}) ***(A1) (C2)***

[2 marks]

d.ii. 84.6 (84.5597...)(km h^{-1}) ***(A1) (C1)***

Note: If the response to part (d)(i) is awarded zero marks, a correct response to part (d)(ii) is awarded ***(C2)***.

20N.1.SL.TZ0.S_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach to find $\overrightarrow{AB}$ ***(M1)***

eg $\overrightarrow{OB} - \overrightarrow{OA}$, A - B

$\overrightarrow{AB} = \begin{pmatrix} 8 \\ m-1 \\ -8 \end{pmatrix}$ ***A1 N2***

[2 marks]

b. valid approach ***(M1)*** *eg* $L = \begin{pmatrix} 9 \\ m \\ -6 \end{pmatrix}$, $\begin{pmatrix} 9 \\ m \\ -6 \end{pmatrix} = \begin{pmatrix} -3 \\ -19 \\ 24 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$

one correct equation ***(A1)*** *eg* $-3+2s=9$, $-6=24-5s$

correct value for s ***A1*** *eg* $s=6$

substituting **their** s value into their expression/equation to find m ***(M1)***

eg $-19+6\times4$ $m=5$ ***A1 N3 [5 marks]***

c. valid approach ***(M1)*** *eg* $\overrightarrow{BC} = \begin{pmatrix} 9p \\ -6 \\ 3 \end{pmatrix}$, $C = 9u + B$, $\overrightarrow{BC} = \begin{pmatrix} x-9 \\ y-5 \\ z+6 \end{pmatrix}$

correct working to find C *(A1)*

eg $\overrightarrow{OC} = \begin{pmatrix} 9p+9 \\ -1 \\ -3 \end{pmatrix}$, $C = 9\begin{pmatrix} p \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix} + \begin{pmatrix} 9 \\ 5 \\ -6 \end{pmatrix}$, $y = -1$ and $z = -3$

correct approach to find u (seen anywhere) *eg* $p^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2$, $\sqrt{p^2 + \frac{4}{9} + \frac{1}{9}}$

recognizing unit vector has magnitude of 1

eg $u = 1$, $\sqrt{p^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = 1$, $p^2 + \frac{5}{9} = 1$ correct working

eg $p^2 = \frac{4}{9}$, $p = \pm\frac{2}{3}$ $p = \frac{2}{3}$ substituting value of p

eg $\begin{pmatrix} x-9 \\ y-5 \\ z+6 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 9 \\ 5 \\ -6 \end{pmatrix}$, $C = 9\begin{pmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix} + \begin{pmatrix} 9 \\ 5 \\ -6 \end{pmatrix}$, $x - 9 = 6$

C(15, −1, −3) (accept $\begin{pmatrix} 15 \\ -1 \\ -3 \end{pmatrix}$)

The marks for finding p are independent of the first two marks.
For example, it is possible to award marks such as or
.

20N.1.SL.TZ0.S_6

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of integration ***(M1)***

eg $\int f'(x)\,dx$, $\int 6e^{2x}$

correct integration (accept missing $+c$) ***(A1)***

eg $\frac{1}{2} \times 6e^{2x}$, $3e^{2x} + c$

substituting initial condition into **their** integrated expression (must have $+c$) ***M1***

eg $3e^{2 \times \ln 4} + c = 20$

Note: Award ***M0*** if candidate has substituted into f' or f'' .

correct application of $\log a^b = b \log a$ rule (seen anywhere)

eg $2 \ln 4 = \ln 16$, $e^{\ln 16}$, $\ln 4^2$

correct application of $e^{\ln a} = a$ rule (seen anywhere)

eg $e^{\ln 16} = 16$, $e^{\ln 4^2} = 4^2$

correct working

eg $3 \times 16 + c = 20$, $3 \times 4^2 + c = 20$, $c = -28$

$f x = 3e^{2x} - 28$

20N.1.SL.TZ0.S_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

recognizing velocity is derivative of displacement ***(M1)***

eg $v = \frac{ds}{dt}$, $\frac{d}{dt} 10 - \frac{7}{4}t^2$

velocity $= -\frac{14}{4}t \quad = -\frac{7}{2}t$ ***A1 N2***

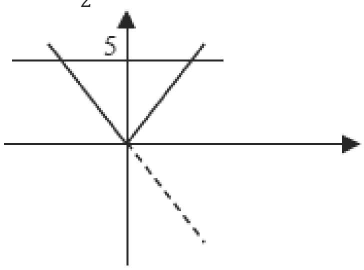
[2 marks]

b. valid approach to find speed of P_2 ***(M1)***

eg $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$, $\sqrt{4^2 + -3^2}$, velocity $= \sqrt{4^2 + -3^2}$ correct speed ***(A1)*** *eg* 5 ms^{-1}

recognizing relationship between speed and velocity (may be seen in inequality/equation) ***R1***

eg $-\frac{7}{2}t$, speed = |velocity| , graph of P_1 speed ,



P_1 speed $= \frac{7}{2}t$, P_2 velocity $= -5$

correct inequality or equation that compares speed or velocity (accept any variable for q) ***A1***

eg $-\frac{7}{2}t > 5$, $-\frac{7}{2}q < -5$, $\frac{7}{2}q > 5$, $\frac{7}{2}q = 5$

$q = \frac{10}{7}$ (seconds) (accept $t > \frac{10}{7}$, do not accept $t = \frac{10}{7}$)

Do not award the last two marks without the .

16N.1.SL.TZ0.T_14

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$2x^3 - 3x$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $2x^3$, award ***(A1)*** for $-3x$.

Award at most ***(A1)(A0)*** if there are any extra terms.

[2 marks]

b. $2x^3 - 3x = -10$ ***(M1)***

Note: Award ***(M1)*** for equating their answer to part (a) to -10. $x = -2$ ***(A1)*****(ft)**

Note: Follow through from part (a). Award ***(M0)(A0)*** for -2 seen without working.

$y = \frac{1}{2}(-2)^4 - \frac{3}{2}(-2)^2 + 7$ ***(M1)***

Note: Award ***(M1)*** substituting their -2 into the original function.

$y = 9$ ***(A1)*****(ft)** ***(C4)*** **Note:** Accept $(-2, 9)$. ***[4 marks]***

19M.1.SL.TZ2.T_15

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

−120 (AUD) ***(A1)*** ***(C1)***

[1 mark]

b. $-\frac{3}{5}x^2 + 14x$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for each correct term. Award at most ***(A1)(A0)*** for extra terms seen.

[2 marks]

c. $-\frac{3}{5}x^2 + 14x = 0$ ***(M1)*** **Note:** Award ***(M1)*** for equating their derivative to zero. **OR** sketch of their derivative (approximately correct shape) with x-intercept seen ***(M1)***

$23\frac{1}{3}\left(23.3, 23.3333\ldots, \frac{70}{3}\right)$ ***(A1)(ft)***

Note: Award ***(C2)*** for $23\frac{1}{3}\left(23.3, 23.3333\ldots, \frac{70}{3}\right)$ seen without working.

23 ***(A1)(ft)*** ***(C3)*** **Note:** Follow through from part (b).

18N.1.SL.TZ0.T_11

(y − (−2)) = 12 (x − (−1)) ***(M1)***

OR

−2 = 12(−1) + c ***(M1)***

Note: Award ***(M1)*** for point **and** their gradient substituted into the equation of a line.

y = 12x + 10 ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (b).

[2 marks]

17N.1.SL.TZ0.T_11

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$x = -2$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $x =$ (a constant) and ***(A1)*** for -2.

[2 marks]

b. $(c =)\ 5$ ***(A1)*** ***(C1)*** ***[1 mark]***

c. $-\frac{b}{2a} = -2$ $a(-2)^2 - 2b + 5 = 3$ or equivalent $a(-4)^2 - 4b + 5 = 5$ or equivalent

$2a(-2) + b = 0$ or equivalent ***(M1)***

Note: Award ***(M1)*** for two of the above equations. $a = 0.5$ ***(A1)*(ft)**

$b = 2$ ***(A1)*(ft)** ***(C3)***

Award at most if the answers are reversed.

Follow through from parts (a) and (b).

18M.1.SL.TZ2.T_13

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$(70 - 75)^2 + 100$ ***(M1)***

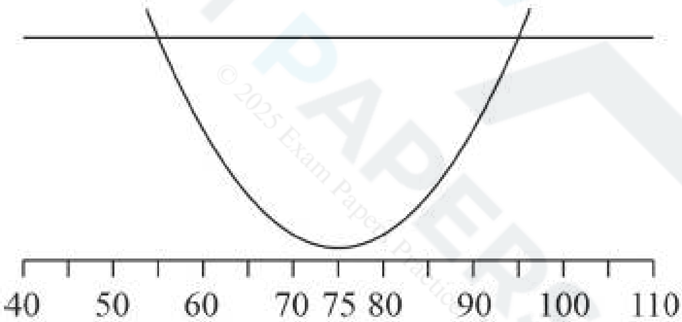
Note: Award ***(M1)*** for substituting in $x = 70$.

125 ***(A1) (C2)***

[2 marks]

b. $(s - 75)^2 + 100 = 500$ ***(M1)***

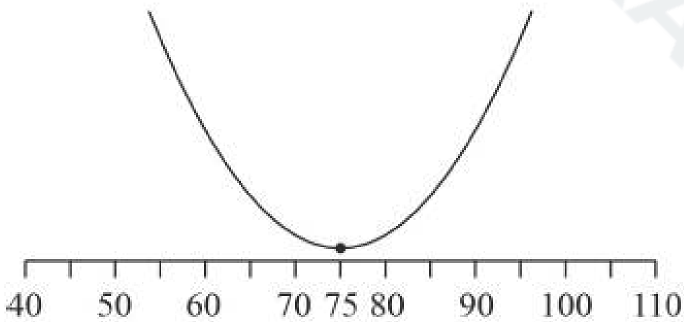
Note: Award ***(M1)*** for equating $C(x)$ to 500. Accept an inequality instead of =. **OR**



(M1)

Note: Award ***(M1)*** for sketching correct graph(s). $(s =)$ 95 ***(A1) (C2)*** ***[2 marks]***

c.



(M1)

Note: Award ***(M1)*** for an attempt at finding the minimum point using graph. **OR**

$\frac{95 + 55}{2}$ ***(M1)***

Note: Award ***(M1)*** for attempting to find the mid-point between their part (b) and 55.

OR $(C'(x) =)\ 2x - 150 = 0$ ***(M1)***

Award for an attempt at differentiation that is correctly equated to zero.

75

20N.1.SL.TZ0.T_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.*

8.97×10^{18} EUR 8.973×10^{18} ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for 8.97 (8.973), ***(A1)*** for $\times10^{18}$. Award ***(A1)(A0)*** for $8.97\text{E}18$. Award ***(A0)(A0)*** for answers of the type 8973×10^{15}.

[2 marks]

b. $\frac{4\times\pi\times113^3}{3}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in volume of sphere formula.

[2 marks]

$6\ 040\ 000\ \text{km}^3$ 6.04×10^6, $\frac{5771588\pi}{3}$, $6\ 043\ 992.82$ ***(A1) (C2)***

c. $\frac{6\ 043\ 992.82 - 6.074\times10^6}{6.074\times10^6}\times100$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the percentage error formula (accept a consistent absence of "$\times10^6$" from all terms).

$0.494\ \%$ $0.494026\ldots\ \%$ ***(A1)(ft) (C2)***

Note: Follow through from their answer to part (b). If the final answer is negative, award at most ***(M1)(A0)***.

[2 marks]

20N.1.SL.TZ0.T_11

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might* not be suitable for all forms of practice.

$x =$ -2 - 4 **OR** $x =$ -2 - 2 - -2 ***(M1)***

Note: Award ***(M1)*** for correct calculation of the left symmetrical point.

$x =$ - 6 ***(A1) (C2)***

[2 marks]

b.

	positive	zero	negative
a			✓
b			✓

(A1)(A1) (C2)

Note: Award ***(A1)*** for each correct row. ***[2 marks]***

c. $x >$ - 2 **OR** $x \geq$ - 2 ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for -2 seen as part of an inequality, ***(A1)*** for completely correct notation. Award ***(A1)(A1)*** for correct equivalent statement in words, for example "decreasing when x is greater than negative 2".

20N.1.SL.TZ0.T_13

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.*

$2x + \frac{k}{x^2}$ ***(A1)(A1)(A1) (C3)***

Note: Award ***(A1)*** for $2x$, ***(A1)*** for $+k$, and ***(A1)*** for x^{-2} or $\frac{1}{x^2}$.
Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

[3 marks]

b. -2.5 $\frac{-5}{2}$ ***(A1) (C1) [1 mark]***

c. $-2.5 = 2 \times -2 + \frac{k}{-2^2}$ ***(M1)***

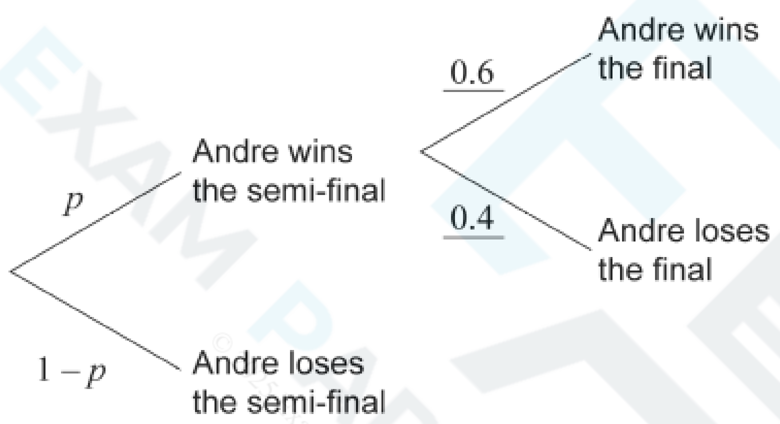
Award for equating their gradient from part (b) to their substituted derivative from part (a).

$k =$ 6 Follow through from parts (a) and (b).

20N.1.SL.TZ0.T_14

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.*



(A1) (C1)

Note: Award ***(A1)*** for the correct pair of probabilities.

[1 mark]

b. $p \times 0.4 + 1 - p = 0.58$ ***(M1)***

Note: Award ***(M1)*** for multiplying and adding correct probabilities for losing equated to 0.58.

OR

$p \times 0.6 = 1 - 0.58$ ***(M1)***

Note: Award ***(M1)*** for multiplying correct probabilities for winning equated to $1 - 0.58$ or 0.42.

$p =$ 0.7 ***(A1)*(ft)*** ***(C2)***

Note: Follow through from their part (a). Award the final ***(A1)*(ft)** only if their p is within nge $0 < p < 1$.

c. $\frac{0.3}{0.58}$ $\frac{1-0.7}{0.58}$

Award for their correct numerator. Follow through from part (b). Award for the correct denominator.

$\frac{0.3}{0.3+0.7\times0.4}$

Award for their correct numerator. Follow through from part (b). Award for their correct calculation of Andre losing the semi-final or winning the semi-final and then losing in the final. Follow through from their parts (a) and (b).

$\frac{15}{29}$ $0.517,\quad 0.517241\ldots,\quad 51.7\%$

Follow through from parts (a) and (b).

20N.1.SL.TZ0.T_12

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.*

$0 = K - 601.2^{0}$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted function equated to zero.

$K =$ 60 ***(A1)*** ***(C2)***

[2 marks]

b. the (vertical) speed that Jean-Pierre is approaching (as t increases) ***(A1)*** ***(C1)***
OR
the limit of the (vertical) speed of Jean-Pierre ***(A1)*** ***(C1)***

Note: Accept "maximum speed" or "terminal speed". ***[1 mark]***

c. $S =$ $60 - 601.2^{-10}$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted function.

$S = \quad 50.3096 \ldots \quad ms^{-1}$ Follow through from part (a).

$181 \quad kmh^{-1} \qquad 181.114 \ldots \quad kmh^{-1}$

Award the final for correct conversion of their speed to $\text{km} \quad \text{h}^{-1}$.

17M.1.SL.TZ2.S_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempt to subtract terms ***(M1)***

eg$d = u_2 - u_1,\ 7 - 3$

$d = 4$ ***A1*** ***N2***

[2 marks]

b. correct approach ***(A1)*** *eg*$u_{10} = 3 + 9(4)$ $u_{10} = 39$ ***A1*** ***N2*** ***[2 marks]***

c. correct substitution into sum ***(A1)*** *eg*$S_{10} = 5(3 + 39),\ S_{10} = \frac{10}{2}(2 \times 3 + 9 \times 4)$

$S_{10} = 210$ ***A1*** ***N2*** ***[2 marks]***

17N.1.SL.TZ0.S_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

subtracting terms ***(M1)***

eg$5 - 8,\ u_2 - u_1$

$d = -3$ ***A1*** ***N2***

[2 marks]

b. correct substitution into formula ***(A1)***

eg$u_{10} = 8 + (10 - 1)(-3),\ 8 - 27,\ -3(10) + 11$ $u_{10} = -19$ ***A1*** ***N2*** ***[2 marks]***

17N.1.SL.TZ0.S_10

recognizing infinite geometric series with squares ***(M1)*** *eg* $k^2 + k^4 + k^6 + \ \dots,\ \frac{k^2}{1-k^2}$

correct substitution into $S_\infty = \frac{9}{16}$ (must substitute into formula) ***(A2)***

eg $\frac{k^2}{1-k^2} = \frac{9}{16}$

correct working ***(A1)***

eg $16k^2 = 9 - 9k^2,\ 25k^2 = 9,\ k^2 = \frac{9}{25}$

$k = \frac{3}{5}$ (seen anywhere) ***A1***

valid approach with segments and CD (may be seen earlier) ***(M1)***

eg $r = k,\ S_\infty = b$

correct expression for b in terms of k (may be seen earlier) ***(A1)***

eg $b = \frac{k}{1-k},\ b = \sum_{n=1}^{\infty} k^n,\ b = k + k^2 + k^3 + \ \dots$

substituting **their** value of k into **their** formula for b ***(M1)***

eg $\frac{\frac{3}{5}}{1-\frac{3}{5}},\ \frac{\left(\frac{3}{5}\right)}{\left(\frac{2}{5}\right)}$

$b = \frac{3}{2}$ ***A1*** ***N3***

[9 marks]

18N.1.SL.TZ0.S_3

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct working ***(A1)***

eg $-5 + (8 - 1)(3)$

$u_8 = 16$ ***A1 N2***

[2 marks]

18M.1.SL.TZ1.S_10

a.i.

valid approach ***(M1)***

eg $\frac{u_2}{u_1},\ \frac{u_1}{u_2}$

$r=\frac{12\sin^2\theta}{18}\left(=\frac{2\sin^2\theta}{3}\right)$ ***A1 N2***

[2 marks]

c. **METHOD 1** (using differentiation) recognizing $\frac{dS_\infty}{d\theta}=0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{dS_\infty}{d\theta}$ ***(A1)***

eg $\frac{0-54\times(-2\sin2\theta)}{(2+\cos2\theta)^2}, -54(2+\cos2\theta)^{-2}(-2\sin2\theta)$ correct working ***(A1)***

eg $\sin 2\theta = 0$ any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***

eg 0, π, ... , sketch of sine curve with *x*-intercept(s) marked both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta=\pi, 3\pi$ (accept values in degrees) both correct answers $\theta=\frac{\pi}{2},\frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

METHOD 2 (using denominator) recognizing when S_∞ is greatest ***(M1)***

eg $2+\cos 2\theta$ is a minimum, $1-r$ is smallest
correct working ***(A1)***

eg minimum value of $2+\cos 2\theta$ is 1, minimum $r=\frac{2}{3}$ correct working ***(A1)***

eg $\cos2\theta=-1, \frac{2}{3}\sin^2\theta=\frac{2}{3}, \sin^2\theta=1$ **EITHER** (using $\cos 2\theta$)

any correct value for $\cos^{-1}(-1)$ (seen anywhere) ***(A1)***

eg π, 3π, ... (accept values in degrees), sketch of cosine curve with *x*-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta=\pi, 3\pi$ (accept values in degrees) **OR** (using $\sin\theta$) $\sin\theta=\pm1$ (A1)

$\sin^{-1}(1)=\frac{\pi}{2}$ (accept values in degrees) (seen anywhere) ***A1*** **THEN**

both correct answers $\theta=\frac{\pi}{2},\frac{3\pi}{2}$ ***A1 N4***

Award if either or both correct answers are given in degrees.
Award if additional values are given.

17M.1.SL.TZ1.S_7

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct use $\log x^n = n\log x$ ***A1***

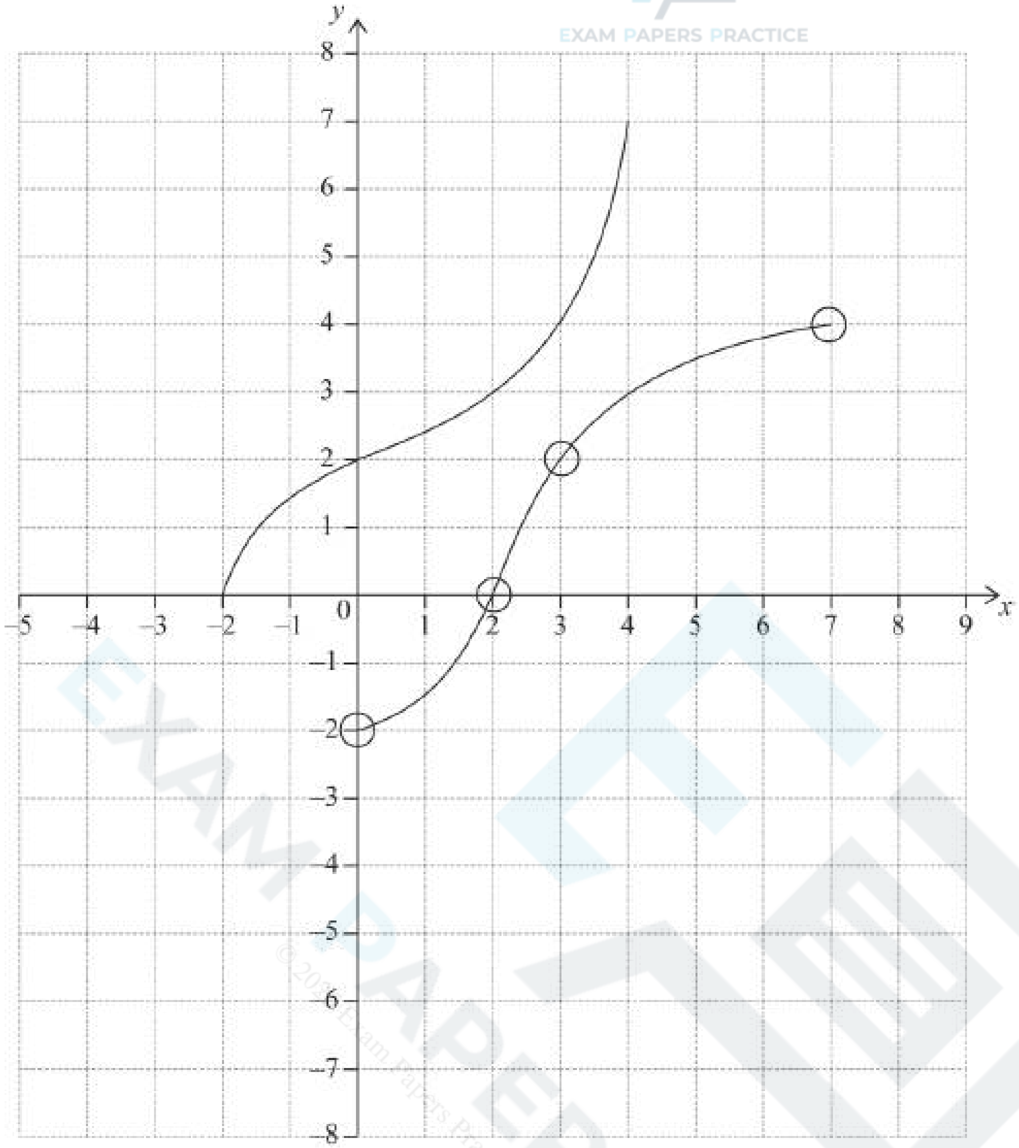
eg $16\ln x$

valid approach to find r ***(M1)***

eg $\frac{u_{n+1}}{u_n}$, $\frac{\ln x^8}{\ln x^{16}}$, $\frac{4\ln x}{8\ln x}$, $\ln x^4 = \ln x^{16} \times r^2$

$r = \frac{1}{2}$

17N.1.SL.TZ0.S_3



Award for both end points within circles,

for images of (2, 3) and (0, 2) within circles,

for approximately correct reflection in $y = x$, concave up then concave down shape (do not accept line segments).

