EXAM PAPERS PRACTICE

Helping you Achieve Highest Grades in IB

IB Mathematics (Analysis and Approaches) Standard Level (SL)

Question Paper

Fully in-lined with the First Assessment Examinations in 2021 & Beyond

Paper: 1 (All Topics)

- **Topic 1 - Number and Algebra**
- **Topic 2 - Functions**
- **Topic 3 - Geometry and Trigonometry**
- **Topic 4 - Statistics and Probability**
- **Topic 5 - Calculus**

Marks: 1566

Total Marks: / 1566

Suitable for SL Students sitting the 2025 exams onwards
However, HL students may also find these resources useful

Questions

SPM.1.SL.TZ0.8

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$.

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

a. Find $f'(x)$. [2]

b. Find the value of a and the value of b. [3]

c.i. Sketch the graph of $y = f'(x)$. [1]

c.ii. Hence explain why the graph of f has a local maximum point at $x = a$. [1]

d.i. Find $f''(b)$. [3]

d.ii.

Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$.

[1]

e.

The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at the point (p, q).

Find the value of p and the value of q. [5]

SPM.1.SL.TZ0.9

Let $f(x) = \frac{\ln 5x}{kx}$ where $x > 0$, $k \in R^+$.

The graph of f has exactly one maximum point P.

The second derivative of f is given by $f''(x) = \frac{2\ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q.

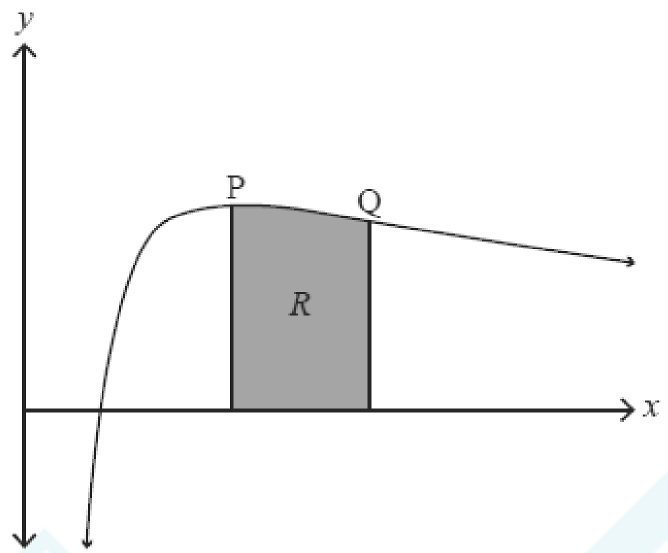
a. Show that $f'(x) = \frac{1 - \ln 5x}{kx^2}$. [3]

b. Find the x-coordinate of P. [3]

c. Show that the x-coordinate of Q is $\frac{1}{5}e^{\frac{3}{2}}$. [3]

d.

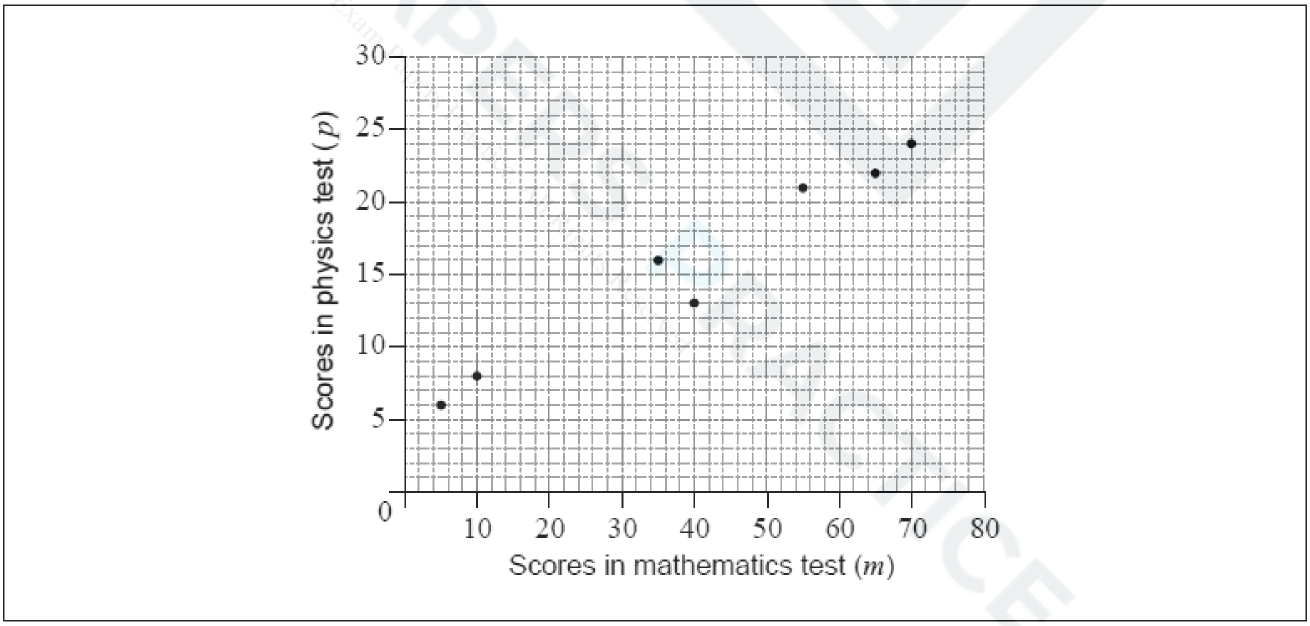
The region R is enclosed by the graph of f, the x-axis, and the vertical lines through the maximum point P and the point of inflexion Q.



Given that the area of R is 3, find the value of k. [7]

18M.1.SL.TZ2.T_1

The following scatter diagram shows the scores obtained by seven students in their mathematics test, m, and their physics test, p.



The mean point, M, for these data is (40, 16).

a. Plot and label the point M $(\bar{m}, \bar{p})$ on the scatter diagram. [2]

b. Draw the line of best fit, by eye, on the scatter diagram. [2]

c.

Using your line of best fit, estimate the physics test score for a student with a score of 20 in their mathematics test.

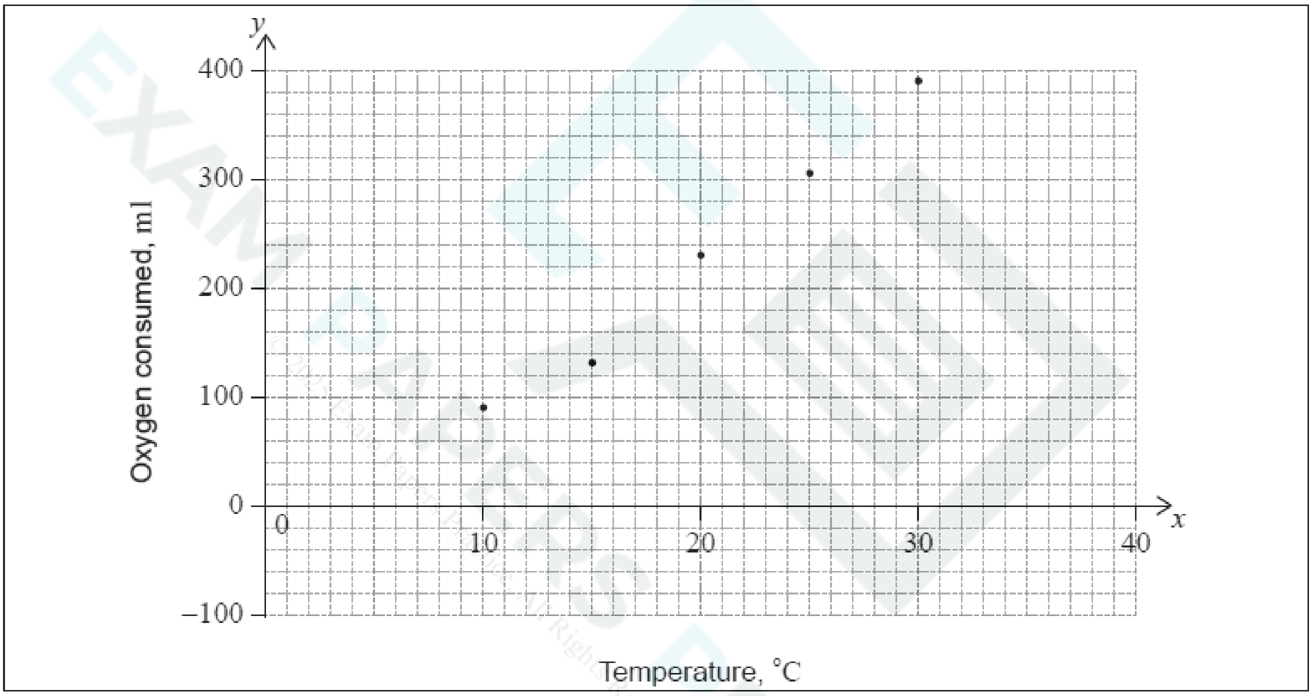
[2]

19M.1.SL.TZ2.T_2

Colorado beetles are a pest, which can cause major damage to potato crops. For a certain Colorado beetle the amount of oxygen, in millilitres (ml), consumed each day increases with temperature as shown in the following table.

Temperature, °C (x)	10	15	20	25	30
Oxygen consumed, ml (y)	90	133	230	306	391

This information has been used to plot a scatter diagram.



The mean point has coordinates (20, 230).

a. Find the equation of the regression line of y on x. [2]

b. Draw the regression line of y on x on the scatter diagram. [2]

c.

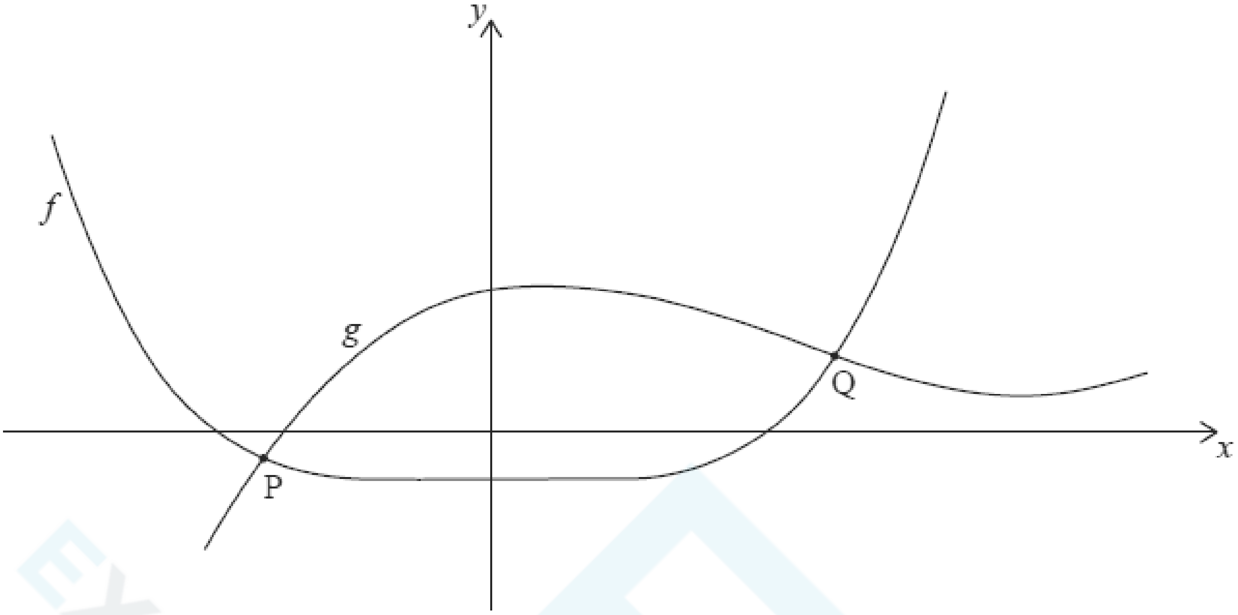
In order to estimate the amount of oxygen consumed, this regression line is considered to be reliable for a temperature x such that $a \leq x \leq b$.

Write down the value of a and of b. [2]

18M.1.SL.TZ1.T_15

Consider the functions $f(x) = x^4 - 2$ and $g(x) = x^3 - 4x^2 + 2x + 6$

The functions intersect at points P and Q. Part of the graph of $y = f(x)$ and part of the graph of $y = g(x)$ are shown on the diagram.



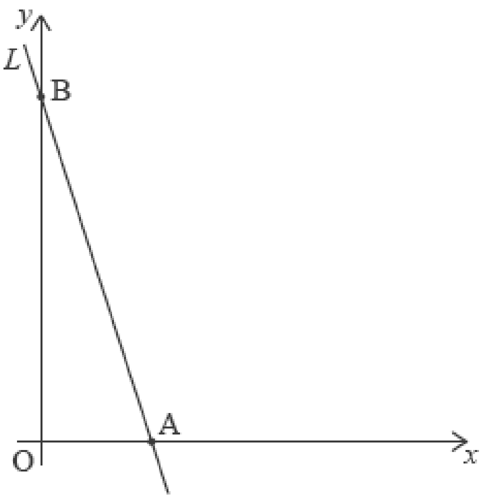
a. Find the range of *f*. [2]

b. Write down the *x*-coordinate of P and the *x*-coordinate of Q. [2]

c. Write down the values of *x* for which $f(x) > g(x)$. [2]

17M.1.SL.TZ2.T_4

Line L intersects the x-axis at point A and the y-axis at point B, as shown on the diagram.



The length of line segment OB is three times the length of line segment OA, where O is the origin.

Point $(2, 6)$ lies on L.

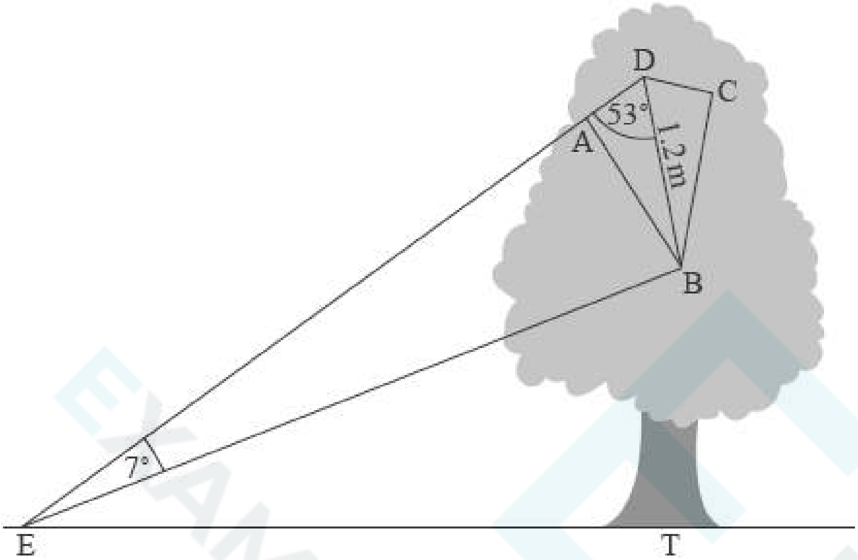
b. Find the equation of L in the form $y = mx + c$. [2]

c. Find the x-coordinate of point A. [2]

17N.1.SL.TZ0.T_10

Emily's kite ABCD is hanging in a tree. The plane ABCDE is vertical.

Emily stands at point E at some distance from the tree, such that EAD is a straight line and angle BED = 7°. Emily knows BD = 1.2 metres and angle BDA = 53°, as shown in the diagram



T is a point at the base of the tree. ET is a horizontal line. The angle of elevation of A from E is 41°.

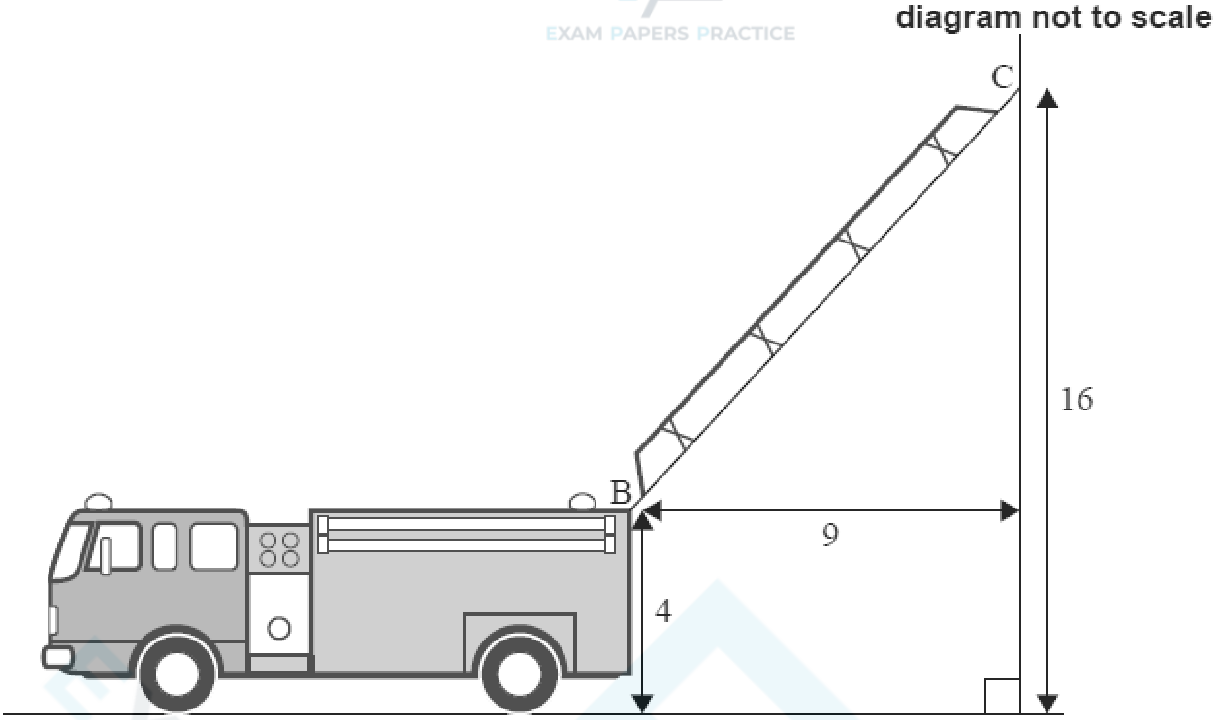
a. Find the length of EB. [3]

b. Write down the angle of elevation of B from E. [1]

c. Find the vertical height of B above the ground. [2]

19M.1.SL.TZ2.T_9

A ladder on a fire truck has its base at point B which is 4 metres above the ground. The ladder is extended and its other end rests on a vertical wall at point C, 16 metres above the ground. The horizontal distance between B and C is 9 metres.



a. Find the angle of elevation from B to C. [3]

b.

A second truck arrives whose ladder, when fully extended, is 30 metres long. The base of this ladder is also 4 metres above the ground. For safety reasons, the maximum angle of elevation that the ladder can make is 70°.

Find the maximum height on the wall that can be reached by the ladder on the second truck.

[3]

EXN.1.SL.TZ0.1

The derivative of a function f is given by $f'x = 3\sqrt{x}$.

Given that $f1 = 3$, find the value of $f4$.

EXN.1.SL.TZ0.2

Solve the equation $2 \ln x = \ln 9 + 4$. Give your answer in the form $x = pe^q$ where $p, \ q \in \mathbb{Z}^+$.

EXN.1.SL.TZ0.3

The following table shows the probability distribution of a discrete random variable X where $x = 1, \ 2, \ 3, \ 4$.

x	1	2	3	4
$\mathrm{P}(X=x)$	k^2	$7k+2$	$-2k$	$3k^2$

Find the value of k, justifying your answer.

EXN.1.SL.TZ0.4

The first three terms of an arithmetic sequence are u_1, $5u_1 - 8$ and $3u_1 + 8$.

a. Show that $u_1 = 4$. [2]

b.

Prove that the sum of the first n terms of this arithmetic sequence is a square number.

[4]

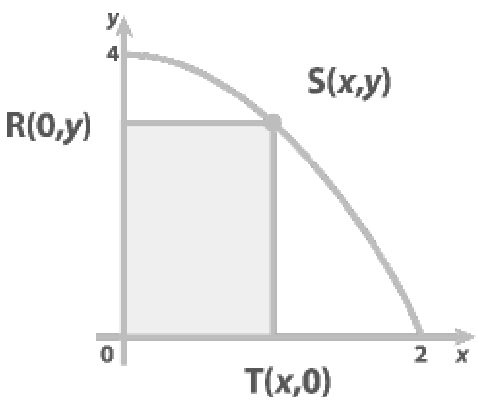
EXN.1.SL.TZ0.5

The functions f and g are defined for $x \in \mathbb{R}$ by $fx = x - 2$ and $gx = ax + b$, where $a, \ b \in \mathbb{R}$.

Given that $f \circ g2 = -3$ and $g \circ f1 = 5$, find the value of a and the value of b.

EXN.1.SL.TZ0.7

The following diagram shows the graph of $y = 4 - x^2$, $0 \le x \le 2$ and rectangle ORST. The rectangle has a vertex at the origin O, a vertex on the y-axis at the point $\mathrm{R}0, \ y$, a vertex on the x-axis at the point $\mathrm{T}x, \ 0$ and a vertex at point $\mathrm{S}x, \ y$ on the graph.



Let P represent the perimeter of rectangle ORST.

Let A represent the area of rectangle ORST.

a. Show that $P = -2x^2 + 2x + 8$. [2]

b.

Find the dimensions of rectangle ORST that has maximum perimeter and determine the value of the maximum perimeter.

[6]

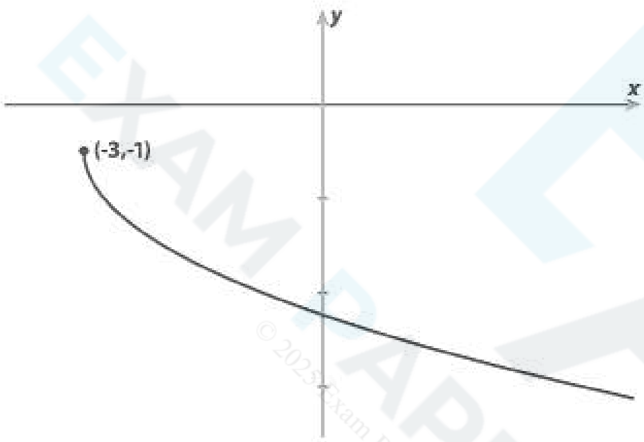
c. Find an expression for A in terms of x. [2]

d. Find the dimensions of rectangle ORST that has maximum area. [5]

e. Determine the maximum area of rectangle ORST. [1]

EXN.1.SL.TZ0.8

The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



A function f is defined by $fx = -1 - \sqrt{x+3}$ for $x \geq -3$.

a.

Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.

[3]

b. State the range of f. [1]

c. Find an expression for $f^{-1}x$, stating its domain. [5]

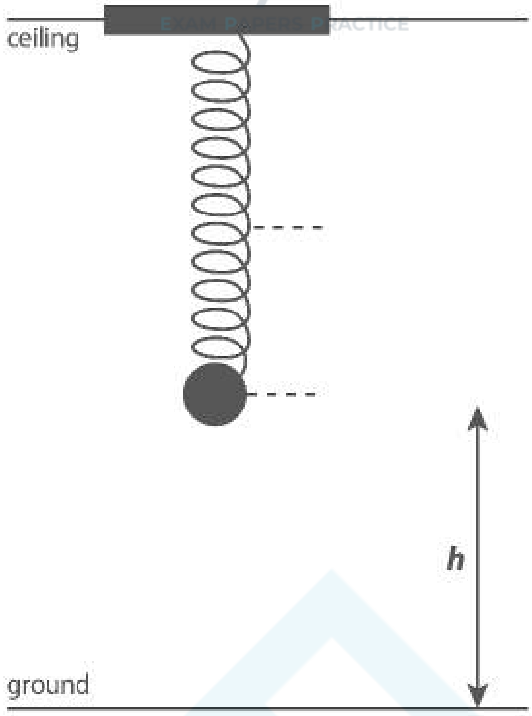
d.

Find the coordinates of the point(s) where the graphs of $y = fx$ and $y = f^{-1}x$ intersect.

[5]

EXN.1.SL.TZ0.9

The following diagram shows a ball attached to the end of a spring, which is suspended from a ceiling.



The height, h metres, of the ball above the ground at time t seconds after being released can be modelled by the function $ht = 0.4 \cos \pi t + 1.8$ where $t \geq 0$.

a. Find the height of the ball above the ground when it is released. [2]

b. Find the minimum height of the ball above the ground. [2]

c.

Show that the ball takes 2 seconds to return to its initial height above the ground for the first time.

[2]

d.

For the first 2 seconds of its motion, determine the amount of time that the ball is less than $1.8 + 0.2\sqrt{2}$ metres above the ground.

[5]

e.

Find the rate of change of the ball's height above the ground when $t = \frac{1}{3}$. Give your answer in the form $p\pi\sqrt{q}$ ms^{-1} where $p \in \mathbb{Q}$ and $q \in \mathbb{Z}^{+}$.

[4]

19N.1.SL.TZ0.S_1

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

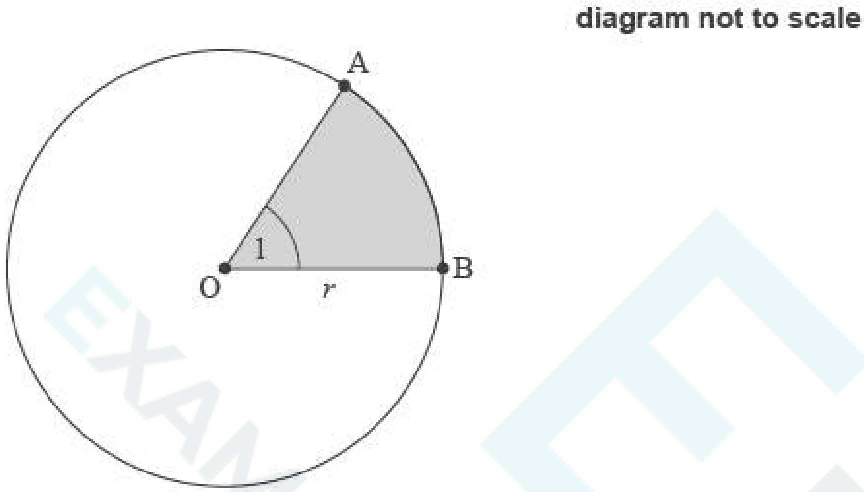
a. Find the common difference. [2]

b. Find the first term. [2]

c. Find the sum of the first 20 terms. [2]

21M.1.SL.TZ2.1

The following diagram shows a circle with centre O and radius r.



Points A and B lie on the circumference of the circle, and $A\hat{O}B = 1$ radian.

The perimeter of the shaded region is 12.

a. Find the value of r. [3]

b. Hence, find the exact area of the **non-shaded** region. [3]

21M.1.SL.TZ2.2

Consider two consecutive positive integers, n and $n+1$.

Show that the difference of their squares is equal to the sum of the two integers.

19N.1.SL.TZ0.S_10

Let $g(x) = p^x + q$, for $x, p, q \in R, p > 1$. The point $A(0, a)$ lies on the graph of g.

Let $f(x) = g^{-1}(x)$. The point B lies on the graph of f and is the reflection of point A in the line $y = x$.

The line L_1 is tangent to the graph of f at B.

a. Write down the coordinates of B. [2]

b. Given that $f'(a) = \frac{1}{\ln p}$, find the equation of L_1 **in terms of** x, p and q. [5]

c. The line L_2 is tangent to the graph of g at A and has equation $y = (\ln p)x + q + 1$.

The line L_2 passes through the point $(-2, -2)$.

The gradient of the normal to g at A is $\frac{1}{\ln\left(\frac{1}{3}\right)}$.

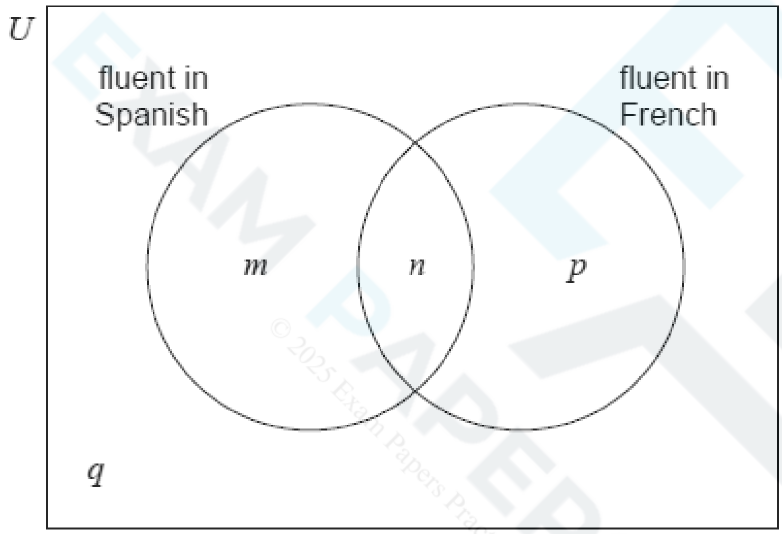
Find the equation of L_1 in terms of x.

[7]

19N.1.SL.TZ0.S_2

In a class of 30 students, 18 are fluent in Spanish, 10 are fluent in French, and 5 are not fluent in either of these languages. The following Venn diagram shows the events "fluent in Spanish" and "fluent in French".

The values m, n, p and q represent numbers of students.



a. Write down the value of q. [1]

b. Find the value of n. [2]

c. Write down the value of m and of p. [3]

19N.1.SL.TZ0.S_3

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of g.

a. Find the value of b. [3]

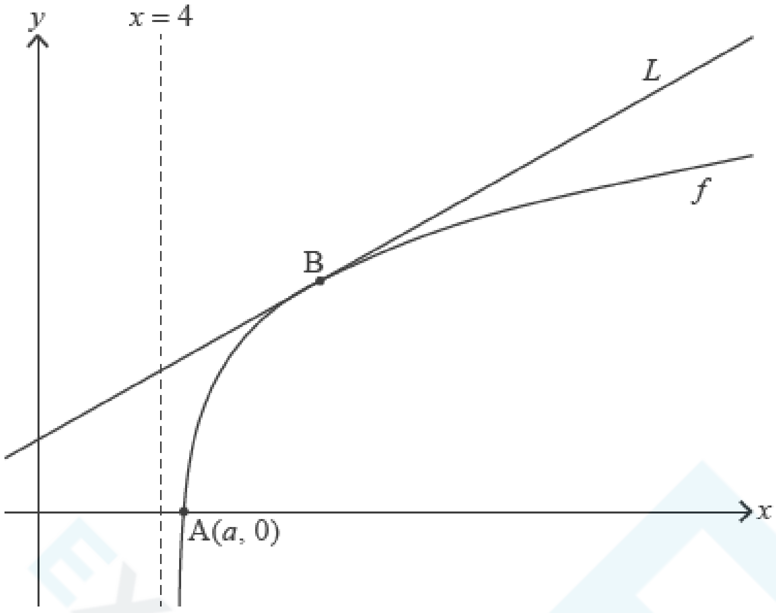
b. The graph of $f(x) = x^2$ is transformed to obtain the graph of g.

Describe this transformation. [4]

21M.1.SL.TZ2.5

Consider the function f defined by $f(x) = \ln(x^2 - 16)$ for $x > 4$.

The following diagram shows part of the graph of f which crosses the x-axis at point A, with coordinates $(a,\quad 0)$. The line L is the tangent to the graph of f at the point B.

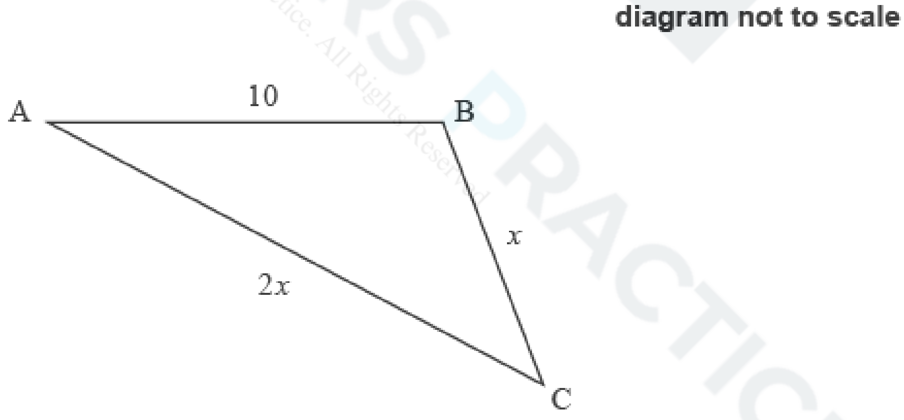


a. Find the exact value of a. [3]

b. Given that the gradient of L is $\frac{1}{3}$, find the x-coordinate of B. [6]

21M.1.SL.TZ2.6

The following diagram shows triangle ABC, with $\text{AB} = 10$, $\text{BC} = x$ and $\text{AC} = 2x$.