17M.1.SL.TZ2.S_7

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct application of $\log a + \log b = \log ab$ ***(A1)***

eg $\log_2(2\sin x\cos x),\ \log 2 + \log(\sin x) + \log(\cos x)$

t equation without logs ***A1***

eg $2\sin x\cos x = 2^{-1},\ \sin x\cos x = \frac{1}{4},\ \sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) ***A1***

eg $\log(\sin 2x),\ 2\sin x\cos x = \sin 2x,\ \sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\ (30°)$ ***(A1)***

correct working ***A1***

eg $x = \frac{\pi}{12} + 2\pi,\ 2x = \frac{25\pi}{6},\ \frac{29\pi}{6},\ 750°,\ 870°,\ x = \frac{\pi}{12}$ $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12},\ \frac{29\pi}{12}$ (do not accept additional values)

17N.1.SL.TZ0.S_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempt to form composite ***(M1)***

eg $g(1 + e^{-x})$

correct function ***A1*** ***N2***

eg $(g \circ f)(x) = 2 + b + 2e^{-x},\ 2(1 + e^{-x}) + b$

[2 marks]

b. evidence of $\lim_{x\to\infty}(2 + b + 2e^{-x}) = 2 + b + \lim_{x\to\infty}(2e^{-x})$ ***(M1)***

eg $2 + b + 2e^{-\infty}$, graph with horizontal asymptote when $x \to \infty$

Note: Award ***M0*** if candidate clearly has incorrect limit, such as $x \to 0,\ e^{\infty},\ 2e^{0}$.

evidence that $e^{-x} \to 0$ (seen anywhere) ***(A1)***

eg $\lim_{x\to\infty}(e^{-x}) = 0,\ 1 + e^{-x} \to 1,\ 2(1) + b = -3,\ e^{\text{large negative number}} \to 0$, graph of $y = e^{-x}$ or $y = 2e^{-x}$ with asymptote $y = 0$, graph of composite function with asymptote $y = -3$

correct working ***(A1)*** *eg* $2 + b = -3$ $b = -5$ ***A1*** ***N2*** ***[4 marks]***

16N.1.SL.TZ0.S_1

(i) $h = 2$ ***A1*** ***N1***

(ii) **METHOD 1**

valid attempt to find k ***(M1)***

eg $f(2)$

correct substitution into **their** function ***(A1)***

eg $(2)^2 - 4(2) + 5$

$k = 1$ ***A1*** ***N2***

METHOD 2

valid attempt to complete the square ***(M1)***

eg $x^2 - 4x + 4$

correct working ***(A1)***

eg $(x^2 - 4x + 4) - 4 + 5,\ (x-2)^2 + 1$

$k = 1$ ***A1*** ***N2***

20N.1.SL.TZ0.T_15

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.*

$u_1 r = 30$ **and** $u_1 r^4 = 240,$ ***(M1)***

Note: Award ***(M1)*** for both the given terms expressed in the formula for u_n.

OR

$30r^3 = 240$ $r^3 = 8$ ***(M1)***

Note: Award ***(M1)*** for a correct equation seen.

$r = 2$ ***(A1)*** ***(C2)***

[2 marks]

b. $u_1 \times 2 = 30$ **OR** $u_1 \times 2^4 = 240$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in geometric sequence formula.

$u_1 = 15$ ***(A1)(ft)*** ***(C2)*** **Note:** Follow through from part (a). ***[2 marks]***

c. $\frac{152^n - 1}{2 - 1} = 61425$

Award for correctly substituted geometric series formula equated to 61425.

$n =$ 12 (slices) Follow through from parts (a) and (b).

20N.1.SL.TZ0.T_2

a.i.

the cost of **each** (large cheese) pizza / **a** pizza / **one** pizza / **per** pizza ***(A1) (C1)***

Note: Award ***(A0)*** for "the cost of (large cheese) pizzas". Do not accept "the **minimum** cost of a pizza".

[1 mark]

a.ii. the (fixed) delivery cost ***(A1) (C1)***

[1 mark]

b. 2 ***(A1) (C1)***

[1 mark]

c. $450 = 34.50n + 8.50$ ***(M1)***

Note: Award ***(M1)*** for equating the cost equation to 450 (may be stated as an inequality).

12 ***(A1)*(ft)** ***(C3)***

12.8 $12.7971\ldots$ ***(A1)***

Note: The final answer must be an integer.
The final ***(A1)*(ft)** is awarded for rounding their answer **down** to a whole number, provided their unrounded answer is seen.

[3 marks]

20N.1.SL.TZ0.T_3

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might* *not be suitable for all forms of practice.*

$400 \leq w < 500$ ***(A1) (C1)***

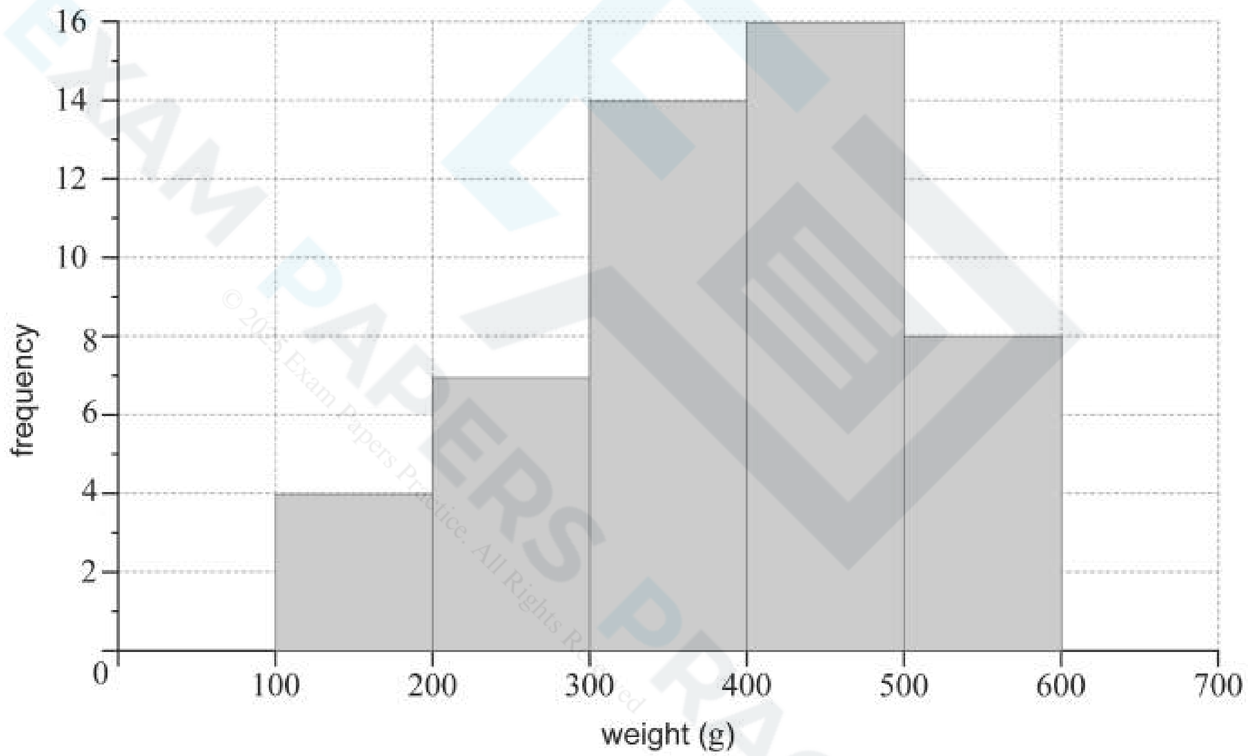
Note: Accept alternative notation $[400,\ \ 500)$ or $[400,\ \ 500[$.
Do not accept "400-500".

[1 mark]

b. 115 $115.265 \ldots$ (g) ***(A2) (C2)***

Note: Award ***(A1)(A0)*** for an answer of 116 $116.459 \ldots$. ***[2 marks]***

c.



(A2)(A1) (C3)

Note: Award ***(A2)*** for all correct heights of bars or ***(A1)*** for three or four correct heights of bars.
Award ***(A1)*** for rectangular bars all with correct left and right end points ($100,\ \ 200,\ \ 300,\ \ 400,\ \ 500$ and 600) and for no gaps; the bars do **not** have to be shaded.
Award at most ***(A2)(A0)*** if a ruler is not used for all lines.

18M.1.SL.TZ2.T_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

20 ***(A1) (C1)***

[1 mark]

b. $\frac{5}{43}$ ($0.11627 \ldots, 11.6279 \ldots \%$) ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator. ***[2 marks]***

c. $\frac{7}{37} \times \frac{12}{36} + \frac{12}{37} \times \frac{7}{36}$ ***(A1)(M1)***

Note: Award ***(A1)*** for first or second correct product seen, ***(M1)*** for adding their two products or for multiplying their product by two.

$= \frac{14}{111}$ ($0.12612 \ldots, 12.6126\%$) ***(A1) (C3)*** ***[3 marks]***

21M.1.SL.TZ1.1

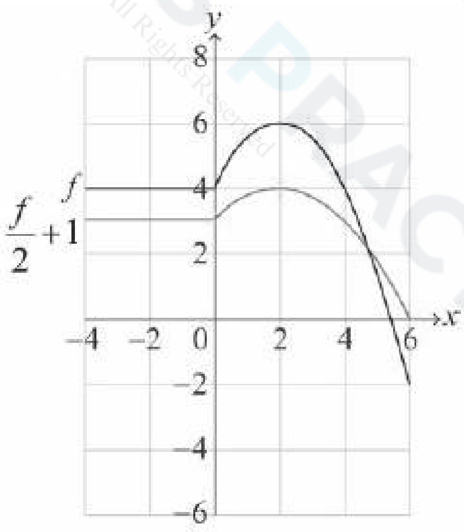
a.i.

$f2 = 6$ ***A1***

[1 mark]

a.ii. $f \circ f2 = -2$ ***A1*** ***[1 mark]***

b.



M1A1A1

Note: Award ***M1*** for an attempt to apply any vertical stretch or vertical translation, ***A1*** for a correct horizontal line segment between -4 and 0 (located roughly at $y = 3$), ***A1*** for a correct concave down parabola including max point at $(2, 4)$ and for correct end points at $(0, 3)$ and $(6, 0)$ (within circles). Points do not need to be labelled.

[3 marks]

20N.1.SL.TZ0.T_4

a.i.

$0 = x + \frac{12}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for equating the function to zero.

$x =$ -2.29 $-2.28942\ldots$ ***(A1) (C2)***

Note: Award ***(C1)*** for a correct x-value given as part of a coordinate pair or alongside an explicitly stated y-value.

[2 marks]

a.ii. $2.88,$ 4.33 $2.88449\ldots,$ $4.32674\ldots$ ***(A1)(A1) (C2)***

Note: Accept $x = 2.88,$ $y = 4.33.$ ***[2 marks]***

b. $3 - x = x + \frac{12}{x^2}$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for equating the functions or for a sketch of the two functions.

$x =$ -1.43 $-1.43080\ldots$ ***(A1) (C2)***

Note: Do not award the final ***(A1)*** if the answer is seen as part of a coordinate pair or a y-value is explicitly stated, unless already penalized in part (a).

[2 marks]

21M.1.SL.TZ1.2

a.

3×10^4 OR 30000 km (accept $3 \cdot 10^4$) ***A1***

[1 mark]

b. $\frac{4}{3}\pi 3 \times 10^{4^3}$ OR $\frac{4}{3}\pi 30000^3$ ***(A1)***

$= \frac{4}{3}\pi \times 27 \times 10^{12}$ $= \pi 36 \times 10^{12}$ OR $= \frac{4}{3}\pi \times 27000000000000$ ***(A1)***

$= \pi 36 \times 10^{13}\ \text{km}^3$ OR $a = 3.6,\quad k = 13$

18N.1.SL.TZ0.S_5

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1 (eliminating *k*)

recognizing parallel vectors are multiples of each other ***(M1)***

eg $\boldsymbol{a} = k\boldsymbol{b}$, $\begin{pmatrix} 3 \\ 2p \end{pmatrix} = k\begin{pmatrix} p+1 \\ 8 \end{pmatrix}$, $\frac{p+1}{3} = \frac{8}{2p}$, $3k = p + 1$ and $2kp = 8$

correct working (must be quadratic) ***(A1)***

eg $2p^2 + 2p = 24$, $p^2 + p - 12$, $3 = \frac{p^2+p}{4}$

valid attempt to solve **their** quadratic equation ***(M1)***

eg factorizing, formula, completing the square

evidence of correct working ***(A1)***

eg $(p + 4)(p - 3)$, $x = \frac{-2 \pm \sqrt{4 - 4(2)(-24)}}{4}$

$p = -4$, $p = 3$ ***A1A1 N4***

METHOD 2 (solving for *k*)

recognizing parallel vectors are multiples of each other ***(M1)***

eg $\boldsymbol{a} = k\boldsymbol{b}$, $\begin{pmatrix} 3 \\ 2p \end{pmatrix} = k\begin{pmatrix} p+1 \\ 8 \end{pmatrix}$, $3k = p + 1$ and $2kp = 8$

correct working (must be quadratic) ***(A1)***

eg $3k^2 - k = 4$, $3k^2 - k - 4$, $4k^2 = 3 - k$

one correct value for *k* ***(A1)***

eg $k = -1$, $k = \frac{4}{3}$, $k = \frac{3}{4}$

substituting **their** value(s) of *k* ***(M1)***

eg $\begin{pmatrix} 3 \\ 2p \end{pmatrix} = \frac{3}{4}\begin{pmatrix} p+1 \\ 8 \end{pmatrix}$, $3\left(\frac{4}{3}\right) = p + 1$ and $2\left(\frac{4}{3}\right)p = 8$, $(-1)\begin{pmatrix} 3 \\ 2p \end{pmatrix} = \begin{pmatrix} p+1 \\ 8 \end{pmatrix}$

$p = -4$, $p = 3$ ***A1A1 N4***

METHOD 3 (working with angles and cosine formula) recognizing angle between parallel vectors is 0 and/or 180°

eg $\cos\theta = \pm 1$, $a \cdot b = |a||b|$

correct substitution of scalar product and magnitudes into equation

eg $\frac{3(p+1)+2p(8)}{\sqrt{3^2+(2p)^2}\sqrt{(p+1)^2+8^2}} = \pm 1$, $19p+3 = \sqrt{4p^2+9}\sqrt{p^2+2p+65}$

correct working (must include both ±)

eg $3(p+1)+2p(8) = \pm\sqrt{3^2+(2p)^2}\sqrt{(p+1)^2+8^2}$,
$19p+3 = \pm\sqrt{4p^2+9}\sqrt{p^2+2p+65}$

correct quartic equation

eg $361p^2+114p+9 = 4p^4+8p^3+269p^2+18p+585$, $4p^4+8p^3-92p^2-96p+576=0$,
$p^4+2p^3-23p^2-24p+144=0$, $(p+4)^2(p-3)^2=0$

$p=-4$, $p=3$

17M.1.SL.TZ2.T_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$18+x+y+22=100$ or equivalent ***(A1) (C1)***

[1 mark]

b. $\frac{18+2x+3y+88}{100} = 2.71$ or equivalent ***(M1)(A1) (C2)***

Note: Award ***(M1)*** for a sum including x and y, divided by 100 and equated to 2.71, ***(A1)*** for a correct equation.

[2 marks]

c. $x+y=60$ and $2x+3y=165$ ***(M1)***

Note: Award ***(M1)*** for obtaining a correct linear equation in one variable from their (a) and their (b).

This may be implied if seen in part (a) or part (b).

$x=15$; $y=45$ ***(A1)*(ft)*(A1)*(ft) *(C3)***

Notes: Follow through from parts (a) and (b), irrespective of working seen provided the answers are positive integers.

21M.1.SL.TZ1.3

METHOD 1 (finding u_1 first, from S_8)

$4u_1 + 8 = 8$ ***(A1)***

$u_1 = -6$ ***A1***

$u_1 + 7d = 8$ OR $42u_1 + 7d = 8$ (may be seen with their value of u_1) ***(A1)***

attempt to substitute their u_1 ***(M1)***

$d = 2$ ***A1***

METHOD 2 (solving simultaneously)

$u_1 + 7d = 8$ ***(A1)***

$4u_1 + 8 = 8$ OR $42u_1 + 7d = 8$ OR $u_1 = -3d$ ***(A1)***

attempt to solve linear or simultaneous equations ***(M1)***

$u_1 = -6, \quad d = 2$ ***A1A1***

[5 marks]

21M.1.SL.TZ1.4

a.

attempt to use definition of outlier

$1.5 \times 20 + Q_3$ ***(M1)***

$1.5 \times 20 + U \geq 75$ ($\Rightarrow U \geq 45$, accept $U > 45$) OR $1.5 \times 20 + Q_3 = 75$ ***A1***

minimum value of $U = 45$ ***A1***

[3 marks]

b. attempt to use interquartile range ***(M1)***

$U - L = 20$ (may be seen in part (a)) OR $L \geq 25$ (accept $L > 25$)

minimum value of $L = 25$ ***A1*** ***[2 marks]***

21M.1.SL.TZ1.5

a.

$f'x = -2x - h$ ***A1***

[1 mark]

b. $g'x = \mathrm{e}^{x-2}$ OR $g'3 = \mathrm{e}^{3-2}$ (may be seen anywhere) ***A1***

Note: The derivative of g must be explicitly seen, either in terms of x or 3.

recognizing $f'3 = g'3$ ***(M1)*** $-23 - h = \mathrm{e}^{3-2}$ $= e$

$-6 + 2h = \mathrm{e}$ OR $3 - h = -\frac{\mathrm{e}}{2}$ ***A1***

Note: The final ***A1*** is dependent on one of the previous marks being awarded.

$h = \frac{\mathrm{e}+6}{2}$ ***AG*** ***[3 marks]***

c. $f3 = g3$ ***(M1)*** $-3 - h^2 + 2k = \mathrm{e}^{3-2} + k$ correct equation in k **EITHER**

$-3 - \frac{\mathrm{e}+6}{2}^2 + 2k = \mathrm{e}^{3-2} + k$ ***A1*** $k = \mathrm{e} + \frac{6-\mathrm{e}-6}{2}^2$ $= \mathrm{e} + \frac{-\mathrm{e}}{2}^2$ ***A1*** **OR**

$k = \mathrm{e} + 3 - \frac{\mathrm{e}+6}{2}^2$ ***A1*** $k = \mathrm{e} + 9 - 3\mathrm{e} - 18 + \frac{\mathrm{e}^2 + 12\mathrm{e} + 36}{4}$ ***A1*** **THEN**

$k = \mathrm{e} + \frac{\mathrm{e}^2}{4}$ ***AG*** ***[3 marks]***

17M.1.SL.TZ1.S_3

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1

evidence of choosing the sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution ***A1***

eg $\frac{x}{\sin 30} = \frac{13}{\sin 45}, \frac{13\sin 30}{\sin 45}$

$\sin 30 = \frac{1}{2}, \sin 45 = \frac{1}{\sqrt{2}}$ ***(A1)(A1)***

correct working ***A1***

eg $\frac{1}{2} \times \frac{13}{\frac{1}{\sqrt{2}}}, \frac{1}{2} \times 13 \times \frac{2}{\sqrt{2}}, 13 \times \frac{1}{2} \times \sqrt{2}$

correct answer ***A1*** ***N3***

$eg \text{PR} = \frac{13\sqrt{2}}{2}, \frac{13}{\sqrt{2}} \text{ (cm)}$

METHOD 2 (using height of ΔPQR)

valid approach to find height of ΔPQR ***(M1)***

$eg \sin 30 = \frac{x}{13}, \cos 60 = \frac{x}{13}$

$\sin 30 = \frac{1}{2} \text{ or } \cos 60 = \frac{1}{2}$

$\text{height} = 6.5$

correct working

$eg \sin 45 = \frac{6.5}{\text{PR}}, \sqrt{6.5^2 + 6.5^2}$

correct working

$eg \sin 45 = \frac{1}{\sqrt{2}}, \cos 45 = \frac{1}{\sqrt{2}}, \sqrt{\frac{169 \times 2}{4}}$

correct answer

$eg \text{PR} = \frac{13\sqrt{2}}{2}, \frac{13}{\sqrt{2}} \text{ (cm)}$

19M.1.SL.TZ2.S_7

recognizing period of g is larger than the period of f ***(M1)***

eg sketch of g with larger period (may be seen on diagram), A at $x = 2\pi$,

image of A when $x > 2\pi$, $\frac{7\pi}{6} \to 2\pi$, $2\sin\left(2\pi p\right) = -1$, $\frac{7\pi}{6} \times k = 2\pi$

correct working ***(A1)***

eg $\frac{7\pi}{6} \cdot \frac{1}{p} = 2\pi$, $2\pi p = \frac{7\pi}{6}$, $\frac{12}{7}$

$p = \frac{7}{12}$ $\left(\text{accept} p < \frac{7}{12} \text{or} p \leqslant \frac{7}{12}\right)$ ***A1 N2***

[3 marks]

18N.1.SL.TZ0.T_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

3 ***(A1) (C1)***

[1 mark]

b. median is 13th position ***(M1)*** CF: 2, 6, 14, 20, 23, 25 ***(M1)***

median = 3 **(A1) (C3)** **[3 marks]**

c.i. 2.5 ***(A1) (C1)***

Award if the sum of parts (c)(i) and (c)(ii) is 4.

c.ii. 1.5

Award if the sum of parts (c)(i) and (c)(ii) is 4.

17N.1.SL.TZ0.S_4

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of choosing the cosine rule ***(M1)***

eg$c^2 = a^2 + b^2 - ab\cos C$

correct substitution into RHS of cosine rule ***(A1)***

eg$3^2 + 8^2 - 2 \times 3 \times 8 \times \cos\frac{\pi}{3}$

evidence of correct value for $\cos\frac{\pi}{3}$ (may be seen anywhere, including in cosine rule) ***A1***

eg$\cos\frac{\pi}{3} = \frac{1}{2},\ \mathrm{AC}^2 = 9 + 64 - \left(48 \times \frac{1}{2}\right),\ 9 + 64 - 24$

correct working clearly leading to answer ***A1***

e*g*$\mathrm{AC}^2 = 49,\ b = \sqrt{49}$

$\mathrm{AC} = 7\ (\mathrm{cm})$ ***AG*** ***N0***

Note: Award no marks if the only working seen is $\mathrm{AC}^2 = 49$ or $\mathrm{AC} = \sqrt{49}$ (or similar).

[4 marks]

b. correct substitution for semicircle ***(A1)*** *eg*$\text{semicircle} = \frac{1}{2}(2\pi \times 3.5),\ \frac{1}{2} \times \pi \times 7,\ 3.5\pi$

valid approach (seen anywhere) ***(M1)***

eg$\text{perimeter} = \mathrm{AB} + \mathrm{BC} + \text{semicircle},\ 3 + 8 + \left(\frac{1}{2} \times 2 \times \pi \times \frac{7}{2}\right),\ 8 + 3 + 3.5\pi$

$11 + \frac{7}{2}\pi\ (= 3.5\pi + 11)\ (\mathrm{cm})$ ***A1*** ***N2*** ***[3 marks]***

19M.1.SL.TZ1.S_3

a.

valid approach ***(M1)***

eg labelled sides on separate triangle, $\sin^2 x + \cos^2 x = 1$

correct working ***(A1)***

eg missing side is 4, $\sqrt{1 - \left(\frac{3}{5}\right)^2}$

$\cos\theta = \frac{4}{5}$ ***A1 N3***

[3 marks]

b. correct substitution into $\cos 2\theta$ ***(A1)*** *eg* $2\left(\frac{16}{25}\right) - 1,\ 1 - 2\left(\frac{3}{5}\right)^2,\ \frac{16}{25} - \frac{9}{25}$

$\cos 2\theta = \frac{7}{25}$ ***A1 N2 [2 marks]***

c. correct working ***(A1)*** *eg* $\frac{7}{25} = \frac{14}{\text{BC}},\ \text{BC} = \frac{14 \times 25}{7}$

$\text{BC} = 50$ (cm) ***A1 N2 [2 marks]***

18M.1.SL.TZ2.S_4

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of correctly substituting into circle formula (may be seen later) ***A1A1***
eg $\frac{1}{2}\theta r^2 = 12, r\theta = 6$

attempt to eliminate one variable ***(M1)***
eg $r = \frac{6}{\theta}, \theta = \frac{1}{r}, \frac{\frac{1}{2}\theta r^2}{r\theta} = \frac{12}{6}$

correct elimination ***(A1)***
eg $\frac{1}{2} \times \frac{6}{r} \times r^2 = 12, \frac{1}{2}\theta \times \left(\frac{6}{\theta}\right)^2 = 12, A = \frac{1}{2} \times r^2 \times \frac{l}{r}, \frac{r^2}{2r} = 2$

correct equation ***(A1)***
eg $\frac{1}{2} \times 6r = 12, \frac{1}{2} \times \frac{36}{\theta} = 12, 12 = \frac{1}{2} \times r^2 \times \frac{6}{r}$

correct working ***(A1)***
eg $3r = 12, \frac{18}{\theta} = 12, \frac{r}{2} = 2, 24 = 6r$

$r = 4$ (cm) ***A1 N2***

[7 marks]

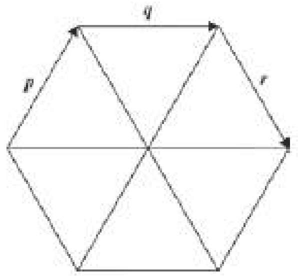
18M.1.SL.TZ1.S_6

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1 (using $|\boldsymbol{p}|\ |2\boldsymbol{q}|\cos\theta$)

finding $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}$ ***(A1)***

eg $2\boldsymbol{q}$,



$|\ \boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}\ | = 2 \times 3\ (= 6)$ (seen anywhere) ***A1***

correct angle between $\boldsymbol{p}$ and $\boldsymbol{q}$ (seen anywhere) ***(A1)***

$\frac{\pi}{3}$ (accept 60°)

substitution of **their** values ***(M1)***

eg $3 \times 6 \times \cos\left(\frac{\pi}{3}\right)$

correct value for $\cos\left(\frac{\pi}{3}\right)$ (seen anywhere) ***(A1)***

eg $\frac{1}{2}, 3 \times 6 \times \frac{1}{2}$

$\boldsymbol{p}\cdot(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1 N3***

METHOD 2 (scalar product using distributive law)

correct expression for scalar distribution ***(A1)***

eg $\boldsymbol{p}\cdot\boldsymbol{p}+\boldsymbol{p}\cdot\boldsymbol{q}+\boldsymbol{p}\cdot\boldsymbol{r}$

three correct angles between the vector pairs (seen anywhere) ***(A2)***

eg 0° between $\boldsymbol{p}$ and $\boldsymbol{p}$, $\frac{\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{q}$, $\frac{2\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{r}$

Note: Award ***A1*** for only two correct angles.

substitution of **their** values ***(M1)***

eg $3.3.\cos 0 + 3.3.\cos\frac{\pi}{3} + 3.3.\cos 120$

one correct value for $\cos 0$, $\cos\left(\frac{\pi}{3}\right)$ or $\cos\left(\frac{2\pi}{3}\right)$ (seen anywhere) ***A1***

eg $\frac{1}{2}, 3 \times 6 \times \frac{1}{2}$

$\boldsymbol{p}\cdot(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1 N3***

(scalar product using relative position vectors)

valid attempt to find one component of or

eg sin 60 = $\frac{x}{3}$, cos 60 = $\frac{x}{3}$, one correct value $\frac{3}{2}, \frac{3\sqrt{3}}{2}, \frac{-3\sqrt{3}}{2}$

one correct vector (two or three dimensions) (seen anywhere)

$$eg\ \ p = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}, q = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, r = \begin{pmatrix} \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}$$

three correct vectors + + = 2

+ + = $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ (seen anywhere, including scalar product)

correct working

eg $\left(\frac{3}{2} \times 6\right) + \left(\frac{3\sqrt{3}}{2} \times 0\right), 9 + 0 + 0$

•(+ +) = 9

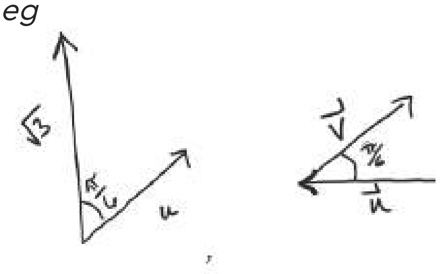
19M.1.SL.TZ1.S_6

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1 (cosine rule)

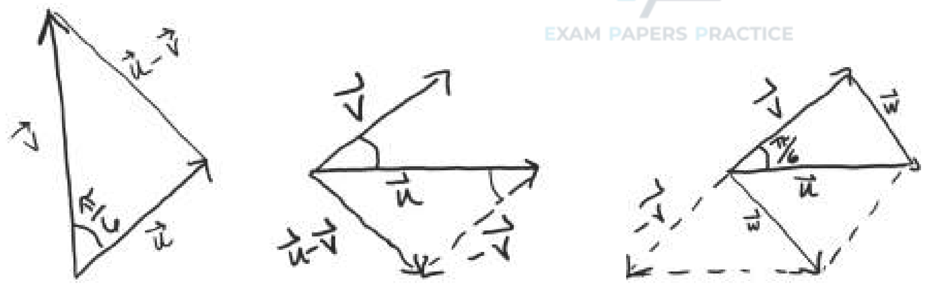
diagram including ***u***, ***v*** and included angle of $\frac{\pi}{6}$ ***(M1)***

eg



sketch of triangle with ***w*** (does not need to be to scale) ***(A1)***

eg



choosing cosine rule ***(M1)***

eg $a^2 + b^2 - 2ab\cos C$

correct substitution ***A1***

eg $4^2 + \left(\sqrt{3}\right)^2 - 2(4)\left(\sqrt{3}\right)\cos\frac{\pi}{6}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ (seen anywhere) ***(A1)***

correct working ***(A1)***

eg 16 + 3 − 12

$|\boldsymbol{w}| = \sqrt{7}$ ***A1 N2***

METHOD 2 (scalar product)

valid approach, in terms of u and v (seen anywhere) ***(M1)***

eg $|\boldsymbol{w}|^2 = (\boldsymbol{u} - \boldsymbol{v})\bullet(\boldsymbol{u} - \boldsymbol{v})$, $|\boldsymbol{w}|^2 = \boldsymbol{u}\bullet\boldsymbol{u} - 2\boldsymbol{u}\bullet\boldsymbol{v} + \boldsymbol{v}\bullet\boldsymbol{v}$, $|\boldsymbol{w}|^2 = \left(u_1 - v_1\right)^2 + \left(u_2 - v_2\right)^2$,

$| \quad | = \sqrt{\left(u_1 - v_1\right)^2 + \left(u_2 - v_2\right)^2 + \left(u_3 - v_3\right)^2}$

correct value for • (seen anywhere)

eg $| \quad |^2 = 16$, • = 16, $u_1{}^2 + u_2{}^2 = 16$

correct value for • (seen anywhere)

eg $| \quad |^2 = 16$, • = 3, $v_1{}^2 + v_2{}^2 + v_3{}^2 = 3$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ (seen anywhere)

• $= 4 \times \sqrt{3} \times \frac{\sqrt{3}}{2}$ (= 6) (seen anywhere)

correct substitution into • − 2 • + • or $u_1{}^2 + u_2{}^2 + v_1{}^2 + v_2{}^2 - 2\left(u_1v_1 + u_2v_2\right)$ (2 or 3 dimensions)

eg 16 − 2(6) + 3 (= 7)

$| \quad | = \sqrt{7}$

19M.1.SL.TZ2.S_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of valid approach ***(M1)***

eg sketch of triangle with sides 3 and 5, $\cos^2\theta = 1 - \sin^2\theta$

correct working ***(A1)***

eg missing side is 4 (may be seen in sketch), $\cos\theta = \frac{4}{5}$, $\cos\theta = -\frac{4}{5}$

$\tan\theta = -\frac{3}{4}$ ***A2 N4***

[4 marks]

b. correct substitution of either gradient **or** origin into equation of line ***(A1)***

(do not accept $y = mx + b$) *eg* $y = x\tan\theta$, $y - 0 = m(x - 0)$, $y = mx$

$y = -\frac{3}{4}x$ ***A2 N4*** **Note:** Award ***A1A0*** for $L = -\frac{3}{4}x$. ***[2 marks]***

d. valid approach to equate **their** gradients ***(M1)***

eg $f' = \tan\theta$, $f' = -\frac{3}{4}$, $e^x\cos x + e^x\sin x - \frac{3}{4} = -\frac{3}{4}$, $e^x(\cos x + \sin x) - \frac{3}{4} = -\frac{3}{4}$

correct equation without e^x ***(A1)*** *eg* $\sin x = -\cos x$, $\cos x + \sin x = 0$, $\frac{-\sin x}{\cos x} = 1$

correct working ***(A1)*** *eg* $\tan\theta = -1$, $x = 135°$

$x = \frac{3\pi}{4}$ (do not accept $135°$) ***A1 N1***

Note: Do not award the final ***A1*** if additional answers are given. ***[4 marks]***

21M.1.SL.TZ1.7

a.

METHOD 1 (discriminant)

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

recognizing $\Delta = 0$ (seen anywhere) ***M1***

$\Delta = -3m^2 - 4m9$ (do not accept only in quadratic formula for x) ***A1***

valid approach to solve quadratic for m ***(M1)***

$9mm - 4 = 0$ OR $m = \frac{36 \pm \sqrt{36^2 - 4 \times 9 \times 0}}{2 \times 9}$

both solutions $m = 0,\ 4$ ***A1***

$m \neq 0$ with a valid reason ***R1***

the two graphs would not intersect OR $0 \neq -9$

$m = 4$ ***AG***

METHOD 2 (equating slopes)

$mx^2 - 2mx = mx - 9$ (seen anywhere) ***(M1)***

$f'x = 2mx - 2m$ ***A1***

equating slopes, $f'x = m$ (seen anywhere) ***M1***

$2mx - 2m = m$

$x = \frac{3}{2}$ ***A1***

substituting their x value ***(M1)***

$\frac{3}{2}^2 m - 2m \times \frac{3}{2} = m \times \frac{3}{2} - 9$

$\frac{9}{4}m - \frac{12}{4}m = \frac{6}{4}m - 9$ ***A1***

$\frac{-9m}{4} = -9$

$m = 4$ ***AG***

METHOD 3 (using $\frac{-b}{2a}$)

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

attempt to find x-coord of vertex using $\frac{-b}{2a}$ ***(M1)***

$\frac{--3m}{2m}$ ***A1***

$x = \frac{3}{2}$ ***A1***

substituting their x value ***(M1)***

$\frac{3}{2}^2 m - 3m \times \frac{3}{2} + 9 = 0$

$\frac{9}{4}m - \frac{9}{2}m + 9 = 0$ ***A1***

$-9m = -36$

$m = 4$ ***AG***

b. $4xx - 2$ $p = 0$ and $q = 2$ OR $p = 2$ and $q = 0$

c. attempt to use valid approach

$\frac{0+2}{2}$, $\frac{--8}{2 \times 4}$, $f1$, $8x - 8 = 0$ OR $4x^2 - 2x + 1 - 1 = 4x - 1^2 - 4$ $h = 1$, $k = -4$

d. recognition $x = h$ to 2 (may be seen on sketch)

recognition that $fx < 0$ and $f'x > 0$ $1 < x < 2$

Award for two correct values, for correct inequality signs.

21M.1.SL.TZ1.8

a.

attempt to use quotient or product rule ***(M1)***

$\frac{dy}{dx} = \frac{x^4 \frac{1}{x} - \ln x 4x^3}{x^{4^2}}$ OR $\ln x \cdot -4x^{-5} + x^{-4} \frac{1}{x}$ ***A1***

correct working ***A1***

$= \frac{x^3 1 - 4 \ln x}{x^8}$ OR cancelling x^3 OR $\frac{-4 \ln x}{x^5} + \frac{1}{x^5}$

$= \frac{1 - 4 \ln x}{x^5}$ ***AG***

[3 marks]

b. $f'x = \frac{dy}{dx} = 0$ ***(M1)*** $\frac{1 - 4 \ln x}{x^5} = 0$ $\ln x = \frac{1}{4}$ ***(A1)*** $x = e^{\frac{1}{4}}$ ***A1***

substitution of their x to find y ***(M1)*** $y = \frac{\ln e^{\frac{1}{4}}}{e^{\frac{1}{4}^4}} = \frac{1}{4e} = \frac{1}{4}e^{-1}$ ***A1*** $Pe^{\frac{1}{4}}, \frac{1}{4e}$

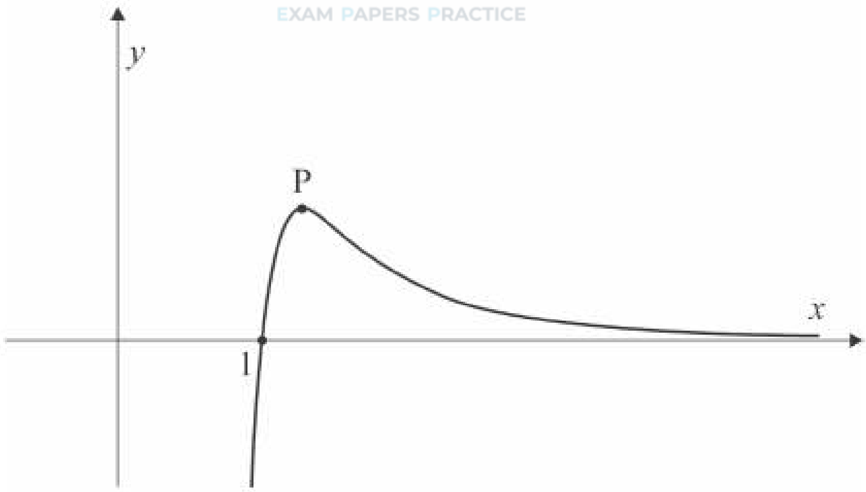
[5 marks]

c. $f'' e^{\frac{1}{4}} = \frac{20 \ln e^{\frac{1}{4}} - 9}{e^{\frac{1}{4}^6}}$ ***(M1)*** $= \frac{5 - 9}{e^{1.5}}$ $= -\frac{4}{e^{1.5}}$ ***A1*** which is negative ***R1***

hence P is a local maximum ***AG***

Note: The ***R1*** is dependent on the previous ***A1*** being awarded. ***[3 marks]***

d. $\ln x > 0$ ***(A1)*** $x > 1$ ***A1*** ***[2 marks]*** e.



Award for one x-intercept only, located at 1

for local maximum, P, in approximately correct position
for curve approaching x-axis as $x \to \infty$ (including change in concavity).

17N.1.SL.TZ0.S_6

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

attempt to find the area of OABC ***(M1)***

eg $\mathrm{OA} \times \mathrm{OC},\ x \times f(x),\ f(x) \times (-x)$

correct expression for area in one variable ***(A1)***

eg $\text{area} = x(15 - x^2),\ 15x - x^3,\ x^3 - 15x$

valid approach to find maximum **area** (seen anywhere) ***(M1)***

eg $A'(x) = 0$

correct derivative ***A1***

eg $15 - 3x^2,\ (15 - x^2) + x(-2x) = 0,\ -15 + 3x^2$

correct working ***(A1)***

eg $15 = 3x^2,\ x^2 = 5,\ x = \sqrt{5}$

$x = -\sqrt{5}\ \left(\text{accept A}\left(-\sqrt{5},\ 0\right)\right)$ ***A2*** ***N3***

[7 marks]

16N.1.SL.TZ0.S_2

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** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of valid approach ***(M1)***

*eg*right triangle, $\cos^2\theta = 1 - \sin^2\theta$

correct working ***(A1)***

*eg*missing side is 2, $\sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2}$

$\cos\theta = \frac{2}{3}$ ***A1 N2***

[3 marks]

b. correct substitution into formula for $\cos 2\theta$ ***(A1)***

eg$2 \times \left(\frac{2}{3}\right)^2 - 1,\ 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2,\ \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$ $\cos 2\theta = -\frac{1}{9}$ ***A1 N2 [2 marks]***

18N.1.SL.TZ0.S_7

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1

correct substitution into formula for $\cos(2x)$ or $\sin(2x)$ ***(A1)***

eg $1 - 2\left(\frac{1}{3}\right)^2,\ 2\left(\frac{\sqrt{8}}{3}\right)^2 - 1,\ 2\left(\frac{1}{3}\right)\left(\frac{\sqrt{8}}{3}\right),\ \left(\frac{\sqrt{8}}{3}\right)^2 - \left(\frac{1}{3}\right)^2$

$\cos(2x) = \frac{7}{9}$ or $\sin(2x) = \frac{2\sqrt{8}}{9}\left(= \frac{\sqrt{32}}{9} = \frac{4\sqrt{2}}{9}\right)$ (may be seen in substitution) ***A2***

recognizing $4x$ is double angle of $2x$ (seen anywhere) ***(M1)***

eg $\cos(2(2x)),\ 2\cos^2(2\theta) - 1,\ 1 - 2\sin^2(2\theta),\ \cos^2(2\theta) - \sin^2(2\theta)$

correct substitution of **their** value of $\cos(2x)$ and/or $\sin(2x)$ into formula for $\cos(4x)$ ***(A1)***

eg $2\left(\frac{7}{9}\right)^2 - 1,\ \frac{98}{81} - 1,\ 1 - 2\left(\frac{2\sqrt{8}}{9}\right)^2,\ 1 - \frac{64}{81},\ \left(\frac{7}{9}\right)^2 - \left(\frac{2\sqrt{8}}{9}\right)^2,\ \frac{49}{81} - \frac{32}{81}$

$\cos(4x) = \frac{17}{81}$ ***A1 N2***

METHOD 2

recognizing $4x$ is double angle of $2x$ (seen anywhere) ***(M1)*** *eg* $\cos(2(2x))$

double angle identity for $2x$

eg $2\cos^2(2\theta)-1,\ 1-2\sin^2(2x),\ \cos^2(2\theta)-\sin^2(2\theta)$

correct expression for $\cos(4x)$ in terms of $\sin x$ and/or $\cos x$

eg $2(1-2\sin^2\theta)^2-1,\ 1-2(2\sin x\cos x)^2,\ (1-2\sin^2\theta)^2-(2\sin\theta\cos\theta)^2$

correct substitution for $\sin x$ and/or $\cos x$

eg $2\left(1-2\left(\frac{1}{3}\right)^2\right)^2-1,\ 2\left(1-4\left(\frac{1}{3}\right)^2+4\left(\frac{1}{3}\right)^4\right)-1,\ 1-2\left(2\times\frac{1}{3}\times\frac{\sqrt{8}}{3}\right)^2$

correct working

eg $2\left(\frac{49}{81}\right)-1,\ 1-2\left(\frac{32}{81}\right),\ \frac{49}{81}-\frac{32}{81}$

$\cos(4x)=\frac{17}{81}$

21M.1.SL.TZ1.9

a.

recognising probabilities sum to 1 ***(M1)***

$p+p+p+\frac{1}{2}p=1$

$p=\frac{2}{7}$ ***A1***

[2 marks]

b. valid attempt to find $\mathrm{E}(X)$ ***(M1)*** $1\times p+2\times p+3\times p+4\times\frac{1}{2}p=8p$

$\mathrm{E}(X)=\frac{16}{7}$ ***A1*** ***[2 marks]***

c.i. $0\le r\le 1$ ***A1*** ***[1 mark]***

c.ii. attempt to find a value of q ***(M1)***

$0\le 1-3q\le 1$ OR $r=0\Rightarrow q=\frac{1}{3}$ OR $r=1\Rightarrow q=0$ $0\le q\le\frac{1}{3}$ ***A1*** ***[2 marks]***

d. $\mathrm{E}(Y)=1\times q+2\times q+3\times q+4\times r$ $(=2+2r$ OR $4-6q)$ ***(A1)***

one correct boundary value ***A1*** $1\times\frac{1}{3}+2\times\frac{1}{3}+3\times\frac{1}{3}+4\times 0=2$ OR

$1\times0+2\times0+3\times0+4\times1 \quad =4$ OR $\ 2+20 \quad =2$ OR $\ 2+21 \quad =4$ OR

$4-60 \quad =4$ OR $4-6\frac{1}{3} \quad =2 \quad 2\le \mathrm{E}(Y)\le 4$ ***A1*** ***[3 marks]***

e. evidence of choosing at least four correct outcomes from

$1\&2,\quad 1\&3,\quad 1\&4,\quad 2\&3,\quad 2\&4,\quad 3\&4$

$\frac{6}{7}q+\frac{6}{7}r$ OR $3pq+3pr$ OR $pq+pq+p1-3q+pq+p1-3q+p1-3q$

solving for either q or r $\quad \frac{6}{7}q+1-3q=\frac{1}{2}$ OR $\frac{6}{7}\frac{1-r}{3}+r=\frac{1}{2}$ OR $3pq+3p1-3q=\frac{1}{2}$

OR $3p\frac{1-r}{3}+3\ \ pr=\frac{1}{2}$ $\quad$ two correct values $\quad q=\frac{5}{24}$ and $r=\frac{3}{8}$

one correct value $\quad q=\frac{5}{24}$ OR $r=\frac{3}{8}$ $\quad$ substituting their value for q or r

$4-6\frac{5}{24}$ OR $2+2\frac{3}{8}$ $\quad \mathrm{E}\left(Y\right)=\frac{11}{4}$ $\quad$ (solving for $\mathrm{E}(Y)$)

evidence of choosing at least four correct outcomes from

$1\&2,\quad 1\&3,\quad 1\&4,\quad 2\&3,\quad 2\&4,\quad 3\&4$

$\frac{6}{7}q+\frac{6}{7}r$ OR $3pq+3pr$ OR $pq+pq+p1-3q+pq+p1-3q+p1-3q$

rearranging to make q the subject $\quad q=\frac{4-\mathrm{E}Y}{6} \quad 3pq+3p1-3q=\frac{1}{2}$

$\frac{6}{7}\times\frac{4-\mathrm{E}(Y)}{6}+\frac{6}{7}1-3\frac{4-\mathrm{E}(Y)}{6}=\frac{1}{2}$ $\quad \frac{2\mathrm{E}(Y)-1}{7}=\frac{1}{2} \quad \mathrm{E}\left(Y\right)=\frac{11}{4}$

19M.1.SL.TZ1.T_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

Bouquet size	Number of roses (n)	Frequency (f)	Cumulative frequency
small	$2\le n\le 4$	15	15
medium	$5\le n\le 8$	25	40
large	$9\le n\le 12$	10	50

(A1)(A1)*(ft) *(C2)

Note: Award ***(A1)*** for 10; ***(A1)*(ft)** for the last column all correct. Follow through from *their* 10 for *their* 50 in the last column.

[2 marks]

b. $\frac{35}{50}\left(0.7,\frac{7}{10},70\%\right)$ ***(A1)*(ft)*(A1)*(ft) *(C2)***

Award for their numerator being 25 + *their* 10, and for their denominator being *their* 50. Follow through from part (a).

c. $\frac{4}{10}\left(0.4, \frac{2}{5}, 40\%\right)$

Award for a numerator of 4 and for *their* 10 as denominator. Follow through from part (a).

17M.1.SL.TZ2.S_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

recognizing $t = 0$ at A ***(M1)***

A is $(4, -1, 3)$ ***A1 N2***

[2 marks]

b.i. **METHOD 1** valid approach ***(M1)*** *eg* $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, $(6, 3, -1)$

correct approach to find $\overrightarrow{AB}$ ***(A1)*** *eg* $AO + OB,\ B - A,\ \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ ***A1 N2***

METHOD 2

recognizing $\overrightarrow{AB}$ is two times the direction vector ***(M1)*** correct working ***(A1)***

eg $\overrightarrow{AB} = 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ ***A1 N2***

[3 marks]

b.ii. correct substitution ***(A1)*** *eg* $\left|\overrightarrow{AB}\right| = \sqrt{2^2 + 4^2 + 4^2}$, $\sqrt{4 + 16 + 16}$, $\sqrt{36}$

$\left|\overrightarrow{AB}\right| = 6$ ***A1 N2*** ***[2 marks]***

c. **METHOD 1 (vector approach)** valid approach involving $\overrightarrow{AB}$ and $\overrightarrow{AC}$ ***(M1)***

eg $\overrightarrow{AB} \cdot \overrightarrow{AC}$, $\frac{\overrightarrow{BA} \cdot \overrightarrow{AC}}{AB \times AC}$ finding scalar product and $\left|\overrightarrow{AC}\right|$ ***(A1)(A1)***

scalar product $2(3) + 4(0) - 4(4)\ (= -10)$ $\left|\overrightarrow{AC}\right| = \sqrt{3^2 + 0^2 + 4^2}\ (= 5)$

substitution of **their** scalar product and magnitudes into cosine formula ***(M1)***

eg $\cos B\hat{A}C = \frac{6+0-16}{6\sqrt{3^2+4^2}}$ $\cos B\hat{A}C = -\frac{10}{30}\left(=-\frac{1}{3}\right)$ ***A1 N2***

METHOD 2 (triangle approach) valid approach involving cosine rule ***(M1)***

eg $\cos B\hat{A}C = \frac{AB^2+AC^2-BC^2}{2\times AB\times AC}$ finding lengths AC and BC ***(A1)(A1)*** $AC=5,\ BC=9$

substitution of **their** lengths into cosine formula ***(M1)*** *eg* $\cos B\hat{A}C = \frac{5^2+6^2-9^2}{2\times 5\times 6}$

$\cos B\hat{A}C = -\frac{20}{60}\left(=-\frac{1}{3}\right)$

d.

Award relevant marks for working seen to find BC in part (c) (if cosine rule used in part (c)).

recognizing need to find BC

choosing cosine rule *eg* $c^2 = a^2+b^2-2ab\cos C$

correct substitution into RHS *eg* $BC^2 = (6)^2+(5)^2-2(6)(5)\left(-\frac{1}{3}\right)$, $36+25+20$

distance is 9 $\overrightarrow{BC}$

recognizing need to find BC valid approach

eg attempt to find $\overrightarrow{OB}$ or $\overrightarrow{OC}$, $\overrightarrow{OB} = \begin{pmatrix}6\\3\\-1\end{pmatrix}$ or $\overrightarrow{OC} = \begin{pmatrix}7\\-1\\7\end{pmatrix}$, $\overrightarrow{BA}+\overrightarrow{AC}$

correct working *eg* $\overrightarrow{BC} = \begin{pmatrix}1\\-4\\8\end{pmatrix}$, $\overrightarrow{CB} = \begin{pmatrix}-1\\4\\-8\end{pmatrix}$, $\sqrt{1^2+4^2+8^2} = \sqrt{81}$

distance is 9

recognizing need to find BC valid approach

eg attempt to find coordinates of B or C, $B(6,\ 3,\ -1)$ or $C(7,\ -1,\ 7)$

correct substitution into distance formula

eg $BC = \sqrt{(6-7)^2+(3-(-1))^2+(-1-7)^2}$, $\sqrt{1^2+4^2+8^2} = \sqrt{81}$

distance is 9

18M.1.SL.TZ1.T_6

a.

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

180 ***(A1) (C1)***
[1 mark]

b. 36, 24 ***(A1)(A1) (C2)***

Note: Award ***(A0)(A1)*** for two incorrect values that add up to 60. ***[2 marks]***

c.i. 125 (accept 125.5) ***(A1)***

c.ii. $\frac{4\times25+36\times75+34\times125+46\times175+24\times225+16\times275}{160}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their mid-interval values, multiplied by their frequencies, into mean formula.