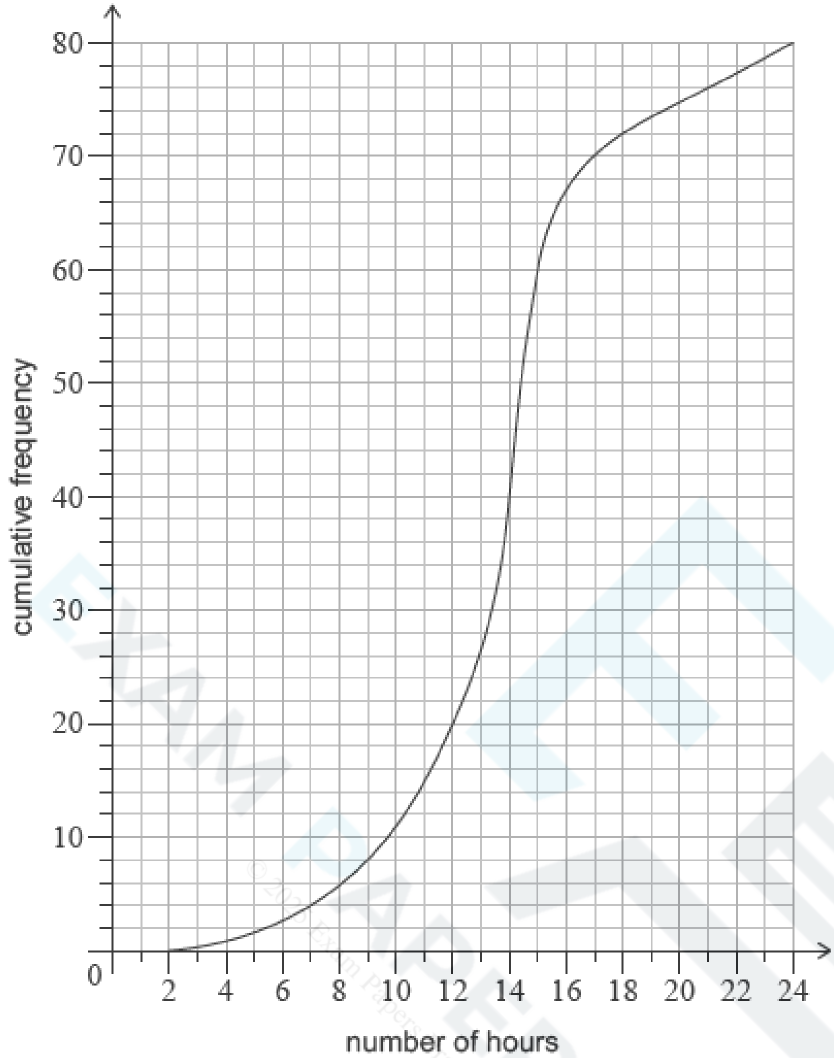
Given that $\cos\ \hat{\text{C}} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p,\quad q \in \mathbb{Z}^{+}$.

21M.1.SL.TZ2.7

A large school has students from Year 6 to Year 12.

A group of 80 students in Year 12 were randomly selected and surveyed to find out how many hours per week they each spend doing homework. Their results are represented by the following cumulative frequency graph.



This same information is represented by the following table.

Hours (h) spent doing homework	$2 < h \leq 7$	$7 < h \leq 15$	$15 < h \leq 21$	$21 < h \leq 24$
Frequency	4	p	16	q

There are 320 students in Year 12 at this school.

a.

Find the median number of hours per week these Year 12 students spend doing homework.

[2]

b.

Given that 10% of these Year 12 students spend more than k hours per week doing homework, find the value of k.

[3]

Find the value of p and the value of q. [4]

d.

Estimate the number of Year 12 students that spend more than 15 hours each week doing homework.

[3]

e.i.

Explain why this sampling method might not provide an accurate representation of the amount of time of the students in the school spend doing homework.

[1]

e.ii. Suggest a more appropriate sampling method. [1]

19N.1.SL.TZ0.S_4

Consider $\begin{pmatrix} 11 \\ a \end{pmatrix} = \frac{11!}{a!9!}$.

a. Find the value of a. [2]

b.

Hence or otherwise find the coefficient of the term in x^9 in the expansion of $(x+3)^{11}$.

[4]

19N.1.SL.TZ0.S_5

Consider the function f, with derivative $f'(x) = 2x^2 + 5kx + 3k^2 + 2$ where $x, k \in R$.

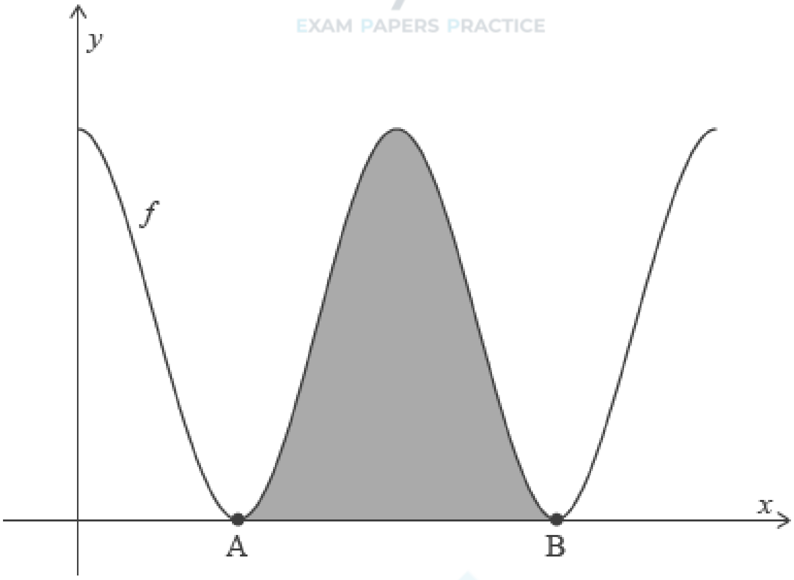
a. Show that the discriminant of $f'(x)$ is $k^2 - 16$. [2]

b. Given that f is an increasing function, find all possible values of k. [4]

21M.1.SL.TZ2.8

Consider the function f defined by $f(x) = 6 + 6\cos x$, for $0 \le x \le 4\pi$.

The following diagram shows the graph of $y = f(x)$.

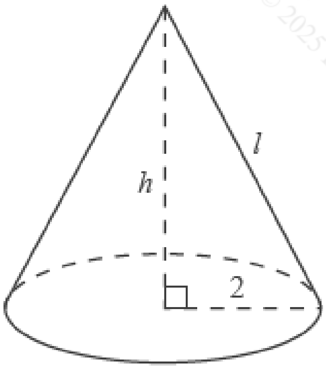


The graph of f touches the x-axis at points A and B, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the x-axis, between the points A and B.

The right cone in the following diagram has a total surface area of 12π, equal to the shaded area in the previous diagram.

The cone has a base radius of 2, height h, and slant height l.

diagram not to scale



a. Find the x-coordinates of A and B. [3]

b. Show that the area of the shaded region is 12π. [5]

c. Find the value of l. [3]

d. Hence, find the volume of the cone. [4]

19N.1.SL.TZ0.S_6

Let $f(x) = 4\cos\left(\frac{x}{2}\right) + 1$, for $0 \leqslant x \leqslant 6\pi$. Find the values of x for which $f(x) > 2\sqrt{2} + 1$.

19N.1.SL.TZ0.S_7

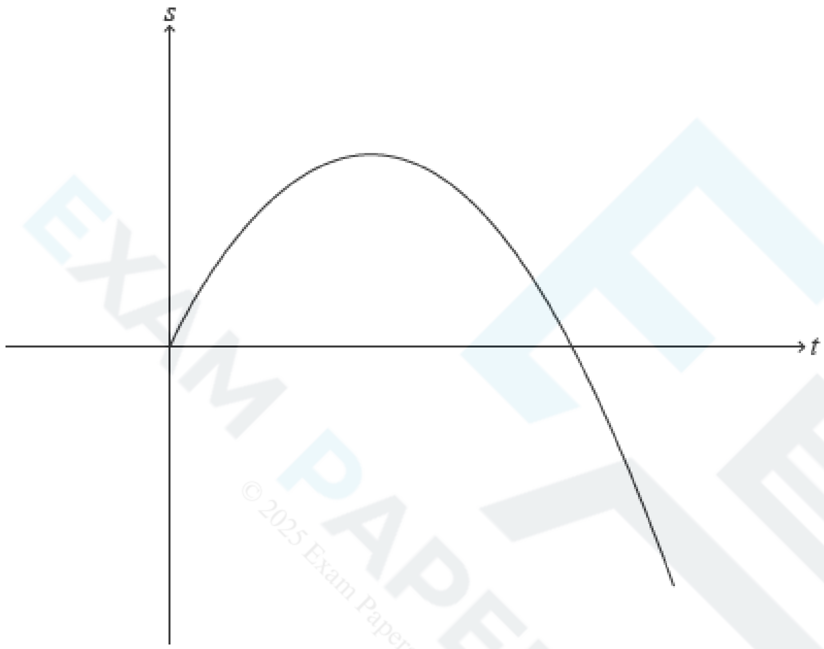
Let X and Y be normally distributed with $X \sim \mathrm{N}(14, a^2)$ and $Y \sim \mathrm{N}(22, a^2)$, $a > 0$.

a. Find b so that $P(X > b) = P(Y < b)$. [2]

b. It is given that $P(X > 20) = 0.112$. Find $P(16 < Y < 28)$. [4]

21M.1.SL.TZ2.9

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

Particle A changes direction when $t = q$.

The total distance travelled by particle A is given by d.

a. Find the value of p. [2]

b.i. Find the value of q. [2]

b.ii. Find the displacement of particle A from the origin when $t = q$. [2]

c. Find the distance of particle A from the origin when $t = 10$. [2]

d. Find the value of d. [2]

e.

A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

When $t = k$, the distance travelled by particle B is equal to d. Find the value of k.

[4]

19N.1.SL.TZ0.S_8

A small cuboid box has a rectangular base of length $3x$ cm and width x cm, where $x > 0$. The height is y cm, where $y > 0$.

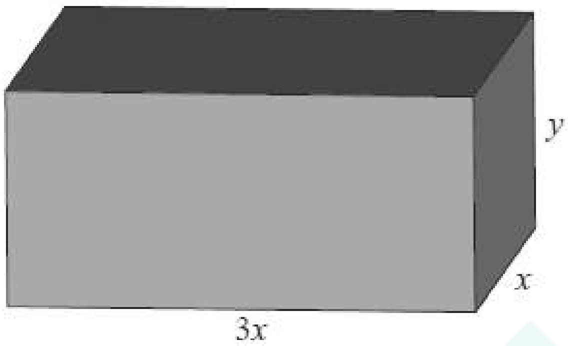


diagram not to scale

The sum of the length, width and height is 12 cm.

The volume of the box is V cm^3.

a. Write down an expression for y in terms of x. [1]

b. Find an expression for V in terms of x. [2]

c. Find $\frac{dV}{dx}$. [2]

d.i. Find the value of x for which V is a maximum. [4]

d.ii. Justify your answer. [3]

e. Find the maximum volume. [2]

19N.1.SL.TZ0.S_9

The points A and B have position vectors $\begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$ respectively.

Point C has position vector $\begin{pmatrix} -1 \\ k \\ 0 \end{pmatrix}$. Let O be the origin.

Find, in terms of k,

a.i. $\overrightarrow{OA} \cdot \overrightarrow{OC}$. [2]

a.ii. $\overrightarrow{OB} \cdot \overrightarrow{OC}$. [1]

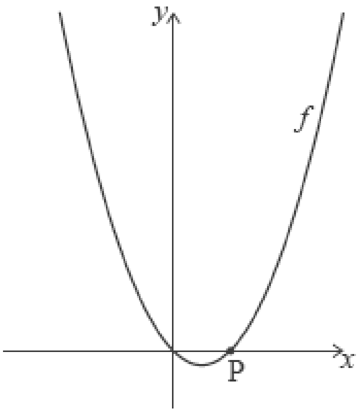
b. Given that $A\hat{O}C = B\hat{O}C$, show that $k = 7$. [8]

c. Calculate the area of triangle AOC. [6]

17N.1.SL.TZ0.S_8

Let $f(x) = x^2 - x$, for $x \in R$. The following diagram shows part of the graph of f.

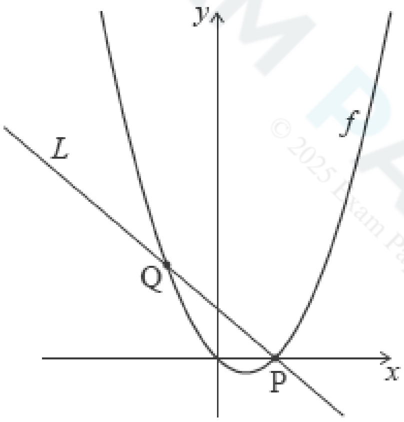
diagram not to scale



The graph of f crosses the x-axis at the origin and at the point $\mathrm{P}(1,\ 0)$.

The line L intersects the graph of f at another point Q, as shown in the following diagram.

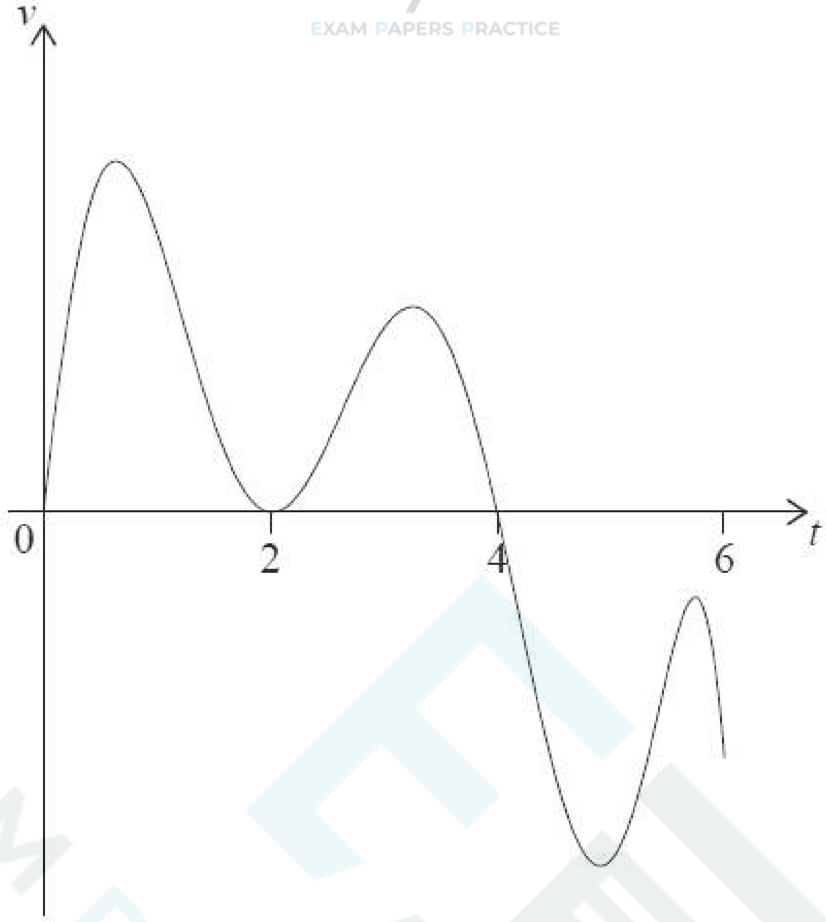
diagram not to scale



Find the area of the region enclosed by the graph of f and the line L.

19M.1.SL.TZ1.S_7

A particle P starts from point O and moves along a straight line. The graph of its velocity, v ms^{-1} after t seconds, for $0 \le t \le 6$, is shown in the following diagram.



The graph of v has t-intercepts when t = 0, 2 and 4.

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_2^4 v\,dt = 9$.

a. Find the value of $s(4) - s(2)$. [2]

b. Find the total distance travelled in the first 5 seconds. [5]

19M.1.SL.TZ1.S_10

Consider $f(x) = \sqrt{x}\sin\left(\frac{\pi}{4}x\right)$ and $g(x) = \sqrt{x}$ for $x \geq 0$. The first time the graphs of f and g intersect is at $x = 0$.

The set of all non-zero values that satisfy $f(x) = g(x)$ can be described as an arithmetic sequence, $u_n = a + bn$ where $n \geq 1$.

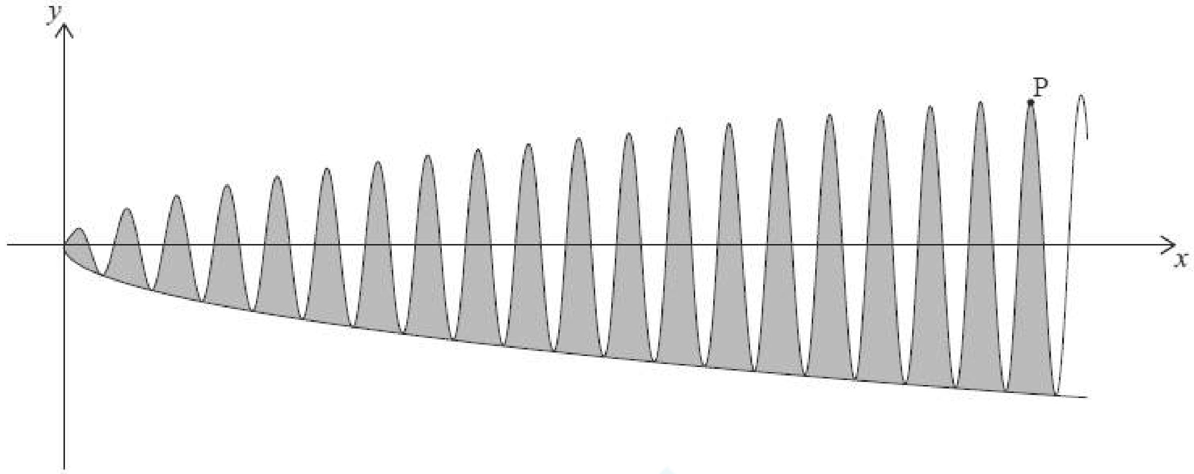
a. Find the **two** smallest non-zero values of x for which $f(x) = g(x)$. [5]

c.

At point P, the graphs of f and g intersect for the 21st time. Find the coordinates of P.

[4]

The following diagram shows part of the graph of g **reflected** in the x-axis. It also shows part of the graph of f and the point P.



Find an expression for the area of the shaded region. Do not calculate the value of the expression.

[4]

17M.1.SL.TZ1.S_10

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$\mathrm{P}(A = a)$	$\cos\theta$	$2\cos 2\theta$

a. Show that $\cos\theta = \frac{3}{4}$. [6]

b. Given that $\tan\theta > 0$, find $\tan\theta$. [3]

c.

Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of $y$between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed.

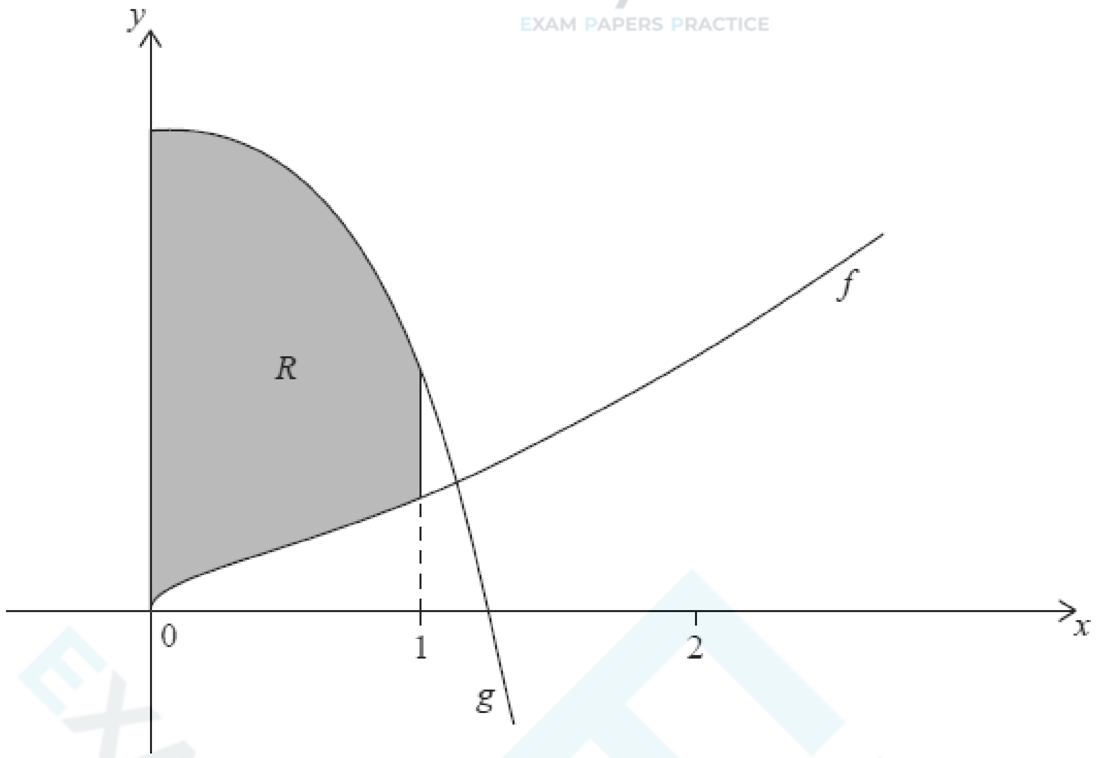
[6]

19M.1.SL.TZ2.S_10

Let $y = \left(x^3 + x\right)^{\frac{3}{2}}$.

Consider the functions $f\left(x\right) = \sqrt{x^3 + x}$ and $g\left(x\right) = 6 - 3x^2\sqrt{x^3 + x}$, for $x \geq 0$.

The graphs of f and g are shown in the following diagram.



The shaded region R is enclosed by the graphs of f, g, the y-axis and $x = 1$.

b. Hence find $\int \left(3x^2 + 1\right)\sqrt{x^3 + x}\,\mathrm{d}x$. [3]

c. Write down an expression for the area of R. [2]

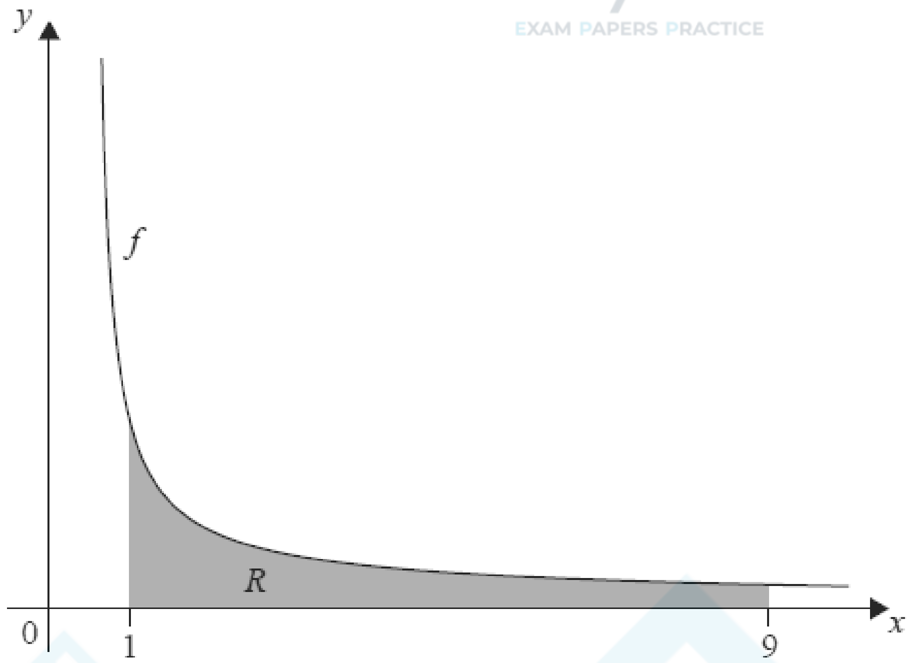
d. Hence find the exact area of R. [6]

18M.1.SL.TZ1.S_5

Let $f\left(x\right) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

a. Find $\int \left(f\left(x\right)\right)^2 \mathrm{d}x$. [3]

b. Part of the graph of f is shown in the following diagram.



The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$ and $x = 9$. Find the volume of the solid formed when R is revolved 360° about the x-axis.

[4]

18N.1.SL.TZ0.T_1

The volume of a hemisphere, V, is given by the formula

$$V = \sqrt{\frac{4S^3}{243\pi}},$$

where S is the total surface area.

The total surface area of a given hemisphere is 350 cm^2.

a. Calculate the volume of this hemisphere in cm^3.

Give your answer correct to **one decimal place**. [3]

b. Write down your answer to part (a) correct to the nearest integer. [1]

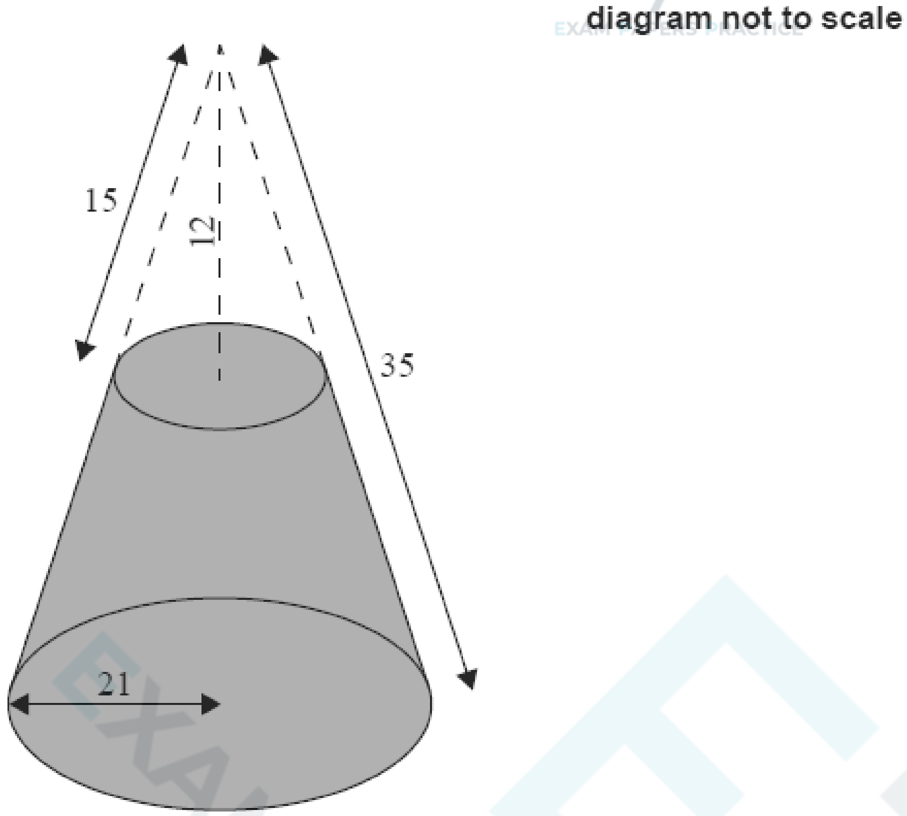
c.

Write down your answer to **part (b)** in the form $a \times 10^k$, where $1 \le a < 10$ and $k \in Z$.

[2]

18M.1.SL.TZ1.T_14

A solid right circular cone has a base radius of 21 cm and a slant height of 35 cm.
A smaller right circular cone has a height of 12 cm and a slant height of 15 cm, and is removed from the top of the larger cone, as shown in the diagram.



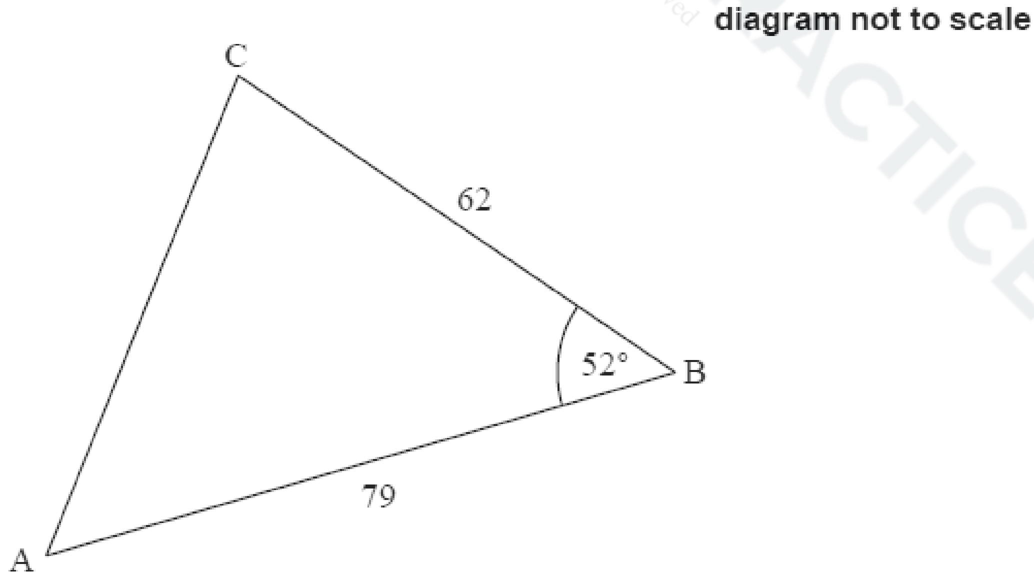
a. Calculate the radius of the base of the cone which has been removed. [2]

b. Calculate the curved surface area of the cone which has been removed. [2]

c. Calculate the curved surface area of the remaining solid. [2]

18M.1.SL.TZ2.T_8

A park in the form of a triangle, ABC, is shown in the following diagram. AB is 79 km and BC is 62 km. Angle $A\hat{B}C$ is 52°.



a. Calculate the length of side AC in km. [3]

b. Calculate the area of the park. [3]

EXM.1.SL.TZ0.2

A set of data comprises of five numbers x_1, x_2, x_3, x_4, x_5 which have been placed in ascending order.

a.

Recalling definitions, such as the Lower Quartile is the $\frac{n+1}{4}th$ piece of data with the data placed in order, find an expression for the Interquartile Range.

[2]

b. Hence, show that a data set with only 5 numbers in it cannot have any outliers.

[5]

c.

Give an example of a set of data with 7 numbers in it that does have an outlier, justify this fact by stating the Interquartile Range.

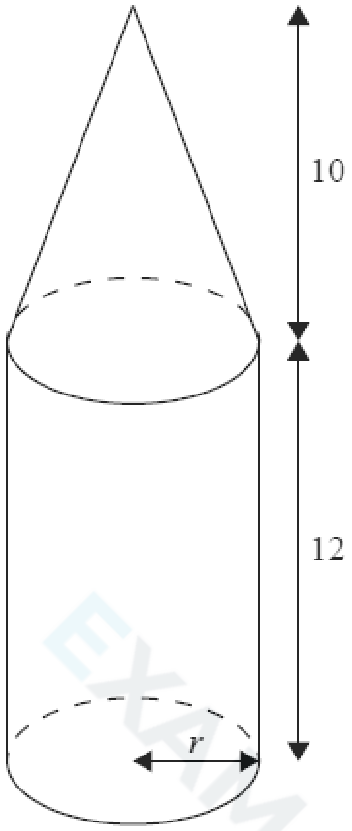
[2]

18M.1.SL.TZ2.T_15

Julio is making a wooden pencil case in the shape of a large pencil. The pencil case consists of a cylinder attached to a cone, as shown.

The cylinder has a radius of *r* cm and a height of 12 cm.

The cone has a base radius of *r* cm and a height of 10 cm.



a. Find an expression for the slant height of the cone . [2]

b.

The total external surface area of the pencil case rounded to 3 significant figures is 570 cm^2.

Using your graphic display calculator, calculate the value of *r*. [4]

17N.1.SL.TZ0.T_3

The speed of light is 300000 kilometres per second. The average distance from the Sun to the Earth is 149.6 million km.

A light-year is the distance light travels in one year and is equal to 9467280 million km. Polaris is a bright star, visible from the Northern Hemisphere. The distance from the Earth to Polaris is 323 light-years.

a. Calculate the time, **in minutes**, it takes for light from the Sun to reach the Earth.

[3]

b.

Find the distance from the Earth to Polaris in millions of km. Give your answer in the form $a \times 10^k$ with $1 \leqslant a < 10$ and $k \in Z$.

[3]

19M.1.SL.TZ2.T_6

A health inspector analysed the amount of sugar in 500 different **snacks** prepared in various school cafeterias. The collected data are shown in the following box-and-whisker diagram.

Amount of sugar per snack in grams

a. State what 13 represents in the given diagram. [1]

b.i. Write down the interquartile range for this data. [2]

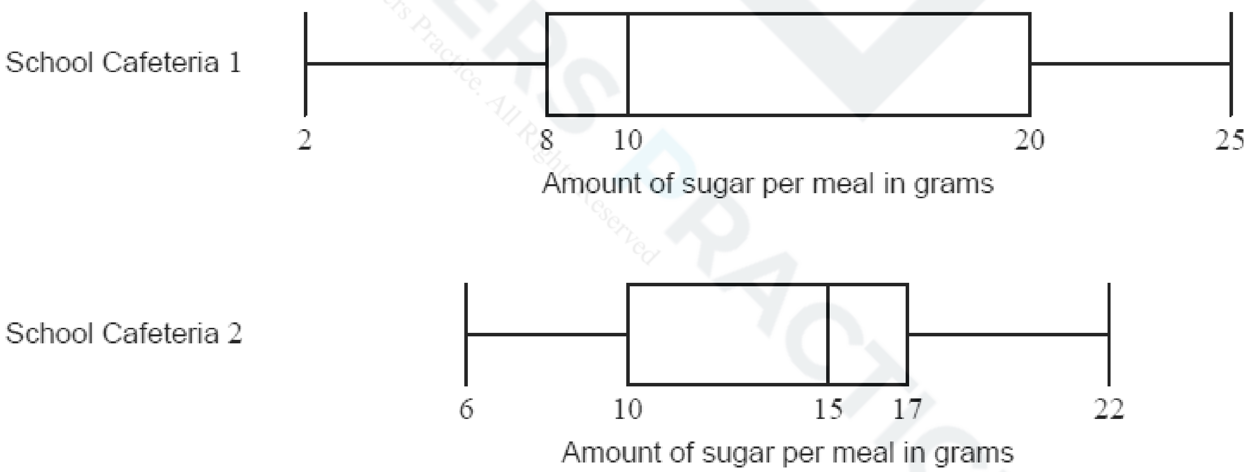
b.ii.