=156 (155.625) ***(A1)*(ft) *(C3)*** **Note:** Follow through from parts (b) and (c)(i). ***[3 ma ks]***

17N.1.SL.TZ0.S_9

a.i.

correct approach ***A1***

eg $\begin{pmatrix}-1\\3\\3\end{pmatrix}-\begin{pmatrix}-3\\4\\2\end{pmatrix}, \begin{pmatrix}3\\-4\\-2\end{pmatrix}+\begin{pmatrix}-1\\3\\3\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}2\\-1\\1\end{pmatrix}$ ***AG N0***

[1 mark]

a.ii. any correct equation in the form $r=a+tb$ (any parameter for t)

where a is $\begin{pmatrix}-3\\4\\2\end{pmatrix}$ or $\begin{pmatrix}-1\\3\\3\end{pmatrix}$ and b is a scalar multiple of $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ ***A2 N2***

eg $r=\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}, (x, y, z)=(-1, 3, 3)+s(-2, 1, -1), r=\begin{pmatrix}-3+2t\\4-t\\2+t\end{pmatrix}$

Note: Award ***A1*** for the form $a+tb$, **A1** for the form $L=a+tb$, **A0** for the form $r=b+ta$.

[2 marks]

b. **METHOD 1 – finding value of parameter** valid approach ***(M1)***

$eg\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $(-1,\ 3,\ 3)+s(-2,\ 1,\ -1)=(3,\ 1,\ p)$

one correct equation (not involving p) ***(A1)***

$eg-3+2t=3,\ -1-2s=3,\ 4-t=1,\ 3+s=1$

correct parameter from their equation (may be seen in substitution) ***A1***

$egt=3,\ s=-2$ correct substitution ***(A1)***

$eg\begin{pmatrix}-3\\4\\2\end{pmatrix}+3\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $3-(-2)$ $p=5\left(\text{accept}\begin{pmatrix}3\\1\\5\end{pmatrix}\right)$ ***A1 N2***

METHOD 2 – eliminating parameter valid approach ***(M1)***

$eg\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $(-1,\ 3,\ 3)+s(-2,\ 1,\ -1)=(3,\ 1,\ p)$

one correct equation (not involving p) ***(A1)***

$eg-3+2t=3,\ -1-2s=3,\ 4-t=1,\ 3+s=1$ correct equation (with p) ***A1***

$eg2+t=p,\ 3-s=p$ correct working to solve for p ***(A1)*** $eg7=2p-3,\ 6=1+p$

$p=5\left(\text{accept}\begin{pmatrix}3\\1\\5\end{pmatrix}\right)$ ***A1 N2***

[5 marks]

c. valid approach to find $\overrightarrow{DC}$ or $\overrightarrow{CD}$

$eg\begin{pmatrix}3\\1\\5\end{pmatrix}-\begin{pmatrix}q^2\\0\\q\end{pmatrix}, \begin{pmatrix}q^2\\0\\q\end{pmatrix}-\begin{pmatrix}3\\1\\5\end{pmatrix}, \begin{pmatrix}q^2\\0\\q\end{pmatrix}-\begin{pmatrix}3\\1\\p\end{pmatrix}$

correct vector for $\overrightarrow{DC}$ or $\overrightarrow{CD}$ (may be seen in scalar product)

$eg\begin{pmatrix}3-q^2\\1\\5-q\end{pmatrix}, \begin{pmatrix}q^2-3\\-1\\q-5\end{pmatrix}, \begin{pmatrix}3-q^2\\1\\p-q\end{pmatrix}$

recognizing scalar product of $\overrightarrow{DC}$ or $\overrightarrow{CD}$ with direction vector of L is zero (seen anywhere)

$eg\begin{pmatrix}3-q^2\\1\\p-q\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix}=0,\ \overrightarrow{DC}\cdot\overrightarrow{AC}=0,\ \begin{pmatrix}3-q^2\\1\\5-q\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\1\end{pmatrix}=0$

correct scalar product in terms of only q

$eg6-2q^2-1+5-q,\ 2q^2+q-10=0,\ 2(3-q^2)-1+5-q$

correct working to solve quadratic $eg(2q+5)(q-2), \frac{-1 \pm \sqrt{1-4(2)(-10)}}{2(2)}$

$q = -\frac{5}{2}, 2$

18M.1.SL.TZ1.S_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct approach ***A1***

eg $\vec{AO} + \vec{OB}, B - A, \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} - \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ ***AG N0***

[1 mark]

b.i. any correct equation in the form ***r*** = ***a*** + *t**b*** (any parameter for *t*) ***A2 N2***

where ***a*** is $\begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix}$ and ***b*** is a scalar multiple of $\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

eg $\boldsymbol{r} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}, (x, y, z) = (2, -4, -4) + t(6, 8, -5), \boldsymbol{r} = \begin{pmatrix} -4+6t \\ -12+8t \\ 1-5t \end{pmatrix}$

Note: Award ***A1*** for the form ***a*** + *t**b***, ***A1*** for the form ***L*** = ***a*** + *t**b***, ***A0*** for the form ***r*** = ***b*** + *t**a***.

[2 marks]

b.ii. **METHOD 1** (solving for *t*) valid approach ***(M1)***

eg $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}, \begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

one correct equation ***A1*** eg $-4 + 8t = 12, -12 + 8t = 12$ correct value for t ***(A1)***

eg $t = 2$ or 3 correct substitution ***A1*** *eg* $2 + 6(2), -4 + 6(3), -[1 + 3(-5)]$

$k = 14$ ***AG N0*** **METHOD 2** (solving simultaneously) valid approach ***(M1)***

eg $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}, \begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

two correct equations in ***A1*** *eg* $k = -4 + 6t, -k = 1 - 5t$ **EITHER** (eliminating *k*) correct value for t ***(A1)*** *eg* $t = 2$ or 3 correct substitution ***A1***

eg $2 + 6(2), -4 + 6(3)$ **OR** (eliminating t) correct equation(s) ***(A1)***

eg $5k + 20 = 30t$ **and** $-6k - 6 = 30t$, $-k = 1 - 5\left(\frac{k+4}{6}\right)$

correct working clearly leading to $k = 14$ ***A1***

eg $-k + 14 = 0$, $-6k = 6 - 5k - 20$, $5k = -20 + 6(1 + k)$ **THEN** $k = 14$ ***AG N0***

[4 marks]

c.i. correct substitution into scalar product ***A1***

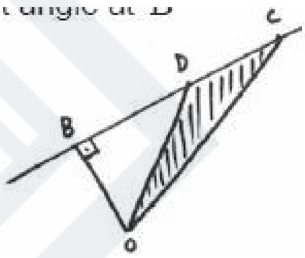
eg $(2)(6) - (4)(8) - (4)(-5)$, $12 - 32 + 20$ $\vec{OB} \cdot \vec{AB} = 0$ ***A1 N0*** ***[2 marks]***

c.ii. $O\hat{B}A = \frac{\pi}{2}, 90^\circ \left(\text{accept} \frac{3\pi}{2}, 270^\circ\right)$ ***A1 N1*** ***[1 marks]***

d. **METHOD 1** ($\frac{1}{2}$ × height × CD)

recognizing that OB is altitude of triangle with base CD (seen anywhere) ***M1***

eg $\frac{1}{2} \times \left|\vec{OB}\right| \times \left|\vec{CD}\right|, \mathrm{OB} \perp \mathrm{CD}$, sketch showing right angle at B



$\vec{CD} = \begin{pmatrix} -6 \\ -8 \\ 5 \end{pmatrix}$ or $\vec{DC} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ (seen anywhere) ***(A1)***

correct magnitudes (seen anywhere) ***(A1)(A1)***

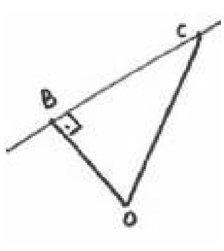
$\left|\vec{OB}\right| = \sqrt{(2)^2 + (-4)^2 + (-4)^2} = \left(\sqrt{36}\right)$

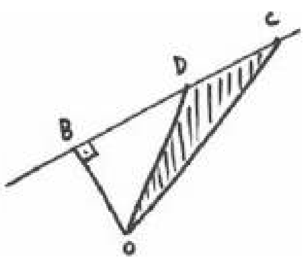
$\left|\vec{CD}\right| = \sqrt{(-6)^2 + (-8)^2 + (5)^2} = \left(\sqrt{125}\right)$ correct substitution into $\frac{1}{2}bh$ ***A1***

eg $\frac{1}{2} \times 6 \times \sqrt{125}$ area $= 3\sqrt{125}, 15\sqrt{5}$ ***A1 N3*** **METHOD 2** (subtracting triangles)

recognizing that OB is altitude of either ΔOBD or ΔOBC(seen anywhere) ***M1***

eg $\frac{1}{2} \times \left|\vec{OB}\right| \times \left|\vec{BD}\right|, \mathrm{OB} \perp \mathrm{BC}$, sketch of triangle showing right angle at B





one correct vector $\vec{BD}$ or $\vec{DB}$ or $\vec{BC}$ or $\vec{CB}$ (seen anywhere)

eg $\vec{BD} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}, \vec{CB} = \begin{pmatrix} -12 \\ -16 \\ 10 \end{pmatrix}$

$\left|\vec{OB}\right| = \sqrt{(2)^2 + (-4)^2 + (-4)^2} = \left(\sqrt{36}\right)$ (seen anywhere)

one correct magnitude of a base (seen anywhere)

$\left|\vec{BD}\right| = \sqrt{(6)^2 + (8)^2 + (5)^2} = \left(\sqrt{125}\right), \left|\vec{BC}\right| = \sqrt{144 + 256 + 100} = \left(\sqrt{500}\right)$

correct working

eg $\frac{1}{2} \times 6 \times \sqrt{500} - \frac{1}{2} \times 6 \times 5\sqrt{5}, \frac{1}{2} \times 6 \times \sqrt{500} \times \sin 90 - \frac{1}{2} \times 6 \times 5\sqrt{5} \times \sin 90$

area $= 3\sqrt{125}, 15\sqrt{5}$ (using $\frac{1}{2}ab \sin C$ with ΔOCD)

two correct side lengths (seen anywhere)

$\left|\vec{OD}\right| = \sqrt{(8)^2 + (4)^2 + (-9)^2} = \left(\sqrt{161}\right), \left|\vec{CD}\right| = \sqrt{(-6)^2 + (-8)^2 + (5)^2} = \left(\sqrt{125}\right),$
$\left|\vec{OC}\right| = \sqrt{(14)^2 + (12)^2 + (-14)^2} = \left(\sqrt{536}\right)$

attempt to find cosine ratio (seen anywhere)
eg $\frac{536 - 286}{-2\sqrt{161}\sqrt{125}}, \frac{\text{OD}\cdot\text{DC}}{|OD||DC|}$

correct working for sine ratio *eg* $\frac{(125)^2}{161 \times 125} + \sin^2 D = 1$

correct substitution into $\frac{1}{2}ab\sin C$ *eg* $0.5 \times \sqrt{161} \times \sqrt{125} \times \frac{6}{\sqrt{161}}$

area $= 3\sqrt{125}, 15\sqrt{5}$

17M.1.SL.TZ2.S_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of scalar product ***M1***

eg $\boldsymbol{a} \cdot \boldsymbol{b}$, $4(k + 3) + 2k$

recognizing scalar product must be zero ***(M1)***

eg $\boldsymbol{a} \cdot \boldsymbol{b} = 0,\ 4k + 12 + 2k = 0$

correct working (must involve combining terms) ***(A1)***

eg $6k + 12, 6k = -12$

$k = -2$

b. attempt to substitute value of k (seen anywhere)

$eg = \begin{pmatrix} -2+3 \\ -2 \end{pmatrix}, 2 = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ correct working

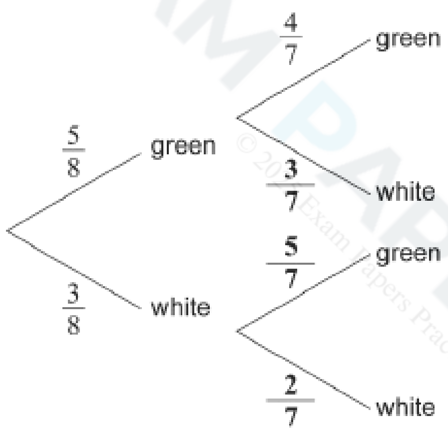
$eg\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}, \begin{pmatrix} 4+2k+6 \\ 2+2k \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

17N.1.SL.TZ0.S_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct probabilities



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** answer.

[3 marks]

b. multiplying along branches ***(M1)*** $eg\frac{5}{8}\times\frac{3}{7}, \frac{3}{8}\times\frac{5}{7}, \frac{15}{56}$

adding probabilities of correct mutually exclusive paths ***(A1)*** $eg\frac{5}{8}\times\frac{3}{7}+\frac{3}{8}\times\frac{5}{7}, \frac{15}{56}+\frac{15}{56}$

$\frac{30}{56}\left(=\frac{15}{28}\right)$ ***A1*** ***N2*** ***[3 marks]***

18M.1.SL.TZ2.T_12

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

12.5 ***(A1) (C1)***

[1 mark]

b.i. 33 + k **OR** 10 + 8 + 5 + 10 + k ***(A1)***

Note: Award ***(A1)*** for "number of customers = 33 + k". ***[1 mark]***

b.ii. $\frac{2.5 \times 10 + 7.5 \times 8 + \ \dots \ + 22.5 \times k}{33 + k} = 12$ ***(M1)(A1)*(ft)**

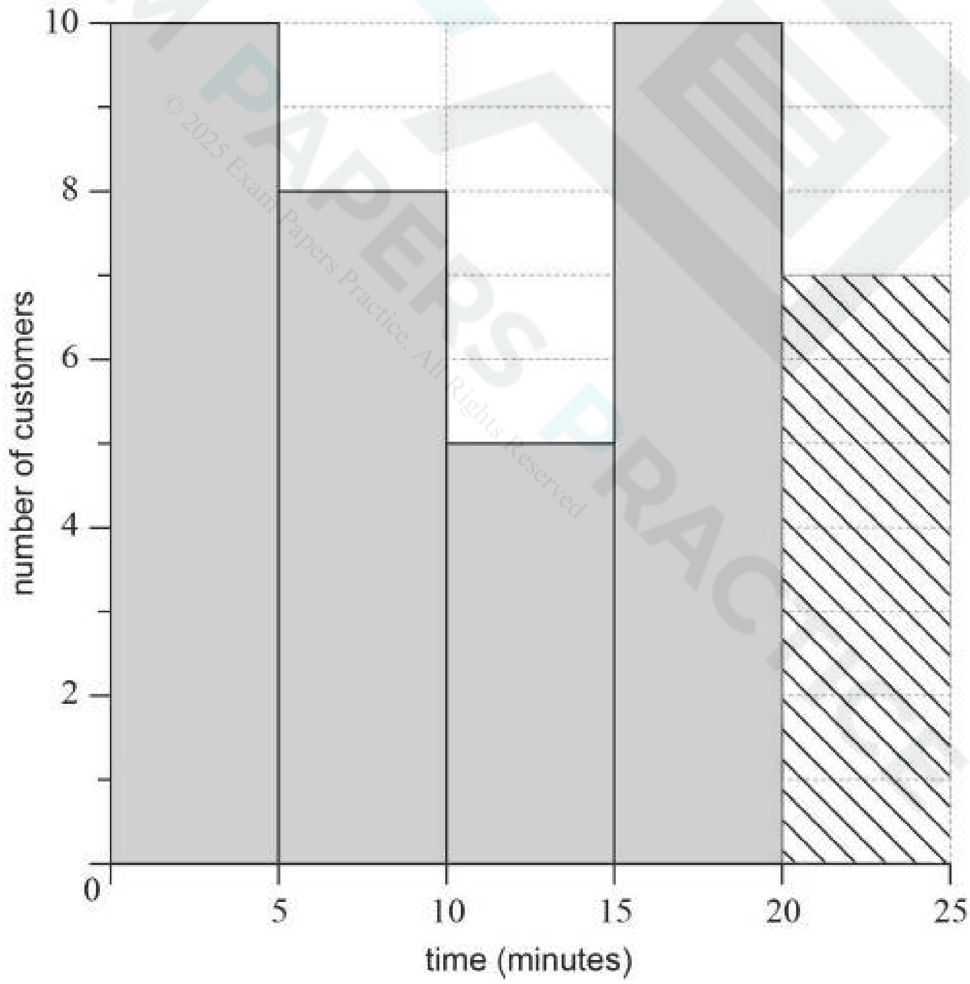
Note: Award ***(M1)*** for substitution into the mean formula and equating to 12, ***(A1)*(ft)** for their correct substitutions.

(k =) 7 ***(A1)*(ft)** ***(C4)***

Note: Follow through from part (b)(i) and their mid-interval values, consistent with part (a). Do not award final ***(A1)*** if answer is not an integer.

[3 marks]

c.



(A1)*(ft)** ***(C1)

Note: Follow through from their part (b)(ii) but only if the value is between 1 and 10, inclusive.

a.i.

valid approach ***(M1)***

eg $A-B,\ -\begin{pmatrix}0\\1\\8\end{pmatrix}+\begin{pmatrix}3\\5\\2\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}3\\4\\-6\end{pmatrix}$ ***A1*** ***N2***

[2 marks]

a.ii. **any** correct equation in the form ***r*** = ***a*** + *t**b*** (any parameter for *t*) ***A2*** ***N2***

where ***a*** is $\begin{pmatrix}0\\1\\8\end{pmatrix}$ or $\begin{pmatrix}3\\5\\2\end{pmatrix}$, and ***b*** is a scalar multiple of $\begin{pmatrix}3\\4\\-6\end{pmatrix}$

*eg**r*** = $\begin{pmatrix}0\\1\\8\end{pmatrix}+t\begin{pmatrix}3\\4\\-6\end{pmatrix}$, ***r*** = $\begin{pmatrix}3+3t\\5+4t\\2-6t\end{pmatrix}$, ***r*** = ***j*** + 8***k*** + *t*(3***i*** + 4***j*** – 6***k***)

Note: Award ***A1*** for the form ***a*** + *t**b***, ***A1*** for the form ***L*** = ***a*** + *t**b***, ***A0*** for the form ***r*** = ***b*** + *t**a***.

[2 marks]

b. valid approach ***(M1)*** *eg*$a\cdot b=0$

choosing correct direction vectors (may be seen in scalar product) ***A1***

eg$\begin{pmatrix}3\\4\\-6\end{pmatrix}$ and $\begin{pmatrix}p\\0\\1\end{pmatrix}$, $\begin{pmatrix}3\\4\\-6\end{pmatrix}\cdot\begin{pmatrix}p\\0\\1\end{pmatrix}=0$ correct working/equation ***A1***

eg$3p-6=0$ $\quad p=2$ ***AG*** ***N0*** ***[3 marks]***

c. valid approach ***(M1)*** *eg*$L_1=\begin{pmatrix}9\\13\\z\end{pmatrix},\ L_1=L_2$

one correct equation (must be different parameters if both lines used) ***(A1)***

eg$3t=9,\ 1+2s=9,\ 5+4t=13,\ 3t=1+2s$ one correct value ***A1***

eg$t=3,\ s=4,\ t=2$ valid approach to substitute their t or s value ***(M1)***

eg$8+3(-6),\ -14+4(1)$ $\quad z=-10$ ***A1*** ***N3*** ***[5 marks]*** d.i. $\left|\vec{d}\right| = \sqrt{2^2+1}\left(=\sqrt{5}\right)$ $\frac{1}{\sqrt{5}}\begin{pmatrix}2\\0\\1\end{pmatrix}\left(\text{accept}\begin{pmatrix}\frac{2}{\sqrt{5}}\\\frac{0}{\sqrt{5}}\\\frac{1}{\sqrt{5}}\end{pmatrix}\right)$

d.ii. valid approach *eg* $\begin{pmatrix}9\\13\\-10\end{pmatrix} \pm \sqrt{5}\hat{d}$

correct working *eg* $\begin{pmatrix}9\\13\\-10\end{pmatrix}+\begin{pmatrix}2\\0\\1\end{pmatrix}, \begin{pmatrix}9\\13\\-10\end{pmatrix}-\begin{pmatrix}2\\0\\1\end{pmatrix}$

one correct point *eg* $(11, 13, -9), (7, 13, -11)$

attempt to use distance between $(1+2s, 13, -14+s)$ and $(9, 13, -10)$

eg $(2s-8)^2+0^2+(s-4)^2=5$

solving $5s^2-40s+75=0$ leading to $s=5$ or $s=3$ one correct point

eg $(11, 13, -9), (7, 13, -11)$

19M.1.SL.TZ1.S_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct equation ***(A1)***

eg $-3+6s=15,\ 6s=18$

$s=3$ ***(A1)***

substitute their s value into z component ***(M1)***

eg $10+3(2),\ 10+6$

$c=16$ ***A1 N3***

[4 marks]

b. $r=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$ $(=(\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k})+t(6\boldsymbol{i}+2\boldsymbol{k}))$ ***A2 N2***

Note: Accept any scalar multiple of $\begin{pmatrix}6\\0\\2\end{pmatrix}$ for the direction vector.

Award for $\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$, for $L_2=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$, for $r=\begin{pmatrix}6\\0\\2\end{pmatrix}+t\begin{pmatrix}1\\2\\3\end{pmatrix}$.

19M.1.SL.TZ2.S_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach ***(M1)***

eg $\boldsymbol{b}=2\boldsymbol{a}$, $a=k\boldsymbol{b}$, $\cos\theta=1$, $\boldsymbol{a}\bullet\boldsymbol{b}=-|\boldsymbol{a}||\boldsymbol{b}|$, $2p=18$

$p=9$ ***A1 N2***

[2 marks]

b. evidence of scalar product ***(M1)*** *eg* $\boldsymbol{a}\bullet\boldsymbol{b}$, $(0)(0)+(3)(6)+p(18)$

recognizing $\boldsymbol{a}\bullet\boldsymbol{b}=0$ (seen anywhere) ***(M1)*** correct working **(A1)**

eg $18+18p=0$, $18p=-18$ ***(A1)*** $p=-1$ ***A1 N3*** ***[4 marks]***

18M.1.SL.TZ2.S_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

any correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\1\\3\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}1\\3\\1\end{pmatrix}$ ***A2 N2***

eg $\boldsymbol{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}=t\begin{pmatrix}1\\3\\1\end{pmatrix}$, $\boldsymbol{r}=2\boldsymbol{i}+\boldsymbol{j}+3\boldsymbol{k}+s(\boldsymbol{i}+3\boldsymbol{j}+\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for the form $L=\boldsymbol{a}+t\boldsymbol{b}$, A0 for the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[2 marks]

b. **METHOD 1** correct scalar product ***(A1)*** *eg* $(1\times 2)+(3\times p)+(1\times 0)$, $2+3p$

evidence of equating **their** scalar product to zero ***(M1)***

eg $\boldsymbol{a}\bullet\boldsymbol{b}=0$, $2+3p=0$, $3p=-2$ $p=-\frac{2}{3}$ ***A1 N3*** **METHOD 2** valid attempt to find angle between vectors

correct substitution into numerator and/or angle

eg $\cos\theta = \frac{(1 \times 2) + (3 \times p) + (1 \times 0)}{|a||b|}$, $\cos\theta = 0$ $p = -\frac{2}{3}$

16N.1.SL.TZ0.S_4

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid attempt to find direction vector ***(M1)***

eg $\overrightarrow{PQ}$, $\overrightarrow{QP}$

correct direction vector (or multiple of) ***(A1)***

eg $6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$

any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (any parameter for t) ***A2*** ***N3***

where $\boldsymbol{a}$ is $\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ or $7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$, and $\boldsymbol{b}$ is a scalar multiple of $6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$

eg $\boldsymbol{r} = 7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k} + t(6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k})$, $\boldsymbol{r} = \begin{pmatrix} 1 + 6s \\ 2 + 1s \\ -1 - 3s \end{pmatrix}$, $r = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} -6 \\ -1 \\ 3 \end{pmatrix}$

Notes: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L} = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[4 marks]

b. correct expression for scalar product ***(A1)*** *eg* $6 \times 2 + 1 \times 0 + (-3) \times n$, $-3n + 12$

setting scalar product equal to zero (seen anywhere) ***(M1)***

eg $\boldsymbol{u} \cdot \boldsymbol{v} = 0$, $-3n + 12 = 0$ $n = 4$ ***A1*** ***N2*** ***[3 marks]***

18M.1.SL.TZ1.S_2

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

recognizing Q_1 or Q_3 (seen anywhere) ***(M1)***

eg 4,11 , indicated on diagram

IQR = 7 ***A1 N2***

[2 marks]

b. recognizing the need to find 1.5 IQR *eg* 1.5 × IQR, 1.5 × 7

valid approach to find k *eg* 10.5 + 11, 1.5 × IQR + Q_3 21.5

k = 22 If no working shown, award for an answer of 21.5.

16N.1.SL.TZ0.T_3

a.

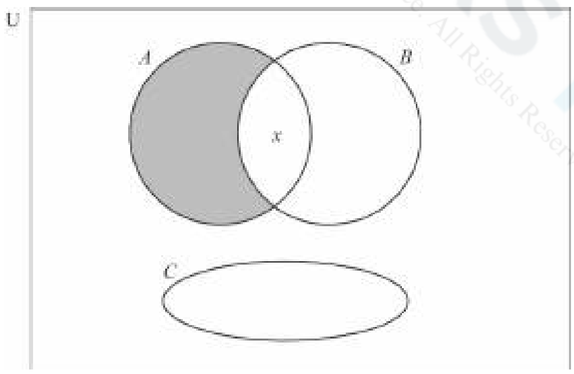
** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

Statement	True or False
$x \in C$	**False**
$x \subset B$	**False**
$A \cup B \neq \varnothing$	**True**
$A \cap B \subset C$	**False**
$A \cap C = \varnothing$	**True**

(A1)(A1)(A1)(A1)(A1) (C5)

[5 marks]

b.



[1 mark]

(A1) (C1)

19M.1.SL.TZ2.S_8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach ***(M1)***

eg 16 + 8, a − 8

24 (hours) ***A1 N2***

[2 marks]

b. valid approach ***(M1)*** *eg* 20 − 15, $Q_3 - Q_1$, 15 − 20 IQR = 5 ***A1 N2***

[2 marks]

c. correct working ***(A1)*** *eg* $\frac{180}{10}$, $\frac{180}{n}$, $\frac{\sum x}{10}$ mean = 18 (hours) ***A1 N2*** ***[2 marks]***

d.i. attempt to find total hours for group B ***(M1)*** *eg* $\bar{x} \times n$

group B total hours = 420 (seen anywhere) ***A1 N2*** ***[2 marks]***

d.ii. attempt to find sum for combined group (may be seen in working) ***(M1)***

eg 180 + 420, 600 correct working ***(A1)*** *eg* $\frac{180+420}{30}$, $\frac{600}{30}$

mean = 20 (hours)

e.i. valid approach to find the new mean *eg* $\frac{1}{2}\mu$, $\frac{1}{2} \times 21$

mean $= \frac{21}{2}$ (= 10.5) hours

17M.1.SL.TZ1.T_4

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

15 ***(A1)*** ***(C1)***

[1 mark]

b. no ***(A1)*** ***(C1)*** **Note:** Accept "it is only offered in Winter and Spring".

[1 mark]

c.i. volleyball, golf, cycling ***(A1)*** ***(C1)***

Note: Responses must list all three sports for the ***(A1)*** to be awarded. ***[1 mark]***

c.ii. 4 ***(A1)*** ***(C1)*** ***[1 mark]***

d. $(F \cup W \cup S)'$ **OR** $F' \cap W' \cap S'$ (or equivalent) ***(A2)*** ***(C2)*** ***[2 marks]***

18M.1.SL.TZ2.S_3

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct approach ***(A1)***

eg $\frac{800}{n} = 20$

40 ***A1 N2***

[2 marks]

b.i. 200 ***A1 N1*** ***[1 mark]***

b.ii. **METHOD 1** recognizing variance = σ^2 ***(M1)*** *eg* $3^2 = 9$

correct working to find new variance ***(A1)*** *eg* $\sigma^2 \times 10^2$, 9×100 900 ***A1 N3***

METHOD 2 new standard deviation is 30 ***(A1)*** recognizing variance = σ^2 ***(M1)***

eg $3^2 = 9$, 30^2 900

17M.1.SL.TZ2.T_3

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\cos 60^\circ = \frac{20}{b}$ **OR** $b = \frac{20}{\cos 60^\circ}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into a correct trig. ratio.

$(b =)\ 40\ (\text{cm})$ ***(A1)*** ***(C2)***

[2 marks]

b. $4 \times 40 + 2\pi(20)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in the circumference of the circle formula, ***(M1)*** for adding 4 times their answer to part (a) to their circumference of the circle.

285.6637… ***(A1)*(ft)**

Note: Follow through from part (a). This ***(A1)*** may be implied by a correct rounded answer.

285.7 (cm) ***(A1)*(ft)** ***(C4)***

Notes: Award ***(A1)*(ft)** for rounding their answer (consistent with their method) to the nearest millimetre, irrespective of unrounded answer seen.

The final ***(A1)(ft)*** is not dependent on any of the previous ***M*** marks. It is for rounding their unrounded answer correctly.

17M.1.SL.TZ2.S_8

a.i.

evidence of median position ***(M1)***

*eg*80th employee

40 hours ***A1*** ***N2***

[2 marks]

a.ii. 130 employees ***A1*** ***N1*** ***[1 mark]***

b.i. £320 ***A1*** ***N1*** ***[1 mark]***

b.ii. splitting into 40 and 3 ***(M1)*** *eg*3 hours more, 3×10 correct working ***(A1)***

eg$320 + 3 \times 10$ £350 ***A1*** ***N3*** ***[3 marks]***

c. valid approach ***(M1)***

*eg*200 is less than 320 so 8 pounds/hour, $200 \div 8,\ 25,\ \frac{200}{320} = \frac{x}{40},$

18 employees ***A2*** ***N3*** ***[3 marks]***

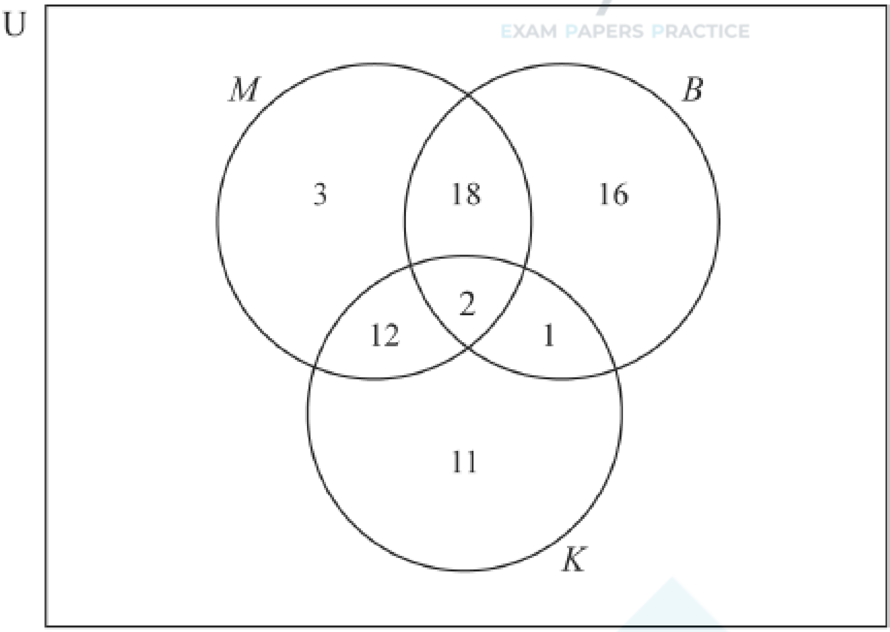
d. valid approach ***(M1)*** *eg*$160 - 10$ 60 hours worked ***(A1)***

correct working ***(A1)*** *eg*$40(8) + 20(10),\ 320 + 200$ $k = 520$ ***A1*** ***N3*** ***[4 marks]***

19M.1.SL.TZ2.T_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1)(A1) (C2)

Note: Award ***(A1)*** for 18, 12 and 1 in correct place on Venn diagram, ***(A1)*** for 3, 16 and 11 in correct place on Venn diagram.

b. 85 − (3 + 16 + 11 + 18 + 12 + 1 + 2)

Award for subtracting the sum of their values from 85. 22

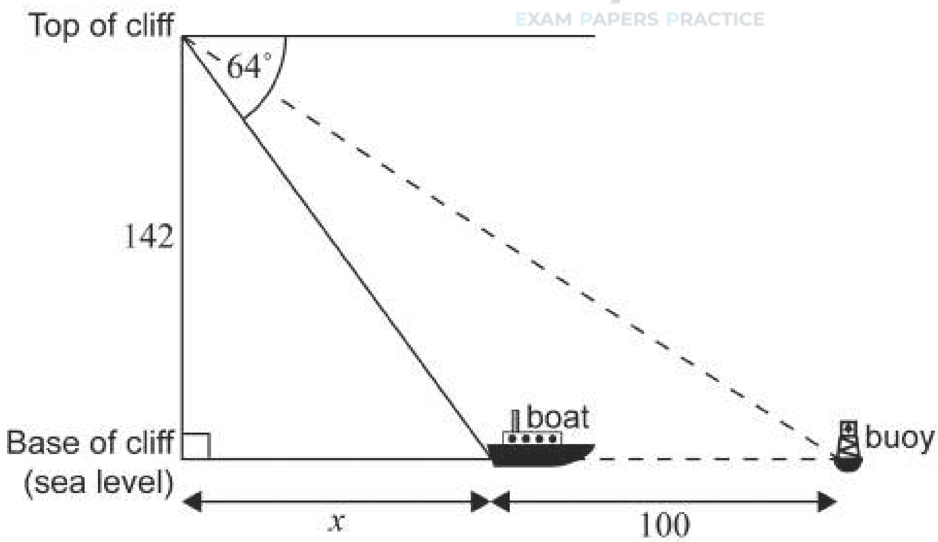
Follow through from their Venn diagram in part (a).
If any numbers that are being subtracted are negative award .

c. $\frac{14}{35}\left(\frac{2}{5}, 0.4, 40\%\right)$

Award for correct numerator; for correct denominator. Follow through from their Venn diagram.

19M.1.SL.TZ1.T_8

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



The horizontal line must be shown and the angle of depression must be labelled. Accept a numerical or descriptive label.

16N.1.SL.TZ0.T_11

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\tan 48° = \frac{CD}{250}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the tangent ratio.

$(CD =)\ 278\ (m)\ (277.653\ ...)$ ***(A1)*** ***(C2)***

[2 marks]

b. $\tan ABC \text{ (or equivalent)} = \frac{\frac{4}{3} \times 277.653\ ...}{250}$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for $\frac{4}{3}$ multiplying their part (a), ***(M1)*** for substitution into the tangent ratio, ***(M1)*** for correct substitution.

OR $90 - \tan^{-1}\left(\frac{250}{\frac{4}{3} \times 277.653\ ...}\right)$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for $\frac{4}{3}$ multiplying their part (a), ***(M1)*** for substitution into the tangent ratio, ***(M1)*** for subtracting from 90 and for correct substitution.

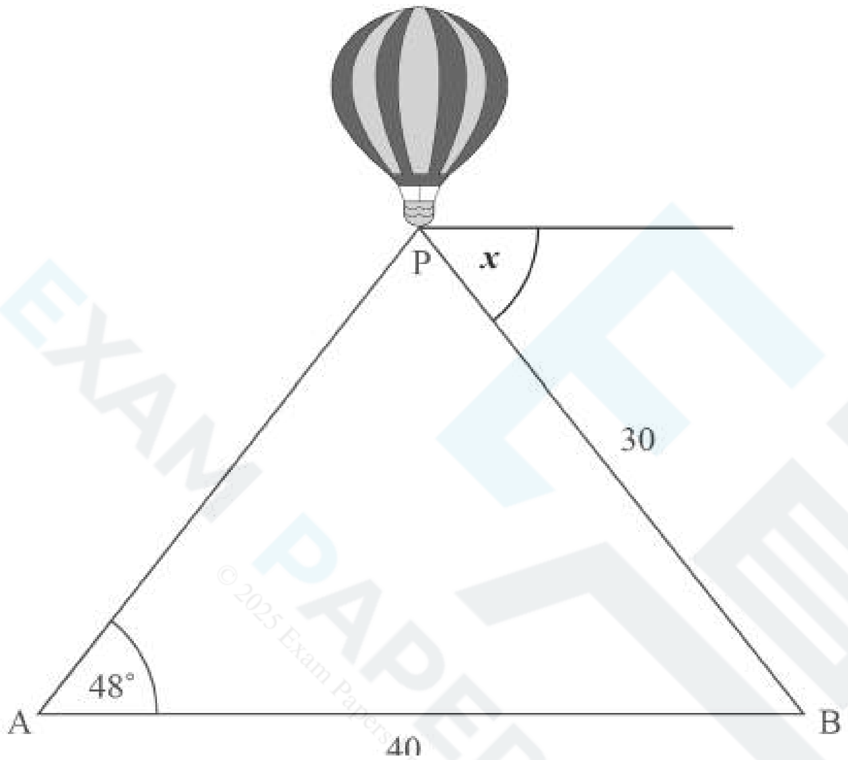
$(\text{angle of depression} =)\ 56.0°\ (55.9687\ ...)$ ***(A1)*****(ft)** ***(C4)***

Follow through from part (a).

18M.1.SL.TZ1.T_8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



(A1) (C1)

[1 mark]

b. $\frac{40}{\sin \text{APB}} = \frac{30}{\sin 48^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into sine rule, ***(A1)*** for correct substitution.

(angle APB =) 82.2° (82.2473…°) ***(A1) (C3) [3 marks]***

c. 180 − 48 − 82.2473… ***(M1)*** 49.8° (49.7526…°) ***(A1)*(ft)** ***(C2)***

Note: Follow through from parts (a) and (b). ***[2 marks]***

19M.1.SL.TZ2.T_12

a.i.

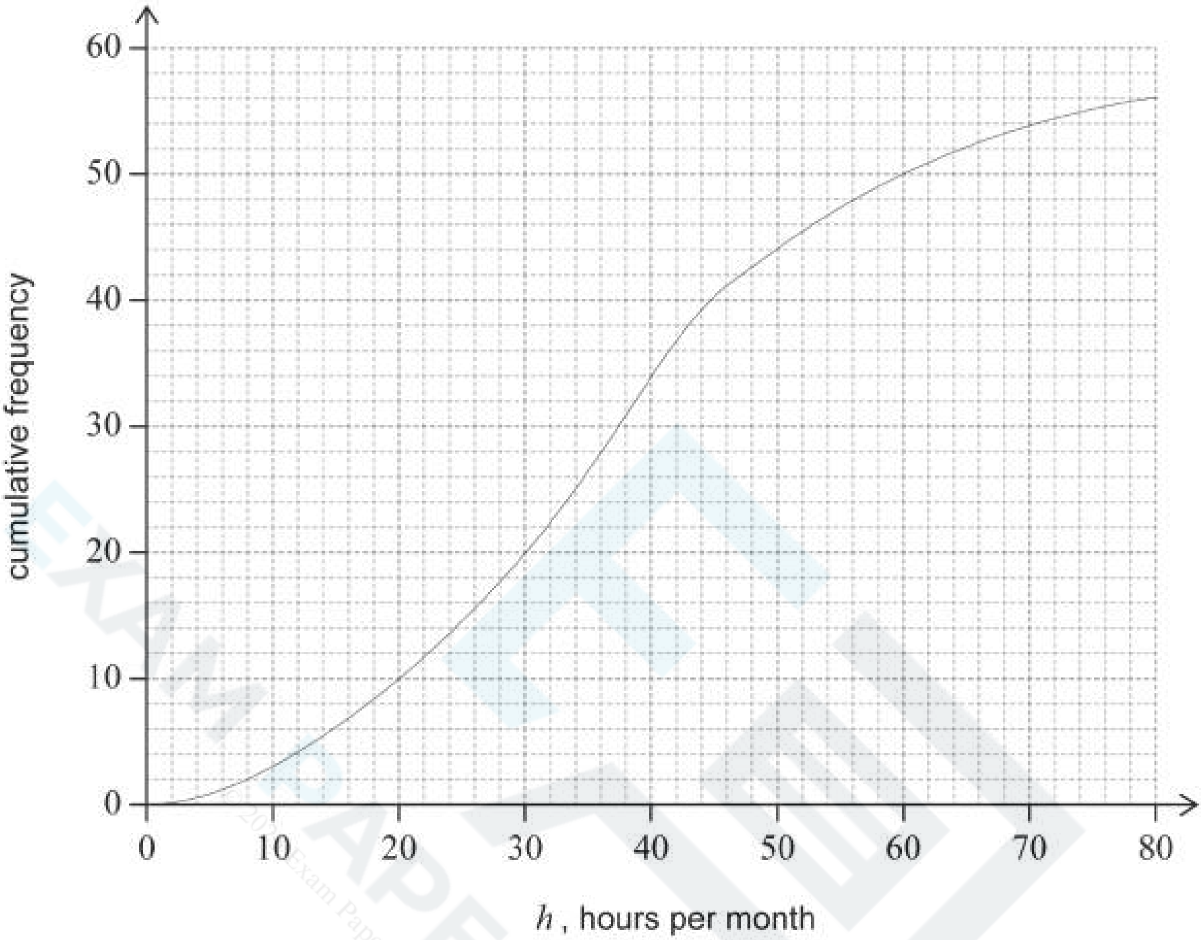
$p = 10$ ***(A1) (C1)***

Note: Award ***(A1)*** for each correct value.

[1 mark]

a.ii. $q = 56$ ***(A1) (C1)*** **Note:** Award ***(A1)*** for each correct value. ***[1 mark]***

b.



(A1)(A1) (C2)

Note: Award ***(A1)*****(ft)** for their 3 correctly plotted points; award ***(A1)*****(ft)** for completing diagram with a smooth curve through their points. The second ***(A1)*****(ft)** can follow through from incorrect points, provided the gradient of the curve is never negative. Award ***(C2)*** for a completely correct smooth curve that goes through the correct points.

[2 marks]

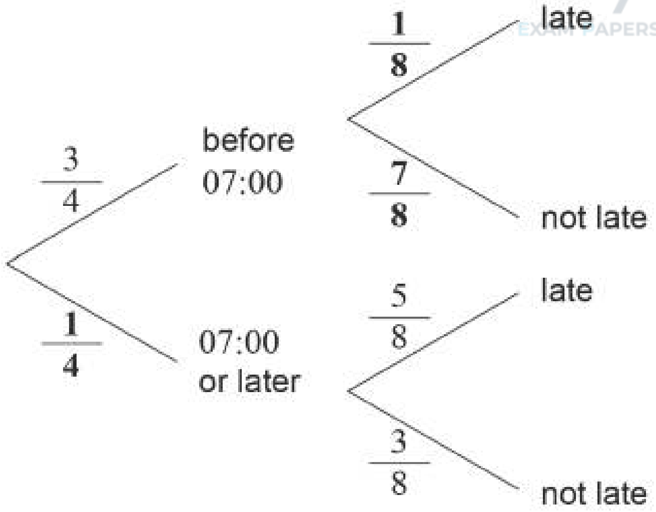
c. a straight vertical line drawn at 35 (accept 35 ± 1)

26 (students) Accept values between 25 and 27 inclusive.

18M.1.SL.TZ2.S_8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*



A1A1A1 N3

Note: Award ***A1*** for each bold fraction.

[3 marks]

b. multiplying along correct branches ***(A1)***
eg $\frac{3}{4} \times \frac{1}{8}$

P(leaves before 07:00 ∩ late) = $\frac{3}{32}$ ***A1 N2*** ***[2 marks]***

c. multiplying along other "late" branch ***(M1)***
eg $\frac{1}{4} \times \frac{5}{8}$

adding probabilities of two mutually exclusive late paths ***(A1)***
eg $\left(\frac{3}{4} \times \frac{1}{8}\right) + \left(\frac{1}{4} \times \frac{5}{8}\right), \frac{3}{32} + \frac{5}{32}$

$\mathrm{P}(L) = \frac{8}{32}\left(= \frac{1}{4}\right)$ ***A1 N2*** ***[3 marks]***

d. recognizing conditional probability (seen anywhere) ***(M1)***
eg $\mathrm{P}(A \mid B), \mathrm{P}(\text{ before 7|late })$

correct substitution of **their** values into formula ***(A1)***
eg $\frac{\frac{3}{32}}{\frac{1}{4}}$

$\mathrm{P}(\text{ left before 07:00|late }) = \frac{3}{8}$ ***A1 N2*** ***[3 marks]***

e. valid approach
eg 1 − P(not late twice), P(late once) + P(late twice)

correct working
eg $1 - \left(\frac{3}{4} \times \frac{3}{4}\right), 2 \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}$ $\frac{7}{16}$

17N.1.SL.TZ0.T_1

a.i.

$$\frac{14 + 2 \times 15 + 7 \times 16 + 17 + 4 \times 18 + 19 + 20 + 3 \times 22}{20}$$ ***(M1)***

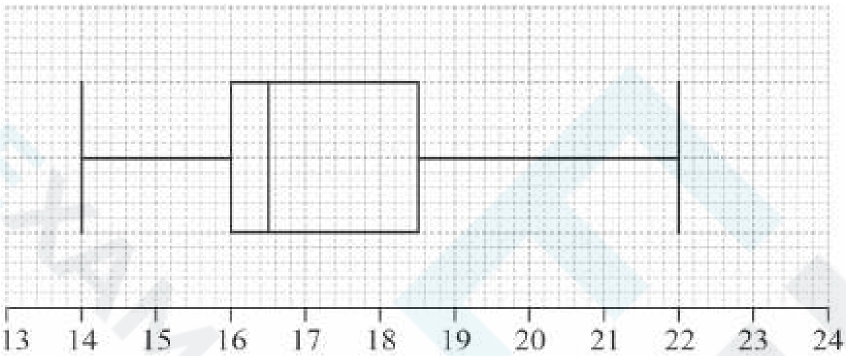
Note: Award ***(M1)*** for correct substitutions into mean formula.

$(=)\ 17.5$ ***(A1)*** ***(C2)***

[2 marks]

a.ii. 16.5 ***(A1)*** ***(C1)*** ***[1 mark]***

b.



(A1)(A1)(A1)*(ft)** ***(C3)

Note: Award ***(A1)*** for correct endpoints, ***(A1)*** for correct quartiles, ***(A1)*(ft)** for their median. Follow through from part (a)(ii), but only if median is between 16 and 18.5. If a horizontal line goes through the box, award at most ***(A1)(A1)(A0)***. Award at most ***(A0)(A1)(A1)*** if a ruler has not been used.

k

19M.1.SL.TZ1.T_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\left(\cos A = \right) \frac{2.6^2 + 3.1^2 - 2.4^2}{2(2.6)(3.1)}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule formula, ***(A1)*** for correct substitutions.

48.8° (48.8381...°) ***(A1)*** ***(C3)***

[3 marks]

b. $\frac{1}{2} \times 2.6 \times 3.1 \times \sin(48.8381\ ...°)$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituted area of a triangle formula, ***(A1)*** for correct substitution.

3.03 (km^2) (3.033997...(km^2)) ***(A1)*(ft)** ***(C3)*** **Note:** Follow through from part (a).

[3 marks]

17M.1.SL.TZ1.S_4

a.i.

t ***A1*** ***N1***

[1 mark]

a.ii. 105 ***A1*** ***N1*** ***[1 mark]***

b. -0.992 ***A2*** ***N2*** ***[2 marks]***

c. valid approach ***(M1)*** *eg* $\frac{\text{dd}}{\text{dt}} = -2.24;\ 2 \times 2.24,\ 2 \times -2.24,\ d(2) = -2 \times 2.24 \times 105,$

finding $d(t_2) - d(t_1)$ where $t_2 = t_1 + 2$ 4.48 (degrees) ***A1*** ***N2***

Notes: Award no marks for answers that **directly** use the table to find the decrease in temperature for 2 minutes *eg* $\frac{105 - 98.4}{2} = 3.3.$

[2 marks]

21N.1.SL.TZ0.3

a.i.

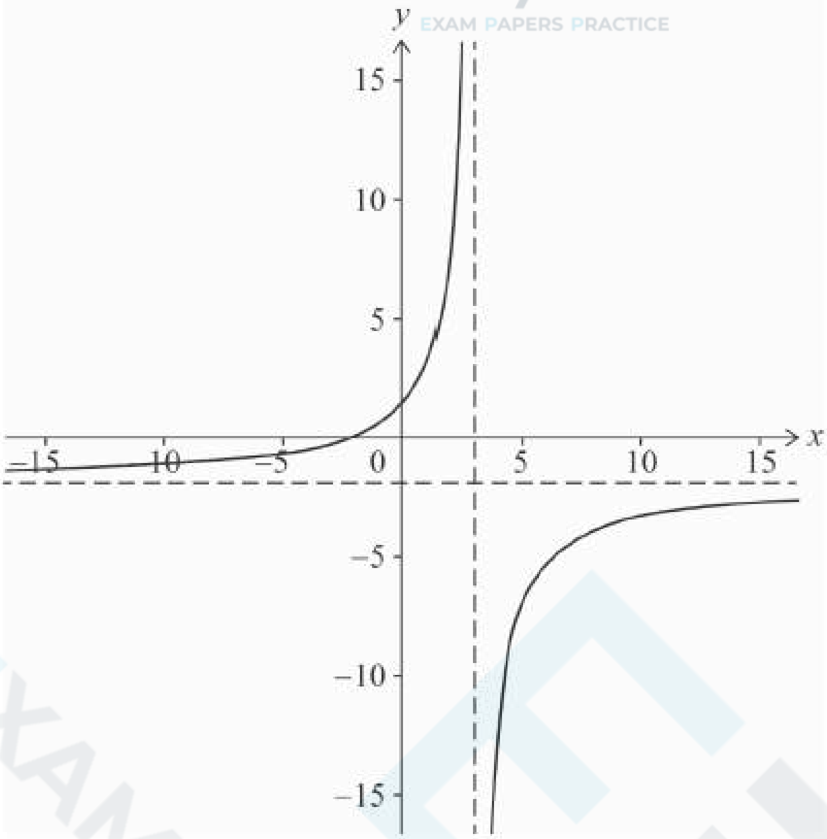
$x = 3$ ***A1***

[1 mark]

a.ii. $y = -2$ ***A1*** ***[1 mark]***

b.i. -2, 0 (accept $x = -2$) ***A1*** ***[1 mark]***

b.ii. $0,\ \frac{4}{3}$ (accept $y = \frac{4}{3}$ and $f0 = \frac{4}{3}$) ***A1*** ***[1 mark]*** c.