Write down the approximate number of snacks whose amount of sugar ranges from 18 to 20 grams.

[1]

c.

The health inspector visits two school cafeterias. She inspects the same number of **meals** at each cafeteria. The data is shown in the following box-and-whisker diagrams.



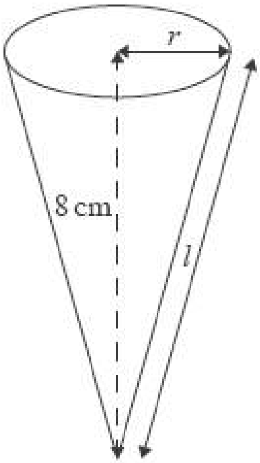
Meals prepared in the school cafeterias are required to have less than 10 grams of sugar.

State, giving a reason, which school cafeteria has more meals that **do not** meet the requirement.

[2]

17M.1.SL.TZ1.T_9

A type of candy is packaged in a right circular cone that has volume $100\ \text{cm}^3$ and vertical height 8 cm.



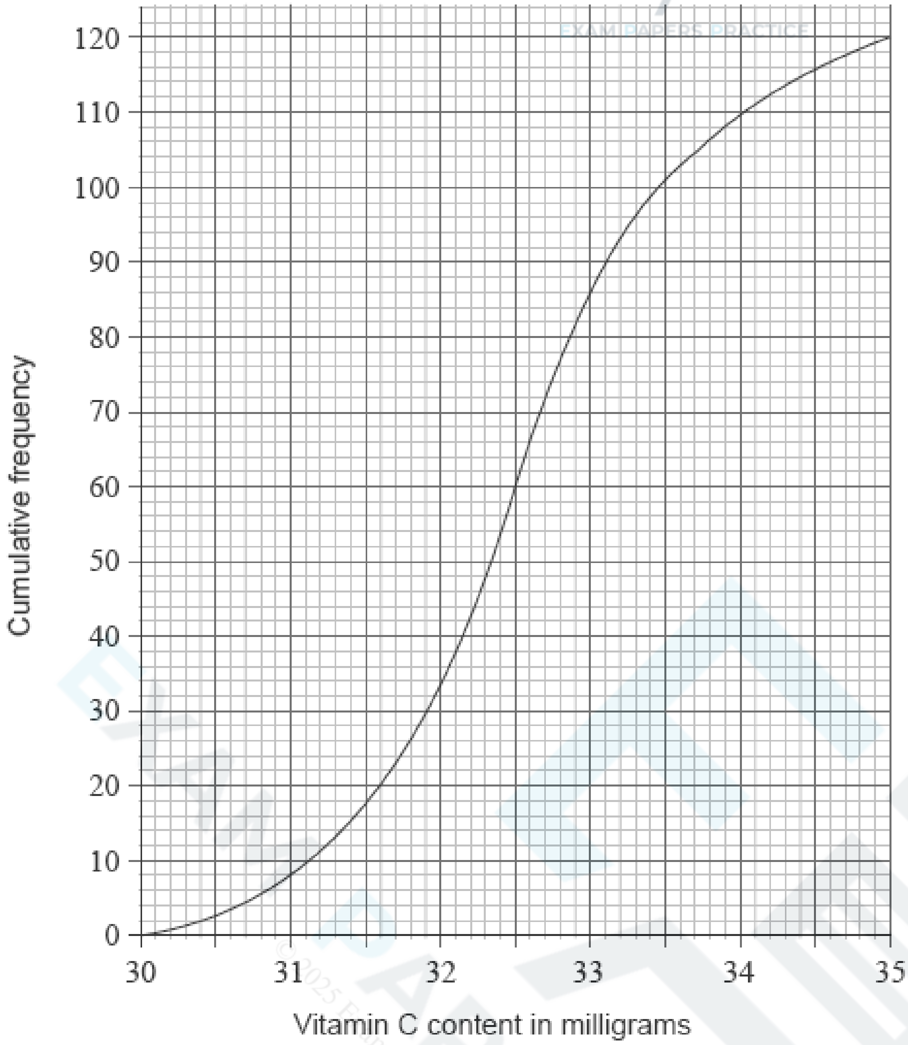
a. Find the radius, r, of the circular base of the cone. [2]

b. Find the slant height, l, of the cone. [2]

c. Find the curved surface area of the cone. [2]

16N.1.SL.TZ0.T_2

A sample of 120 oranges was tested for Vitamin C content. The cumulative frequency curve below represents the Vitamin C content, in milligrams, of these oranges.



The minimum level of Vitamin C content of an orange in the sample was 30.1 milligrams.
The maximum level of Vitamin C content of an orange in the sample was 35.0 milligrams.

a. Giving your answer to one decimal place, write down the value of

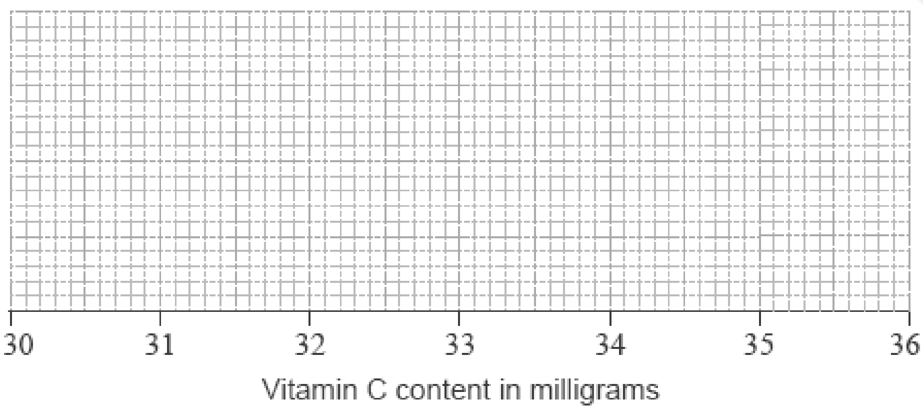
(i) the median level of Vitamin C content of the oranges in the sample;

(ii) the lower quartile; (iii) the upper quartile. [3]

b.

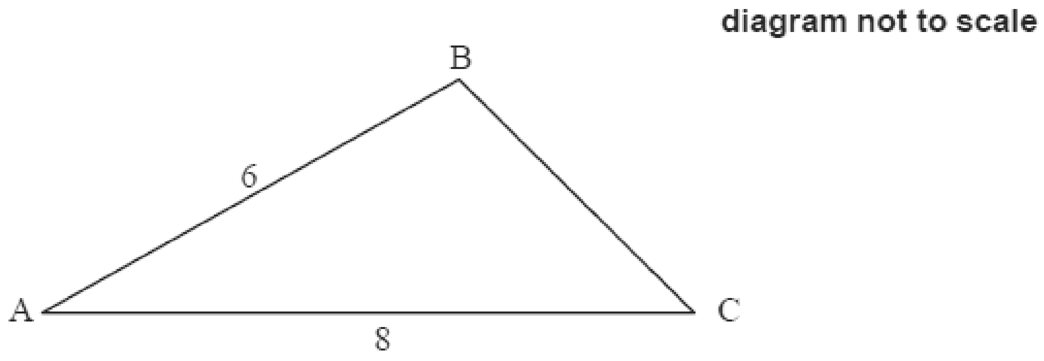
Draw a box-and-whisker diagram on the grid below to represent the Vitamin C content, in milligrams, for this sample.

[3]



SPM.1.SL.TZ0.1

The following diagram shows triangle ABC, with AB = 6 and AC = 8.



a. Given that $\cos\hat{A} = \frac{5}{6}$ find the value of $\sin\hat{A}$. [3]

b. Find the area of triangle ABC. [2]

18M.1.SL.TZ1.T_2

Each month the number of days of rain in Cardiff is recorded.
The following data was collected over a period of 10 months.

11 13 8 11 8 7 8 14 x 15

For these data the **median** number of days of rain per month is 10.

a. Find the value of *x*. [2]

b.i. Find the standard deviation [2]

b.ii. Find the interquartile range. [2]

SPM.1.SL.TZ0.2

Let A and B be events such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \cup B) = 0.6$.

Find $\mathrm{P}(A|B)$.

SPM.1.SL.TZ0.4

Let $f'(x) = \frac{8x}{\sqrt{2x^2+1}}$. Given that $f(0) = 5$, find $f(x)$.

SPM.1.SL.TZ0.5

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

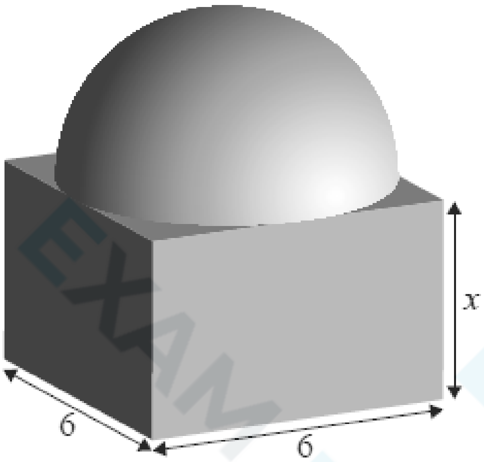
a. Show that $(g \circ f)(x) = 2x + 11$. [2]

b. Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

18N.1.SL.TZ0.T_9

A solid glass paperweight consists of a hemisphere of diameter 6 cm on top of a cuboid with a square base of length 6 cm, as shown in the diagram.

diagram not to scale



The height of the cuboid, x cm, is equal to the height of the hemisphere.

a.i. Write down the value of x. [1]

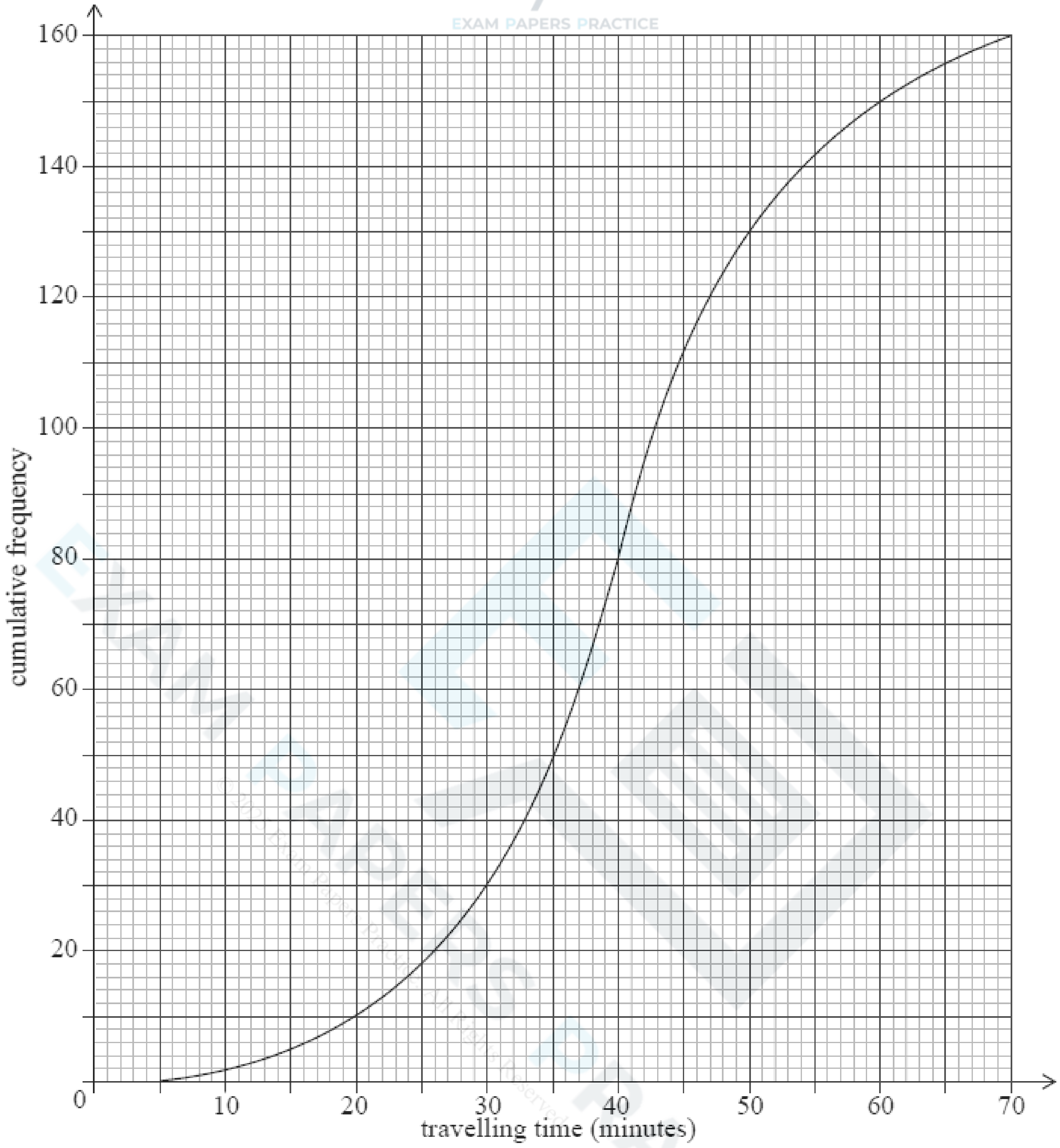
a.ii. Calculate the volume of the paperweight. [3]

b. $1\ \text{cm}^3$ of glass has a mass of 2.56 grams.

Calculate the mass, in grams, of the paperweight. [2]

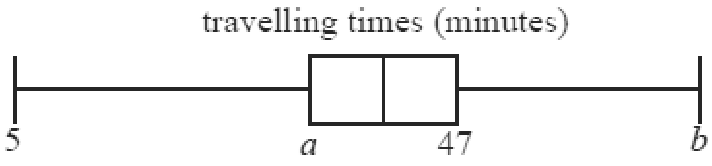
SPM.1.SL.TZ0.7

A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



Only 10% of the employees spent more than k minutes traveling to work.

The results of the survey can also be displayed on the following box-and-whisker diagram.



a. Find the median number of minutes spent traveling to work. [2]

b.

Find the number of employees whose travelling time is within 15 minutes of the median.

[3]

c. Find the value of k. [3]

d. Write down the value of b. [1]

e.i. Find the value of a. [2]

e.ii. Hence, find the interquartile range. [2]

f. Travelling times of less than p minutes are considered outliers. Find the value of p.

[2]

19M.1.SL.TZ1.T_1

A calculator fits into a cuboid case with height 29 mm, width 98 mm and length 186 mm.

Find the volume, in cm^3, of this calculator case.

19M.1.SL.TZ2.T_1

A sphere with diameter 3 474 000 metres can model the shape of the Moon.

a.

Use this model to calculate the circumference of the Moon in **kilometres**. Give your full calculator display.

[3]

b. Give your answer to part (a) correct to three significant figures. [1]

c. Write your answer to **part (b)** in the form $a \times 10^k$, where $1 \le a < 10$, $k \in Z$. [2]

19M.1.SL.TZ2.T_11

Consider the following sets:

The universal set U consists of all positive integers less than 15;
A is the set of all numbers which are multiples of 3;
B is the set of all even numbers.

a. Write down the elements that belong to $A \cap B$. [3]

b.i. Write down the elements that belong to $A \cap B'$. [2]

b.ii. Write down $n\left(A \cap B'\right)$. [1]

18M.1.SL.TZ2.T_5

In this question, give all answers to two decimal places.

Karl invests 1000 US dollars (USD) in an account that pays a nominal annual interest of 3.5%, **compounded quarterly**. He leaves the money in the account for 5 years.

a.i. Calculate the amount of money he has in the account after 5 years. [3]

a.ii. Write down the amount of **interest** he earned after 5 years. [1]

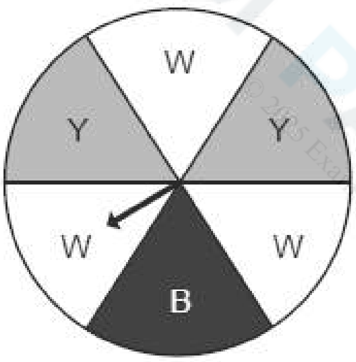
b.

Karl decides to donate this **interest** to a charity in France. The charity receives 170 euros (EUR). The exchange rate is 1 USD = t EUR.

Calculate the value of t. [2]

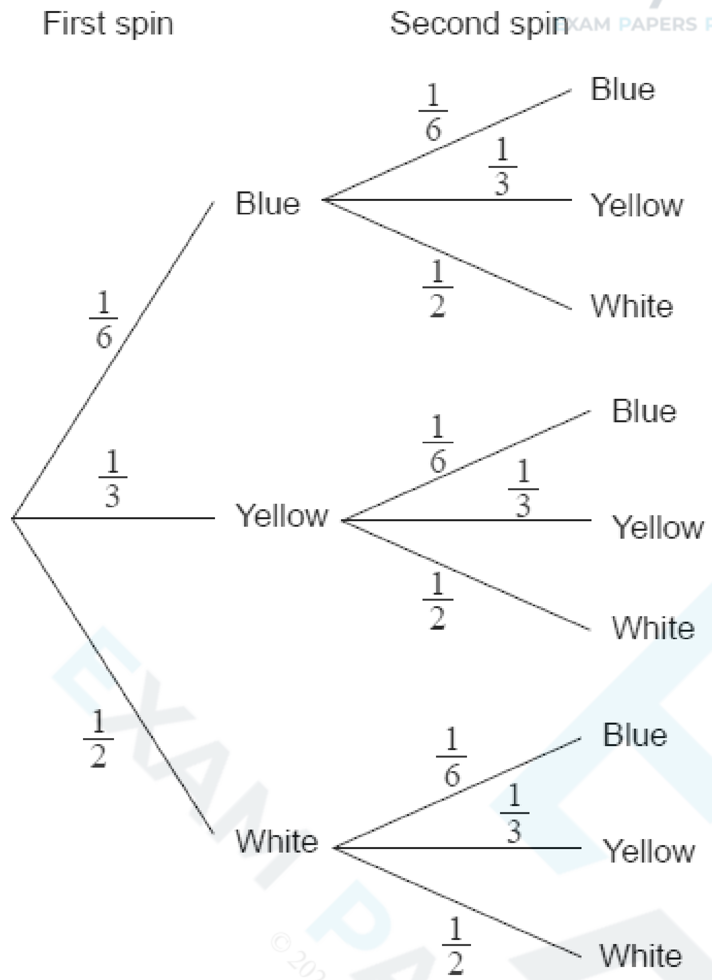
19M.1.SL.TZ1.T_12

The diagram shows a circular horizontal board divided into six equal sectors. The sectors are labelled white (W), yellow (Y) and blue (B).



A pointer is pinned to the centre of the board. The pointer is to be spun and when it stops the colour of the sector on which the pointer stops is recorded. The pointer is equally likely to stop on any of the six sectors.

Eva will spin the pointer twice. The following tree diagram shows all the possible outcomes.



a. Find the probability that both spins are yellow. [2]

b. Find the probability that at least one of the spins is yellow. [3]

c.

Write down the probability that the second spin is yellow, given that the first spin is blue.

[1]

17M.1.SL.TZ2.T_10

The Home Shine factory produces light bulbs, 7% of which are found to be defective.

Francesco buys two light bulbs produced by Home Shine.

The Bright Light factory also produces light bulbs. The probability that a light bulb produced by Bright Light is not defective is a.

Deborah buys three light bulbs produced by Bright Light.

a.

Write down the probability that a light bulb produced by Home Shine is not defective.

[1]

b.i. Find the probability that both light bulbs are not defective. [2]

b.ii. Find the probability that at least one of Francesco's light bulbs is defective. [2]

c.

Write down an expression, in terms of a, for the probability that at least one of Deborah's three light bulbs is defective.

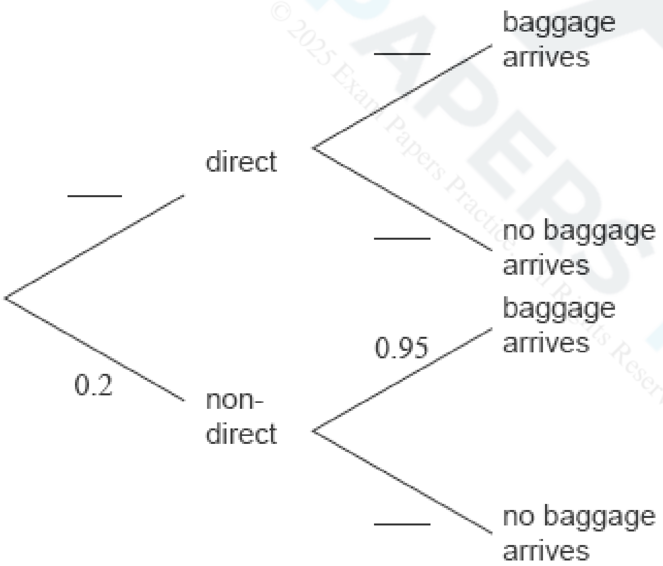
[1]

17M.1.SL.TZ1.T_7

Sara regularly flies from Geneva to London. She takes either a direct flight or a non-directflight that goes via Amsterdam.

If she takes a direct flight, the probability that her baggage does not arrive in London is 0.01.
If she takes a non-direct flight the probability that her baggage arrives in London is 0.95.

The probability that she takes a non-direct flight is 0.2.



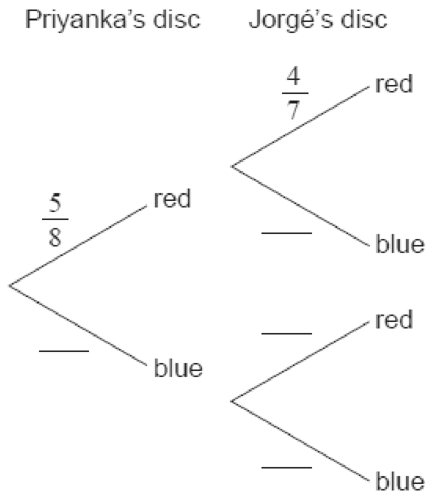
a. Complete the tree diagram. [3]

b. Find the probability that Sara's baggage arrives in London. [3]

18N.1.SL.TZ0.T_8

A bag contains 5 red and 3 blue discs, all identical except for the colour. First, Priyanka takes a disc at random from the bag and then Jorgé takes a disc at random from the bag.

a. Complete the tree diagram.



[3]

b. Find the probability that Jorgé chooses a red disc. [3]

19M.1.SL.TZ2.T_14

The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

a.ii. Find the price that is two standard deviations above the mean price. [1]

b.

Find the probability that the price of a kilogram of tomatoes, chosen at random, will be between 2.00 and 3.00 euro.

[2]

c.

To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the lowest 20 %.

Find the highest price that a seller can charge and still receive a free permit. [2]

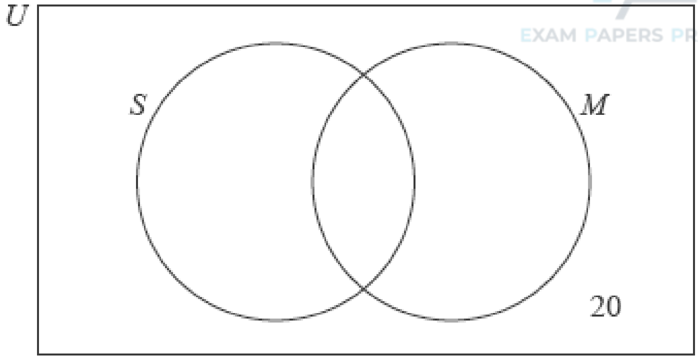
17N.1.SL.TZ0.T_7

Rosewood College has 120 students. The students can join the sports club (S) and the music club (M).

For a student chosen at random from these 120, the probability that they joined both clubs is $\frac{1}{4}$ and the probability that they joined the music club is $\frac{1}{3}$.

There are 20 students that did not join either club.

a. Complete the Venn diagram for these students.



[2]

b.

One of the students who joined the sports club is chosen at random. Find the probability that this student joined both clubs.

[2]

c. Determine whether the events S and M are independent. [2]

16N.1.SL.TZ0.T_12

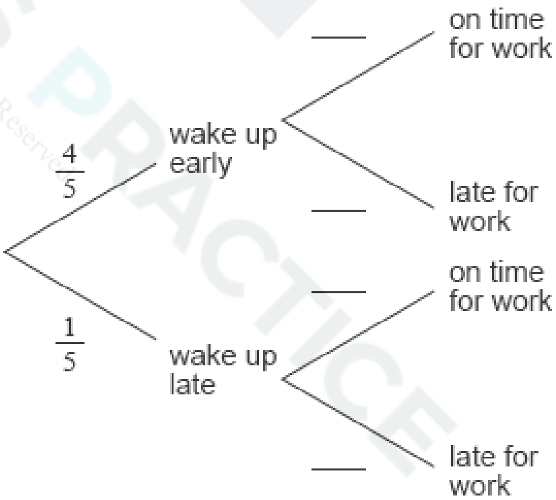
On a work day, the probability that Mr Van Winkel wakes up early is $\frac{4}{5}$.

If he wakes up early, the probability that he is on time for work is p.

If he wakes up late, the probability that he is on time for work is $\frac{1}{4}$.

The probability that Mr Van Winkel arrives on time for work is $\frac{3}{5}$.

a. Complete the tree diagram below. [2]



b. Find the value of p. [4]

18M.1.SL.TZ1.T_10

A group of 60 sports enthusiasts visited the PyeongChang 2018 Winter Olympic games to watch a variety of sporting events.

The most popular sports were snowboarding (S), figure skating (F) and ice hockey (H).

For this group of 60 people:

4 did not watch any of the most popular sports,
x watched all three of the most popular sports,
9 watched snowboarding only,
11 watched figure skating only,
15 watched ice hockey only,
7 watched snowboarding and figure skating,
13 watched figure skating and ice hockey,
11 watched snowboarding and ice hockey.

Find the value of x.

18M.1.SL.TZ1.T_13

Malthouse school opens at 08:00 every morning.

The daily arrival times of the 500 students at Malthouse school follow a normal distribution. The mean arrival time is 52 minutes after the school opens and the standard deviation is 5 minutes.

a.i.

Find the probability that a student, chosen at random arrives at least 60 minutes after the school opens.

[2]

a.ii.

Find the probability that a student, chosen at random arrives between 45 minutes and 55 minutes after the school opens.

[2]

b.

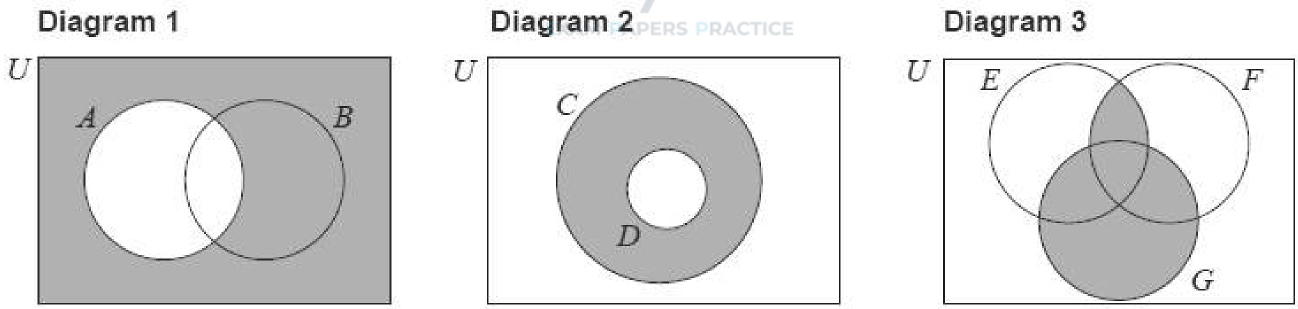
A second school, Mulberry Park, also opens at 08:00 every morning. The arrival times of the students at this school follows exactly the same distribution as Malthouse school.

Given that, on one morning, 15 students arrive at least 60 minutes after the school opens, estimate the number of students at Mulberry Park school.

[2]

18M.1.SL.TZ2.T_9

Consider the following Venn diagrams.



a.i.

Write down an expression, in set notation, for the **shaded** region represented by Diagram 1.

[1]

a.ii.

Write down an expression, in set notation, for the **shaded** region represented by Diagram 2.

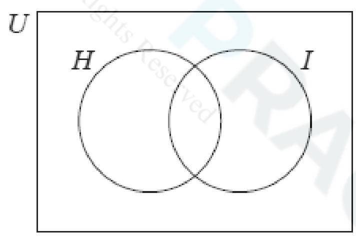
[1]

a.iii.

Write down an expression, in set notation, for the shaded region represented by Diagram 3.

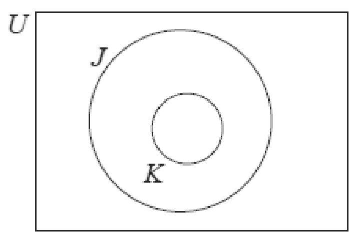
[2]

b.i. Shade, on the Venn diagram, the region represented by the set $(H \cup I)'$.



[1]

b.ii. Shade, on the Venn diagram, the region represented by the set $J \cap K$.



[1]

17N.1.SL.TZ0.T_13

Applicants for a job had to complete a mathematics test. The time they took to complete the test is normally distributed with a mean of 53 minutes and a standard deviation of 16.3. One of the applicants is chosen at random.

For 11% of the applicants it took longer than k minutes to complete the test.

There were 400 applicants for the job.

a.

Find the probability that this applicant took at least 40 minutes to complete the test.

[2]

b. Find the value of k. [2]

c.

Estimate the number of applicants who completed the test in less than 25 minutes.

[2]

17M.1.SL.TZ2.T_11

The mass of a certain type of Chilean corncob follows a normal distribution with a mean of 400 grams and a standard deviation of 50 grams.

A farmer labels one of these corncobs as premium if its mass is greater than a grams. 25% of these corncobs are labelled as premium.

a.

Write down the probability that the mass of one of these corncobs is greater than 400 grams.

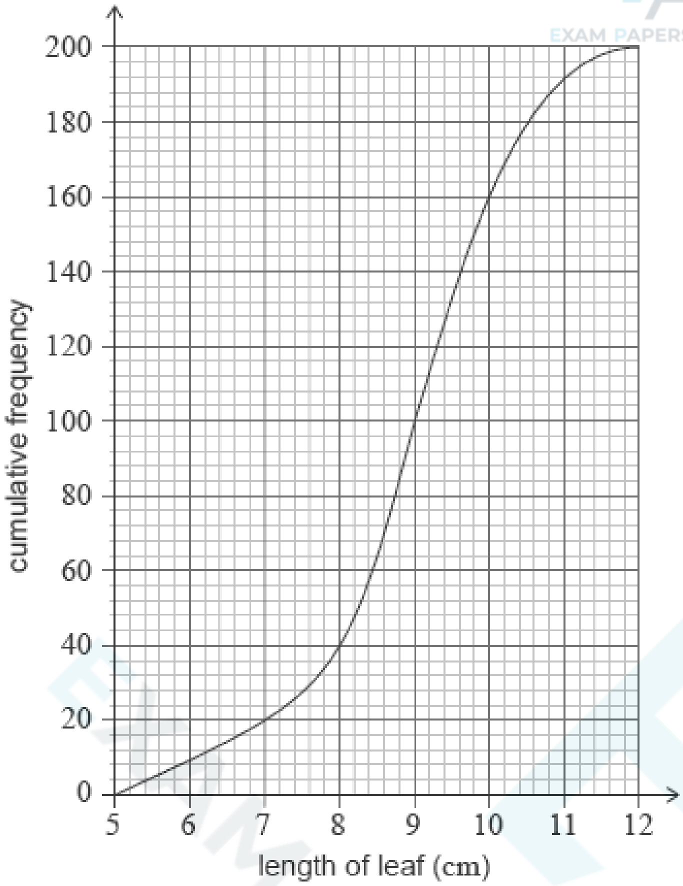
[1]

b. Find the value of a. [2]

c. Estimate the interquartile range of the distribution. [3]

17M.1.SL.TZ2.T_6

For a study, a researcher collected 200 leaves from oak trees. After measuring the lengths of the leaves, in cm, she produced the following cumulative frequency graph.



The researcher finds that 10% of the leaves have a length greater than k cm.

a. Write down the median length of these leaves. [1]

b. Write down the number of leaves with a length less than or equal to 8 cm. [1]

c.i. Use the graph to find the value of k. [2]

c.ii.

Before measuring, the researcher estimated k to be approximately 9.5 cm. Find the percentage error in her estimate.

[2]

17M.1.SL.TZ2.T_2

All the children in a summer camp play at least one sport, from a choice of football (F) or basketball (B). 15 children play both sports.

The number of children who play only football is double the number of children who play only basketball.

Let x be the number of children who play only football.