Award for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

21N.1.SL.TZ0.4

a.

valid approach to find $\mathrm{P}R$ ***(M1)***

tree diagram (must include probabilty of picking box) with correct required probabilities

OR $\mathrm{P}R \cap B_1 + \mathrm{P}R \cap B_2$ OR $\mathrm{P}R \quad B_1 \mathrm{P}B_1 + \mathrm{P}R \quad B_2 \mathrm{P}B_2$

$\frac{5}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2}$ ***(A1)***

$\mathrm{P}R = \frac{9}{14}$ ***A1***

[3 marks]

b. events A and R are not independent, since $\frac{9}{14} \cdot \frac{1}{2} \neq \frac{5}{14}$ OR $\frac{5}{7} \neq \frac{9}{14}$ OR $\frac{5}{9} \neq \frac{1}{2}$

OR an explanation e.g. different number of red balls in each box ***A2***

Note: Both conclusion and reasoning are required. Do not split the ***A2***. ***[2 marks]***

17M.1.SL.TZ1.T_13

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\frac{\text{BC}}{\sin 34°} = \frac{5}{\sin 120°}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule formula, ***(A1)*** for correct substitutions.

$\text{BC} = 3.23 \text{ (cm)} \left(3.22850 \ldots \text{ (cm)}\right)$ ***(A1)*** ***(C3)***

[3 marks]

b. $\frac{1}{2}(5)(3.22850)\sin 26°$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituted area of a triangle formula, ***(A1)*** for correct substitutions.

$= 3.54 \text{ (cm}^2\text{)} \left(3.53820 \ldots \text{ (cm}^2\text{)}\right)$ ***(A1)*(ft)** ***(C3)***

Note: Follow through from part (a). ***[3 marks]***

17M.1.SL.TZ2.T_12

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$\pi \times 8^2 \times 12$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the volume of a cylinder formula.

2410 cm^3 $(2412.74 \ldots \text{ cm}^3, 768\pi \text{ cm}^3)$ ***(A1)*** ***(C2)***

[2 marks]

b. $\frac{4}{3}\pi \times 2.9^3 + 768\pi = \pi \times 8^2 h$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into the volume of a sphere formula (this may be implied by seeing 102.160…), ***(M1)*** for adding their volume of the ball to their part (a), ***(M1)*** for equating **a** volume to the volume of a cylinder with a height of h.

OR $\frac{4}{3}\pi \times 2.9^3 = \pi \times 8^2(h - 12)$ ***(M1)(M1)(M1)***

Award for correct substitution into the volume of a sphere formula (this may be implied by seeing 102.160...), for equating to the volume of a cylinder, for the height of the water level increase, $h-12$. Accept h for $h-12$ if adding 12 is implied by their answer.

$(h=)\ 12.5\ (\text{cm})\ \left(12.5081\ ...\ (\text{cm})\right)$

If 3 sf answer used, answer is 12.5 (12.4944...). Follow through from part (a) if first method is used.

20N.1.SL.TZ0.T_6

a.i.

$\frac{3}{9}$ $\frac{1}{3}$, 0.333, $0.333333\ ...$, 33.3% ***(A1) (C1)***

[1 mark]

a.ii. $\frac{5}{9}$ 0.556, $0.555555\ ...$, 55.6% ***(A1) (C1)***

[1 mark]

b. $\frac{3}{8}$ 0.375, 37.5% ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for correct numerator, ***(A1)*** for correct denominator. ***[2 marks]***

c. $\frac{2}{9}\times\frac{1}{8}$ ***(M1)***

Note: Award ***(M1)*** for a correct compound probability calculation seen.

$\frac{2}{72}$ $\frac{1}{36}$, 0.0278, $0.0277777\ ...$, 2.78% ***(A1) (C2) [2 marks]***

19M.1.SL.TZ1.T_15

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$A=2\pi r(12-r)$ **OR** $A=24\pi r-2\pi r^2$ ***(A1)(M1) (C2)***

Note: Award ***(A1)*** for $r+h=12$ or $h=12-r$ seen. Award ***(M1)*** for correctly substituting into curved surface area of a cylinder. Accept $A=2\pi r(12-r)$ **OR** $A=24\pi r-2\pi r^2$.

[2 marks]

b. $24\pi - 4\pi r$

Award for 24π and for $-4\pi r$. Follow through from part (a). Award at most if additional terms are seen.

c. $24\pi - 4\pi r = 0$ Award for setting *their* part (b) equal to zero.

6 (cm) Follow through from part (b).

20N.1.SL.TZ0.T_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.*

$24 - 8$ **OR** $24 - 32 - 24$ **OR** $24 = \frac{32+h}{2}$ ***(M1)***

Note: Award ***(M1)*** for subtracting 8 from the median, or equivalent.

16 cm ***(A1) (C2)***

[2 marks]

b. $q - p = 50$ (or equivalent) ***(A1) (C1)***

[1 mark]

c. $\frac{p+16+32+q}{4} = 27$ **OR** $p + q = 60$ (or equvalent) ***(A1)*(ft) *(C1)***

Note: Follow through from part (a). ***[1 mark]***

d.i. 5 cm ***(A1)*(ft) *(C1)***

Note: Follow through from parts (b) and (c). ***[1 mark]***

d.ii. 55 cm ***(A1)*(ft) *(C1)***

Note: Follow through from parts (b) and (c). ***[1 mark]***

16N.1.SL.TZ0.T_7

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

Units are required in parts (a) and (b).

$\frac{4}{3}\pi \times 6^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into volume of sphere formula.

$= 905 \text{ cm}^3 \ (288\pi \text{ cm}^3, \ 904.778 \ldots \ \text{cm}^3)$ ***(A1)*** ***(C2)***

Note: Answers derived from the use of approximations of π (3.14; 22/7) are awarded ***(A0)***.

[2 marks]

b. **Units are required in parts (a) and (b).**

$\frac{140}{100} \times 904.778 \ldots = \frac{4}{3}\pi r^3$ **OR** $\frac{140}{100} \times 288\pi = \frac{4}{3}\pi r^3$ **OR** $1266.69 \ldots = \frac{4}{3}\pi r^3$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplying their part (a) by 1.4 or equivalent, ***(M1)*** for equating to the volume of a sphere formula.

$r^3 = \frac{3 \times 1266.69 \ldots}{4\pi}$ **OR** $r = \sqrt[3]{\frac{3 \times 1266.69 \ldots}{4\pi}}$ **OR** $r = \sqrt[3]{(1.4) \times 6^3}$ **OR** $r^3 = 302.4$ ***(M1)***

Award for isolating r. $(r =)\ 6.71 \text{ cm} \ (6.71213 \ldots)$

Follow through from part (a).

21N.1.SL.TZ0.1

a.i.

setting $fx = 0$ ***(M1)***

$x = 1, \quad x = -3$ (accept $1, 0, -3, 0$) ***A1***

[2 marks]

a.ii. **METHOD 1** $x = -1$ ***A1*** substituting their x-coordinate into f ***(M1)***

$y = 8$ ***A1*** $-1, 8$ **METHOD 2** attempt to complete the square ***(M1)***

$-2x + 1^2 - 4$ ***(M1)*** $x = -1, \quad y = 8$ ***A1A1*** $-1, 8$ ***[3 marks]***

b. $h = -1$ ***A1*** $k = 8$ ***A1*** ***[2 marks]***

20N.1.SL.TZ0.T_8

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.*

$FV = \quad 25 \ \ 000 \times 1 + \frac{3.6}{100 \times 12}^{12 \times 5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted compound interest formula, ***(A1)*** for correct substitutions.

OR

$N = 5$
$I\% = 3.6$
$PV = \mp 25 \ \ 000$
$P / Y = 1$
$C / Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C / Y = 12$ seen, ***(M1)*** for **all** other correct entries.

OR

$N = 60$
$I\% = 3.6$
$PV = \mp 25 \ \ 000$
$P / Y = 12$
$C / Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C / Y = 12$ seen, ***(M1)*** for **all** other correct entries.

$FV = \quad 29 \ \ 922 \ \ \text{SGD}$ ***(A1)*** ***(C3)***

Note: Do not award the final ***(A1)*** if answer is not given correct to the nearest integer.

[3 marks]

b. $20 \ \ 000 = PV \times 1 + \frac{5.7}{100 \times 2}^{2 \times 1.5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted compound interest equated to $20 \ \ 000$. Award ***(A1)*** for correct substitutions.

OR

$N = 1.5$
$I\% = 5.7$
$FV = \pm 20 \ \ 000$
$P / Y = 1$
$C / Y = 2$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C / Y = 2$ seen, ***(M1)*** for **all** other correct entries.

OR

$N = 3$
$I\% = 5.7$
$FV = \pm 20 \ \ 000$
$P / Y = 2$
$C / Y = 2$ ***(A1)(M1)***

Award for $C / Y = 2$ seen, for other correct entries.

$x = \quad 18 \;\; 383 \quad \text{SGD}$

Do not award the final if answer is not given correct to the nearest integer (unless already penalized in part(a)).

17M.1.SL.TZ2.T_14

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

400 (USD) ***(A1)*** ***(C1)***

[1 mark]

b. $8500\,(0.95)^t = 400 \times t + 2000$ ***(M1)***

Note: Award ***(M1)*** for equating $8500(0.95)^t$ to $400 \times t + 2000$ or for comparing the difference between the two expressions to zero or for showing a sketch of both functions.

$(t =)\ 8.64\ \text{(months)}\ \left(8.6414\ \ldots\ \text{(months)}\right)$ ***(A1)*** ***(C2)***

Note: Accept 9 months. ***[2 marks]***

c. $8500(0.95)^2 - (400 \times 2 + 2000)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution of $t = 2$ into equation for P, ***(M1)*** for finding the difference between a value/expression for P and a value/expression for S. The first ***(M1)*** is implied if 7671.25 seen.

4870 (USD) (4871.25) ***(A1)*** ***(C3)*** **Note:** Accept 4871.3. ***[3 marks]***

21N.1.SL.TZ0.2

METHOD 1

recognition that $y = \int \cos x - \frac{\pi}{4}\,\mathrm{d}\,x$ ***(M1)***

$y = \sin x - \frac{\pi}{4} + c$ ***(A1)***

substitute both x and y values into their integrated expression including c ***(M1)***

$2 = \sin\frac{\pi}{2} + c$

$c = 1$

$y = \sin x - \frac{\pi}{4} + 1$ ***A1***

METHOD 2

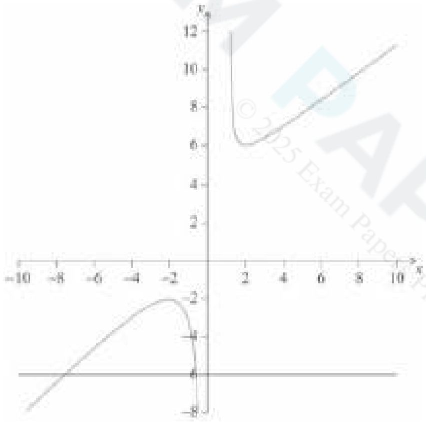
$\int_2^y \mathrm{d}\, y = \int_{\frac{3\pi}{4}}^{x} \cos x - \frac{\pi}{4} \,\mathrm{d}\, x$

$y - 2 = \sin x - \frac{\pi}{4} - \sin\frac{\pi}{2}$

$y = \sin x - \frac{\pi}{4} + 1$

17M.1.SL.TZ1.T_12

b.i.



(A1) ***(C1)***

Note: The command term "Draw" states: "A ruler (straight edge) should be used for straight lines"; do not accept a freehand $y = -6$ line.

[1 mark]

b.ii. 2 ***(A1)*(ft)** ***(C1)*** **Note:** Follow through from part (b)(i). ***[1 mark]***

c. $-2 < k < 6$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for both end points correct and ***(A1)*** for correct **strict** inequalities.

Award at most ***(A1)(A0)*** if the stated variable is different from k or y for example $-2 < x < 6$ is ***(A1)(A0)***.

[2 marks]

17M.1.SL.TZ1.S_1

a.i.

valid approach ***(M1)***

eg $p + 3 = 13,\ 13 - 3$

$p = 10$ ***A1*** ***N2***

[2 marks]

a.ii. valid approach ***(M1)*** *eg* $p + 3 + 5 + q = 20,\ 10 - 10 - 8$ $q = 2$ ***A1*** ***N2***

[2 marks]

b. valid approach ***(M1)*** *eg* $20 - p - q - 3,\ 1 - \frac{15}{20},\ n(E \cap H') = 5$ $\frac{5}{20}\left(\frac{1}{4}\right)$ ***A1*** ***N2***

[2 marks]

18N.1.SL.TZ0.S_9

a.i.

$\frac{2}{n}$ ***A1 N1***

[1 mark]

a.ii. correct probability for one of the draws ***A1***

eg P(not blue first) = $\frac{n-2}{n}$, blue second = $\frac{2}{n-1}$ valid approach ***(M1)***

eg recognizing loss on first in order to win on second, P(B' then B), P(B') × P($B \mid B'$), tree diagram

correct expression in terms of n ***A1 N3*** *eg* $\frac{n-2}{n} \times \frac{2}{n-1},\ \frac{2n-4}{n^2-n},\ \frac{2(n-2)}{n(n-1)}$ ***[3 marks]***

b.i. correct working ***(A1)*** *eg* $\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$ $\frac{12}{60}\left(= \frac{1}{5}\right)$ ***A1 N2*** ***[2 marks]***

b.ii. correct working ***(A1)*** *eg* $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}$ $\frac{6}{60}\left(= \frac{1}{10}\right)$ ***A1 N2*** ***[2 marks]***

c. correct probabilities (seen anywhere) ***(A1)(A1)***

eg $\mathrm{P}(1) = \frac{2}{5},\ \mathrm{P}(2) = \frac{6}{20}$ (may be seen on tree diagram)

valid approach to find E (M) or expected winnings using **their** probabilities ***(M1)***

eg $\mathrm{P}(1) \times (0) + \mathrm{P}(2) \times (20) + \mathrm{P}(3) \times (8k) + \mathrm{P}(4) \times (12k)$,

$\mathrm{P}(1) \times (-20) + \mathrm{P}(2) \times (0) + \mathrm{P}(3) \times (8k - 20) + \mathrm{P}(4) \times (12k - 20)$

correct working to find E (M) or expected winnings

eg $\frac{2}{5}(0)+\frac{3}{10}(20)+\frac{1}{5}(8k)+\frac{1}{10}(12k)$,

$\frac{2}{5}(-20)+\frac{3}{10}(0)+\frac{1}{5}(8k-20)+\frac{1}{10}(12k-20)$ correct equation for fair game

eg $\frac{3}{10}(20)+\frac{1}{5}(8k)+\frac{1}{10}(12k)=20,\ \frac{2}{5}(-20)+\frac{1}{5}(8k-20)+\frac{1}{10}(12k-20)=0$

correct working to combine terms in k

eg $-8+\frac{14}{5}k-4-2=0,\ 6+\frac{14}{5}k=20,\ \frac{14}{5}k=14$ $k=5$

Do not award the final if the candidate's probabilities do not sum to 1.

19M.1.SL.TZ1.S_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach ***(M1)***

eg 0.30 − 0.1, p + 0.1 = 0.3

p = 0.2 ***A1 N2***

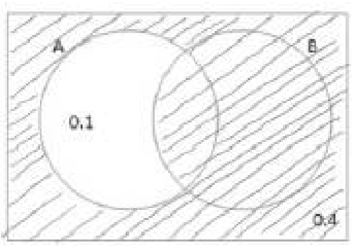
[2 marks]

b. valid approach ***(M1)*** *eg* 1 − (0.3 + 0.4), 1 − 0.4 − 0.1 − p q = 0.3 ***A1 N2***

[2 marks]

c. valid approach ***(M1)***

eg $0.7+0.5-0.3,\ p+q+0.4,\ 1-0.1,\ \mathrm{P}\left(A'\cup B\right)=\mathrm{P}\left(A'\right)+\mathrm{P}(B)-\mathrm{P}\left(A'\cap B\right)$



$\mathrm{P}\left(A'\cup B\right)=0.9$ ***A1 N2*** ***[2 marks]***

16N.1.SL.TZ0.S_5

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid interpretation (may be seen on a Venn diagram) ***(M1)***

eg$\mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B),\ 0.2 + 0.6$

$\mathrm{P}(B) = 0.8$ ***A1*** ***N2***

[2 marks]

b. valid attempt to find $\mathrm{P}(A)$ ***(M1)*** *eg*$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B),\ 0.8 \times A = 0.2$

correct working for $\mathrm{P}(A)$ ***(A1)*** *eg*$0.25,\ \frac{0.2}{0.8}$ correct working for $\mathrm{P}(A \cup B)$ ***(A1)***

eg$0.25 + 0.8 - 0.2,\ 0.6 + 0.2 + 0.05$ $\mathrm{P}(A \cup B) = 0.85$

22M.1.SL.TZ2.2

a.

$u_1 = 12$ ***A1***

[1 mark]

b. $15 - 3n = -33$ ***(A1)*** $n = 16$ ***A1*** ***[2 marks]***

c. valid approach to find d ***(M1)***

$u_2 - u_1 = 9 - 12$ OR recognize gradient is -3 OR attempts to solve $-33 = 12 + 15d$

$d = -3$ ***A1*** ***[2 marks]***

22M.1.SL.TZ2.4

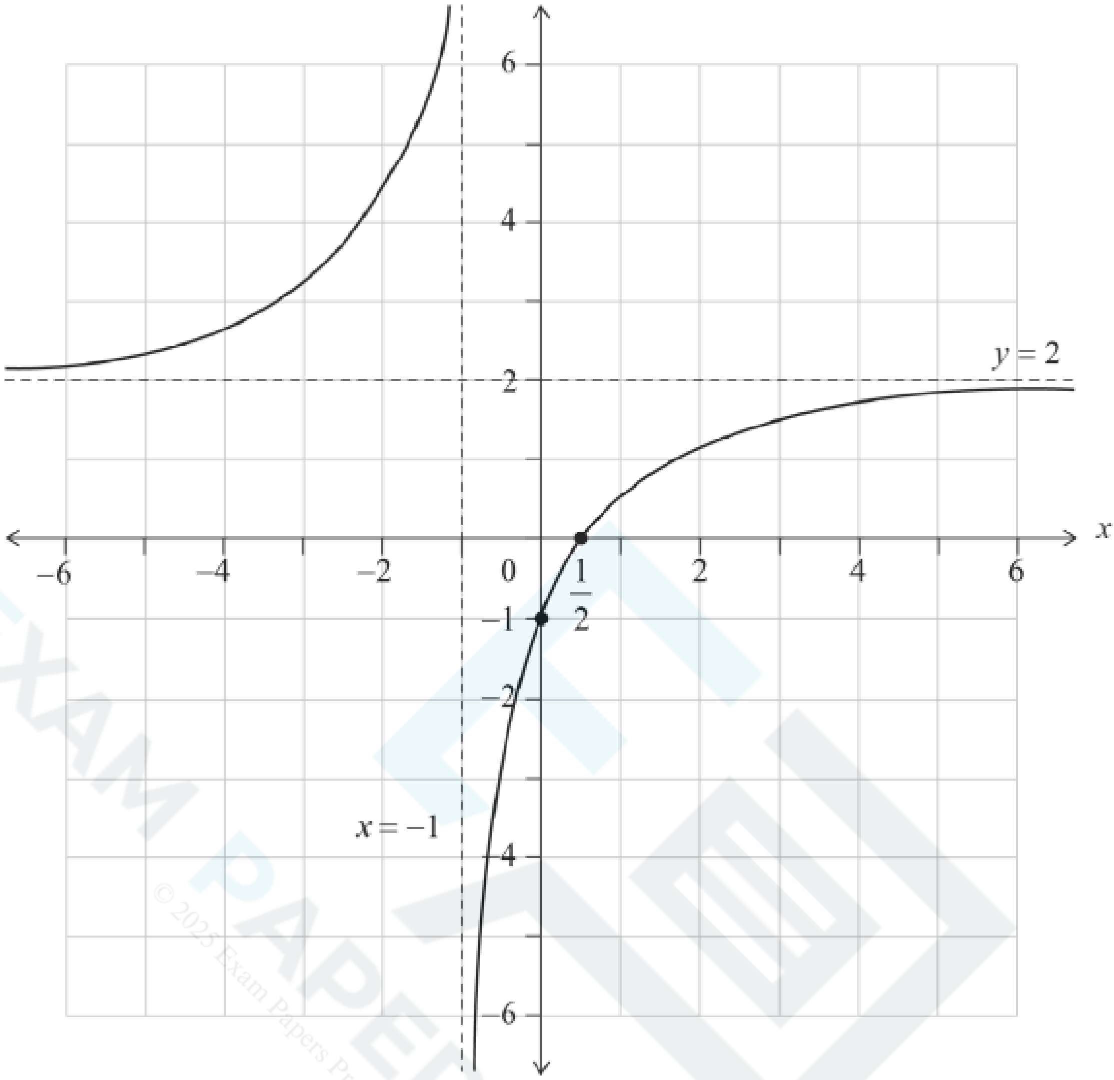
a.i.

$x = -1$ ***A1***

[1 mark]

a.ii. $y = 2$ ***A1*** ***[1 mark]***

b.



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown

The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x = -1$ and $y = 2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x = \frac{1}{2}$ and $y = -1$

c. $x > \frac{1}{2}$

Accept correct alternative correct notation, such as $\frac{1}{2}, \quad \infty$ and $\left]\frac{1}{2}, \infty\right[$.

22M.1.SL.TZ2.3

a.

$n - 1 + n + n + 1$ ***(A1)***

$= 3n$ ***A1***

which is always divisible by 3 ***AG***

[2 marks]

b. $n - 1^2 + n^2 + n + 1^2 \quad = n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$ ***A1***

attempts to expand either $n - 1^2$ or $n + 1^2$ (do not accept $n^2 - 1$ or $n^2 + 1$) ***(M1)***

$= 3n^2 + 2$ ***A1***

demonstrating recognition that 2 is not divisible by 3 or $\frac{2}{3}$ seen after correct expression divided by 3 ***R1***

$3n^2$ is divisible by 3 and so $3n^2 + 2$ is never divisible by 3

OR the first term is divisible by 3, the second is not OR $3n^2 + \frac{2}{3}$ OR $\frac{3n^2+2}{3} = n^2 + \frac{2}{3}$

hence the sum of the squares is never divisible by 3

22M.1.SL.TZ2.5

determines $\frac{\pi}{4}$ (or 45°) as the first quadrant (reference) angle ***(A1)***

attempts to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}$ ***(M1)***

Note: Award ***M1*** for attempting to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}, \frac{7\pi}{4}, \ldots$

$\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4} \Rightarrow x < 0$ and so $\frac{\pi}{4}$ is rejected ***(R1)***

$\frac{x}{2} + \frac{\pi}{3} = 2\pi - \frac{\pi}{4} \quad = \frac{7\pi}{4}$ ***A1***

$x = \frac{17\pi}{6}$ (must be in radians) ***A1***

[5 marks]

22M.1.SL.TZ2.7

a.

$x = 3$ ***A1***

Note: Must be an equation in the form "$x =$". Do not accept 3 or $\frac{-b}{2a} = 3$.

[1 mark]

b.i. $h = 3$, $k = 4$ (accept $ax - 3^2 + 4$) ***A1A1*** ***[2 marks]***

b.ii. attempt to substitute coordinates of Q ***(M1)***

$12 = a5 - 3^2 + 4$, $4a + 4 = 12$ $a = 2$ ***A1*** ***[2 marks]***

c. recognize need to find derivative of f ***(M1)***

$f'x = 4x - 3$ or $f'x = 4x - 12$ ***A1***

$f'5 = 8$ (may be seen as gradient in their equation) ***(A1)***

$y - 12 = 8x - 5$ or $y = 8x - 28$ ***A1*** **Note:** Award ***A0*** for $L = 8x - 28$.

[4 marks]

d. **METHOD 1** Recognizing that for g to be increasing, $fx - d > 0$, or $g' > 0$ ***(M1)***

The vertex must be above the x-axis, $4 - d > 0$, $d - 4 < 0$ ***(R1)*** $d < 4$ ***A1***

METHOD 2 attempting to find discriminant of g' ***(M1)*** $-12^2 - 4222 - d$

recognizing discriminant must be negative ***(R1)*** $-32 + 8d < 0$ OR $\Delta < 0$

$d < 4$ ***A1*** ***[3 marks]***

e. recognizing that for g to be concave up, $g'' > 0$

$g'' > 0$ when $f' > 0$, $4x - 12 > 0$, $x - 3 > 0$ $x > 3$

22M.1.SL.TZ2.6

a.

EITHER

recognises the required term (or coefficient) in the expansion ***(M1)***

$bx^5 = C_{27}x^5 1^2$ OR $b = C_{27}$ OR C_{57}

$b = \frac{7!}{2!5!}$ $= \frac{7!}{2!7-2!}$

correct working ***A1***

$\frac{7\times6\times5\times4\times3\times2\times1}{2\times1\times5\times4\times3\times2\times1}$ OR $\frac{7\times6}{2!}$ OR $\frac{42}{2}$

OR

lists terms from row 7 of Pascal's triangle ***(M1)***

$1, \quad 7, \quad 21, \ldots$ ***A1***

THEN

$b = 21$ ***AG***

b. $a = 7$ correct equation $21x^5 = \frac{ax^6 + 35x^4}{2}$ OR $21x^5 = \frac{7x^6 + 35x^4}{2}$

correct quadratic equation

$7x^2 - 42x + 35 = 0$ OR $x^2 - 6x + 5 = 0$ (or equivalent)

valid attempt to solve quadratic $x - 1x - 5 = 0$ OR $x = \frac{6 \pm \sqrt{-6^2 - 415}}{21}$

$x = 1, \quad x = 5$ Award final for obtaining $x = 0, \quad x = 1, \quad x = 5.$

22M.1.SL.TZ2.9

a.

uses $\sum \mathrm{P}X = x = 1$ to form a linear equation in p and q ***(M1)***

correct equation in terms of p and q from summing to 1 ***A1***

$p + 0.3 + q + 0.1 = 1$ OR $p + q = 0.6$ (or equivalent)

uses $\mathrm{E}X = 2$ to form a linear equation in p and q ***(M1)***

correct equation in terms of p and q from $\mathrm{E}X = 2$ ***A1***

$p + 0.6 + 3q + 0.4 = 2$ OR $p + 3q = 1$ (or equivalent)

Note: The marks for using $\sum \mathrm{P}X = x = 1$ and the marks for using $\mathrm{E}X = 2$ may be awarded independently of each other.

evidence of correctly solving these equations simultaneously ***A1***

for example, $2q = 0.4 \Rightarrow q = 0.2$ or $p + 3 \times 0.6 - p = 1 \Rightarrow p = 0.4$

so $p = 0.4$ and $q = 0.2$ ***AG***

[5 marks]

b. valid approach ***(M1)***

$\mathrm{P}(X>2)=\mathrm{P}(X=3)+\mathrm{P}(X=4)$ OR $\mathrm{P}(X>2)=1-\mathrm{P}(X=1)-\mathrm{P}(X=2)$

$=0.3$ ***A1*** ***[2 marks]***

c.

recognises at least one of the valid scores (6, 7, or 8) required to win the game ***(M1)***

Note: Award ***M0*** if candidate also considers scores other than 6, 7, or 8 (such as 5).

let T represent the score on the last two rolls

a score of 6 is obtained by rolling $2,4$, $4,2$ or $3,3$

$PT=6=20.30.1+0.2^2 \quad =0.1$ ***A1*** a score of 7 is obtained by rolling $3,4$ or $4,3$

$PT=7=20.20.1 \quad =0.04$ ***A1*** a score of 8 is obtained by rolling $4,4$

$PT=8=0.1^2 \quad =0.01$ ***A1***

Note: The above 3 ***A1*** marks are independent of each other.

$\mathrm{P}(\text{Nicky wins})=0.1+0.04+0.01 \quad =0.15$ ***A1*** ***[5 marks]***

d. $3+b=8$ ***(M1)*** $b=5$ ***A1*** ***[2 marks]***

e. $PS=5=\frac{4}{16}$ $PS=a+2=\frac{4}{16}$ $\Rightarrow a+2=5$

$PS=6=\frac{3}{16}$ $PS=a+3=\frac{2}{16}$ and $PS=5+1=\frac{1}{16}$ $\Rightarrow a+3=6$

$PS=4=\frac{3}{16}$ $PS=a+1=\frac{2}{16}$ and $PS=1+3=\frac{1}{16}$ $\Rightarrow a+1=4$

$\Rightarrow a=3$

Award for $a=3$ obtained without working/reasoning/justification.

correctly lists a relevant part of the sample space

for example, $S=4=3,1,1,a,1,a$ or $S=5=2,a,2,a,2,a,2,a$ or $S=6=3,a,3,a,1,5$

$a+3=6$ eliminates possibilities (exhaustion) for $a<5$

convincingly shows that $a\neq 2,4$

$a\neq 4$, for example, $PS=7=\frac{2}{16}$ from $2,5,2,5$ and so $3,a,3,a\Rightarrow a+3\neq 7$

$\Rightarrow a=3$

22M.1.SL.TZ2.8

a.

$\frac{1}{x-4}+1=x-3$ ***(M1)***

$x^2-8x+15=0$ OR $x-4^2=1$ ***(A1)***

valid attempt to solve **their** quadratic ***(M1)***

$x-3x-5=0$ OR $x=\frac{8\pm\sqrt{8^2-4115}}{21}$ OR $x-4=\pm 1$

$x=5$ $x=3,$ $x=5$ (may be seen in answer) ***A1***

$\mathrm{B}5,$ 2 (accept $x=5,$ $y=2$) ***A1***

[5 marks]

b.

recognizing two correct regions from $x=3$ to $x=5$ and from $x=5$ to $x=k$ ***(R1)***

triangle $+\int_5^k fx\,\mathrm{d}\,x$ OR $\int_3^5 gx\,\mathrm{d}\,x+\int_5^k fx\,\mathrm{d}\,x$ OR $\int_3^5 x-3\,\mathrm{d}\,x+\int_5^k \frac{1}{x-4}+1\,\mathrm{d}\,x$

area of triangle is 2 OR $\frac{2\cdot 2}{2}$ OR $\frac{5^2}{2}-35-\frac{3^2}{2}-33$ ***(A1)***

correct integration ***(A1)(A1)*** $\int\frac{1}{x-4}+1\,\mathrm{d}\,x=\ln x-4+x$ $+C$

Note: Award ***A1*** for $\ln x-4$ and ***A1*** for x.
Note: The first three ***A*** marks may be awarded independently of the ***R*** mark.

substitution of **their** limits (for x) into **their** integrated function (in terms of x) ***(M1)***

$\ln k-4+k-\ln\ 1+5$ $\ln x-4+x_5^k=\ln k-4+k-5$ ***A1***

adding **their** two areas (in terms of k) and equating to $\ln\ p+8$ ***(M1)***

$2+\ln k-4+k-5=\ln\ p+8$

equating **their** non-log terms to 8 (equation must be in terms of k) ***(M1)*** $k-3=8$

$k=11$ ***A1*** $11-4=p$ $p=7$ ***A1*** ***[10 marks]***

19M.1.SL.TZ1.S_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

recognizing area under curve = 1 ***(M1)*** *eg* $a+x+b=1,\ 100-a-b,\ 1-a+b$

$\text{P}(-1.6<z<2.4)=1-a-b(=1-(a+b))$ ***A1 N2***

[2 marks]

b. $\text{P}(z>-1.6)=1-a$ (seen anywhere) ***(A1)***

recognizing conditional probability ***(M1)*** *eg* $\text{P}(A|B),\ \text{P}(B|A)$

correct working ***(A1)*** *eg* $\frac{\text{P}(z<2.4\cap z>-1.6)}{\text{P}(z>-1.6)},\ \frac{\text{P}(-1.6<z<2.4)}{\text{P}(z>-1.6)}$

$\text{P}(z<2.4|z>-1.6)=\frac{1-a-b}{1-a}$ ***A1 N4***

Note: Do not award the final ***A1*** if correct answer is seen followed by incorrect simplification.

[4 marks]

c. $z=-1.6$ (may be seen in part (d)) ***A1 N1***

Note: Depending on the candidate's interpretation of the question, they may give $\frac{1-m}{s}$ as the answer to part (c). Such answers should be awarded the first ***(M1)*** in part (d), even when part (d) is left blank. If the candidate goes on to show $z=-1.6$ as part of their working in part (d), the ***A1*** in part (c) may be awarded.

[1 mark]

d. attempt to standardize x (do not accept $\frac{x-\mu}{\sigma}$) ***(M1)***

eg $\frac{1-m}{s}$ (may be seen in part (c)), $\frac{m-2}{s},\ \frac{x-m}{\sigma}$

correct equation with each z-value ***(A1)(A1)***

eg $-1.6=\frac{1-m}{s},\ 2.4=\frac{2-m}{s},\ m+2.4s=2$

valid approach (to set up equation in one variable) ***M1***

eg $2.4=\frac{2-(1.6s+1)}{s},\ \frac{1-m}{-1.6}=\frac{2-m}{2.4}$ correct working

eg $1.6s+1=2-2.4s,\ 4s=1,\ m=\frac{7}{5}\quad s=\frac{1}{4}$

20N.1.SL.TZ0.S_1

a.i.

valid approach to find t ***(M1)***

eg $t+3=19,\quad 19 - 3$

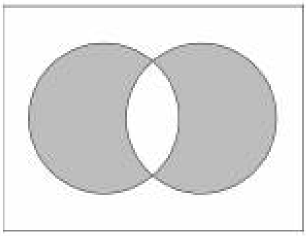
$t=16$ (may be seen on Venn diagram) ***A1 N2***

[2 marks]

a.ii. valid approach to find v ***(M1)*** *eg* $t+3+v+6=30$, $30-19-6$

$v=5$ (may be seen on Venn diagram) ***A1 N2*** ***[2 marks]***

b. valid approach ***(M1)*** *eg* $16+5$, 21 students, $1-\frac{3+6}{30}$, $\frac{21}{30}$ $=\frac{7}{10}$ ***A1 N2***



21N.1.SL.TZ0.5

a.

$f'(4)=6$ ***A1***

[1 mark]

b. $f(4)=6\times 4-1=23$ ***A1*** ***[1 mark]***

c. $h4=fg4$ ***(M1)*** $h4=f4^2-3\times 4=f4$ $h4=23$ ***A1*** ***[2 marks]***

d. attempt to use chain rule to find h' ***(M1)*** $f'gx\times g'x$ OR $x^2-3x'\times f'x^2-3x$

$h'4=2\times 4-3f'4^2-3\times 4$ ***A1*** $=30$

$y-23=30x-4$ OR $y=30x-97$ ***A1*** ***[3 marks]***

21N.1.SL.TZ0.7

a.i.

valid approach to find turning point ($v'=0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4-6t=0$ OR $-\frac{4}{2\text{-}3}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t=\frac{2}{3}$ (s) ***A1***

[2 marks]

a.ii. attempt to integrate v ***(M1)***

$\int v \quad dt=\int 4+4t-3t^2 \quad dt=4t+2t^2-t^3+c$ ***A1A1***

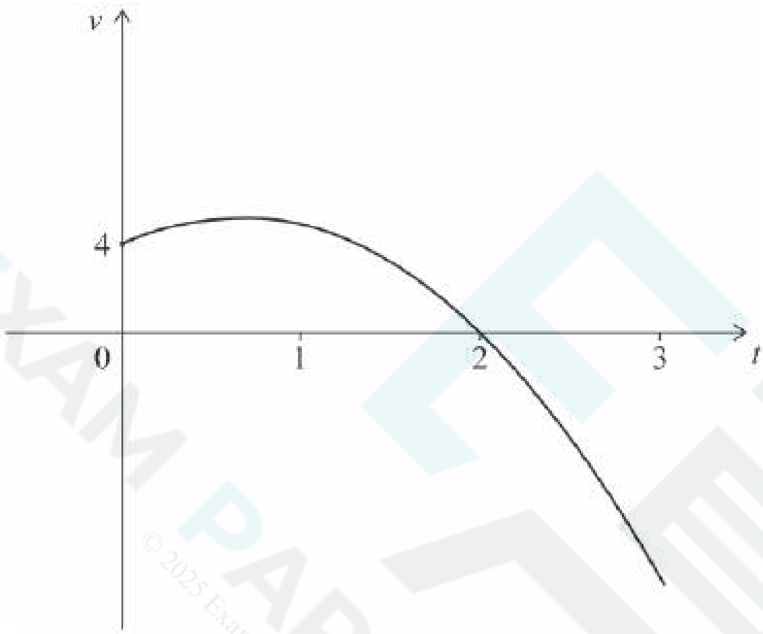
Note: Award ***A1*** for $4t + 2t^2$, ***A1*** for $-t^3$.

attempt to substitute their t into their solution for the integral ***(M1)***

distance $= 4\frac{2}{3} + 2\frac{2}{3}^2 - \frac{2}{3}^3 = \frac{8}{3} + \frac{8}{9} - \frac{8}{27}$ (or equivalent) ***A1***

$= \frac{88}{27}$ (m) ***AG*** ***[5 marks]***

b.



valid approach to solve $4 + 4t - 3t^2 = 0$ (may be seen in part (a)) ***(M1)***

$2 - t2 + 3t$ OR $\frac{-4 \pm \sqrt{16 + 48}}{-6}$ correct x- intercept on the graph at $t = 2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0, 4)$ ***A1***

Note: The 3 must be clearly indicated.

[4 marks]

vertex in approximately correct place for $t = \frac{2}{3}$ and $v > 4$ ***A1***

c.

recognising to integrate between 0 and 2, or 2 and 3 OR $\int_0^3 4 + 4t - 3t^2 \; \mathrm{d}t$

$\int_0^2 4 + 4t - 3t^2 \; \mathrm{d}t = 8$ $\int_2^3 4 + 4t - 3t^2 \; \mathrm{d}t = -5$

lid approach to sum the two areas (seen anywhere)

$\int_0^2 v \, dt - \int_2^3 v \, dt$ OR $\int_0^2 v \, dt + \int_2^3 v \, dt$

total distance travelled $= 13$ (m)

21N.1.SL.TZ0.8

a.

$f\frac{2}{3} = 4$ OR $a^{\frac{2}{3}} = 4$ ***(M1)***

$a = 4^{\frac{3}{2}}$ OR $a = 2^{2^{\frac{3}{2}}}$ OR $a^2 = 64$ OR $\sqrt[3]{a} = 2$ ***A1***

$a = 8$ ***AG***

[2 marks]

b. $f^{-1}x = \log_8 x$ ***A1***

[1 mark]

Note: Accept $f^{-1}x = \log_a x$.
Accept any equivalent expression for f^{-1} e.g. $f^{-1}x = \frac{\ln x}{\ln 8}$.

c. correct substitution ***(A1)*** $\log_8 \sqrt{32}$ OR $8^x = 32^{\frac{1}{2}}$

correct working involving log/index law ***(A1)***

$\frac{1}{2}\log_8 32$ OR $\frac{5}{2}\log_8 2$ OR $\log_8 2 = \frac{1}{3}$ OR $\log_2 2^{\frac{5}{2}}$ OR $\log_2 8 = 3$ OR $\frac{\ln 2^{\frac{5}{2}}}{\ln 2^3}$ OR $2^{3x} = 2^{\frac{5}{2}}$

$f^{-1}\sqrt{32} = \frac{5}{6}$ ***A1*** ***[3 marks]***

d.i. **METHOD 1** equating a pair of differences ***(M1)*** $u_2 - u_1 = u_4 - u_3 = u_3 - u_2$

$\log_8 p - \log_8 27 = \log_8 125 - \log_8 q$ $\log_8 125 - \log_8 q = \log_8 q - \log_8 p$

$\log_8 \frac{p}{27} = \log_8 \frac{125}{q}$, $\log_8 \frac{125}{q} = \log_8 \frac{q}{p}$ ***A1A1*** $\frac{p}{27} = \frac{125}{q}$ and $\frac{125}{q} = \frac{q}{p}$ ***A1***

$27, \ p, \ q$ and 125 are in geometric sequence ***AG***

Note: If candidate assumes the sequence is geometric, award no marks for part (i). If $r = \frac{5}{3}$ has been found, this will be awarded marks in part (ii).

METHOD 2 expressing a pair of consecutive terms, in terms of d ***(M1)***

$p = 8^d \times 27$ and $q = 8^{2d} \times 27$ OR $q = 8^{2d} \times 27$ and $125 = 8^{3d} \times 27$

two correct pairs of consecutive terms, in terms of d ***A1***

$\frac{8^d \times 27}{27} = \frac{8^{2d} \times 27}{8^d \times 27} = \frac{8^{3d} \times 27}{8^{2d} \times 27}$ (must include 3 ratios)

all simplify to 8^d $27, \quad p, \quad q$ and 125 are in geometric sequence

d.ii. $r \quad u_4 = u_1 r^3$ OR $125 = 27r^3$

$r = \frac{5}{3}$ (seen anywhere) $p = 27r$ OR $\frac{125}{q} = \frac{5}{3}$

$p = 45, \quad q = 75$

$u_4 = u_1 + 3d$ OR $\log_8 \ 125 = \log_8 \ 27 + 3d$

$d = \log_8 \frac{5}{3}$ (seen anywhere)

$\log_8 \ p = \log_8 \ 27 + \log_8 \frac{5}{3}$ OR $\log_8 \ q = \log_8 \ 27 + 2 \ \log_8 \frac{5}{3}$

$p = 45, \quad q = 75$

recognizing proportion $pq = 125 \times 27$ OR $q^2 = 125p$ OR $p^2 = 27q$

two correct proportion equations

attempt to eliminate either p or q

$q^2 = 125 \times \frac{125 \times 27}{q}$ OR $p^2 = 27 \times \frac{125 \times 27}{p}$ $p = 45, \quad q = 75$

19M.1.SL.TZ2.S_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of using $\sum p = 1$ ***(M1)***

correct working ***(A1)***

eg $\frac{3}{13} + \frac{1}{13} + \frac{4}{13} + k = 1, \ 1 - \frac{8}{13}$

$k = \frac{5}{13}$ ***A1 N2***

[3 marks]

b. valid approach to find $\mathrm{E}(X)$ ***(M1)***

eg $1 \times \frac{1}{13} + 2 \times \frac{4}{13} + 3 \times k, \ 0 \times \frac{3}{13} + 1 \times \frac{1}{13} + 2 \times \frac{4}{13} + 3 \times \frac{5}{13}$ correct working ***(A1)***

eg $\frac{1}{13} + \frac{8}{13} + \frac{15}{13}$ $\mathrm{E}(X) = \frac{24}{13}$ ***A1 N2*** ***[3 marks]***

18M.1.SL.TZ1.S_7

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

recognizing the need to find h' ***(M1)***

recognizing the need to find $h'(3)$ (seen anywhere) ***(M1)***

evidence of choosing chain rule ***(M1)***

eg $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}, f'(g(3)) \times g'(3), f'(g) \times g'$

correct working ***(A1)***

eg $f'(7) \times 4, -5 \times 4$

$h'(3) = -20$ ***(A1)***

evidence of taking **their** negative reciprocal for normal ***(M1)***

eg $-\frac{1}{h'(3)}, m_1 m_2 = -1$

gradient of normal is $\frac{1}{20}$ ***A1 N4***

[7 marks]

17M.1.SL.TZ2.S_3

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

$P(X > 107) = 0.24\left(= \frac{6}{25}, 24\%\right)$ ***A1 N1***

[1 mark]

b. valid approach ***(M1)*** *eg* $P(X > 100) = 0.5,\ P(X > 100) - P(X > 107)$

correct working ***(A1)*** *eg* $0.5 - 0.24,\ 0.76 - 0.5$

$P(100 < X < 107) = 0.26\left(= \frac{13}{50}, 26\%\right)$ ***A1 N2*** ***[3 marks]***

c. valid approach ***(M1)*** *eg* $2 \times 0.26,\ 1 - 2(0.24),\ P(93 < X < 100) = P(100 < X < 107)$

$P(93 < X < 107) = 0.52\left(= \frac{13}{25}, 52\%\right)$ ***A1 N2*** ***[2 marks]***

18N.1.SL.TZ0.S_10

b.i.

$f^{'} = 3x^2 - 4x + a$ ***A2 N2***

[2 marks]

b.ii. valid approach ***(M1)*** *eg* $f^{'}(0)$ correct working ***(A1)***

eg $3(0)^2 - 4(0) + a$, slope = a, $f^{'}(0) = a$

attempt to substitute gradient and coordinates into linear equation ***(M1)***

eg $y - 6 = a(x - 0)$, $y - 0 = a(x - 6)$, $6 = a(0) + c$, $L = ax + 6$

correct equation ***A1 N3*** *eg* $y = ax + 6$, $y - 6 = ax$, $y - 6 = a(x - 0)$ ***[4 marks]***

c. valid approach to find intersection ***(M1)*** *eg* $f(x) = L$ correct equation ***(A1)***

eg $x^3 - 2x^2 + ax + 6 = ax + 6$ correct working ***(A1)***

eg $x^3 - 2x^2 = 0$, $x^2(x - 2) = 0$ $x = 2$ at Q ***(A1)***

valid approach to find minimum ***(M1)*** *eg* $f^{'}(x) = 0$ correct equation ***(A1)***

eg $3x^2 - 4x + a = 0$

substitution of **their** value of x at Q into **their** $f^{'}(x) = 0$ equation ***(M1)***

eg $3(2)^2 - 4(2) + a = 0$, $12 - 8 + a = 0$ $a = -4$ ***A1 N0*** ***[8 marks]***

18M.1.SL.TZ2.S_9

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

correct equation for volume ***(A1)***
eg $\pi r^2 h = 20\pi$

$h = \frac{20}{r^2}$ ***A1 N2***

[2 marks]

b. attempt to find formula for cost of parts ***(M1)***
eg 10 × two circles, 8 × curved side

correct expression for cost of two circles in terms of *r* (seen anywhere) ***A1***
eg $2\pi r^2 \times 10$

correct expression for cost of curved side (seen anywhere) ***(A1)***
eg $2\pi r \times h \times 8$

correct expression for cost of curved side in terms of r $\quad C = 20\pi r^2 + \frac{320\pi}{r}$
eg $8 \times 2\pi r \times \frac{20}{r^2}, \frac{320\pi}{r^2}$

c. recognize $C' = 0$ at minimum
eg $C' = 0, \frac{\mathrm{d}C}{\mathrm{d}r} = 0$

correct differentiation (may be seen in equation) $\quad C' = 40\pi r - \frac{320\pi}{r^2}$

correct equation
eg $40\pi r - \frac{320\pi}{r^2} = 0, 40\pi r\frac{320\pi}{r^2}$

correct working
eg $40r^3 = 320, r^3 = 8$

r = 2 (m)

attempt to substitute value of r into C
eg $20\pi \times 4 + 320 \times \frac{\pi}{2}$

correct working
eg $80\pi + 160\pi$

240π (cents) $\quad$ Do not accept 753.6, 753.98 or 754, even if 240π is seen.

21N.1.SL.TZ0.9

a.

Special note: In this question if candidates use the word 'gradient' in their reasoning. e.g. gradient is positive, it must be clear whether this is the gradient of f or the gradient of f' to earn the ***R*** mark.

f increases when $p < x < 0$ $\quad$ ***A1***

f increases when $f'x > 0$ OR f' is above the x-axis $\quad$ ***R1***

Note: Do not award ***A0R1***.

[2 marks]

b.

Special note: In this question if candidates use the word 'gradient' in their reasoning. e.g. gradient is positive, it must be clear whether this is the gradient of f or the gradient of f' to earn the ***R*** mark.

$x = 0$ $\quad$ ***A1*** $\quad$ ***[1 mark]***

c.i.

Special note: In this question if candidates use the word 'gradient' in their reasoning. e.g. gradient is positive, it must be clear whether this is the gradient of f or the gradient of f' to earn the ***R*** mark.

f is minimum when $x = p$ ***A1***

because $f'p = 0$, $f'x < 0$ when $x < p$ and $f'x > 0$ when $x > p$

(may be seen in a sign diagram clearly labelled as f')

OR because f' changes from negative to positive at $x = p$

OR $f'p = 0$ and slope of f' is positive at $x = p$ ***R1*** **Note:** Do not award ***A0 R1***

[2 marks]

c.ii.

Special note: In this question if candidates use the word 'gradient' in their reasoning. e.g. gradient is positive, it must be clear whether this is the gradient of f or the gradient of f' to earn the ***R*** mark.

f has points of inflexion when $x = q$, $x = r$ and $x = t$ ***A2***

OR

f' has turning points at $x = q$, $x = r$ and $x = t$

$f''q = 0$, $f''r = 0$ and $f''t = 0$ and f' changes from increasing to decreasing or vice versa at each of these x-values (may be seen in a sign diagram clearly labelled as f'' and f')

Award if any incorrect answers are given. Do not award

d.

In this question if candidates use the word 'gradient' in their reasoning. e.g. gradient is positive, it must be clear whether this is the gradient of f or the gradient of f' to earn the mark.

recognizing area from p to t (seen anywhere) $\int_p^t f'x \, \mathrm{d}x$

recognizing to negate integral for area below x-axis

$\int_p^0 f'x \, \mathrm{d}x - \int_0^t f'x \, \mathrm{d}x$ OR $\int_p^0 f'x \, \mathrm{d}x + \int_t^0 f'x \, \mathrm{d}x$

$\int_m^n f'x \, \mathrm{d}x = fn - fm$ (for any integral)

$f0 - fp - ft - f0$ OR $f0 - fp + f0 - ft$

$2f0 - ft + fp = 20$, $2f0 - 4 = 20$ $f0 = 12$

16N.1.SL.TZ0.S_10

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in arking or structure.*

(i) $f^{'}(x) = -\sin x,\ f^{''}(x) = -\cos x,\ f^{(3)}(x) = \sin x,\ f^{(4)}(x) = \cos x$ ***A2*** ***N2***

(ii) valid approach ***(M1)***

*eg*recognizing that 19 is one less than a multiple of 4, $f^{(19)}(x) = f^{(3)}(x)$

$f^{(19)}(x) = \sin x$ ***A1*** ***N2***

[4 marks]

b. (i) $g^{'}(x) = kx^{k-1}$

$g^{''}(x) = k(k-1)x^{k-2},\ g^{(3)}(x) = k(k-1)(k-2)x^{k-3}$ ***A1A1*** ***N2*** (ii) **METHOD 1**

correct working that leads to the correct answer, involving the correct expression for the 19th derivative ***A2***

eg$k(k-1)(k-2)\ \ldots\ (k-18) \times \frac{(k-19)!}{(k-19)!},\ {}_kP_{19}$ $p = 19$ (accept $\frac{k!}{(k-19)!}x^{k-19}$) ***A1*** ***N1***

METHOD 2

correct working involving recognizing patterns in coefficients of first three derivatives (may be seen in part (b)(i)) leading to a general rule for 19th coefficient ***A2***

eg$g^{''} = 2!\begin{pmatrix} k \\ 2 \end{pmatrix},\ k(k-1)(k-2) = \frac{k!}{(k-3)!},\ g^{(3)}(x) = {}_k P_3(x^{k-3})$

$g^{(19)}(x) = 19!\begin{pmatrix} k \\ 19 \end{pmatrix},\ 19! \times \frac{k!}{(k-19)! \times 19!},\ {}_kP_{19}$ $p = 19$ (accept $\frac{k!}{(k-19)!}x^{k-19}$) ***A1*** ***N1***

[5 marks]

c. (i) valid approach using product rule ***(M1)*** *eg*$uv^{'} + vu^{'},\ f^{(19)}g^{(20)} + f^{(20)}g^{(19)}$

correct 20th derivatives (must be seen in product rule) ***(A1)(A1)***

eg$g^{(20)}(x) = \frac{21!}{(21-20)!}x,\ f^{(20)}(x) = \cos x$

$h^{'}(x) = \sin x(21!x) + \cos x\left(\frac{21!}{2}x^2\right)$ $\left(\text{accept } \sin x\left(\frac{21!}{1!}x\right) + \cos x\left(\frac{21!}{2!}x^2\right)\right)$ ***A1*** ***N3***

(ii) substituting $x = \pi$ (seen anywhere) ***(A1)***

eg$f^{(19)}(\pi)g^{(20)}(\pi) + f^{(20)}(\pi)g^{(19)}(\pi),\ \sin\pi\frac{21!}{1!}\pi + \cos\pi\frac{21!}{2!}\pi^2$

evidence of one correct value for $\sin\pi$ or $\cos\pi$ (seen anywhere) ***(A1)***

eg$\sin\pi = 0,\ \cos\pi = -1$ evidence of correct values substituted into $h^{'}(\pi)$ ***A1***

eg$21!(\pi)\left(0 - \frac{\pi}{2!}\right),\ 21!(\pi)\left(-\frac{\pi}{2}\right),\ 0 + (-1)\frac{21!}{2}\pi^2$

Note: If candidates write only the first line followed by the answer, award ***A1A0A0***.