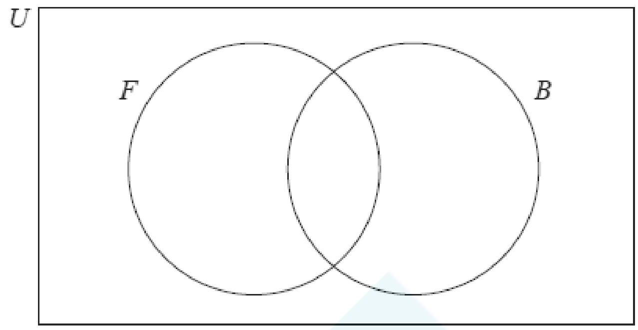
There are 120 children in the summer camp.

a.

Write down an expression, in terms of x, for the number of children who play only basketball.

[1]

b. Complete the Venn diagram using the above information.



[2]

c. Find the number of children who play only football. [2]

d. Write down the value of $n(F)$. [1]

16N.1.SL.TZ0.T_1

Let $p = \frac{\cos x + \sin y}{\sqrt{w^2 - z}}$,

where $x = 36°$, $y = 18°$, $w = 29$ and $z = 21.8$.

a. Calculate the value of p. Write down your full calculator display. [2]

b. Write your answer to part (a) (i) correct to two decimal places;

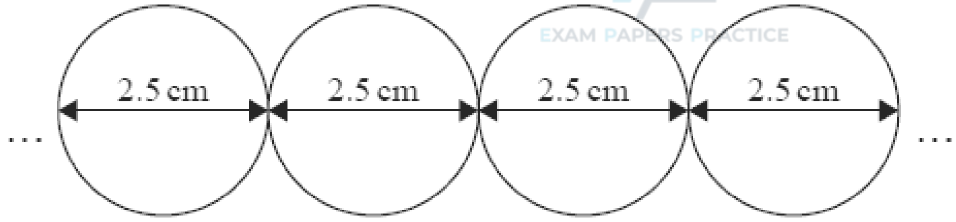
(ii) correct to three significant figures. [2]

c. Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \leqslant a < 10$, $k \in \mathbb{Z}$. [2]

18M.1.SL.TZ2.T_3

Last year a South American candy factory sold 4.8×10^8 spherical sweets. Each sweet has a diameter of 2.5 cm.

The factory is producing an advertisement showing all of these sweets placed in a straight line.



The advertisement claims that the length of this line is x times the length of the Amazon River. The length of the Amazon River is 6400 km.

a.

Find the length, in cm, of this line. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

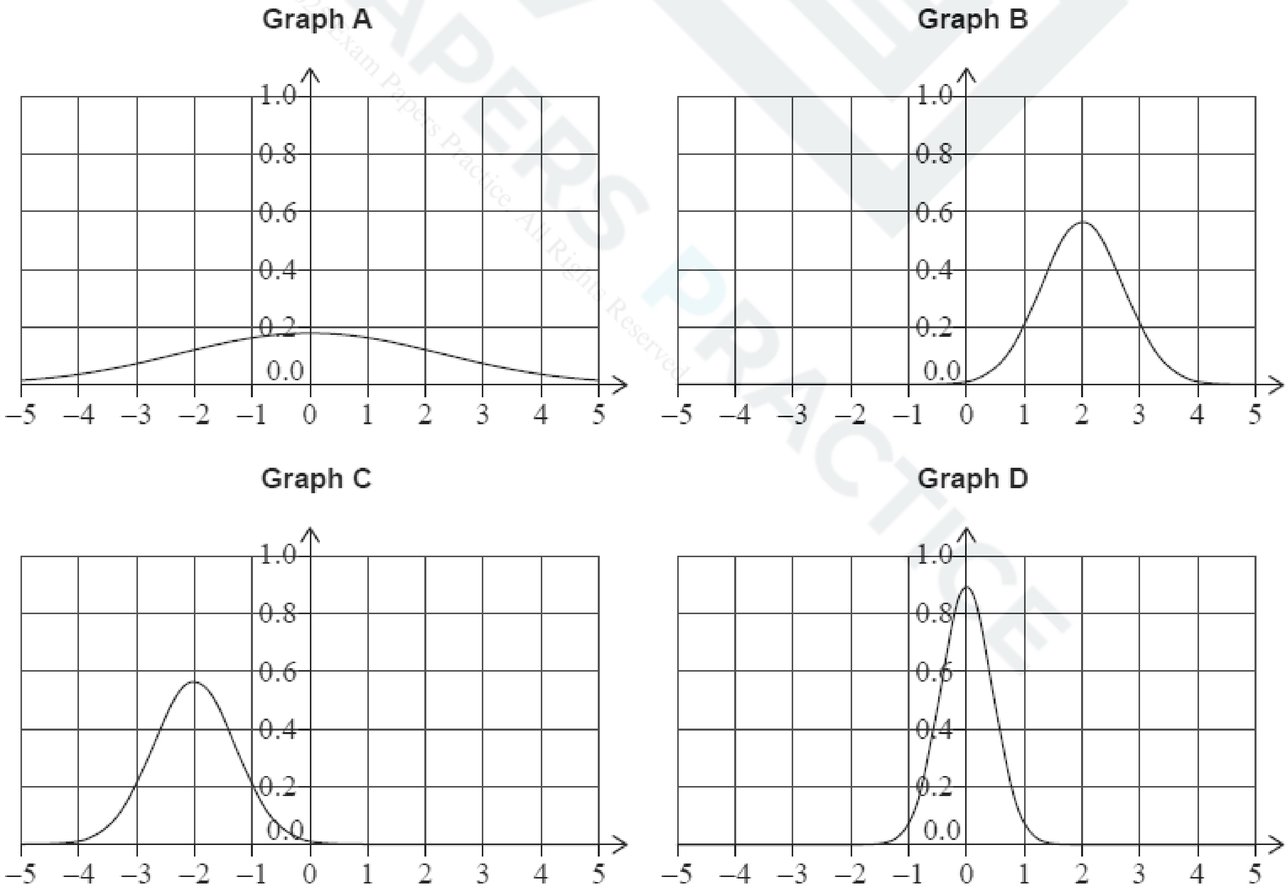
[3]

b.i. Write down the length of the Amazon River in cm. [1]

b.ii. Find the value of x. [2]

19M.1.SL.TZ1.T_11

Consider the following graphs of normal distributions.



At an airport, the weights of suitcases (in kg) were measured. The weights are normally distributed with a mean of 20 kg and standard deviation of 3.5 kg.

In the following table, write down the letter of the corresponding graph next to the given mean and standard deviation.

Mean and standard deviation	Graph
Mean $= -2$; standard deviation $= 0.707$	
Mean $= 0$; standard deviation $= 0.447$	

[2]

b. Find the probability that a suitcase weighs less than 15 kg. [2]

c. Any suitcase that weighs more than k kg is identified as excess baggage. 19.6 % of the suitcases at this airport are identified as excess baggage.

Find the value of k. [2]

17M.1.SL.TZ1.T_2

In the Canadian city of Ottawa:

97% of the population speak English,
38% of the population speak French,
36% of the population speak both English and French.

The total population of Ottawa is 985000.

a.

Calculate the percentage of the population of Ottawa that speak English but not French.

[2]

b. Calculate the number of people in Ottawa that speak both English and French.

[2]

c.

Write down your answer to part (b) in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in Z$.

[2]

18M.1.SL.TZ1.T_7

Sergei is training to be a weightlifter. Each day he trains at the local gym by lifting a metal bar that has heavy weights attached. He carries out successive lifts. After each lift, the same amount of weight is **added** to the bar to increase the weight to be lifted.

ts of each of Sergei's lifts form an arithmetic sequence.

Sergei's friend, Yuri, records the weight of each lift. Unfortunately, last Monday, Yuri misplaced all but two of the recordings of Sergei's lifts.

On that day, Sergei lifted 21 kg on the third lift and 46 kg on the eighth lift.

a.i. For that day find how much weight was added after each lift. [2]

a.ii. For that day find the weight of Sergei's first lift. [2]

b.

On that day, Sergei made 12 successive lifts. Find the total combined weight of these lifts.

[2]

18M.1.SL.TZ1.T_4

A scientist measures the concentration of dissolved oxygen, in milligrams per litre (*y*) , in a river. She takes 10 readings at different temperatures, measured in degrees Celsius (*x*).

The results are shown in the table.

Temperature, °C (x)	20	24	25	28	29	32	27	25	23	21
Dissolved Oxygen, mg l^{-1} (y)	10.9	9.7	9.2	7.6	7.3	6.4	7.9	8.4	9.4	9.9

It is believed that the concentration of dissolved oxygen in the river varies linearly with the temperature.

a.i. For these data, find Pearson's product-moment correlation coefficient, *r*. [2]

a.ii. For these data, find the equation of the regression line *y* on *x*. [2]

b.

Using the equation of the regression line, estimate the concentration of dissolved oxygen in the river when the temperature is 18 °C.

[2]

17M.1.SL.TZ2.T_1

Consider the numbers $p = 2.78 \times 10^{11}$ and $q = 3.12 \times 10^{-3}$.

a. Calculate $\sqrt[3]{\frac{p}{q}}$. Give your full calculator display. [2]

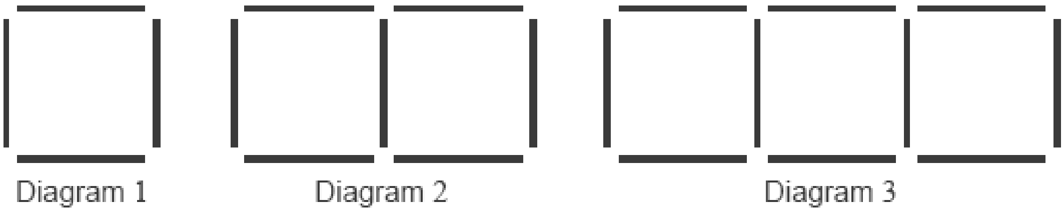
b.i. Write down your answer to part (a) correct to two decimal places; [1]

b.ii. Write down your answer to part (a) correct to three significant figures. [1]

c. Write your answer to in the form $a \times 10^k$, where $1 \leqslant a < 10,\ k \in Z$. [2]

17M.1.SL.TZ2.T_5

Tomás is playing with sticks and he forms the first three diagrams of a pattern. These diagrams are shown below.



Tomás continues forming diagrams following this pattern.

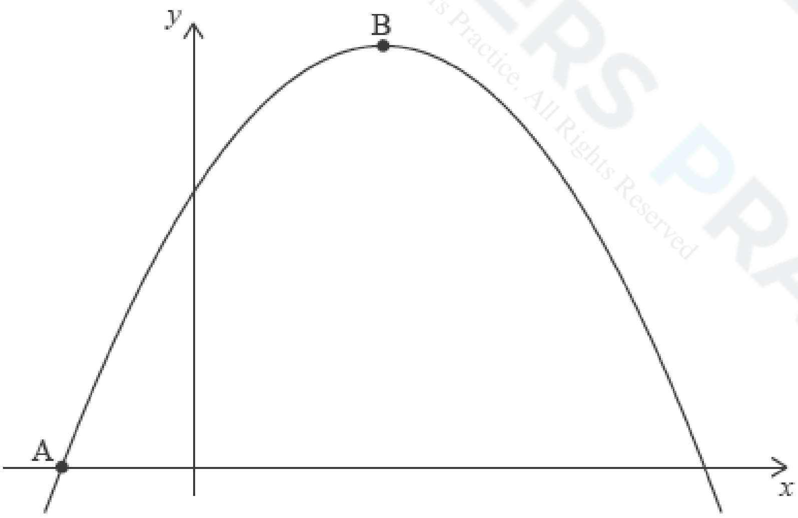
Tomás forms a total of 24 diagrams.

a. Diagram n is formed with 52 sticks. Find the value of n. [3]

b. Find the total number of sticks used by Tomás for all 24 diagrams. [3]

16N.1.SL.TZ0.T_9

The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the x-axis at the point $\mathrm{A}(-1,\ 0)$ and has its vertex at the point $\mathrm{B}(3,\ 16)$.



a. Write down the equation of the axis of symmetry for this graph. [2]

b. Find the value of b. [2]

c. Write down the range of $f(x)$. [2]

17M.1.SL.TZ1.T_5

The company Snakezen's Ladders makes ladders of different lengths. All the ladders that the company makes have the same design such that:

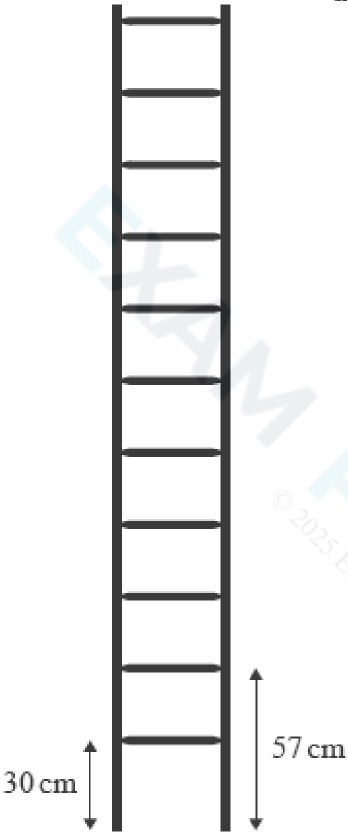
the first rung is 30 cm from the base of the ladder,

the second rung is 57 cm from the base of the ladder,

the distance between the first and second rung is equal to the distance between all adjacent rungs on the ladder.

The ladder in the diagram was made by this company and has eleven equally spaced rungs.

diagram not to scale



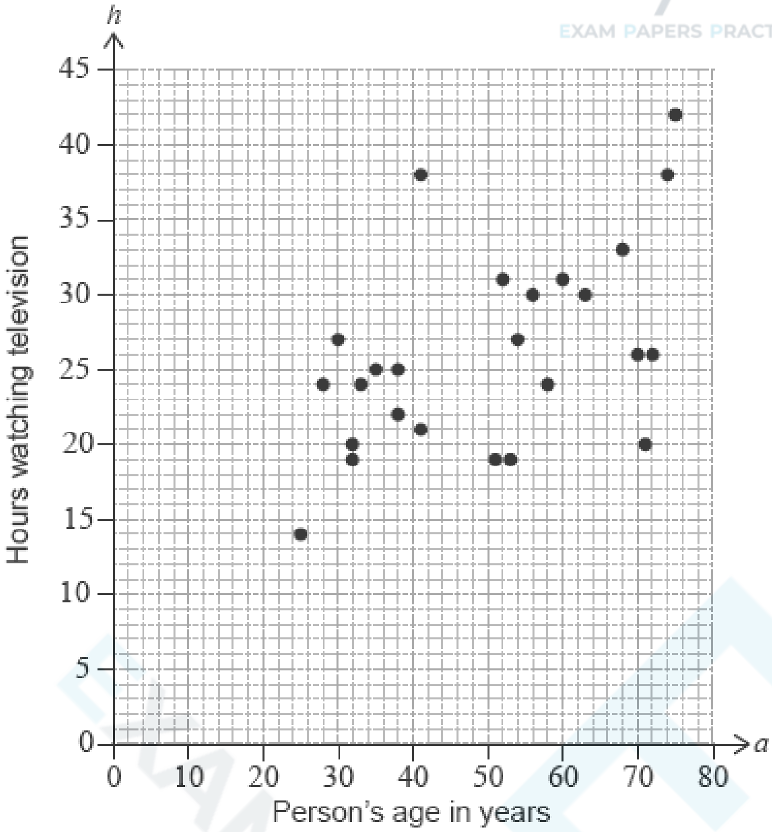
a. Find the distance from the base of this ladder to the top rung. [3]

b. The company also makes a ladder that is 1050 cm long.

Find the maximum number of rungs in this 1050 cm long ladder. [3]

17N.1.SL.TZ0.T_5

A survey was carried out to investigate the relationship between a person's age in years (a) and the number of hours they watch television per week (h). The scatter diagram represents the results of the survey.



The mean age of the people surveyed was 50.

For these results, the equation of the regression line h on a is $h = 0.22a + 15$.

a.

Find the mean number of hours that the people surveyed watch television per week.

[2]

b. Draw the regression line on the scatter diagram. [2]

c.

By placing a tick (✓) in the correct box, determine which of the following statements is true:

The correlation between h and a is positive.	
The correlation between h and a is negative.	
There is no correlation between h and a.	

[1]

d.

Diogo is 18 years old. Give a reason why the regression line should not be used to estimate the number of hours Diogo watches television per week.

[1]

16N.1.SL.TZ0.T_13

A comet orbits the Sun and is seen from Earth every 37 years. The comet was first seen from Earth in the year 1064.

a. Find the year in which the comet was seen from Earth for the fifth time. [3]

b.

Determine how many times the comet has been seen from Earth up to the year 2014.

[3]

17N.1.SL.TZ0.T_15

Maria owns a cheese factory. The amount of cheese, in kilograms, Maria sells in one week, Q , is given by

$$Q = 882 - 45p,$$

where p is the price of a kilogram of cheese in euros (EUR).

Maria earns $(p - 6.80)$ EUR for each kilogram of cheese sold.

To calculate her weekly profit W, in EUR, Maria multiplies the amount of cheese she sells by the amount she earns per kilogram.

a.

Write down how many kilograms of cheese Maria sells in one week if the price of a kilogram of cheese is 8 EUR.

[1]

b.

Find how much Maria earns in one week, from selling cheese, if the price of a kilogram of cheese is 8 EUR.

[2]

c. Write down an expression for W in terms of p. [1]

d. Find the price, p, that will give Maria the highest weekly profit. [2]

17M.1.SL.TZ2.T_9

Consider the geometric sequence $u_1 = 18,\ u_2 = 9,\ u_3 = 4.5,\ \ldots$.

a. Write down the common ratio of the sequence. [1]

b. Find the value of u_5. [2]

c. Find the smallest value of n for which u_n is less than 10^{-3}. [3]

17M.1.SL.TZ1.T_10

The first three terms of a geometric sequence are $u_1 = 486,\ u_2 = 162,\ u_3 = 54$.

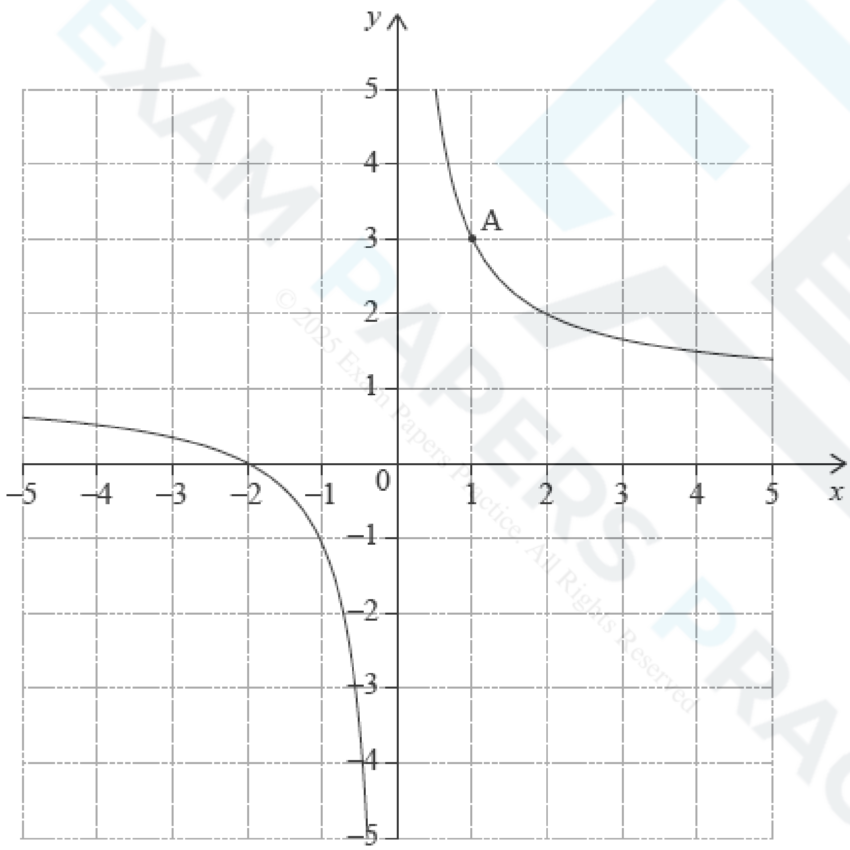
a. Find the value of r, the common ratio of the sequence. [2]

b. Find the value of n for which $u_n = 2$. [2]

c. Find the sum of the first 30 terms of the sequence. [2]

17M.1.SL.TZ2.T_13

The diagram shows part of the graph of a function $y = f(x)$. The graph passes through point $\mathrm{A}(1,\ 3)$.



The tangent to the graph of $y = f(x)$ at A has equation $y = -2x + 5$. Let N be the normal to the graph of $y = f(x)$ at A.

a. Write down the value of $f(1)$. [1]

b.

Find the equation of N. Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in Z$.

[3]

c. Draw the line N on the diagram above. [2]

16N.1.SL.TZ0.T_10

A hydraulic hammer drives a metal post vertically into the ground by striking the top of the post. The distance that the post is driven into the ground, by the nth strike of the hammer, is d_n.

The distances $d_1, d_2, d_3 \ \dots, d_n$ form a geometric sequence.

The distance that the post is driven into the ground by the first strike of the hammer, d_1, is 64 cm.

The distance that the post is driven into the ground by the second strike of the hammer, d_2, is 48 cm.

a. Find the value of the common ratio for this sequence. [2]

b.

Find the distance that the post is driven into the ground by the eighth strike of the hammer.

[2]

c.

Find the **total depth** that the post has been driven into the ground after 10 strikes of the hammer.

[2]

17N.1.SL.TZ0.T_9

Juan buys a bicycle in a sale. He gets a discount of 30% off the original price and pays 560 US dollars (USD).

To buy the bicycle, Juan takes a loan of 560 USD for 6 months at a nominal annual interest rate of 75%, **compounded monthly**. Juan believes that the total amount he will pay will be less than the original price of the bicycle.

a. Calculate the original price of the bicycle. [2]

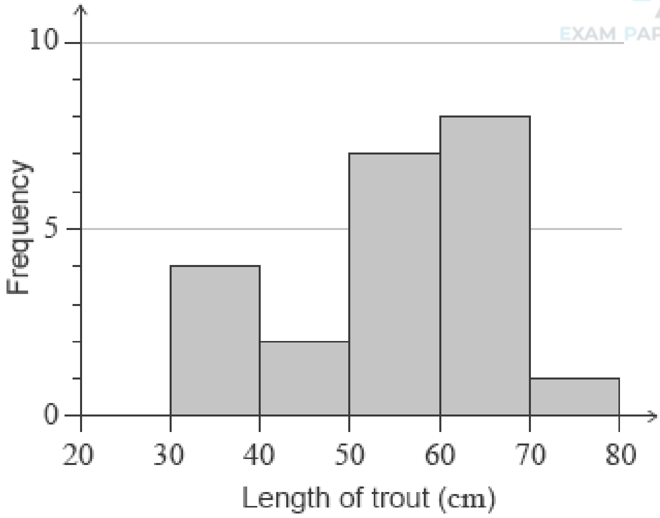
b.

Calculate the difference between the original price of the bicycle and the total amount Juan will pay.

[4]

17M.1.SL.TZ1.T_1

The lengths of trout in a fisherman's catch were recorded over one month, and are represented in the following histogram.



a. Complete the following table. [2]

Length of trout	Frequency
20 cm < trout length ≤ 30 cm	0
30 cm < trout length ≤ 40 cm	
40 cm < trout length ≤ 50 cm	
50 cm < trout length ≤ 60 cm	
60 cm < trout length ≤ 70 cm	
70 cm < trout length ≤ 80 cm	1

b. State whether **length of trout** is a continuous or discrete variable. [1]

c. Write down the modal class. [1]

d. Any trout with length 40 cm or less is returned to the lake.

Calculate the percentage of the fisherman's catch that is returned to the lake. [2]

18M.1.SL.TZ1.T_5

The point A has coordinates (4 , −8) and the point B has coordinates (−2 , 4).

The point D has coordinates (−3 , 1).

a. Write down the coordinates of C, the midpoint of line segment AB. [2]

b. Find the gradient of the line DC. [2]

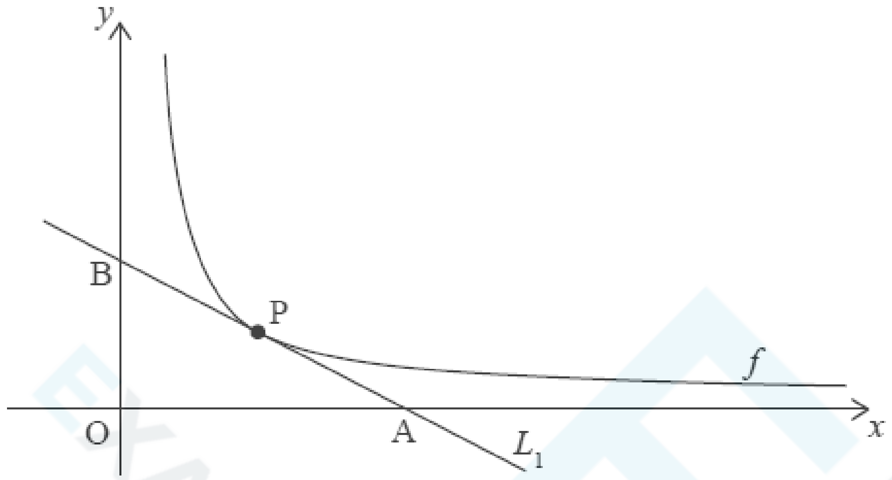
c.

Find the equation of the line DC. Write your answer in the form $ax + by + d = 0$ where a , b and d are integers.

[2]

The following diagram shows part of the graph of $f x = \frac{k}{x}$, for $x > 0, \quad k > 0$.

Let $\text{P}p, \quad \frac{k}{p}$ be any point on the graph of f. Line L_1 is the tangent to the graph of f at P.



Line L_1 intersects the x-axis at point $\text{A}2p, \quad 0$ and the y-axis at point B.

a.i. Find $f'p$ in terms of k and p. [2]

a.ii. Show that the equation of L_1 is $kx + p^2y - 2pk = 0$. [2]

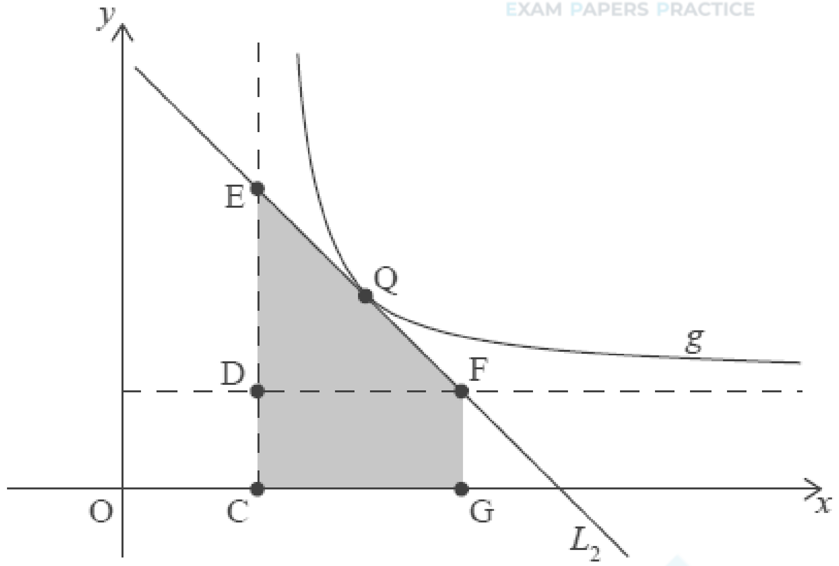
b. Find the area of triangle AOB in terms of k. [5]

c. The graph of f is translated by $\begin{matrix} 4 \\ 3 \end{matrix}$ to give the graph of g.

In the following diagram:

point Q lies on the graph of g
points C, D and E lie on the vertical asymptote of g
points D and F lie on the horizontal asymptote of g
point G lies on the x-axis such that FG is parallel to DC.

Line L_2 is the tangent to the graph of g at Q, and passes through E and F.

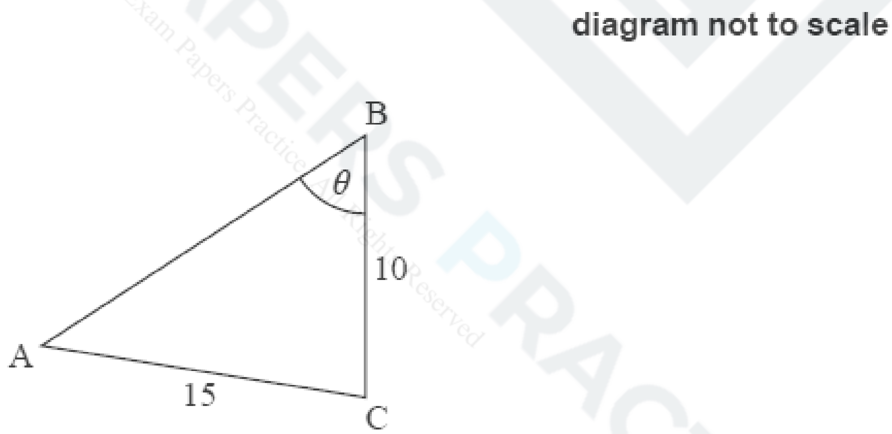


Given that triangle EDF and rectangle CDFG have equal areas, find the gradient of L_2 in terms of p.

[6]

20N.1.SL.TZ0.S_2

The following diagram shows a triangle ABC.



$\text{AC} = 15\ \text{cm}$, $\text{BC} = 10\ \text{cm}$, and $\text{A}\hat{\text{B}}\text{C} = \theta$.

Let $\sin\ \text{C}\hat{\text{A}}\text{B} = \frac{\sqrt{3}}{3}$.

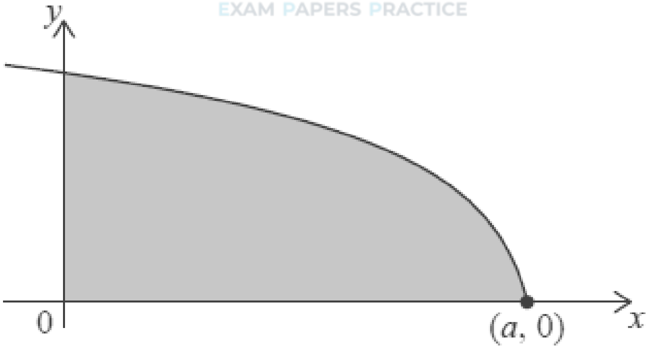
a. Given that $\text{A}\hat{\text{B}}\text{C}$ is acute, find $\sin\ \theta$. [3]

b. Find $\cos\ 2 \times \text{C}\hat{\text{A}}\text{B}$. [3]

20N.1.SL.TZ0.S_3

Let $f x = \sqrt{12 - 2x}$, $x \le a$. The following diagram shows part of the graph of f.

The shaded region is enclosed by the graph of f, the x-axis and the y-axis.



The graph of f intersects the x-axis at the point $a, \quad 0$.

a. Find the value of a. [2]

b.

Find the volume of the solid formed when the shaded region is revolved $360°$ about the x-axis.

[5]

20N.1.SL.TZ0.S_5

Let $fx = -x^2 + 4x + 5$ and $gx = -fx + k$.

Find the values of k so that $gx = 0$ has no real roots.

20N.1.SL.TZ0.S_4

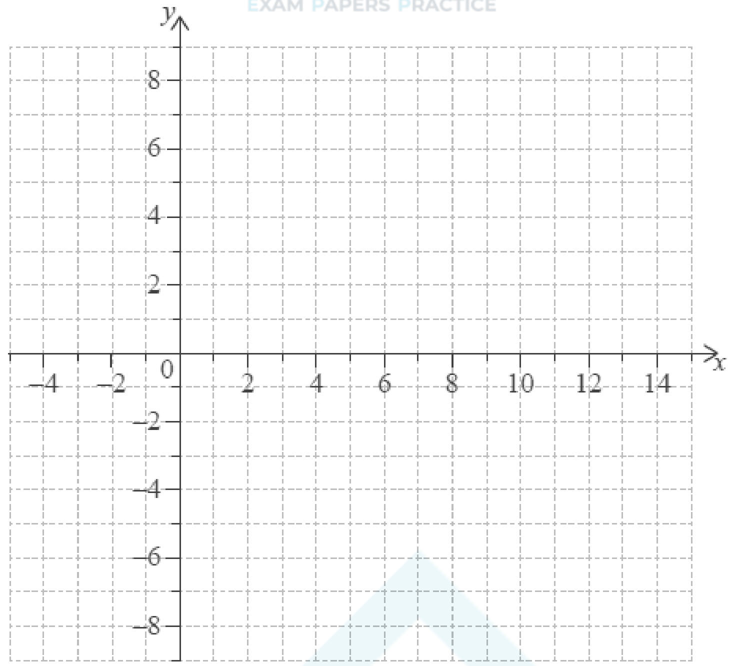
Let $fx = a \quad \log_3 x - 4$, for $x > 4$, where $a > 0$.

Point $\mathrm{A}13, \quad 7$ lies on the graph of f.

a. Find the value of a. [3]

b. The x-intercept of the graph of f is $5, \quad 0$.

On the following grid, sketch the graph of f.