$\frac{-21!}{2}\pi^2$

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1 – using discriminant

correct equation without logs ***(A1)***

eg $6x-3x^2=k^2$

valid approach ***(M1)***

eg $-3x^2+6x-k^2=0,\ 3x^2-6x+k^2=0$

recognizing discriminant must be zero (seen anywhere) ***M1***

eg $\Delta=0$

correct discriminant ***(A1)***

eg $6^2-4(-3)(-k^2),\ 36-12k^2=0$

correct working ***(A1)***

eg $12k^2=36,\ k^2=3$

$k=\sqrt{3}$ ***A2*** ***N2***

METHOD 2 – completing the square

correct equation without logs ***(A1)***

eg $6x-3x^2=k^2$

valid approach to complete the square ***(M1)***

eg $3(x^2-2x+1)=-k^2+3,\ x^2-2x+1-1+\frac{k^2}{3}=0$

correct working ***(A1)***

eg $3(x-1)^2=-k^2+3,\ (x-1)^2-1+\frac{k^2}{3}=0$

recognizing conditions for one solution ***M1***

eg $(x-1)^2=0,\ -1+\frac{k^2}{3}=0$

correct working ***(A1)***

eg $\frac{k^2}{3}=1,\ k^2=3$

$k=\sqrt{3}$ ***A2*** ***N2***

[7 marks]

17M.1.SL.TZ2.S_5

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach ***(M1)***

eg $\int f' \mathrm{d}x,\ \int \frac{3x^2}{(x^3+1)^5}\mathrm{d}x$

correct integration by substitution/inspection ***A2***

eg $f(x) = -\frac{1}{4}(x^3+1)^{-4} + c,\ \frac{-1}{4(x^3+1)^4}$

correct substitution into **their** integrated function (must include c) ***M1***

eg $1 = \frac{-1}{4(0^3+1)^4} + c,\ -\frac{1}{4} + c = 1$

Note: Award ***M0*** if candidates substitute into f' or f''.

$c = \frac{5}{4}$ ***(A1)***

$f(x) = -\frac{1}{4}(x^3+1)^{-4} + \frac{5}{4}\left(= \frac{-1}{4(x^3+1)^4} + \frac{5}{4},\ \frac{5(x^3+1)^4 - 1}{4(x^3+1)^4}\right)$ ***A1 N4***

[6 marks]

18N.1.SL.TZ0.S_6

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

METHOD 1 (limits in terms of x)

valid approach to find x-intercept ***(M1)***

eg $f(x) = 0,\ \frac{6-2x}{\sqrt{16+6x-x^2}} = 0,\ 6-2x = 0$

x-intercept is 3 ***(A1)***

valid approach using substitution or inspection ***(M1)***

eg $u = 16+6x-x^2,\ \int_0^3 \frac{6-2x}{\sqrt{u}}\mathrm{d}x,\ \mathrm{d}u = 6-2x,\ \int \frac{1}{\sqrt{u}},\ 2u^{\frac{1}{2}},$

$u = \sqrt{16+6x-x^2},\ \frac{\mathrm{d}u}{\mathrm{d}x} = (6-2x)\frac{1}{2}(16+6x-x^2)^{-\frac{1}{2}},\ \int 2\mathrm{d}u,\ 2u$

$\int f(x)\mathrm{d}x = 2\sqrt{16+6x-x^2}$ ***(A2)***

substituting **both** of **their** limits into **their** integrated function and subtracting ***(M1)*** *eg* $2\sqrt{16+6(3)-3^2}-2\sqrt{16+6(0)^2-0^2}$, $2\sqrt{16+18-9}-2\sqrt{16}$

Note: Award ***M0*** if they substitute into original or differentiated function. Do not accept only "– 0" as evidence of substituting lower limit.

correct working ***(A1)***

eg $2\sqrt{25}-2\sqrt{16}$, $10-8$

area = 2 ***A1 N2***

METHOD 2 (limits in terms of u)

valid approach to find x-intercept ***(M1)***

eg $f(x)=0$, $\frac{6-2x}{\sqrt{16+6x-x^2}}=0$, $6-2x=0$

x-intercept is 3 ***(A1)***

valid approach using substitution or inspection ***(M1)***

eg $u=16+6x-x^2$, $\int_0^3 \frac{6-2x}{\sqrt{u}}\mathrm{d}x$, $\mathrm{d}u=6-2x$, $\int \frac{1}{\sqrt{u}}$,

$u=\sqrt{16+6x-x^2}$, $\frac{\mathrm{d}u}{\mathrm{d}x}=(6-2x)\frac{1}{2}(16+6x-x^2)^{-\frac{1}{2}}$, $\int 2\mathrm{d}u$

correct integration ***(A2)***

eg $\int \frac{1}{\sqrt{u}}\mathrm{d}u=2u^{\frac{1}{2}}$, $\int 2\mathrm{d}u=2u$

both correct limits for u ***(A1)***

eg u = 16 and u = 25, $\int_{16}^{25}\frac{1}{\sqrt{u}}\mathrm{d}u$, $\left[2u^{\frac{1}{2}}\right]_{16}^{25}$, u = 4 and u = 5, $\int_4^5 2\mathrm{d}u$, $[2u]_4^5$

substituting of limits for u (do not accept 0 and 3) into integrated function and subtracting

eg $2\sqrt{25}-2\sqrt{16}$, $10-8$

Award if they substitute into original or differentiated function, or if they have not attempted to find limits for u.

area = 2

16N.1.SL.TZ0.S_6

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

evidence of integration ***(M1)***

eg $\int f'(x)\mathrm{d}x$

correct integration (accept missing C) ***(A2)***

eg $\frac{1}{2}\times\frac{\sin^4(2x)}{4},\ \frac{1}{8}\sin^4(2x)+C$

substituting initial condition into their integrated expression (must have $+C$) ***M1***

eg $1=\frac{1}{8}\sin^4\left(\frac{\pi}{2}\right)+C$

Note: Award ***M0*** if they substitute into the original or differentiated function.

recognizing $\sin\left(\frac{\pi}{2}\right)=1$ ***(A1)***

eg $1=\frac{1}{8}(1)^4+C$

$C=\frac{7}{8}$ ***(A1)***

$f(x)=\frac{1}{8}\sin^4(2x)+\frac{7}{8}$ ***A1 N5***

[7 marks]

19M.1.SL.TZ1.S_5

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

recognizing to integrate ***(M1)***

eg $\int f',\ \int 2\mathrm{e}^{-3x}\mathrm{d}x,\ \mathrm{d}u=-3$

correct integral (do not penalize for missing $+C$) ***(A2)***

eg $-\frac{2}{3}\mathrm{e}^{-3x}+C$

substituting $\left(\frac{1}{3},5\right)$ (in any order) into **their** integrated expression (must have $+C$) ***M1***

eg $-\frac{2}{3}\mathrm{e}^{-3(1/3)}+C=5$

Award if they substitute into original or differentiated function.

$f(x) = -\frac{2}{3}e^{-3x} + 5 + \frac{2}{3}e^{-1}$ (or *any* equivalent form, *eg* $-\frac{2}{3}e^{-3x} + 5 - \frac{2}{-3e}$)

22M.1.SL.TZ1.7

a.

EITHER

attempt to use $x = -\frac{b}{2a}$ ***(M1)***

$q = -\frac{-12}{2 \times 3}$

OR

attempt to complete the square ***(M1)***

$3x - 2^2 - 12 + p$

OR

attempt to differentiate and equate to 0 ***(M1)***

$f''x = 6x - 12 = 0$

THEN

$q = 2$ ***A1***

[2 marks]

b.i. discriminant $= 0$ ***A1*** ***[1 mark]***

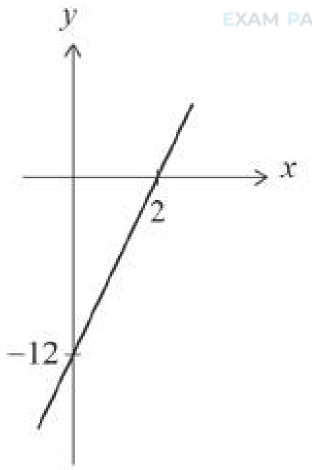
b.ii. **EITHER** attempt to substitute into $b^2 - 4ac$ ***(M1)*** $-12^2 - 4 \times 3 \times p = 0$ ***A1***

OR $f'(2) = 0$ ***(M1)*** $-12 + p = 0$ ***A1***

THEN $p = 12$ ***A1*** ***[3 marks]***

c. $f''x = 6x - 12$ ***A1*** attempt to find $f''0$ ***(M1)*** $= 6 \times 0 - 12$

gradient $= -12$ ***A1*** ***[3 marks]*** d.

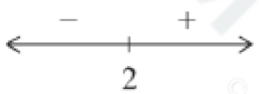


Award   for line with positive gradient,   for correct intercepts.

e.i. $a = 2$

e.ii. $x < 2$ $f''\ x < 0$ (for $x < 2$) OR the f'' is below the x-axis (for $x < 2$)

OR



f'' (sign diagram must be labelled f'')

22M.1.SL.TZ1.8

a.i.

EITHER

attempt to use a ratio from consecutive terms ***M1***

$\frac{p\ \ln\ x}{\ln\ x} = \frac{\frac{1}{3}\ln\ x}{p\ \ln\ x}$ OR $\frac{1}{3}\ln\ x = \ln\ x r^2$ OR $p\ \ln\ x = \ln\ x \frac{1}{3p}$

Note: Candidates may use $\ln\ x^1 + \ln\ x^p + \ln\ x^{\frac{1}{3}}$ ... and consider the powers of x in geometric sequence

Award ***M1*** for $\frac{p}{1} = \frac{\frac{1}{3}}{p}$.

OR

$r = p$ and $r^2 = \frac{1}{3}$ ***M1***

THEN

$p^2 = \frac{1}{3}$ OR $r = \pm\frac{1}{\sqrt{3}}$ ***A1***

$p = \pm\frac{1}{\sqrt{3}}$ ***AG***

Note: Award ***M0A0*** for $r^2 = \frac{1}{3}$ or $p^2 = \frac{1}{3}$ with no other working seen.

[2 marks]

a.ii. $\frac{\ln x}{1 - \frac{1}{\sqrt{3}}} = 3 + \sqrt{3}$ ***(A1)***

$\ln x = 3 - \frac{3}{\sqrt{3}} + \sqrt{3} - \frac{\sqrt{3}}{\sqrt{3}}$ OR $\ln x = 3 - \sqrt{3} + \sqrt{3} - 1$ $\Rightarrow \ln x = 2$ ***A1*** $x = e^2$ ***A1***

[3 marks]

b.i. **METHOD 1** attempt to find a difference from consecutive terms or from u_2 ***M1***

correct equation ***A1***

$p \ln x - \ln x = \frac{1}{3}\ln x - p \ln x$ OR $\frac{1}{3}\ln x = \ln x + 2p \ln x - \ln x$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \ldots$ and consider the powers of x in arithmetic sequence.

Award ***M1A1*** for $p - 1 = \frac{1}{3} - p$ $2p \ln x = \frac{4}{3}\ln x$ $\Rightarrow 2p = \frac{4}{3}$ ***A1*** $p = \frac{2}{3}$ ***AG***

METHOD 2 attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ ***M1***

$p \ln x = \frac{\ln x + \frac{1}{3}\ln x}{2}$ ***A1*** $2p \ln x = \frac{4}{3}\ln x$ $\Rightarrow 2p = \frac{4}{3}$ ***A1*** $p = \frac{2}{3}$ ***AG***

METHOD 3 attempt to find difference using u_3 ***M1***

$\frac{1}{3}\ln x = \ln x + 2d$ $\Rightarrow d = -\frac{1}{3}\ln x$

$u_2 = \ln x + \frac{1}{2}\frac{1}{3}\ln x - \ln x$ OR $p \ln x - \ln x = -\frac{1}{3}\ln x$ ***A1*** $p \ln x = \frac{2}{3}\ln x$ ***A1***

$p = \frac{2}{3}$ ***AG*** ***[3 marks]***

b.ii. $d = -\frac{1}{3}\ln x$ ***A1*** ***[1 mark]***

b.iii. **METHOD 1** $S_n = \frac{n}{2}2 \ln x + n - 1 \times -\frac{1}{3}\ln x$

attempt to substitute into S_n and equate to $-3 \ln x$ ***(M1)***

$\frac{n}{2}2 \ln x + n - 1 \times -\frac{1}{3}\ln x = -3 \ln x$ correct working with S_n (seen anywhere) ***(A1)***

$\frac{n}{2}2 \ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x$ OR $n \ln x - \frac{nn - 1}{6}\ln x$ OR $\frac{n}{2}\ln x + \frac{4 - n}{3}\ln x$

correct equation without $\ln x$ $\frac{n}{2}\frac{7}{3} - \frac{n}{3} = -3$ OR $n - \frac{nn-1}{6} = -3$ or equivalent

Award as above if the series $1 + p + \frac{1}{3} + \ldots$ is considered leading to $\frac{n}{2}\frac{7}{3} - \frac{n}{3} = -3$.

$n^2 - 7n - 18 = 0$

attempt to form a quadratic $= 0$

attempt to solve their quadratic $n - 9n + 2 = 0$ $n = 9$

listing the first 7 terms of the sequence

$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + \ldots$

recognizing first 7 terms sum to 0 8^{th} term is $-\frac{4}{3}\ln x$

9^{th} term is $-\frac{5}{3}\ln x$ sum of 8^{th} and 9^{th} term $= -3 \ln x$

$n = 9$

22M.1.SL.TZ1.9

a.i.

EITHER

attempt to use binomial expansion *(M1)*

$1 + C_{13} \times 1 \times -a + C_{23} \times 1 \times -a^2 + 1 \times -a^3$

OR

$1 - a1 - a1 - a$

$= 1 - a1 - 2a + a^2$ *(M1)*

THEN

$= 1 - 3a + 3a^2 - a^3$ *A1*

[2 marks]

a.ii. $a = \cos\ 2x$ *(A1)* So, $1 - 3\ \cos\ 2x + 3\ \cos^2\ 2x - \ \cos^3\ 2x =$

$1 - \cos\ 2x^3$ *A1*

attempt to substitute any double angle rule for $\cos\ 2x$ into $1 - \cos\ 2x^3$ *(M1)*

$= 2\ \sin^2\ x^3$ *A1* $= 8\ \sin^6\ x$ *AG*

Note: Allow working RHS to LHS.

[4 marks]

b.i. recognizing to integrate $\int 4\ \cos\ x \times 8\ \sin^6\ x\ \mathrm{d}\ x$ *(M1)*

EITHER

applies integration by inspection *(M1)* $32 \int \cos\ x \times \sin\ x^6\ \mathrm{d}\ x$

$= \frac{32}{7}\sin^7\ x\ +c$ *A1* $\frac{32}{7}\sin^7\ x{}_0^m$ $= \frac{32}{7}\sin^7\ m - \frac{32}{7}\sin^7\ 0$ *A1*

OR

$u = \sin\ x \Rightarrow \frac{\mathrm{d}\ u}{\mathrm{d}\ x} = \cos\ x$ *(M1)* $\int 32\ \cos\ x\ \sin^6\ x\ \mathrm{d}\ x = \int 32u^6\ \ \mathrm{d}\ u$

$= \frac{32}{7}u^7\ +c$ *A1* $\frac{32}{7}\sin^7\ x{}_0^m$ OR $\frac{32}{7}u^7{}_0^{\sin\ m}$ $= \frac{32}{7}\sin^7\ m - \frac{32}{7}\sin^7\ 0$ *A1*

THEN

$= \frac{32}{7}\sin^7\ m$ *AG* ***[4 marks]***

b.ii. **EITHER** $\int_m^{\frac{\pi}{2}} fx\ \mathrm{d}\ x = \frac{32}{7}\sin^7\ x{}_m^{\frac{\pi}{2}} = \frac{32}{7}\sin^7\ \frac{\pi}{2} - \frac{32}{7}\sin^7\ m$ *M1*

$\frac{32}{7}\sin^7\ \frac{\pi}{2} - \frac{32}{7}\sin^7\ m = \frac{127}{28}$ OR $\frac{32}{7}1 - \sin^7\ m = \frac{127}{28}$ *(M1)*

OR

$\int^{\frac{\pi}{2}} fx\ \mathrm{d}\ x = \int_0^m fx\ \mathrm{d}\ x + \int_m^{\frac{\pi}{2}} fx\ \mathrm{d}\ x$ *M1* $\frac{32}{7} = \frac{32}{7}\sin^7\ m + \frac{127}{28}$ *(M1)*

$\sin^7 m = \frac{1}{128} \quad = \frac{1}{2^7} \quad \sin m = \frac{1}{2}$

$m = \frac{\pi}{6}$

22M.1.SL.TZ1.1

a.

$m_{BC} = \frac{12 - 6}{-14 - 4} \quad = -\frac{1}{3}$ ***(A1)***

finding $m_L = \frac{-1}{m_{BC}}$ using their m_{BC} ***(M1)***

$m_L = 3$

$y - 20 = 3x + 2, \quad y = 3x + 26$ ***A1***

Note: Do not accept $L = 3x + 26$

[3 marks]

b. substituting $(k, \ 2)$ into their L ***(M1)*** $2 - 20 = 3k + 2$ OR $2 = 3k + 26$

$k = -8$ ***A1*** ***[2 marks]***

22M.1.SL.TZ1.3

a.

$\text{IQR} = 10 - 6 = 4$ ***(A1)***

attempt to find $Q_3 + 1.5 \times \text{IQR}$ ***(M1)***

$10 + 6$

16 ***A1***

[3 marks]

b.i. choosing $c = \frac{1}{2}a - 9$ ***(M1)*** $\frac{1}{2} \times 42 - 9 \quad = 12$ (years old) ***A1***

[2 marks]

b.ii. attempt to solve system by substitution or elimination ***(M1)***

34 (years old) ***A1*** ***[2 marks]***

22M.1.SL.TZ1.4

a.

$(f \circ g)(x) = f2x$ ***(A1)***

$f2x = \sqrt{3}\sin\ 2x + \cos\ 2x$ ***A1***

[2 marks]

b. $\sqrt{3}\sin\ 2x + \cos\ 2x = 2\ \cos\ 2x$ $\sqrt{3}\sin\ 2x = \cos\ 2x$

recognising to use tan or cot ***M1***

$\tan\ 2x = \frac{1}{\sqrt{3}}$ OR $\cot\ 2x = \sqrt{3}$ (values may be seen in right triangle) ***(A1)***

$\arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$ (seen anywhere) (accept degrees) ***(A1)*** $2x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$ ***A1A1***

Note: Do not award the final ***A1*** if any additional solutions are seen. ***[5 marks]***
Award ***A1A0*** for correct answers in degrees.
Award ***A0A0*** for correct answers in degrees with additional values.

22M.1.SL.TZ1.5

evidence of using product rule ***(M1)***

$\frac{dy}{dx} = 2x - 1 \times ke^{kx} + 2 \times e^{kx}$ $= e^{kx}2kx - k + 2$ ***A1***

correct working for one of (seen anywhere) ***A1***

$\frac{dy}{dx}$ at $x = 1 \Rightarrow ke^{k} + 2e^{k}$

OR

slope of tangent is $5e^{k}$

their $\frac{dy}{dx}$ at $x = 1$ equals the *slope* of $y = 5e^{k}x$ $= 5e^{k}$ (seen anywhere) ***(M1)***

$ke^{k} + 2e^{k} = 5e^{k}$

$k = 3$ ***A1***

[5 marks]

22M.1.SL.TZ1.6

a.

translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction ***A1***

translation (shift) by q upwards/positive vertical direction ***A1***

Note: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

b. **METHOD 1** minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) ***(M1)***

$-4 + 2.5 + q \geq 7$ $q \geq 8.5$ (accept $q = 8.5$) ***A1***

substituting $x = 0$ and their $q = 8.5$ to find r ***(M1)*** $r = 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + 8.5$

$4 + 2.5 + 8.5$ ***(A1)*** smallest value of r is 15 ***A1*** **METHOD 2**

substituting $x = 0$ to find an expression (for r) in terms of q ***(M1)***

$g(0) = r = 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + q$ $r = 6.5 + q$ ***A1***

minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is -4 ***(M1)*** $-4 + 2.5 + q \geq 7$

$-4 + 2.5 + r - 6.5 \geq 7$ (accept $=$) ***(A1)*** smallest value of r is 15 ***A1***

METHOD 3 $4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q = 4\cos(x) + 2.5 + q$ ***A1***

y-intercept of $4\cos(x) + 2.5 + q$ is a maximum ***(M1)*** amplitude of $g(x)$ is 4 ***(A1)***

attempt to find least maximum ***(M1)*** $r = 2 \times 4 + 7$ smallest value of r is 15 ***A1***

[5 marks]

22M.1.SL.TZ2.1

a.

$g(0) = -2$ ***A1***

[1 mark]

b. evidence of using composite function ***(M1)*** $fg0$ OR $f\text{-}2$

$(f \circ g)(0) = 8$ ***A1*** ***[2 marks]***

c. $x = 3$ ***A2*** ***[2 marks]***

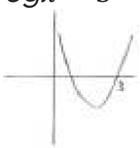
17M.1.SL.TZ1.S_9

a.

METHOD 1 (using *x*-intercept)

determining that 3 is an x-intercept ***(M1)***

eg $x - 3 = 0$,



valid approach ***(M1)***

eg $3 - 2.5,\ \frac{p+3}{2} = 2.5$

$p = 2$ ***A1*** ***N2***

METHOD 2 (expanding *f*(*x*))

correct expansion (accept absence of a) ***(A1)***

eg $ax^2 - a(3+p)x + 3ap,\ x^2 - (3+p)x + 3p$

valid approach involving equation of axis of symmetry ***(M1)***

eg $\frac{-b}{2a} = 2.5,\ \frac{a(3+p)}{2a} = \frac{5}{2},\ \frac{3+p}{2} = \frac{5}{2}$

$p = 2$ ***A1*** ***N2***

METHOD 3 (using derivative)

correct derivative (accept absence of a) ***(A1)***

eg $a(2x - 3 - p),\ 2x - 3 - p$

valid approach ***(M1)***

eg $f'(2.5) = 0$

$p = 2$ ***A1*** ***N2***

[3 marks]

b. attempt to substitute $(0, -6)$

eg $-6 = a(0-2)(0-3),\ a(0)^2 - 5a(0) + 6a = -6$ correct working

eg $-6 = 6a$ $a = -1$

c. recognizing tangent intersects curve once

recognizing one solution when discriminant = 0

attempt to set up equation *eg* $g = f,\ kx - 5 = -x^2 + 5x - 6$

rearranging their equation to equal zero *eg* $x^2 - 5x + kx + 1 = 0$

correct discriminant (if seen explicitly, not just in quadratic formula)

eg $(k-5)^2 - 4,\ 25 - 10k + k^2 - 4$ correct working

eg $k - 5 = \pm 2,\ (k-3)(k-7) = 0,\ \frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$ $k = 3,\ 7$

attempt to set up equation

eg $g = f,\ kx - 5 = -x^2 + 5x - 6$ recognizing derivative/slope are equal

eg $f' = m_T,\ f' = k$ correct derivative of f *eg* $-2x + 5$

attempt to set up equation in terms of either x or k

eg $(-2x + 5)x - 5 = -x^2 + 5x - 6,\ k\left(\frac{5-k}{2}\right) - 5 = -\left(\frac{5-k}{2}\right)^2 + 5\left(\frac{5-k}{2}\right) - 6$

rearranging their equation to equal zero *eg* $x^2 - 1 = 0,\ k^2 - 10k + 21 = 0$

correct working *eg* $x = \pm 1,\ (k-3)(k-7) = 0,\ \frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3,\ 7$