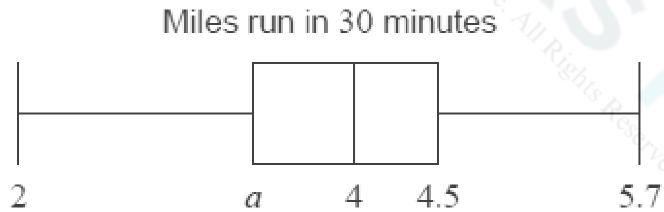
[3]

20N.1.SL.TZ0.S_8

Each athlete on a running team recorded the distance (M miles) they ran in 30 minutes.

The median distance is 4 miles and the interquartile range is 1.1 miles.

This information is shown in the following box-and-whisker plot.

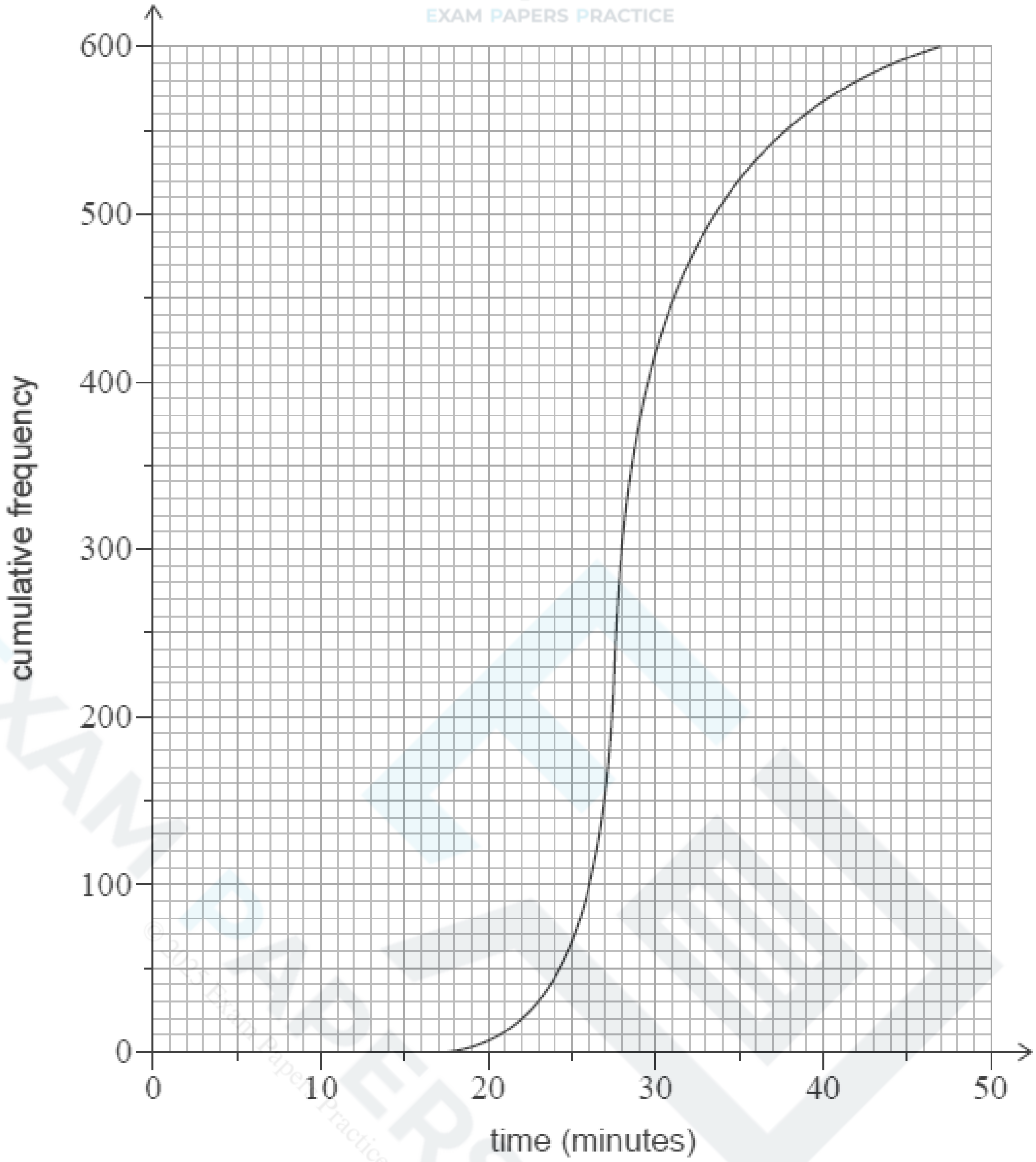


The distance in miles, M, can be converted to the distance in kilometres, K, using the formula $K = \frac{8}{5}M$.

The variance of the distances run by the athletes is $\frac{16}{9}\ \text{km}^2$.

The standard deviation of the distances is b miles.

A total of 600 athletes from different teams compete in a $5\ \text{km}$ race. The times the 600 athletes took to run the $5\ \text{km}$ race are shown in the following cumulative frequency graph.



There were 400 athletes who took between 22 and m minutes to complete the 5 km race.

a. Find the value of a. [2]

b. Write down the value of the median distance in kilometres (km). [1]

c. Find the value of b. [4]

d. Find m. [3]

e. The first 150 athletes that completed the race won a prize.

Given that an athlete took between 22 and m minutes to complete the 5 km race, calculate the probability that they won a prize.

[5]

19M.1.SL.TZ1.T_2

The fastest recorded speeds of eight animals are shown in the following table.

Animal	Speed ($km\ h^{-1}$)
Golden eagle	300
Swordfish	97
Hare	80
Lion	80
Horse	71
Zebra	64
Komodo dragon	21
Tiger beetle	6

a. State whether **speed** is a continuous or discrete variable. [1]

b. Write down the median speed for these animals. [1]

c. Write down the range of the animal speeds. [1]

d.i. For these eight animals find the mean speed. [2]

d.ii. For these eight animals write down the standard deviation. [1]

20N.1.SL.TZ0.S_9

Points A and B have coordinates $1,\ 1,\ 2$ and $9,\ m,\ -6$ respectively.

The line L, which passes through B, has equation $r = \begin{pmatrix} -3 \\ -19 \\ 24 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$.

a. Express $\overrightarrow{AB}$ in terms of m. [2]

b. Find the value of m. [5]

c. Consider a unit vector u, such that $u = pi - \frac{2}{3}j + \frac{1}{3}k$, where $p > 0$.

Point C is such that $\overrightarrow{BC} = 9u$. Find the coordinates of C. [8]

20N.1.SL.TZ0.S_6

The graph of a function f passes through the point $\ln\ 4,\ 20$.

Given that $f'x = 6e^{2x}$, find fx.

20N.1.SL.TZ0.S_7

In this question, all lengths are in metres and time is in seconds.

Consider two particles, P_1 and P_2, which start to move at the same time.

Particle P_1 moves in a straight line such that its displacement from a fixed-point is given by $st = 10 - \frac{7}{4}t^2$, for $t \geq 0$.

a. Find an expression for the velocity of P_1 at time t. [2]

b.

Particle P_2 also moves in a straight line. The position of P_2 is given by $r = \begin{matrix} -1 \\ 6 \end{matrix} + t \begin{matrix} 4 \\ -3 \end{matrix}$.

The speed of P_1 is greater than the speed of P_2 when $t > q$. Find the value of q. [5]

16N.1.SL.TZ0.T_14

The equation of a curve is $y = \frac{1}{2}x^4 - \frac{3}{2}x^2 + 7$.

The gradient of the tangent to the curve at a point P is -10.

a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

b. Find the coordinates of P. [4]

19M.1.SL.TZ2.T_15

A potter sells x vases per month.

His monthly profit in Australian dollars (AUD) can be modelled by

$$P(x) = -\frac{1}{5}x^3 + 7x^2 - 120, x \geqslant 0.$$

a. Find the value of P if no vases are sold. [1]

b. Differentiate $P(x)$. [2]

c. **Hence**, find the number of vases that will maximize the profit. [3]

18N.1.SL.TZ0.T_11

Consider the curve $y = 5x^3 - 3x$.

The curve has a tangent at the point P(−1, −2).

Find the equation of this tangent. Give your answer in the form $y = mx + c$.

17N.1.SL.TZ0.T_11

A quadratic function f is given by $f(x) = ax^2 + bx + c$. The points $(0,\ 5)$ and $(-4,\ 5)$ lie on the graph of $y = f(x)$.

The y-coordinate of the minimum of the graph is 3.

a. Find the equation of the axis of symmetry of the graph of $y = f(x)$. [2]

b. Write down the value of c. [1]

c. Find the value of a and of b. [3]

18M.1.SL.TZ2.T_13

A factory produces shirts. The cost, *C*, in Fijian dollars (FJD), of producing *x* shirts can be modelled by

$$C(x) = (x - 75)^2 + 100.$$

The cost of production should not exceed 500 FJD. To do this the factory needs to produce at least 55 shirts and at most *s* shirts.

a. Find the cost of producing 70 shirts. [2]

b. Find the value of *s*. [2]

c. Find the number of shirts produced when the cost of production is lowest. [2]

20N.1.SL.TZ0.T_1

Iron in the asteroid *16 Psyche* is said to be valued at 8973 quadrillion euros EUR, where one quadrillion $= 10^{15}$.

James believes the asteroid is approximately spherical with radius $113\ \text{km}$. He uses this information to estimate its volume.

a. Write down the value of the iron in the form $a \times 10^k$ where $1 \le a < 10\ ,\ k \in \mathbb{Z}$.

[2]

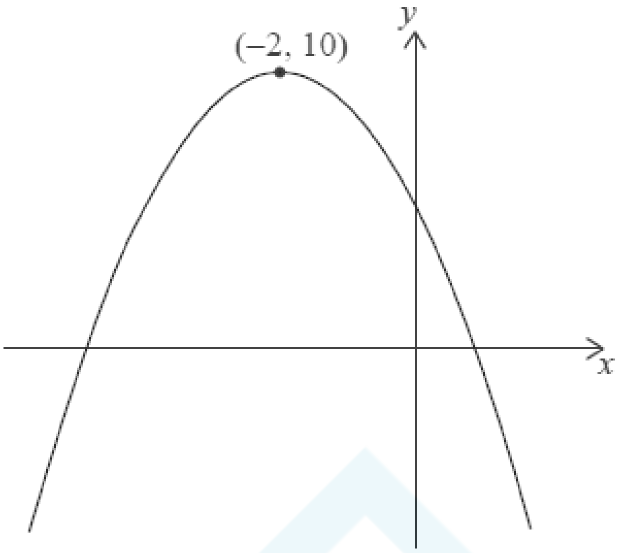
b. Calculate James's estimate of its volume, in km^3. [2]

c. The actual volume of the asteroid is found to be $6.074 \times 10^6\ \text{km}^3$.

Find the percentage error in James's estimate of the volume. [2]

20N.1.SL.TZ0.T_11

The diagram shows the graph of the quadratic function $f(x) = ax^2 + bx + c$, with vertex $-2, \quad 10$.



The equation $f(x) = k$ has two solutions. One of these solutions is $x = 2$.

a. Write down the other solution of $f(x) = k$. [2]

b.

Complete the table below placing a tick (✓) to show whether the unknown parameters a and b are positive, zero or negative. The row for c has been completed as an example.

[2]

	positive	zero	negative
a			
b			
c	✓		

c. State the values of x for which $f(x)$ is decreasing. [2]

20N.1.SL.TZ0.T_13

Consider the graph of the function $f x = x^2 - \frac{k}{x}$.

The equation of the tangent to the graph of $y = f x$ at $x = -2$ is $2y = 4 - 5x$.

a. Write down $f'(x)$. [3]

b. Write down the gradient of this tangent. [1]

c. Find the value of k. [2]

20N.1.SL.TZ0.T_14

Andre will play in the semi-final of a tennis tournament.

If Andre wins the semi-final he will progress to the final. If Andre loses the semi-final, he will **not** progress to the final.

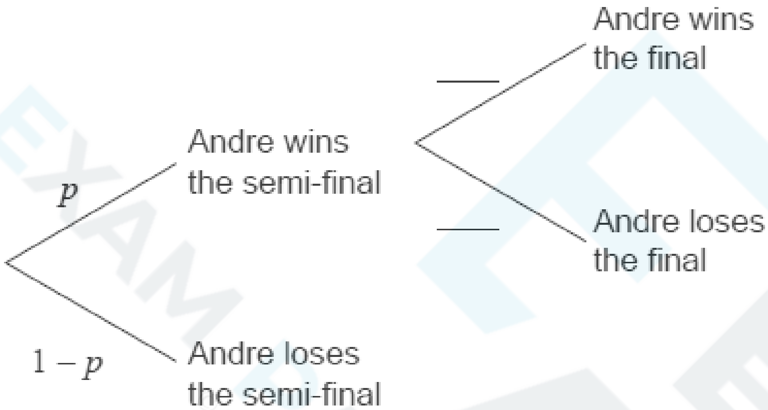
If Andre wins the final, he will be the champion.

The probability that Andre will win the semi-final is p. If Andre wins the semi-final, then the probability he will be the champion is 0.6.

The probability that Andre will not be the champion is 0.58.

a. Complete the values in the tree diagram.

[1]



b. Find the value of p. [2]

c.

Given that Andre did not become the champion, find the probability that he lost in the semi-final.

[3]

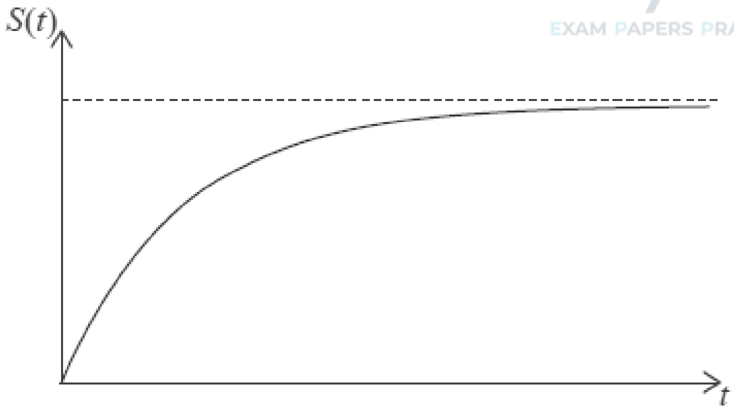
20N.1.SL.TZ0.T_12

Jean-Pierre jumps out of an airplane that is flying at constant altitude. Before opening his parachute, he goes through a period of freefall.

Jean-Pierre's vertical speed during the time of freefall, S, in ms^{-1}, is modelled by the following function.

$$St = K - 601.2^{-t} \quad , \quad t \geq 0$$

where t, is the number of seconds after he jumps out of the airplane, and K is a constant. A sketch of Jean-Pierre's vertical speed against time is shown below.



Jean-Pierre's initial vertical speed is $0\ \text{ms}^{-1}$.

a. Find the value of K. [2]

b. In the context of the model, state what the horizontal asymptote represents. [1]

c. Find Jean-Pierre's vertical speed after 10 seconds. Give your answer in km h^{-1}. [3]

17M.1.SL.TZ2.S_1

In an arithmetic sequence, the first term is 3 and the second term is 7.

a. Find the common difference. [2]

b. Find the tenth term. [2]

c. Find the sum of the first ten terms of the sequence. [2]

17N.1.SL.TZ0.S_2

In an arithmetic sequence, the first term is 8 and the second term is 5.

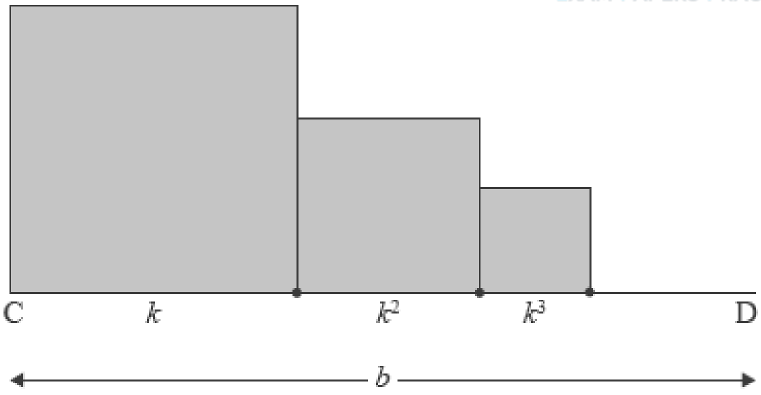
a. Find the common difference. [2]

b. Find the tenth term. [2]

17N.1.SL.TZ0.S_10

The following diagram shows [CD], with length b cm, where $b > 1$. Squares with side lengths k cm, k^2 cm, k^3 cm, ... , where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.

diagram not to scale



The sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.

18N.1.SL.TZ0.S_3

In an arithmetic sequence, $u_1 = -5$ and $d = 3$.

Find u_8.

18M.1.SL.TZ1.S_10

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

a.i. Find an expression for r in terms of θ. [2]

c. Find the values of θ which give the greatest value of the sum. [6]

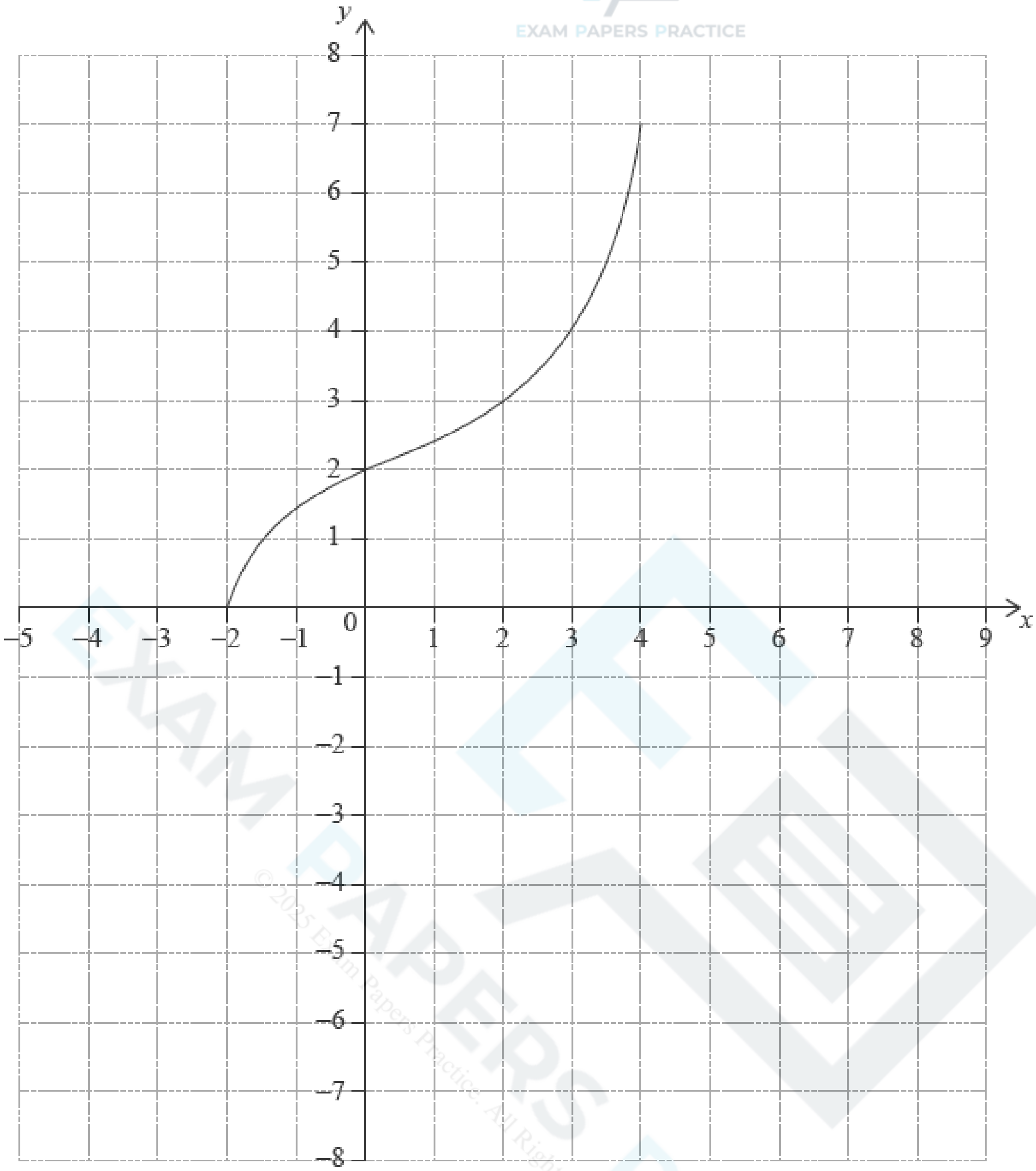
17M.1.SL.TZ1.S_7

The first three terms of a geometric sequence are $\ln x^{16}$, $\ln x^8$, $\ln x^4$, for $x > 0$.

Find the common ratio.

17N.1.SL.TZ0.S_3

The following diagram shows the graph of a function f, with domain $-2 \leqslant x \leqslant 4$.



The points $(-2,\ 0)$ and $(4,\ 7)$ lie on the graph of f.

On the grid, sketch the graph of f^{-1}.

17M.1.SL.TZ2.S_7

Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

17N.1.SL.TZ0.S_5

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in R$, where b is a constant.

a. Find $(g \circ f)(x)$. [2]

b. Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. [4]

16N.1.SL.TZ0.S_1

Let $f(x) = x^2 - 4x + 5$.

The function can also be expressed in the form $f(x) = (x - h)^2 + k$.

(i) Write down the value of h. (ii) Find the value of k.

20N.1.SL.TZ0.T_15

Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence.

The second smallest slice has a volume of 30 cm^3. The fifth smallest slice has a volume of 240 cm^3.

a. Find the common ratio of the sequence. [2]

b. Find the volume of the smallest slice of pie. [2]

c. The apple pie has a volume of $61\ 425 \text{ cm}^3$.

Find the total number of slices Mia can cut from this pie. [2]

20N.1.SL.TZ0.T_2

Olava's Pizza Company supplies and delivers large cheese pizzas.

The total cost to the customer, C, in Papua New Guinean Kina (PGK), is modelled by the function

$$Cn = 34.50n + 8.50 \quad , \quad n \geq 2 \quad , \quad n \in \mathbb{Z},$$

where n, is the number of large cheese pizzas ordered. This total cost includes a fixed cost for delivery.

a.i. State, in the context of the question, what the value of 34.50 represents. [1]

a.ii. State, in the context of the question, what the value of 8.50 represents. [1]

b. Write down the minimum number of pizzas that can be ordered. [1]

c. Kaelani has 450 PGK.

Find the maximum number of large cheese pizzas that Kaelani can order from Olava's Pizza Company.

[3]

20N.1.SL.TZ0.T_3

Hafizah harvested 49 mangoes from her farm. The weights of the mangoes, w, in grams, are shown in the following grouped frequency table.

Weight (g)	$100 \le w < 200$	$200 \le w < 300$	$300 \le w < 400$	$400 \le w < 500$	$500 \le w < 600$
Frequency	4	7	14	16	8

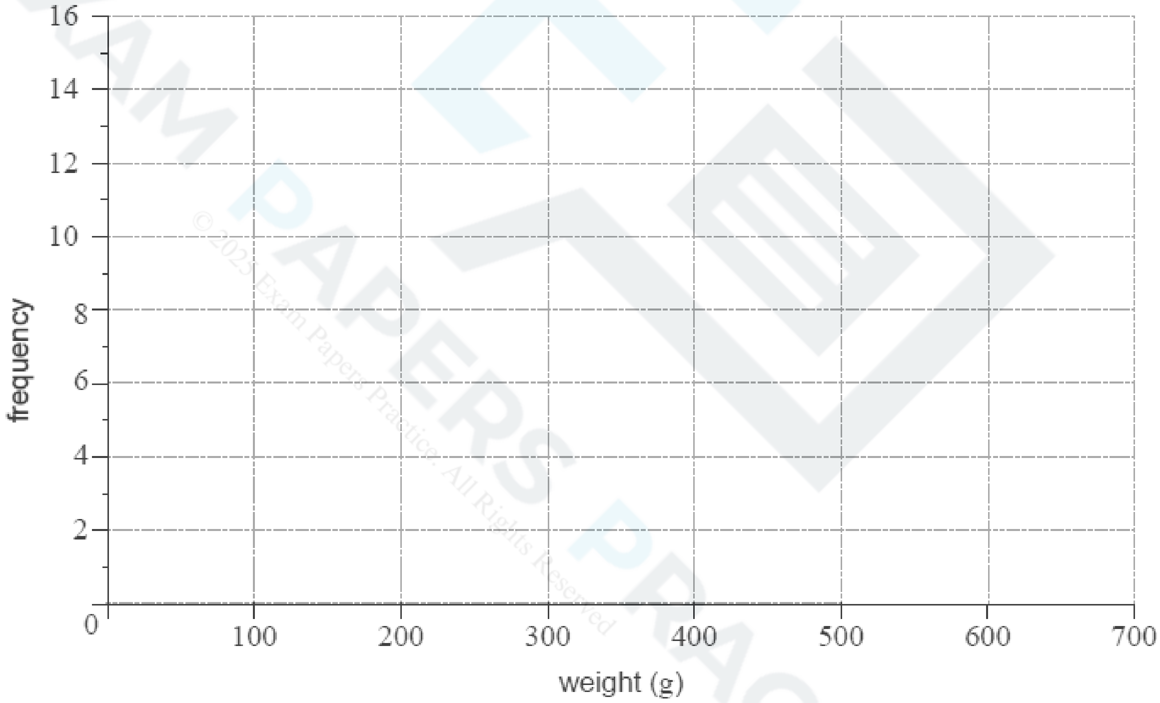
a. Write down the modal group for these data. [1]

b.

Use your graphic display calculator to find an estimate of the standard deviation of the weights of mangoes from this harvest.

[2]

c. On the grid below, draw a histogram for the data in the table.



[3]

18M.1.SL.TZ2.T_7

In an international competition, participants can answer questions in **only one** of the three following languages: Portuguese, Mandarin or Hindi. 80 participants took part in the competition. The number of participants answering in Portuguese, Mandarin or Hindi is shown in the table.

		Languages			
		Portuguese	Mandarin	Hindi	**Total**
Participants	Boys	20	18	5	**43**
	Girls	18	7	12	**37**
	Total	**38**	**25**	**17**	**80**

A boy is chosen at random.

a. State the number of boys who answered questions in Portuguese. [1]

b. Find the probability that the boy answered questions in Hindi. [2]

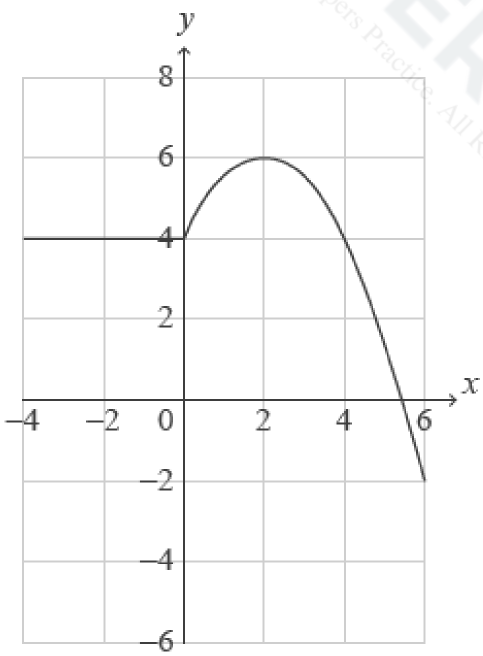
c. Two girls are selected at random.

Calculate the probability that one girl answered questions in Mandarin and the other answered questions in Hindi.

[3]

21M.1.SL.TZ1.1

The graph of $y = fx$ for -4 $\leq x \leq 6$ is shown in the following diagram.



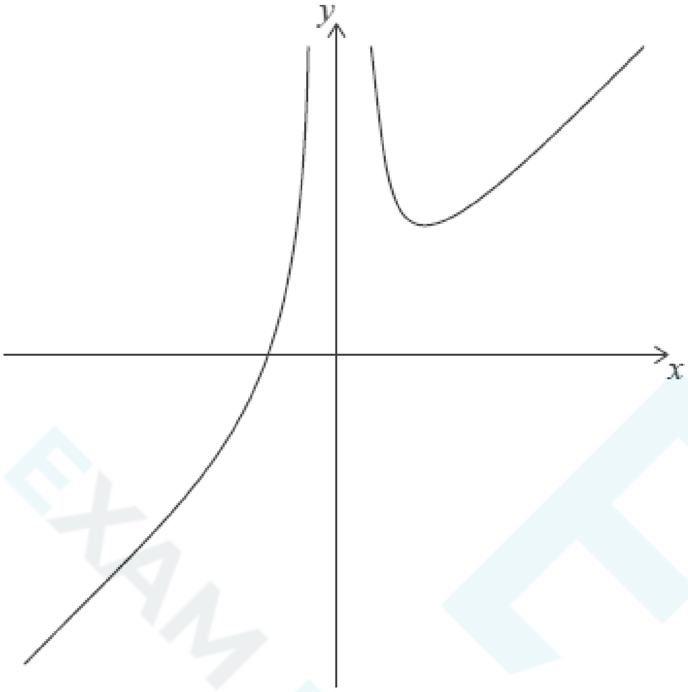
a.i. Write down the value of $f2$. [1]

a.ii. Write down the value of $f \circ f2$. [1]

b. Let $gx = \frac{1}{2}fx + 1$ for -4 $\leq x \leq 6$. On the axes above, sketch the graph of g. [3]

20N.1.SL.TZ0.T_4

Consider the graph of the function $f x = x + \frac{12}{x^2}, \quad x \neq 0$.



a.i. Write down the zero of $f x$. [2]

a.ii. Write down the coordinates of the local minimum point. [2]

b. Consider the function $g x = 3 - x$. Solve $f x = g x$. [2]

21M.1.SL.TZ1.2

The diameter of a spherical planet is 6×10^4 km.

a. Write down the radius of the planet. [1]

b.

The volume of the planet can be expressed in the form $\pi a \times 10^k$ km^3 where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

Find the value of a and the value of k. [3]

18N.1.SL.TZ0.S_5

Consider the vectors ***a*** = $\begin{pmatrix} 3 \\ 2p \end{pmatrix}$ and ***b*** = $\begin{pmatrix} p+1 \\ 8 \end{pmatrix}$.

Find the possible values of *p* for which ***a*** and ***b*** are parallel.

17M.1.SL.TZ2.T_7

A tetrahedral (four-sided) die has written on it the numbers 1, 2, 3 and 4. The die is rolled many times and the scores are noted. The table below shows the resulting frequency distribution.

Score	1	2	3	4
Frequency	18	x	y	22

The die was rolled a total of 100 times.

The mean score is 2.71.

a.

Write down an equation, in terms of x and y, for the total number of times the die was rolled.

[1]

b. Using the mean score, write down a second equation in terms of x and y. [2]

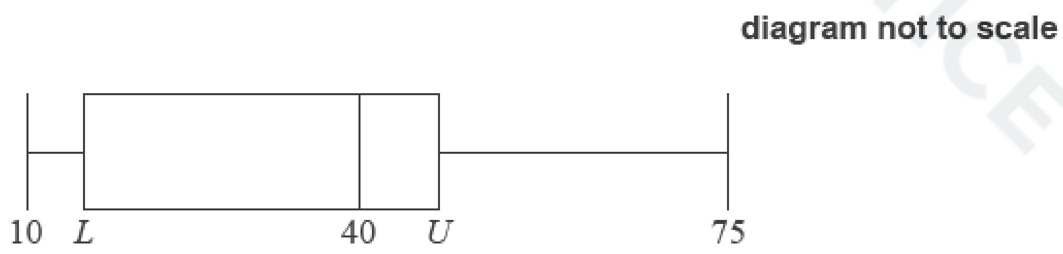
c. Find the value of x and of y. [3]

21M.1.SL.TZ1.3

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d.

21M.1.SL.TZ1.4

A research student weighed lizard eggs in grams and recorded the results. The following box and whisker diagram shows a summary of the results where L and U are the lower and upper quartiles respectively.



The interquartile range is 20 grams and there are no outliers in the results.

a. Find the minimum possible value of U. [3]

b. Hence, find the minimum possible value of L. [2]

21M.1.SL.TZ1.5

Consider the functions $f x = -x - h^2 + 2k$ and $g x = e^{x-2} + k$ where $h, \ k \in \mathbb{R}$.

The graphs of f and g have a common tangent at $x = 3$.

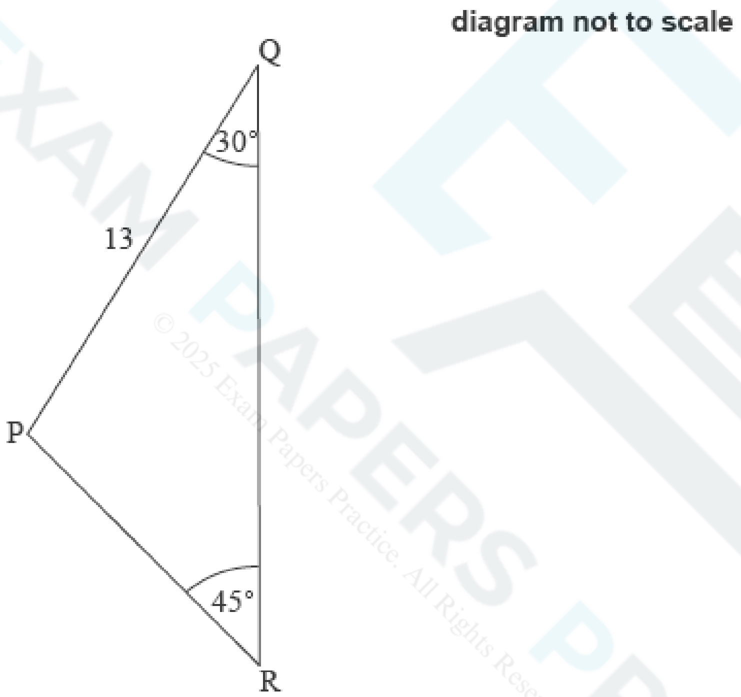
a. Find $f'x$. [1]

b. Show that $h = \frac{e+6}{2}$. [3]

c. Hence, show that $k = e + \frac{e^2}{4}$. [3]

17M.1.SL.TZ1.S_3

The following diagram shows triangle PQR.

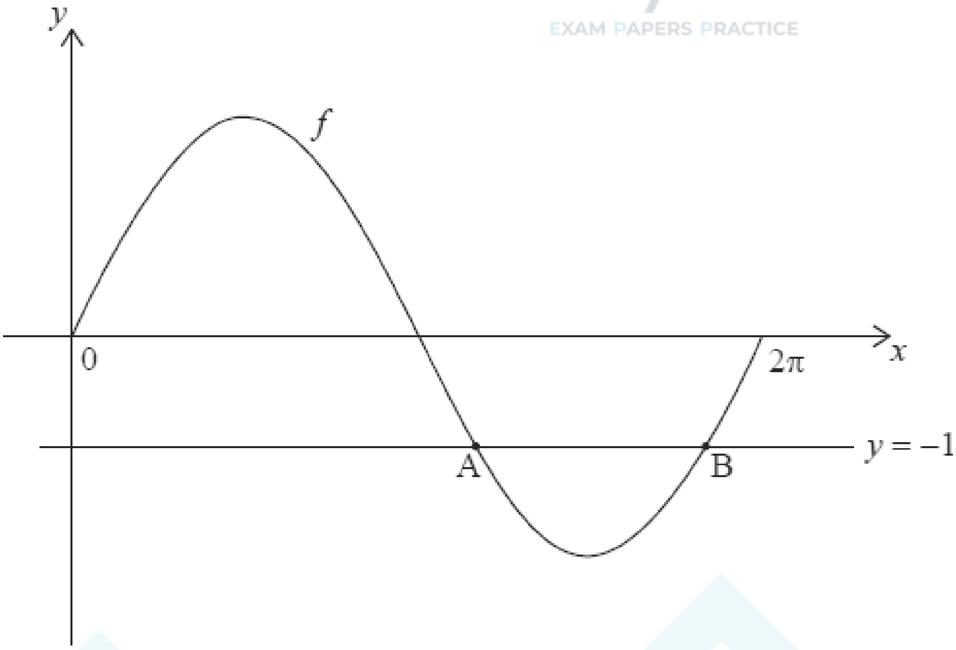


$P\hat{Q}R = 30°$, $Q\hat{R}P = 45°$ and $PQ = 13$ cm.

Find PR.

19M.1.SL.TZ2.S_7

Consider the graph of the function $f(x) = 2\sin x$, $0 \le x < 2\pi$. The graph of f intersects the line $y = -1$ exactly twice, at point A and point B. This is shown in the following diagram.

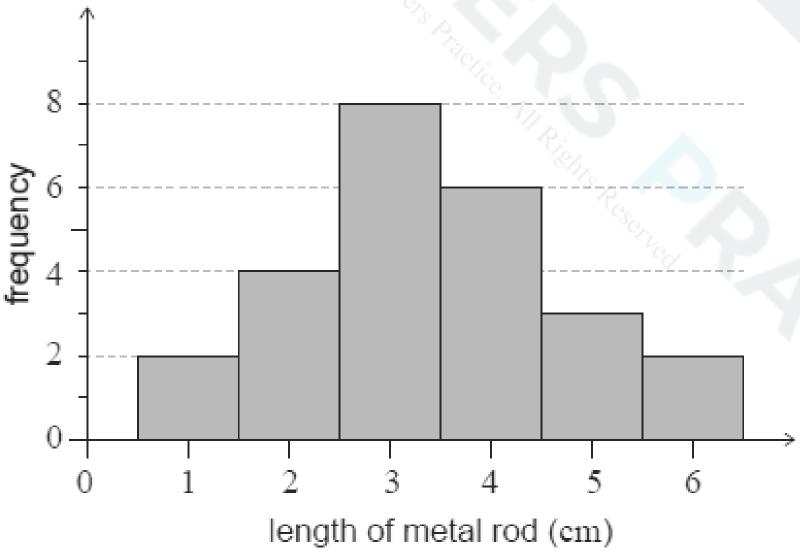


Consider the graph of $g(x) = 2\sin px$, $0 \leq x < 2\pi$, where $p > 0$.

Find the greatest value of p such that the graph of g does not intersect the line $y = -1$.

18N.1.SL.TZ0.T_2

The histogram shows the lengths of 25 metal rods, each measured correct to the nearest cm.



The upper quartile is 4 cm.

a. Write down the modal length of the rods. [1]

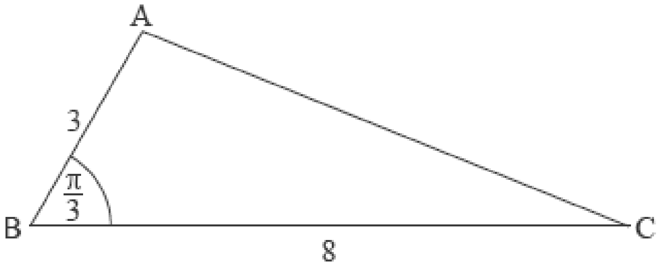
b. Find the median length of the rods. [3]

c.i. Calculate the lower quartile. [1]

c.ii. Calculate the interquartile range. [1]

17N.1.SL.TZ0.S_4

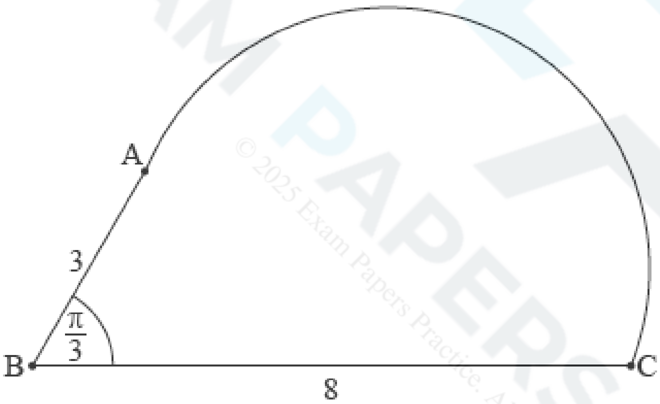
The following diagram shows triangle ABC, with $AB = 3$ cm, $BC = 8$ cm, and $A\hat{B}C = \frac{\pi}{3}$.



a. Show that $AC = 7$ cm. [4]

b.

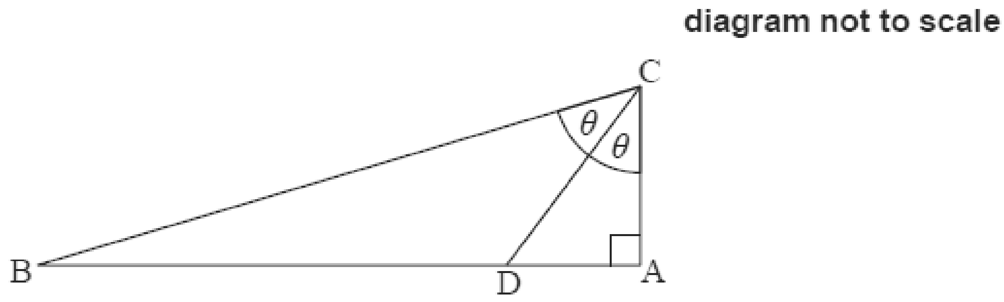
The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle.



Find the exact perimeter of this shape. [3]

19M.1.SL.TZ1.S_3

The following diagram shows a right triangle ABC. Point D lies on AB such that CD bisects $A\hat{C}B$.



$A\hat{C}D = \theta$ and AC = 14 cm

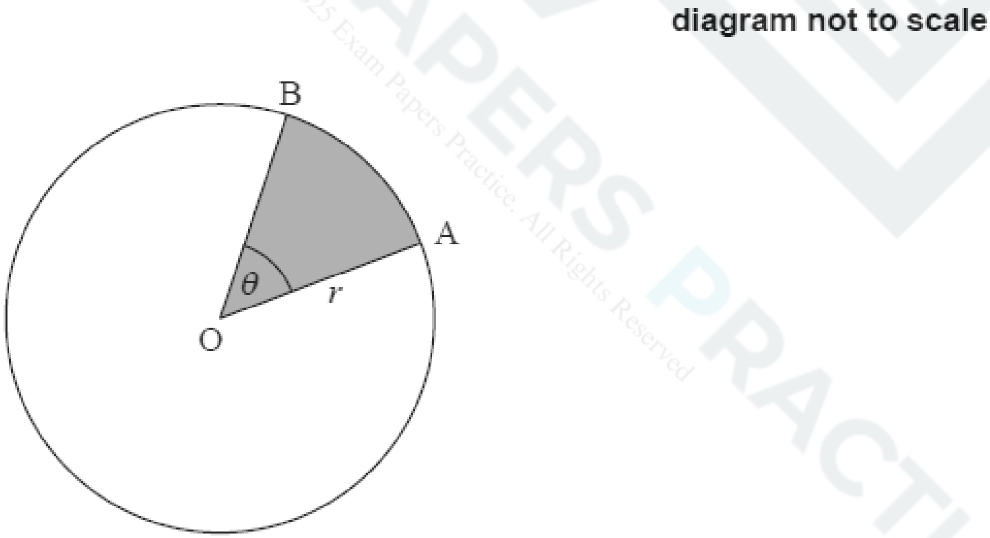
a. Given that $\sin\theta = \frac{3}{5}$, find the value of $\cos\theta$. [3]

b. Find the value of $\cos 2\theta$. [3]

c. Hence or otherwise, find BC. [2]

18M.1.SL.TZ2.S_4

The following diagram shows a circle with centre O and radius *r* cm.



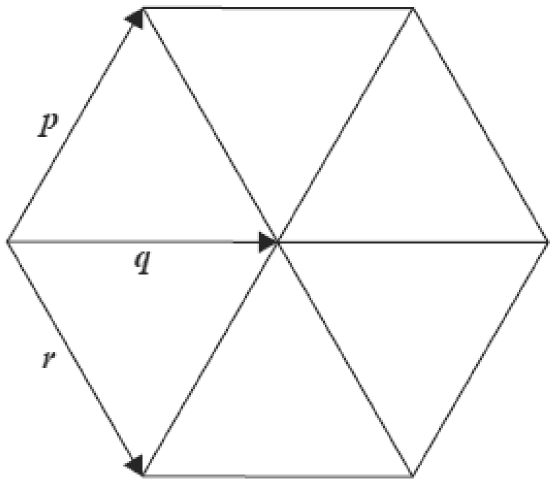
The points A and B lie on the circumference of the circle, and $A\hat{O}B = \theta$. The area of the shaded sector AOB is 12 cm^2 and the length of arc AB is 6 cm.

Find the value of *r*.

18M.1.SL.TZ1.S_6

Six equilateral triangles, each with side length 3 cm, are arranged to form a hexagon. This is shown in the following diagram.

diagram not to scale



The vectors , and are shown on the diagram.

Find •(+ +).

19M.1.SL.TZ1.S_6

The magnitudes of two vectors, ***u*** and ***v***, are 4 and $\sqrt{3}$ respectively. The angle between ***u*** and ***v*** is $\frac{\pi}{6}$.

Let ***w*** = ***u*** − ***v***. Find the magnitude of ***w***.

19M.1.SL.TZ2.S_9

Let θ be an **obtuse** angle such that $\sin\theta = \frac{3}{5}$.

Let $f(x) = e^x \sin x - \frac{3x}{4}$.

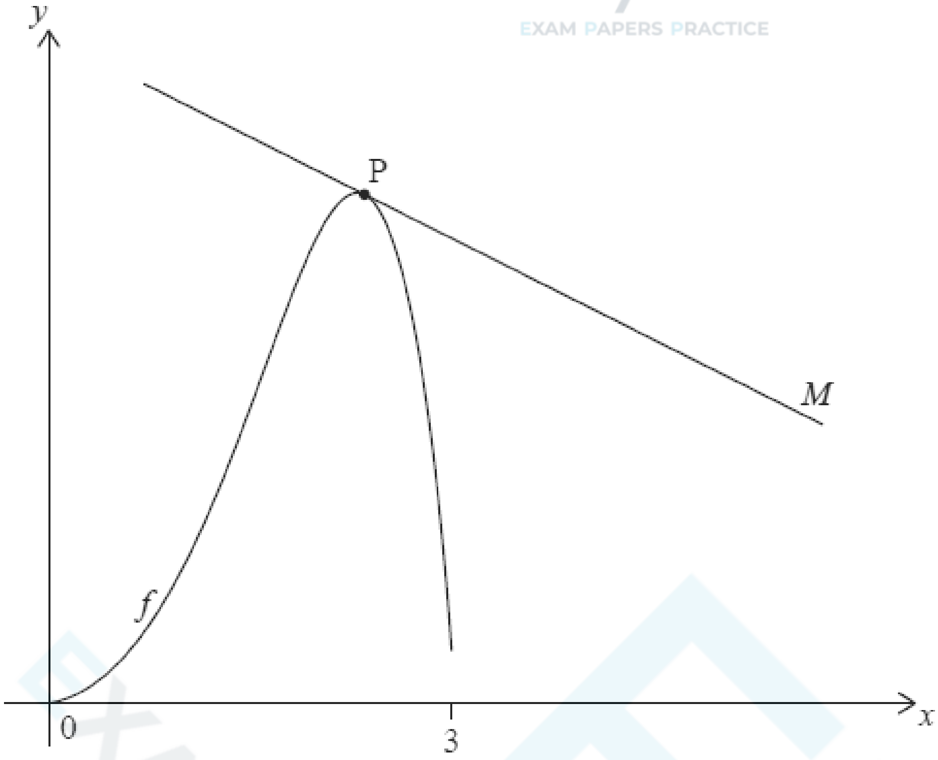
a. Find the value of $\tan\theta$. [4]

b. Line L passes through the origin and has a gradient of $\tan\theta$. Find the equation of L.

[2]

d.

The following diagram shows the graph of f for $0 \le x \le 3$. Line M is a tangent to the graph of f at point P.



Given that M is parallel to L, find the x-coordinate of P. [4]

21M.1.SL.TZ1.7

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

The function f can be expressed in the form $f(x) = 4(x - p)(x - q)$, where $p, \ q \in \mathbb{R}$.

The function f can also be expressed in the form $f(x) = 4(x - h)^2 + k$, where $h, \ k \in \mathbb{R}$.

a. Show that $m = 4$. [6]

b. Find the value of p and the value of q. [2]

c. Find the value of h and the value of k. [3]

d. Hence find the values of x where the graph of f is both negative and increasing. [3]

21M.1.SL.TZ1.8

Let $y = \frac{\ln x}{x^4}$ for $x > 0$.

Consider the function defined by $f(x) = \frac{\ln x}{x^4}$ for $x > 0$ and its graph $y = f(x)$.

a. Show that $\frac{dy}{dx} = \frac{1 - 4\ln x}{x^5}$. [3]

b. The graph of f has a horizontal tangent at point P. Find the coordinates of P. [5]

c. Given that $f''(x) = \frac{20\ln x - 9}{x^6}$, show that P is a local maximum point. [3]

d. Solve $f(x) > 0$ for $x > 0$. [2]

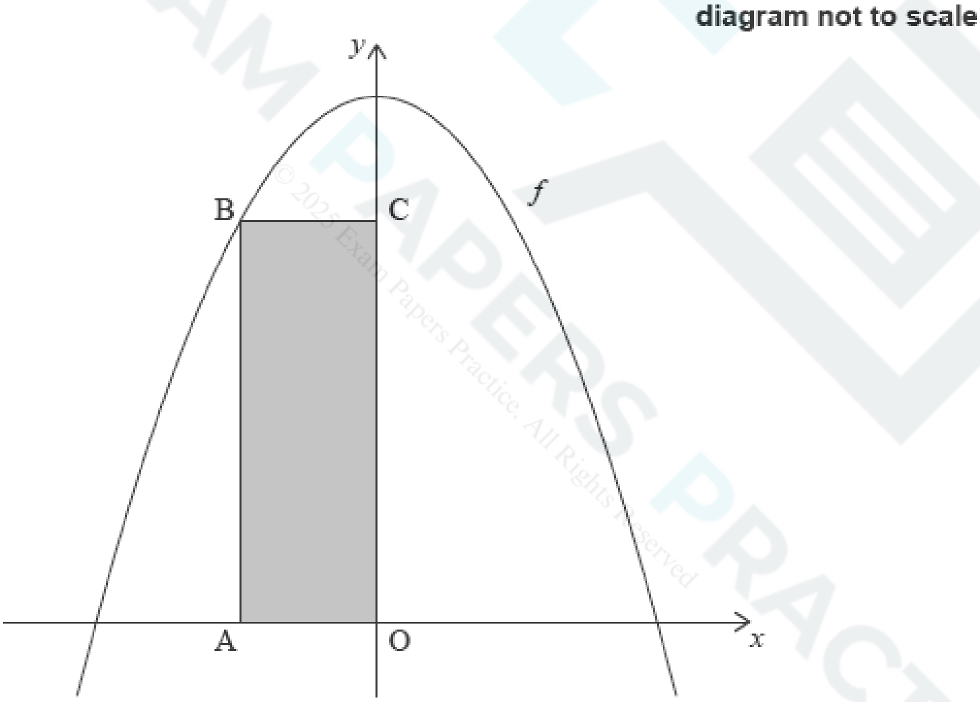
e.

Sketch the graph of f, showing clearly the value of the x-intercept and the approximate position of point P.

[3]

17N.1.SL.TZ0.S_6

Let $f(x) = 15 - x^2$, for $x \in R$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis.



Find the x-coordinate of A that gives the maximum area of OABC.

16N.1.SL.TZ0.S_2

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

a. Find $\cos\theta$. [3]

b. Find $\cos 2\theta$. [2]

18N.1.SL.TZ0.S_7

Given that $\sin x = \frac{1}{3}$, where $0 < x < \frac{\pi}{2}$, find the value of $\cos 4x$.

21M.1.SL.TZ1.9

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$\mathrm{P}(X=x)$	p	p	p	$\frac{1}{2}p$

A second biased four-sided die, B, is rolled. Let Y be the score obtained when die B is rolled.
The probability distribution for Y is given in the following table.

y	1	2	3	4
$\mathrm{P}(Y=y)$	q	q	q	r

a. Find the value of p. [2]

b. Hence, find the value of $\mathrm{E}(X)$. [2]

c.i. State the range of possible values of r. [1]

c.ii. Hence, find the range of possible values of q. [2]

d. Hence, find the range of possible values for $\mathrm{E}(Y)$. [3]

e.

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is $\frac{1}{2}$.

Find the value of $\mathrm{E}(Y)$. [6]

19M.1.SL.TZ1.T_5

A florist sells bouquets of roses. The florist recorded, in **Table 1**, the number of roses in each bouquet sold to customers.

Table 1

Number of roses in a bouquet (n)	2	3	4	5	6	7	8	9	10	11	12
Number of customers (f)	9	2	4	5	7	3	10	2	3	1	4

The roses can be arranged into bouquets of size small, medium or large. The data from **Table 1** has been organized into a cumulative frequency table, **Table 2**.

Table 2

Bouquet size	Number of roses (n)	Frequency (f)	Cumulative frequency
small	$2 \le n \le 4$	15	
medium	$5 \le n \le 8$	25	
large	$9 \le n \le 12$		

a. Complete the cumulative frequency table. [2]

b. Write down the probability that a bouquet of roses sold is small. [2]

c. A customer buys a large bouquet.

Find the probability that there are 12 roses in this bouquet. [2]

17M.1.SL.TZ2.S_9

Note: In this question, distance is in metres and time is in seconds.

Two particles P_1 and P_2 start moving from a point A at the same time, along different straight lines.

After t seconds, the position of P_1 is given by $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

Two seconds after leaving A, P_1 is at point B.

Two seconds after leaving A, P_2 is at point C, where $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

a. Find the coordinates of A. [2]

b.i. Find $\overrightarrow{AB}$; [3]

b.ii. Find $\left|\overrightarrow{AB}\right|$. [2]

c. Find $\cos B\hat{A}C$. [5]

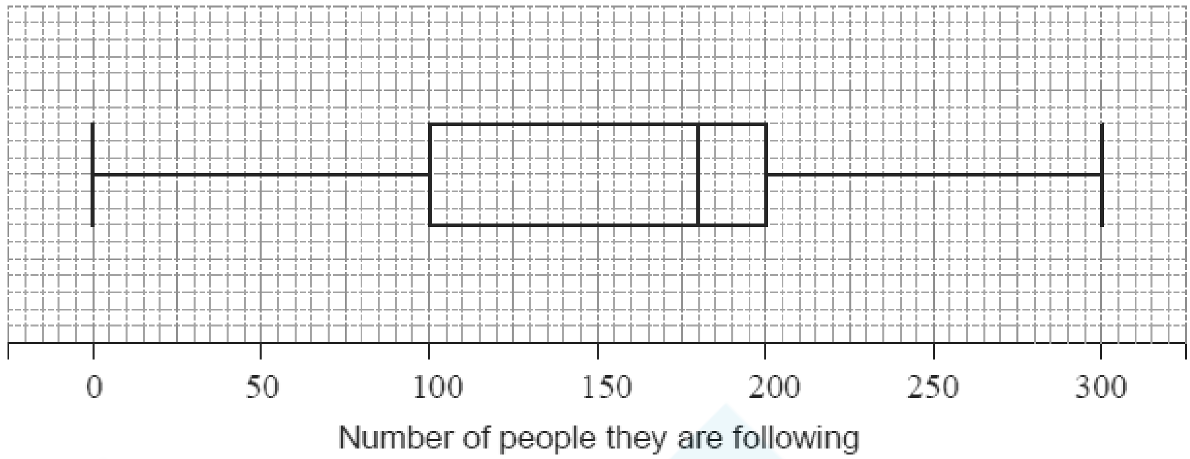
d.

Hence or otherwise, find the distance between P_1 and P_2 two seconds after they leave A.

[4]

18M.1.SL.TZ1.T_6

In a high school, 160 students completed a questionnaire which asked for the number of people they are following on a social media website. The results were recorded in the following box-and-whisker diagram.



The following incomplete table shows the distribution of the responses from these 160 students.

Number of people they are following (x)	Number of high school students
$0 \le x \le 50$	4
$50 < x \le 100$	
$100 < x \le 150$	34
$150 < x \le 200$	46
$200 < x \le 250$	
$250 < x \le 300$	16

a. Write down the median. [1]

b. Complete the table. [2]

c.i. Write down the mid-interval value for the $100 < x \le 150$ group. [1]

c.ii.

Using the table, calculate an estimate for the mean number of people being followed on the social media website by these 160 students.

[2]

17N.1.SL.TZ0.S_9

A line L passes through points $\mathrm{A}(-3, 4, 2)$ and $\mathrm{B}(-1, 3, 3)$.

The line L also passes through the point $\mathrm{C}(3, 1, p)$.

a.i. Show that $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. [1]

a.ii. Find a vector equation for L. [2]

b. Find the value of p. [5]

c.

The point D has coordinates $(q^2, 0, q)$. Given that $\overrightarrow{DC}$ is perpendicular to L, find the possible values of q.

[7]

18M.1.SL.TZ1.S_9

Point A has coordinates (−4, −12, 1) and point B has coordinates (2, −4, −4).

The line *L* passes through A and B.

a. Show that $\vec{AB} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$ [1]

b.i. Find a vector equation for *L*. [2]

b.ii. Point *C* (*k* , 12 , −*k*) is on *L*. Show that *k* = 14. [4]

c.i. Find $\vec{OB} \cdot \vec{AB}$. [2]

c.ii. Write down the value of angle OBA. [1]

d. Point D is also on *L* and has coordinates (8, 4, −9). Find the area of triangle OCD.

[6]

17M.1.SL.TZ2.S_2

The vectors $\boldsymbol{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} k+3 \\ k \end{pmatrix}$ are perpendicular to each other.

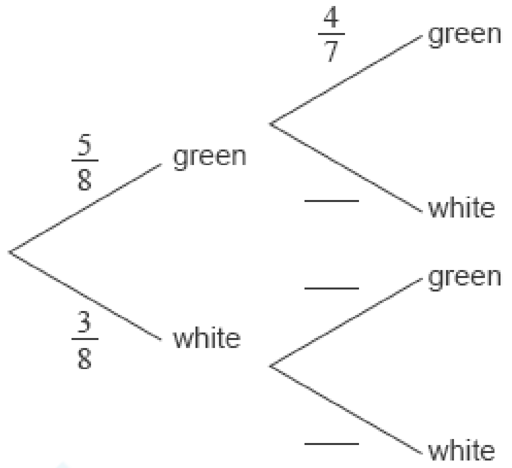
a. Find the value of k. [4]

b. Given that $\boldsymbol{c} = \boldsymbol{a} + 2\boldsymbol{b}$, find $\boldsymbol{c}$. [3]

17N.1.SL.TZ0.S_1

A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

a. Complete the following tree diagram. [3]

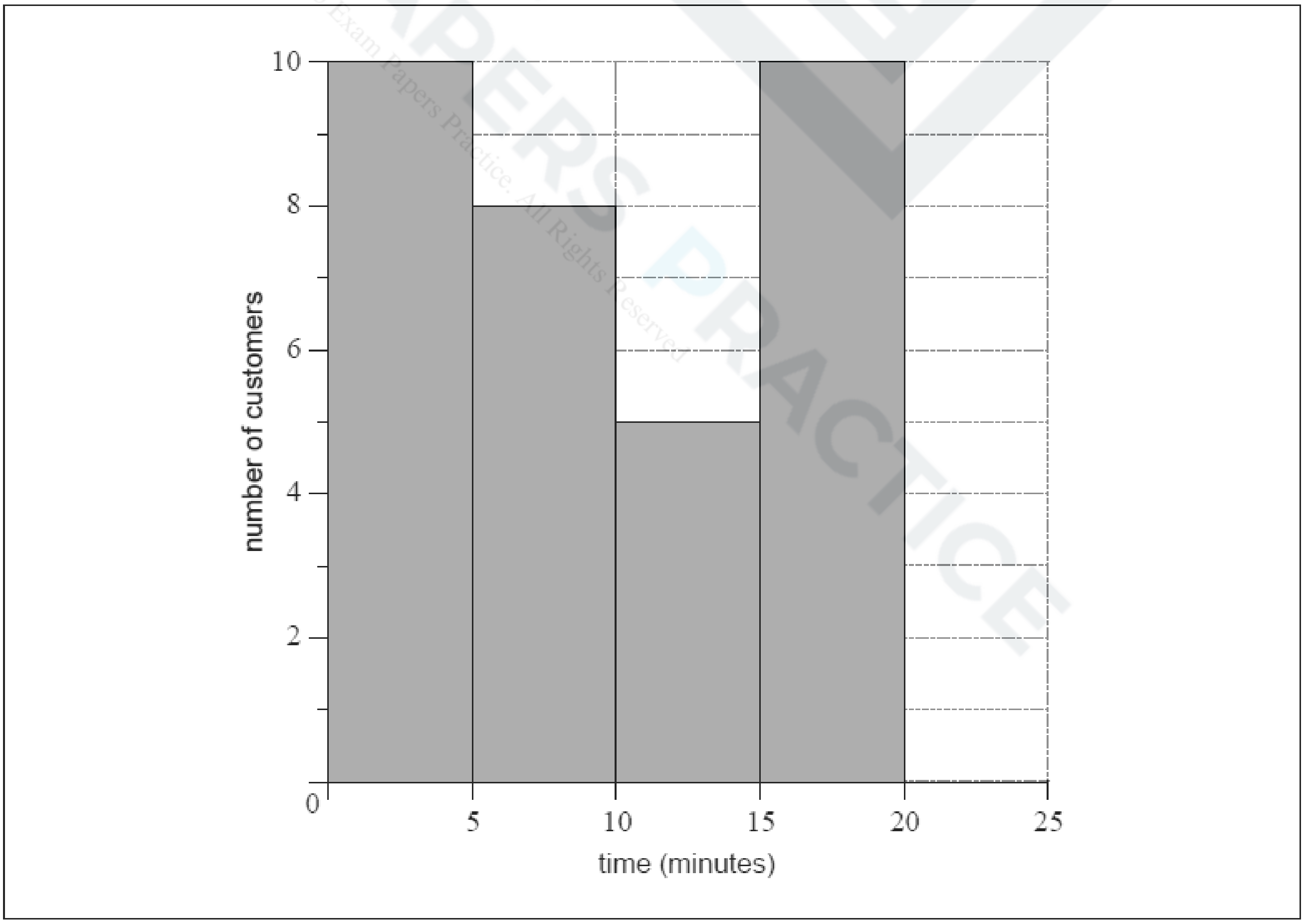


b. Find the probability that exactly one of the selected balls is green. [3]

18M.1.SL.TZ2.T_12

The histogram shows the time, t, in minutes, that it takes the customers of a restaurant to eat their lunch on one particular day. Each customer took less than 25 minutes.

The histogram is incomplete, and only shows data for $0 \leq t < 20$.



The mean time it took **all** customers to eat their lunch was estimated to be 12 minutes.

It was found that k customers took between 20 and 25 minutes to eat their lunch.

a. Write down the mid-interval value for $10 \le t < 15$. [1]

b.i. Write down the total number of customers in terms of k. [1]

b.ii. Calculate the value of k. [3]

c. Hence, complete the histogram. [1]

17M.1.SL.TZ1.S_8

A line L_1 passes through the points $\mathrm{A}(0,\ 1,\ 8)$ and $\mathrm{B}(3,\ 5,\ 2)$.

Given that L_1 and L_2 are perpendicular, show that $p = 2$.

a.i. Find $\overrightarrow{AB}$. [2]

a.ii. Hence, write down a vector equation for L_1. [2]

b. A second line L_2, has equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 13 \\ -14 \end{pmatrix} + s\begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix}$.

Given that L_1 and L_2 are perpendicular, show that $p = 2$. [3]

c. The lines L_1 and L_1 intersect at $C(9,\ 13,\ z)$. Find z. [5]

d.i. Find a unit vector in the direction of L_2. [2]

d.ii. Hence or otherwise, find one point on L_2 which is $\sqrt{5}$ units from C. [3]

19M.1.SL.TZ1.S_2

A line, L_1, has equation $r = \begin{pmatrix} -3 \\ 9 \\ 10 \end{pmatrix} + s\begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$. Point $\mathrm{P}\,(15{,}9{,}c)$ lies on L_1.

a. Find c. [4]

b. A second line, L_2, is parallel to L_1 and passes through (1, 2, 3).

Write down a vector equation for L_2. [2]

19M.1.SL.TZ2.S_2

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 0 \\ 3 \\ p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 0 \\ 6 \\ 18 \end{pmatrix}$.

Find the value of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are

a. parallel. [2]

b. perpendicular. [4]

18M.1.SL.TZ2.S_1

Let $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, where O is the origin. L_1 is the line that passes through A and B.

a. Find a vector equation for L_1. [2]

b. The vector $\begin{pmatrix} 2 \\ p \\ 0 \end{pmatrix}$ is perpendicular to $\overrightarrow{AB}$. Find the value of p. [3]

16N.1.SL.TZ0.S_4

The position vectors of points P and Q are $\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ respectively.

a. Find a vector equation of the line that passes through P and Q. [4]

b.

The line through P and Q is perpendicular to the vector $2\boldsymbol{i} + n\boldsymbol{k}$. Find the value of n.

[3]

18M.1.SL.TZ1.S_2

The following box-and-whisker plot shows the number of text messages sent by students in a school on a particular day.

a. Find the value of the interquartile range. [2]

b.

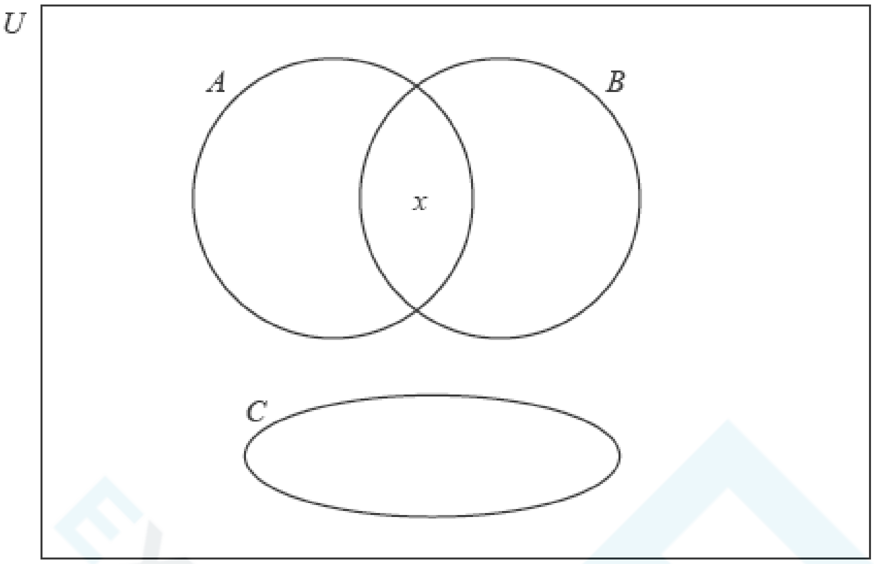
One student sent k text messages, where $k > 11$. Given that k is an outlier, find the least value of k.

[4]

16N.1.SL.TZ0.T_3

The following Venn diagram shows the sets A, B, C and U.

x is an element of U.



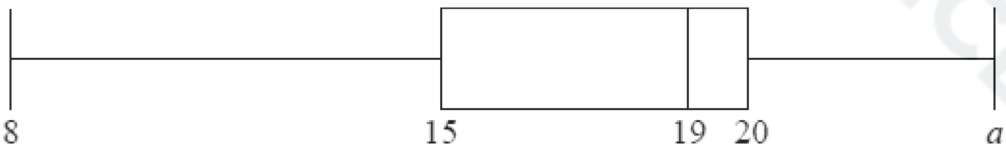
a. In the table indicate whether the given statements are True or False. [5]

Statement	True or False
$x \in C$	
$x \subset B$	
$A \cup B \neq \varnothing$	
$A \cap B \subset C$	
$A \cap C = \varnothing$	

b. On the Venn diagram, shade the region $A \cap (B \cup C)'$. [1]

19M.1.SL.TZ2.S_8

A group of 10 girls recorded the number of hours they spent watching television during a particular week. Their results are summarized in the box-and-whisker plot below.



The group of girls watched a total of 180 hours of television.

A group of 20 boys also recorded the number of hours they spent watching television that same week. Their results are summarized in the table below.

$\bar{x} = 21$	$\sigma = 3$

The following week, the group of boys had exams. During this exam week, the boys spent half as much time watching television compared to the previous week.

For this exam week, find

a. The range of the data is 16. Find the value of a. [2]

b. Find the value of the interquartile range. [2]

c.

Find the mean number of hours that the girls in this group spent watching television that week.

[2]

d.i.

Find the total number of hours the group of boys spent watching television that week.

[2]

d.ii.

Find the mean number of hours that girls and boys spent watching television that week.

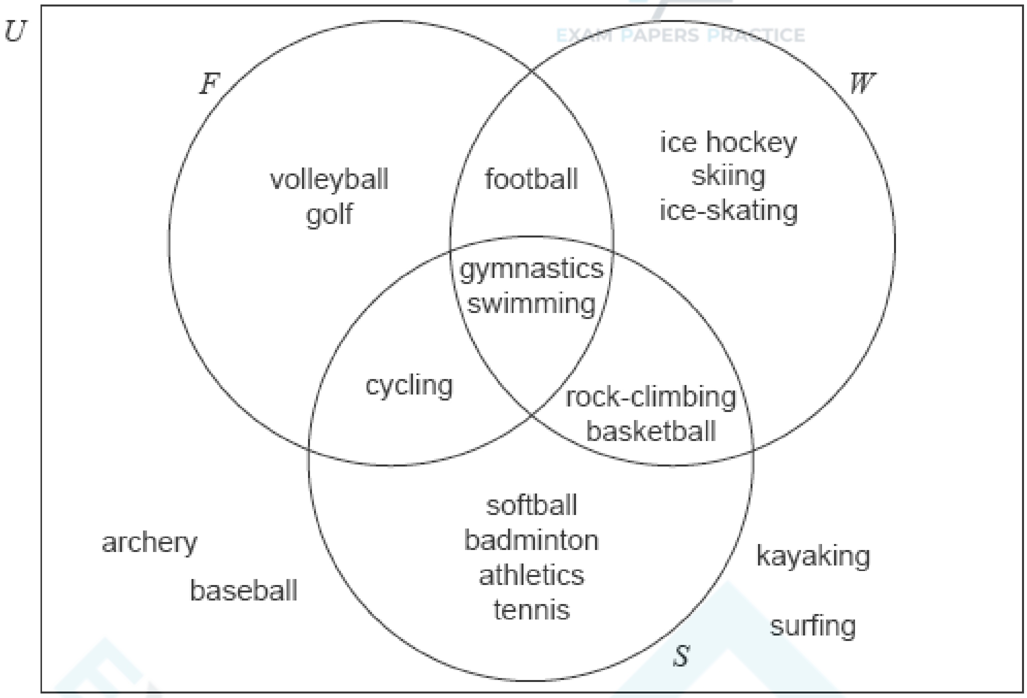
[3]

e.i. the mean number of hours that the group of boys spent watching television. [2]

17M.1.SL.TZ1.T_4

Dune Canyon High School organizes its **school year** into three trimesters: fall/autumn (F), winter (W) and spring (S). The school offers a variety of sporting activities during and outside the school year.

The activities offered by the school are summarized in the following Venn diagram.



a.

Write down the number of sporting activities offered by the school during its **school**
.

[1]

b.

Determine whether rock-climbing is offered by the school in the fall/autumn trimester.

[1]

c.i. Write down the elements of the set $F \cap W'$; [1]

c.ii. Write down $n(W \cap S)$. [1]

d.

Write down, in terms of F, W and S, an expression for the set which contains only archery, baseball, kayaking and surfing.

[2]

18M.1.SL.TZ2.S_3

A data set has *n* items. The sum of the items is 800 and the mean is 20.

The standard deviation of this data set is 3. Each value in the set is multiplied by 10.

a. Find *n*. [2]

b.i. Write down the value of the new mean. [1]

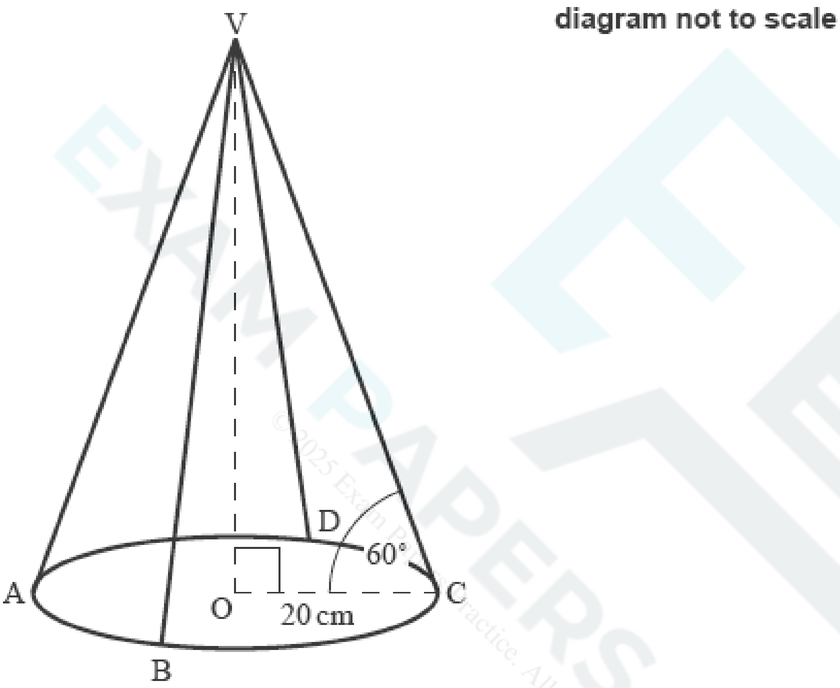
b.ii. Find the value of the new variance. [3]

17M.1.SL.TZ2.T_3

A lampshade, in the shape of a cone, has a wireframe consisting of a circular ring and four straight pieces of equal length, attached to the ring at points A, B, C and D.

The ring has its centre at point O and its radius is 20 centimetres. The straight pieces meet at point V, which is vertically above O, and the angle they make with the base of the lampshade is 60°.

This information is shown in the following diagram.



a. Find the length of one of the straight pieces in the wireframe. [2]

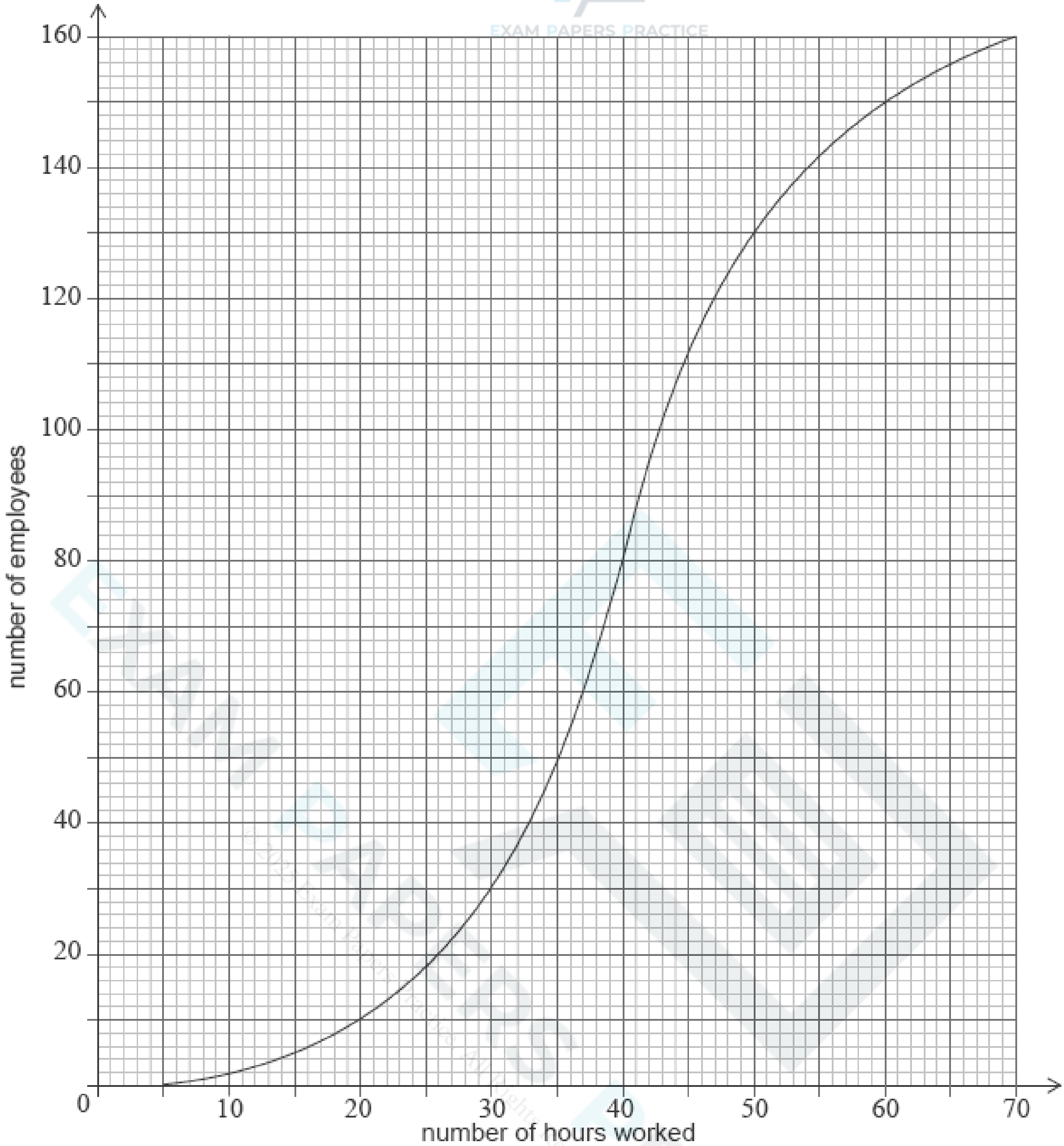
b.

Find the total length of wire needed to construct this wireframe. Give your answer in centimetres correct to the nearest millimetre.

[4]

17M.1.SL.TZ2.S_8

A city hired 160 employees to work at a festival. The following cumulative frequency curve shows the number of hours employees worked during the festival.



The city paid each of the employees £8 per hour for the first 40 hours worked, and £10 per hour for each hour they worked after the first 40 hours.

a.i. Find the median number of hours worked by the employees. [2]

a.ii. Write down the number of employees who worked 50 hours or less. [1]

b.i. Find the amount of money an employee earned for working 40 hours; [1]

b.ii. Find the amount of money an employee earned for working 43 hours. [3]

c. Find the number of employees who earned £200 or less. [3]

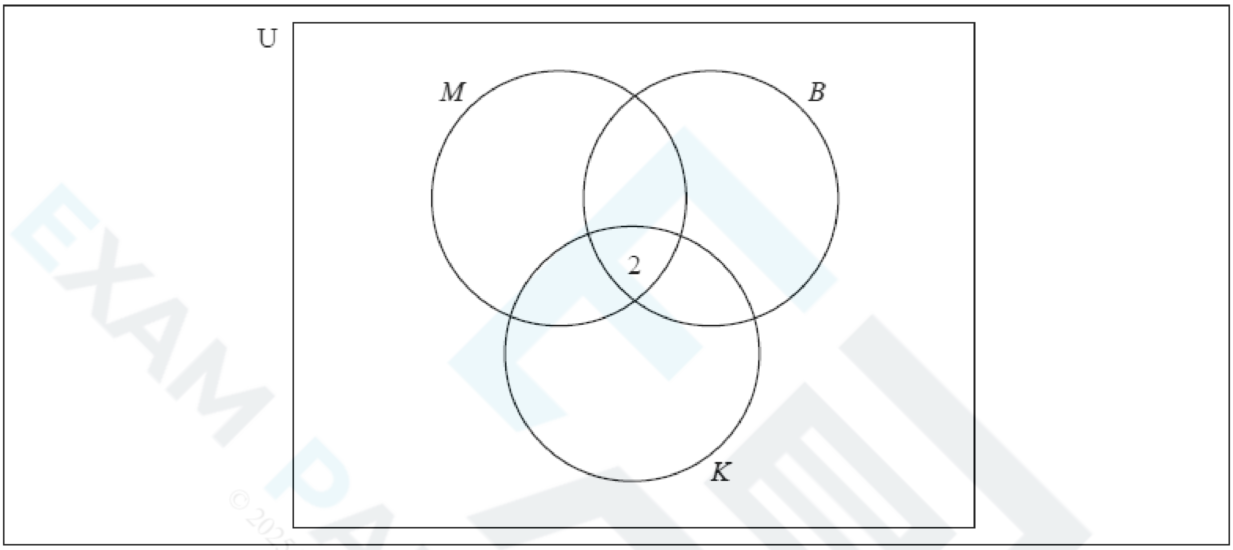
d. Only 10 employees earned more than £k. Find the value of k. [4]

19M.1.SL.TZ2.T_5

A school café sells three flavours of smoothies: mango (M), kiwi fruit (K) and banana (B). 85 students were surveyed about which of these three flavours they like.

35 students liked mango, 37 liked banana, and 26 liked kiwi fruit
2 liked all three flavours
20 liked both mango and banana
14 liked mango and kiwi fruit
3 liked banana and kiwi fruit

a. Using the given information, complete the following Venn diagram.



[2]

b. Find the number of surveyed students who did not like any of the three flavours.

[2]

c. A student is chosen at random from the surveyed students.

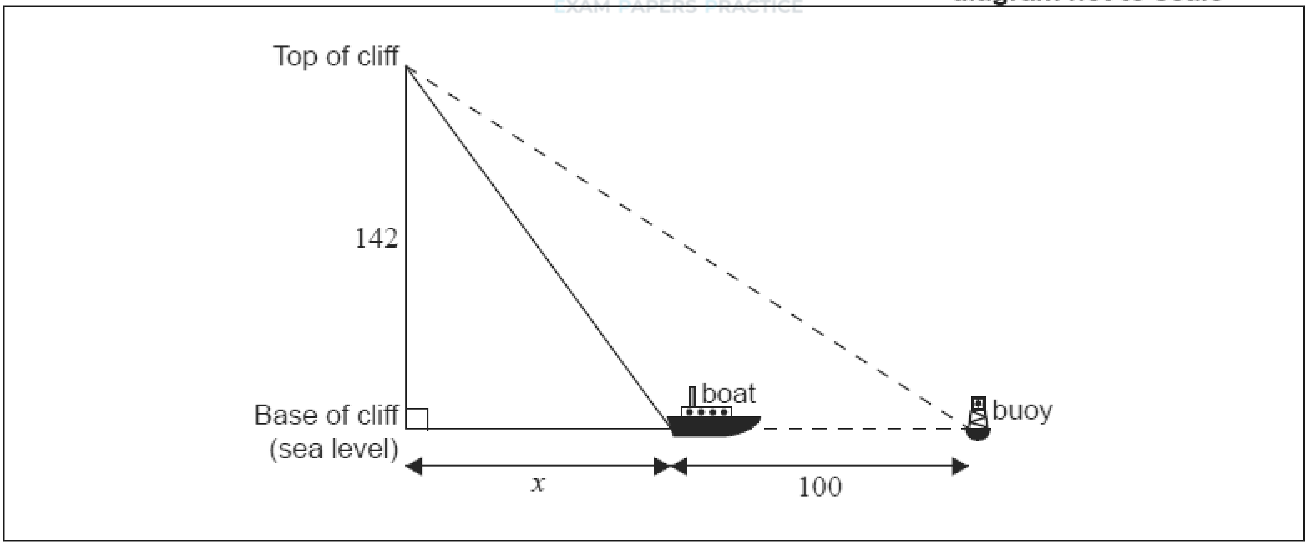
Find the probability that this student likes kiwi fruit smoothies given that they like mango smoothies.

[2]

19M.1.SL.TZ1.T_8

A buoy is floating in the sea and can be seen from the top of a vertical cliff. A boat is travelling from the base of the cliff directly towards the buoy.

The top of the cliff is 142 m above sea level. Currently the boat is 100 metres from the buoy and the angle of depression from the top of the cliff to the boat is 64°.

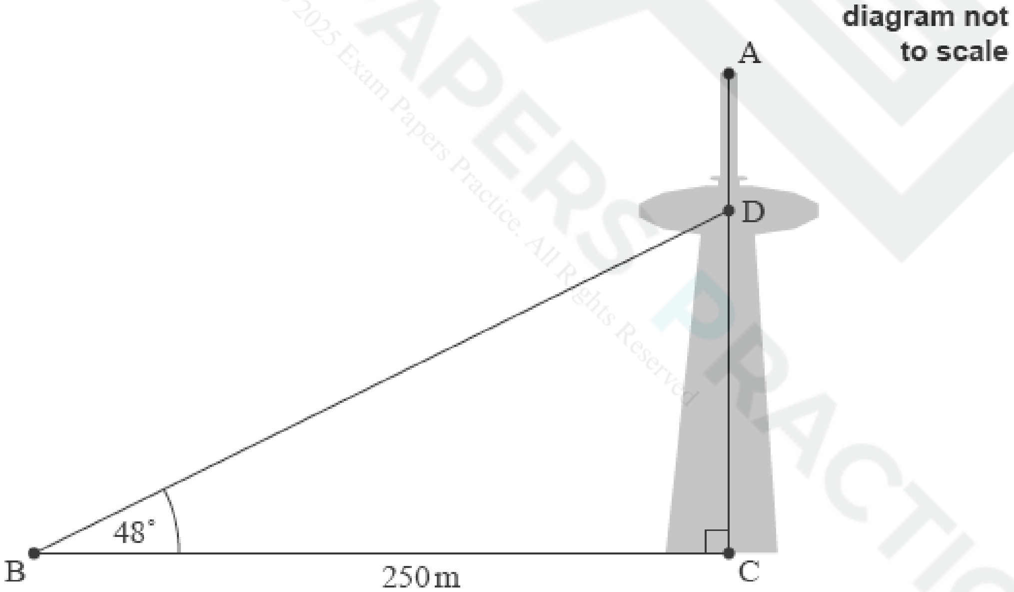


Draw and label the angle of depression on the diagram.

16N.1.SL.TZ0.T_11

AC is a vertical communications tower with its base at C.

The tower has an observation deck, D, three quarters of the way to the top of the tower, A.



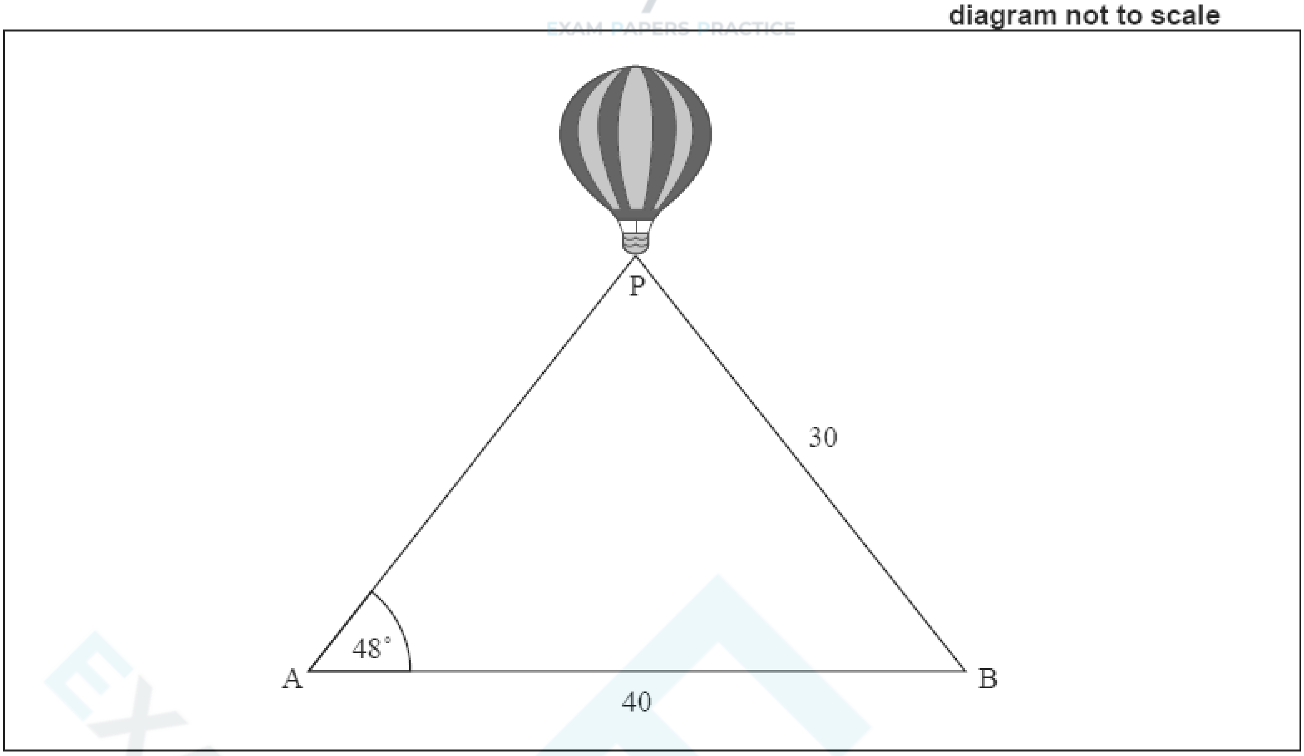
From a point B, on horizontal ground 250 m from C, the angle of elevation of D is 48°.

a. Calculate CD, the height of the observation deck above the ground. [2]

b. Calculate the angle of depression from A to B. [4]

18M.1.SL.TZ1.T_8

Two fixed points, A and B, are 40 m apart on horizontal ground. Two straight ropes, AP and BP, are attached to the same point, P, on the base of a hot air balloon which is vertically above the line AB. The length of BP is 30 m and angle BAP is 48°.



Angle APB is acute.

a. On the diagram, draw and label with an *x* the angle of depression of B from P. [1]

b. Find the size of angle APB. [3]

c. Find the size of the angle of depression of B from P. [2]

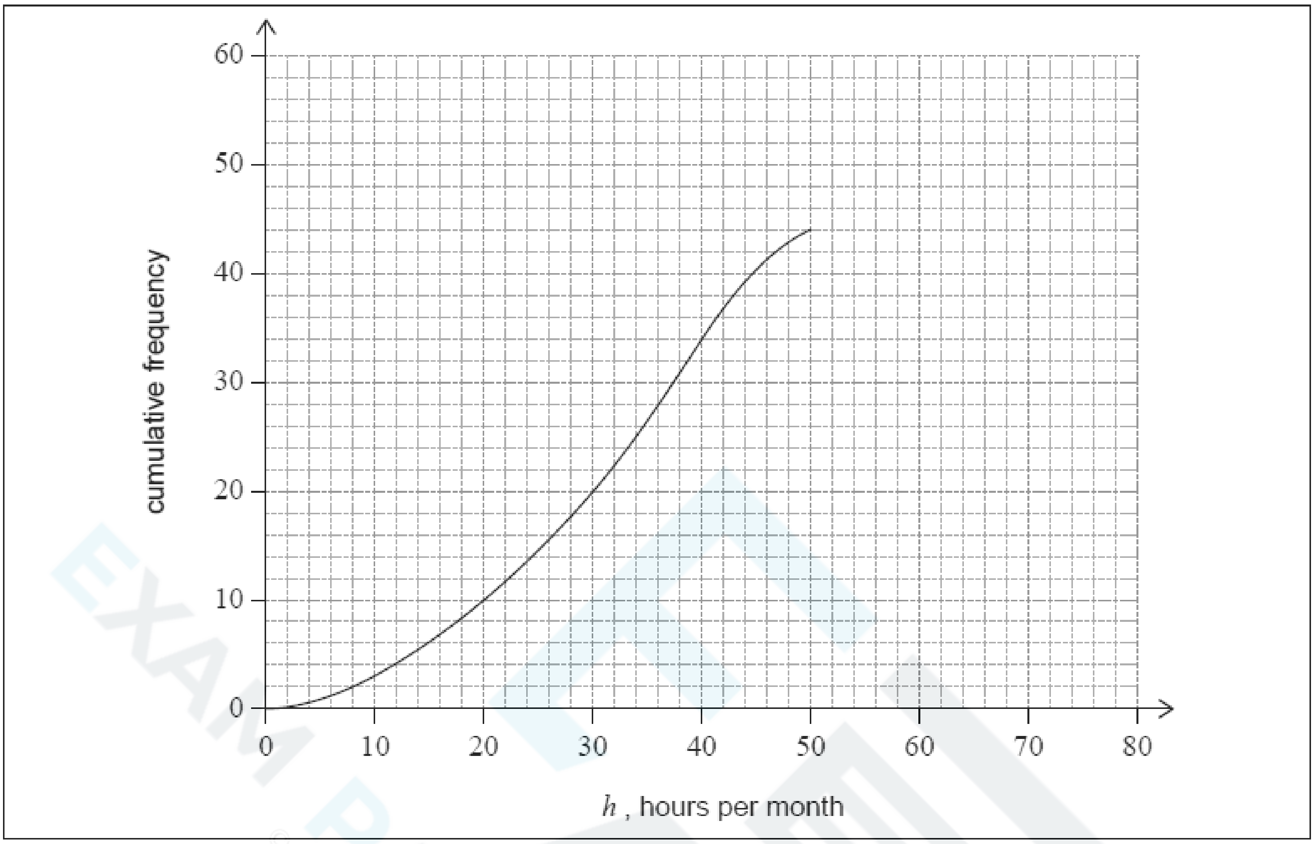
19M.1.SL.TZ2.T_12

University students were surveyed and asked how many hours, h , they worked each month. The results are shown in the following table.

Hours per month, h	Frequency	Cumulative frequency
$0 < h \leq 10$	3	3
$10 < h \leq 20$	7	10
$20 < h \leq 30$	10	20
$30 < h \leq 40$	14	34
$40 < h \leq 50$	p	44
$50 < h \leq 60$	6	50
$60 < h \leq 70$	4	54
$70 < h \leq 80$	2	q

Use the table to find the following values.

The first five class intervals, indicated in the table, have been used to draw part of a cumulative frequency curve as shown.



a.i. p. [1]

a.ii. q. [1]

b. On the same grid, complete the cumulative frequency curve for these data. [2]

c.

Use the cumulative frequency curve to find an estimate for the number of students who worked at most 35 hours per month.

[2]

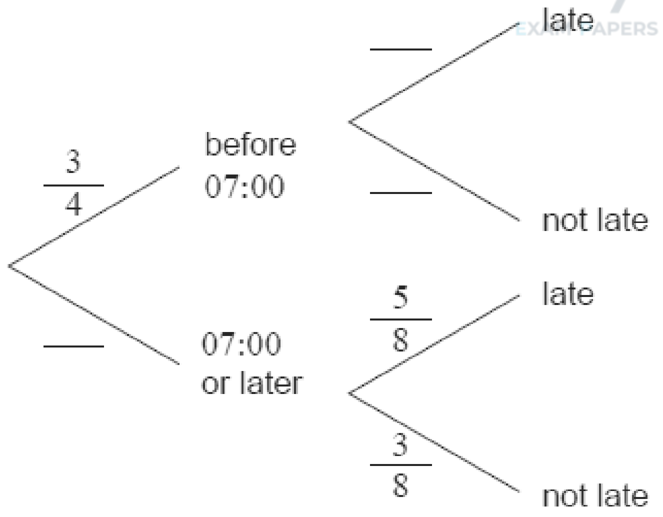
18M.1.SL.TZ2.S_8

Pablo drives to work. The probability that he leaves home before 07:00 is $\frac{3}{4}$.

If he leaves home before 07:00 the probability he will be late for work is $\frac{1}{8}$.

If he leaves home at 07:00 or later the probability he will be late for work is $\frac{5}{8}$.

a. **Copy** and complete the following tree diagram.



[3]

b. Find the probability that Pablo leaves home before 07:00 and is late for work. [2]

c. Find the probability that Pablo is late for work. [3]

d.

Given that Pablo is late for work, find the probability that he left home before 07:00.

[3]

e.

Two days next week Pablo will drive to work. Find the probability that he will be late at least once.

[3]

17N.1.SL.TZ0.T_1

A group of 20 students travelled to a gymnastics tournament together. Their ages, in years, are given in the following table.

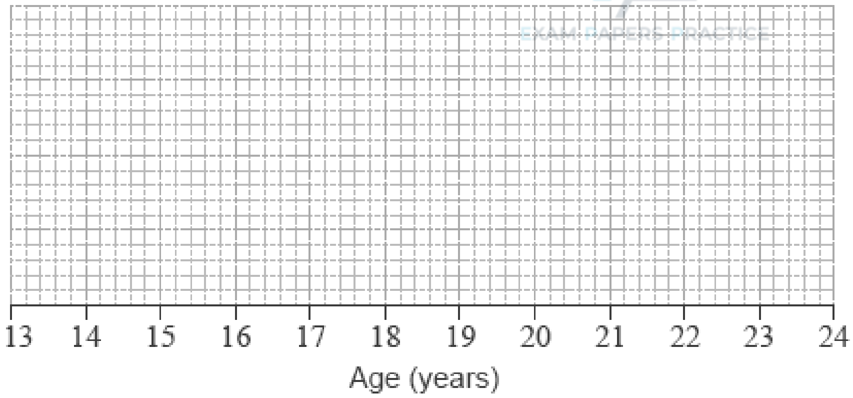
Age (years)	14	15	16	17	18	19	20	22
Frequency	1	2	7	1	4	1	1	3

The lower quartile of the ages is 16 and the upper quartile is 18.5.

a.i. For the students in this group find the mean age; [2]

a.ii. For the students in this group write down the median age. [1]

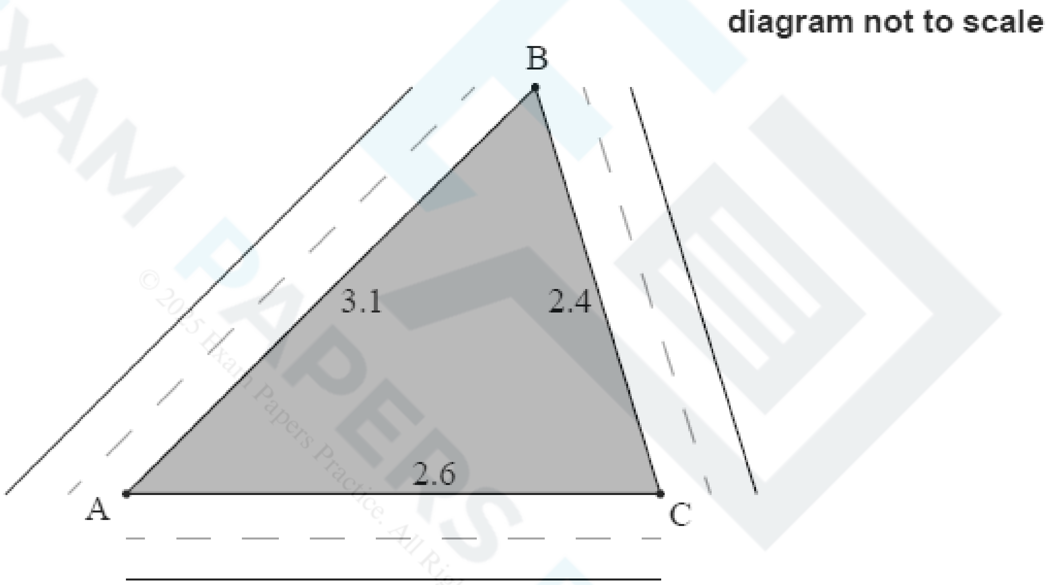
b. Draw a box-and-whisker diagram, for these students' ages, on the following grid.



[3]

19M.1.SL.TZ1.T_10

Three airport runways intersect to form a triangle, ABC. The length of AB is 3.1 km, AC is 2.6 km, and BC is 2.4 km.



A company is hired to cut the grass that grows in triangle ABC, but they need to know the area.

a. Find the size, in degrees, of angle BÂC. [3]

b. Find the area, in km^2, of triangle ABC. [3]

17M.1.SL.TZ1.S_4

Jim heated a liquid until it boiled. He measured the temperature of the liquid as it cooled. The following table shows its temperature, d degrees Celsius, t minutes after it boiled.

t (min)	0	4	8	12	16	20
d (°C)	105	98.4	85.4	74.8	68.7	62.1

Jim believes that the relationship between d and t can be modelled by a linear regression equation.

a.i. Write down the independent variable. [1]

a.ii. Write down the boiling temperature of the liquid. [1]

b.

Jim describes the correlation as **very strong**. Circle the value below which best represents the correlation coefficient.

0.992 0.251 0 -0.251 -0.992 [2]

c.

Jim's model is $d = -2.24t + 105$, for $0 \leqslant t \leqslant 20$. Use his model to predict the decrease in temperature for any 2 minute interval.

[2]

21N.1.SL.TZ0.3

The function f is defined by $fx = \frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

Write down the equation of

Find the coordinates where the graph of f crosses

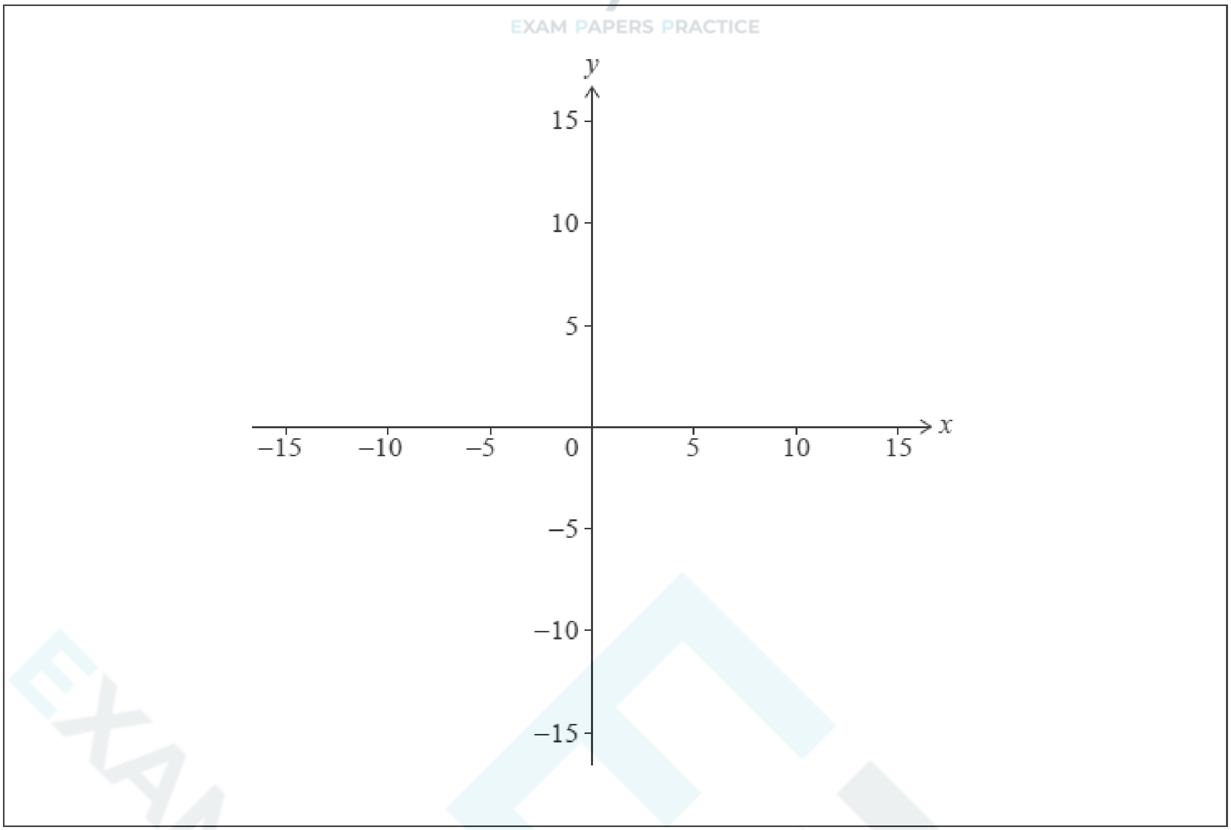
a.i. the vertical asymptote of the graph of f. [1]

a.ii. the horizontal asymptote of the graph of f. [1]

b.i. the x-axis. [1]

b.ii. the y-axis. [1]

c. Sketch the graph of f on the axes below.



[1]

21N.1.SL.TZ0.4

Box 1 contains 5 red balls and 2 white balls.

Box 2 contains 4 red balls and 3 white balls.

a.

A box is chosen at random and a ball is drawn. Find the probability that the ball is red.

[3]

b.

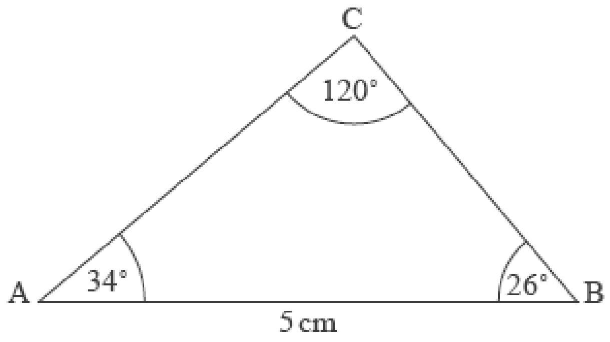
Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

Determine whether events A and R are independent. [2]

17M.1.SL.TZ1.T_13

A triangular postage stamp, ABC, is shown in the diagram below, such that $\mathrm{AB} = 5\text{ cm}, \mathrm{B\hat{A}C} = 34°, \mathrm{A\hat{B}C} = 26°$ and $\mathrm{A\hat{C}B} = 120°$.

diagram not to scale



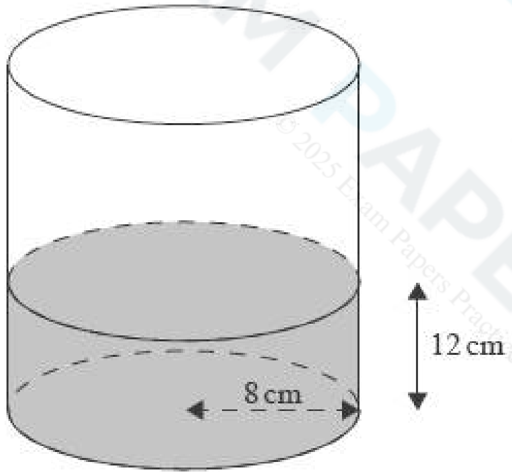
a. Find the length of BC. [3]

b. Find the area of the postage stamp. [3]

17M.1.SL.TZ2.T_12

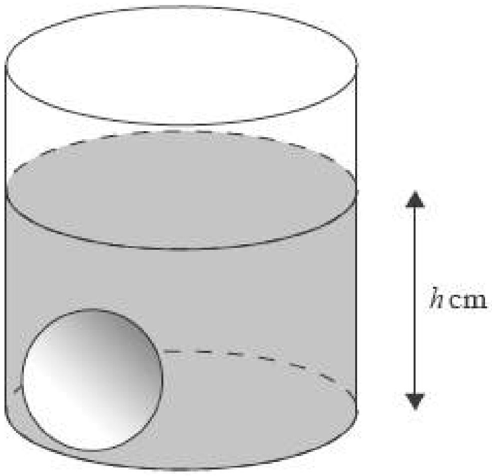
A cylindrical container with a radius of 8 cm is placed on a flat surface. The container is filled with water to a height of 12 cm, as shown in the following diagram.

diagram not to scale



A heavy ball with a radius of 2.9 cm is dropped into the container. As a result, the height of the water increases to h cm, as shown in the following diagram.

diagram not to scale

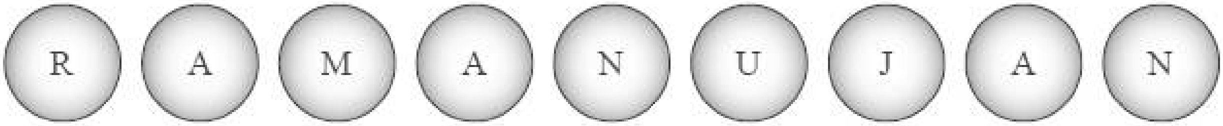


a. Find the volume of water in the container. [2]

b. Find the value of h. [4]

20N.1.SL.TZ0.T_6

Srinivasa places the nine labelled balls shown below into a box.



Srinivasa then chooses two balls at random, one at a time, from the box. The first ball is **not replaced** before he chooses the second.

a.i. Find the probability that the first ball chosen is labelled A. [1]

a.ii. Find the probability that the first ball chosen is labelled A or labelled N. [1]

b.

Find the probability that the second ball chosen is labelled A, given that the first ball chosen was labelled N.

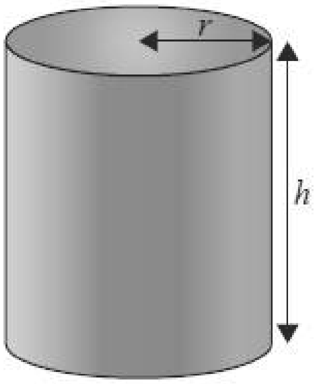
[2]

c. Find the probability that both balls chosen are labelled N. [2]

19M.1.SL.TZ1.T_15

A cylinder with radius r and height h is shown in the following diagram.

diagram not to scale



The sum of r and h for this cylinder is 12 cm.

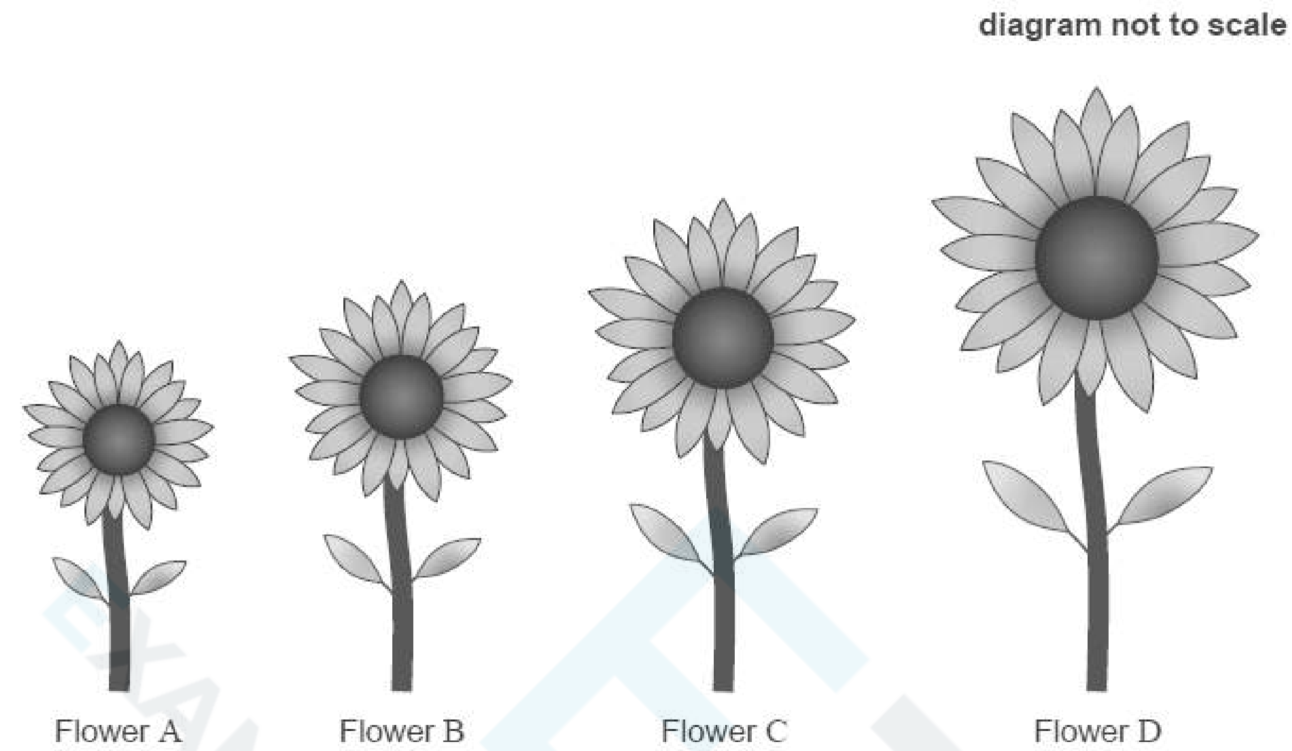
a. Write down an equation for the area, A, of the **curved** surface in terms of r. [2]

b. Find $\frac{dA}{dr}$. [2]

c. Find the value of r when the area of the curved surface is maximized. [2]

20N.1.SL.TZ0.T_7

Anne-Marie planted four sunflowers in order of height, from shortest to tallest.



Flower C is 32 cm tall.

The median height of the flowers is 24 cm.

The range of the heights is 50 cm. The height of Flower A is p cm and the height of Flower D is q cm.

The mean height of the flowers is 27 cm.

a. Find the height of Flower null. [2]

b. Using this information, write down an equation in p and q. [1]

c. Write down a second equation in p and q. [1]

d.i. Using your answers to **parts (b)** and **(c)**, find the height of Flower A. [1]

d.ii. Using your answers to **parts (b)** and **(c)**, find the height of Flower D. [1]

16N.1.SL.TZ0.T_7

A balloon in the shape of a sphere is filled with helium until the radius is 6 cm.

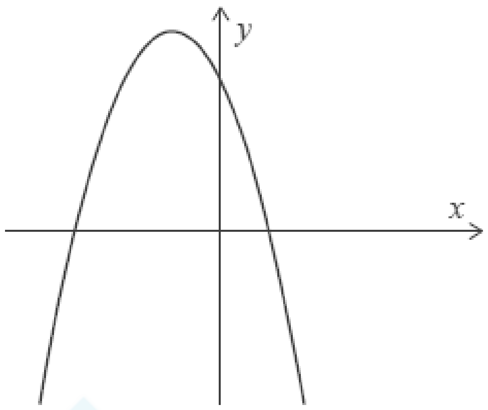
The volume of the balloon is increased by 40%.

a. Calculate the volume of the balloon. [2]

b. Calculate the radius of the balloon following this increase. [4]

21N.1.SL.TZ0.1

Consider the function $f x = -2x-1x+3$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.



For the graph of f

a.i. find the x-coordinates of the x-intercepts. [2]

a.ii. find the coordinates of the vertex. [3]

b. The function f can be written in the form $f x = -2x-h^2+k$.

Write down the value of h and the value of k. [2]

20N.1.SL.TZ0.T_8

Give your answers in this question correct to the nearest whole number.

Imon invested $25\ 000$ Singapore dollars (SGD) in a fixed deposit account with a nominal annual interest rate of 3.6%, compounded **monthly**.

a. Calculate the value of Imon's investment after 5 years. [3]

b.

At the end of the 5 years, Imon withdrew x SGD from the fixed deposit account and reinvested this into a super-savings account with a nominal annual interest rate of 5.7%, compounded **half-yearly**.

The value of the super-savings account increased to $20\ 000$ SGD after 18 months.

Find the value of x. [3]

17M.1.SL.TZ2.T_14

Jashanti is saving money to buy a car. The price of the car, in US Dollars (USD), can be modelled by the equation

$$P = 8500\,(0.95)^t.$$

Jashanti's savings, in USD, can be modelled by the equation

$$S = 400t + 2000.$$

In both equations t is the time in months since Jashanti started saving for the car.

Jashanti does not want to wait too long and wants to buy the car two months after she started saving. She decides to ask her parents for the extra money that she needs.

a. Write down the amount of money Jashanti saves per month. [1]

b.

Use your graphic display calculator to find how long it will take for Jashanti to have saved enough money to buy the car.

[2]

c. Calculate how much extra money Jashanti needs. [3]

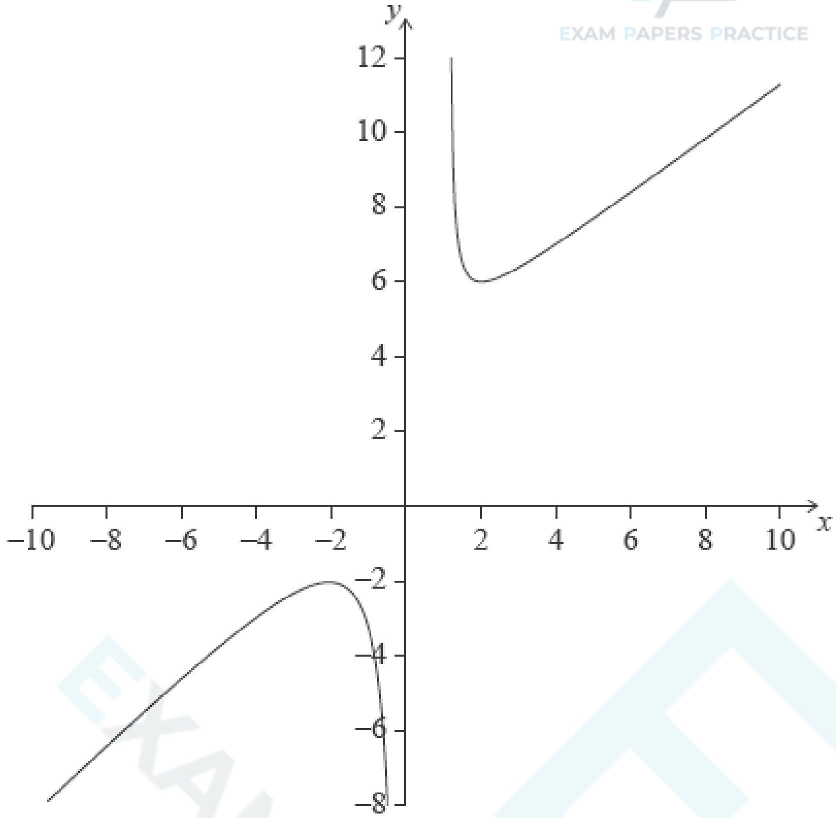
21N.1.SL.TZ0.2

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x - \frac{\pi}{4}$ and $y = 2$ when $x = \frac{3\pi}{4}$, find y in terms of x.

17M.1.SL.TZ1.T_12

The function f is of the form $f(x) = ax + b + \frac{c}{x}$, where a , b and c are positive integers.

Part of the graph of $y = f(x)$ is shown on the axes below. The graph of the function has its local maximum at $(-2, -2)$ and its local minimum at $(2, 6)$.



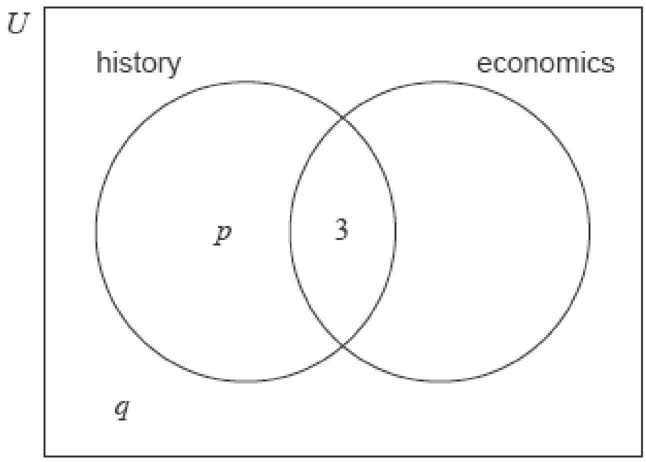
b.i. Draw the line $y = -6$ on the axes. [1]

b.ii. Write down the number of solutions to $f(x) = -6$. [1]

c. Find the range of values of k for which $f(x) = k$ has no solution. [2]

17M.1.SL.TZ1.S_1

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



a.i. Find the value of p; [2]

a.ii. Find the value of q. [2]

b.

A girl is selected at random. Find the probability that she takes economics but not history.

[2]

18N.1.SL.TZ0.S_9

A bag contains n marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

Let n = 5. Find the probability that the game will end on her

a.i. Find the probability, in terms of n, that the game will end on her first draw. [1]

a.ii. Find the probability, in terms of n, that the game will end on her second draw.

[3]

b.i. third draw. [2]

b.ii. fourth draw. [2]

c.

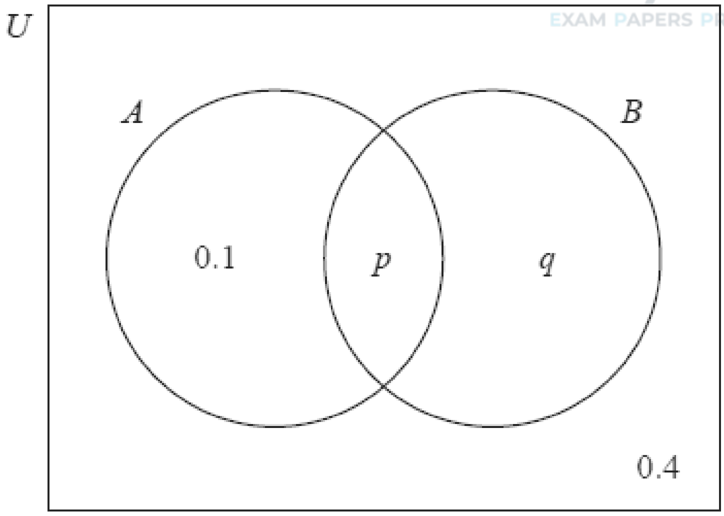
Hayley plays the game when n = 5. She pays \$20 to play and can earn money back depending on the number of draws it takes to obtain a blue marble. She earns no money back if she obtains a blue marble on her first draw. Let M be the amount of money that she earns back playing the game. This information is shown in the following table.

Number of draws	1	2	3	4
Money earned back (\$$M$)	0	20	$8k$	$12k$

Find the value of k so that this is a fair game. [7]

19M.1.SL.TZ1.S_1

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.3$. The values shown are probabilities.



a. Find the value of p. [2]

b. Find the value of q. [2]

c. Find $\mathrm{P}\left(A' \cup B\right)$. [2]

16N.1.SL.TZ0.S_5

Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

a. Find $\mathrm{P}(B)$. [2]

b. Find $\mathrm{P}(A \cup B)$. [4]

22M.1.SL.TZ2.2

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

a. State the value of the first term, u_1. [1]

b. Given that the n^{th} term of this sequence is -33, find the value of n. [2]

c. Find the common difference, d. [2]

22M.1.SL.TZ2.4

A function f is defined by $fx = \frac{2x - 1}{x + 1}$, where $x \in \mathbb{R}, \quad x \neq -1$.

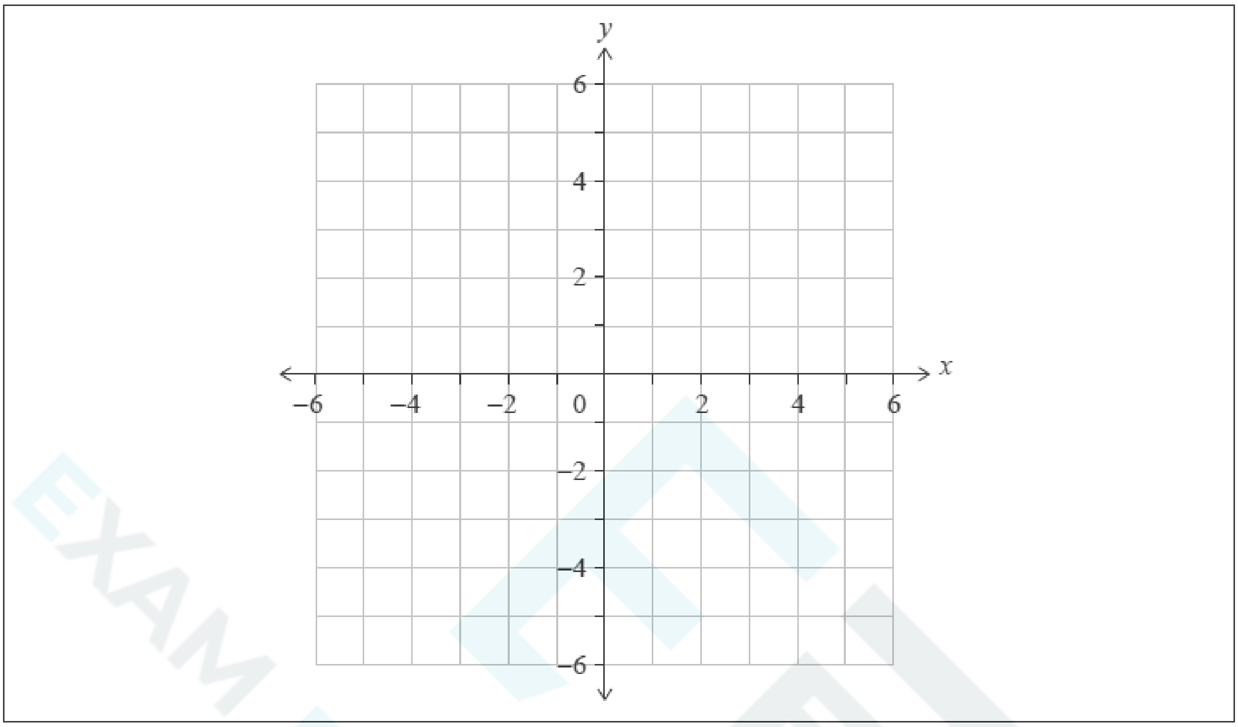
The graph of $y = f\left(x\right)$ has a vertical asymptote and a horizontal asymptote.

a.i. Write down the equation of the vertical asymptote. [1]

a.ii. Write down the equation of the horizontal asymptote. [1]

b. On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

c. Hence, solve the inequality $0 < \frac{2x - 1}{x + 1} < 2$. [1]

22M.1.SL.TZ2.3

Consider any three consecutive integers, $n - 1$, n and $n + 1$.

a. Prove that the sum of these three integers is always divisible by 3. [2]

b. Prove that the sum of the squares of these three integers is never divisible by 3.

[4]

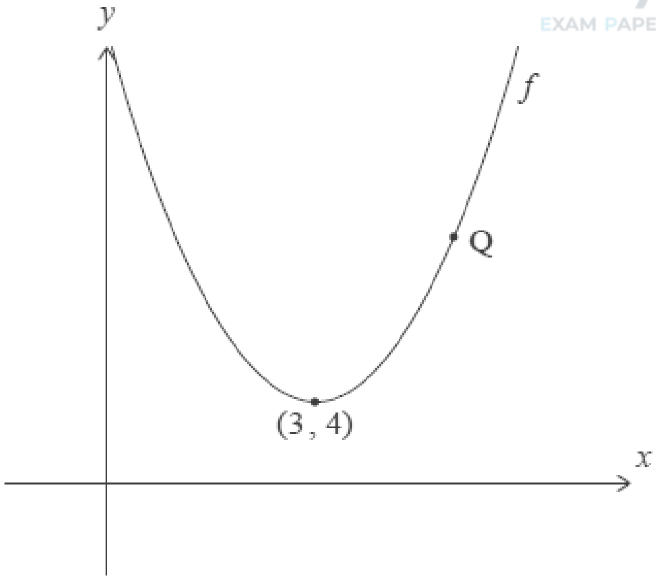
22M.1.SL.TZ2.5

Find the least positive value of x for which $\cos\frac{x}{2} + \frac{\pi}{3} = \frac{1}{\sqrt{2}}$.

22M.1.SL.TZ2.7

The following diagram shows part of the graph of a quadratic function f.

The graph of f has its vertex at $(3, \ 4)$, and it passes through point Q as shown.



The function can be written in the form $f\left(x\right) = a\left(x - h\right)^2 + k$.

The line L is tangent to the graph of f at Q.

Now consider another function $y = g\left(x\right)$. The derivative of g is given by $g'\left(x\right) = f\left(x\right) - d$, where $d \in \mathbb{R}$.

a. Write down the equation of the axis of symmetry. [1]

b.i. Write down the values of h and k. [2]

b.ii. Point Q has coordinates $\left(5, \quad 12\right)$. Find the value of a. [2]

c. Find the equation of L. [4]

d. Find the values of d for which g is an increasing function. [3]

e. Find the values of x for which the graph of g is concave-up. [3]

22M.1.SL.TZ2.6

Consider the binomial expansion $\left(x + 1\right)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \ \dots \ + 1$ where $x \neq 0$ and $a, \quad b \in \mathbb{Z}^+$.

a. Show that $b = 21$. [2]

b.

The third term in the expansion is the mean of the second term and the fourth term in the expansion.

Find the possible values of x. [5]

22M.1.SL.TZ2.9

A biased four-sided die with faces labelled $1, \; 2, \; 3$ and 4 is rolled and the result recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$\mathrm{P}(X=x)$	p	0.3	q	0.1

For this probability distribution, it is known that $\mathrm{E}(X) = 2$.

Nicky plays a game with this four-sided die. In this game she is allowed a maximum of five rolls. Her score is calculated by adding the results of each roll. Nicky wins the game if her score is at least ten.

After three rolls of the die, Nicky has a score of four.

David has two pairs of unbiased four-sided dice, a yellow pair and a red pair.

Both yellow dice have faces labelled $1, \; 2, \; 3$ and 4. Let S represent the sum obtained by rolling the two yellow dice. The probability distribution for S is shown below.

s	2	3	4	5	6	7	8
$\mathrm{P}(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

The first red die has faces labelled $1, \; 2, \; 2$ and 3. The second red die has faces labelled $1, \; a, \; a$ and b, where $a < b$ and $a, \; b \in \mathbb{Z}^+$. The probability distribution for the sum obtained by rolling the red pair is the same as the distribution for the sum obtained by rolling the yellow pair.

a. Show that $p = 0.4$ and $q = 0.2$. [5]

b. Find $\mathrm{P}(X > 2)$. [2]

c.

Assuming that rolls of the die are independent, find the probability that Nicky wins the game.

[5]

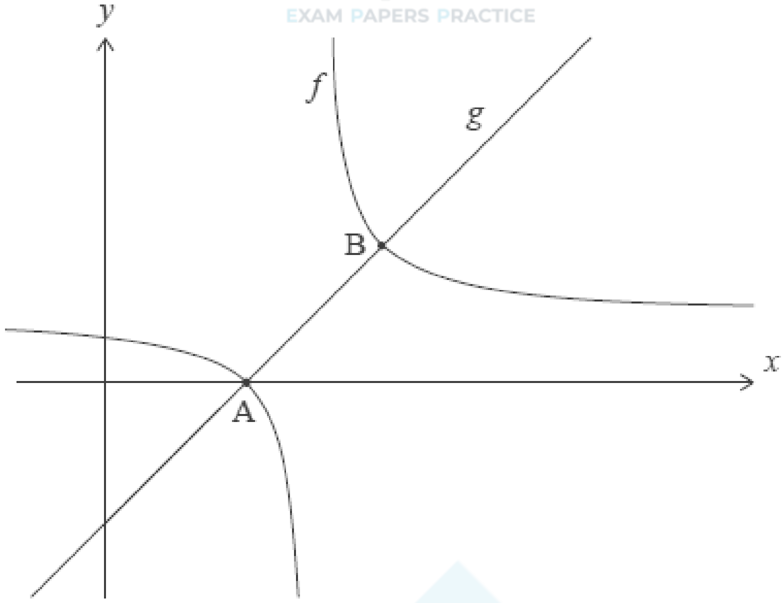
d. Determine the value of b. [2]

e. Find the value of a, providing evidence for your answer. [2]

22M.1.SL.TZ2.8

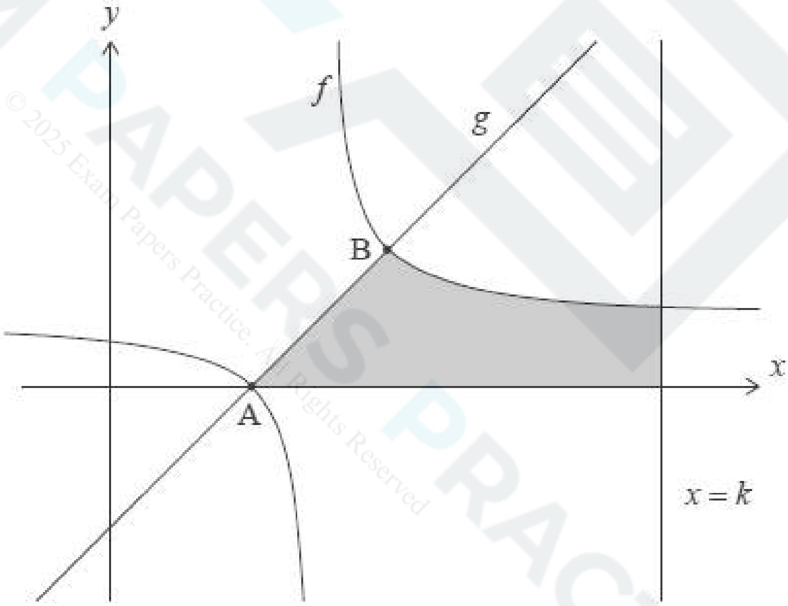
Consider the functions $f x = \frac{1}{x - 4} + 1$, for $x \neq 4$, and $g x = x - 3$ for $x \in \mathbb{R}$.

The following diagram shows the graphs of f and g.



The graphs of f and g intersect at points A and B. The coordinates of A are $(3,\ 0)$.

In the following diagram, the shaded region is enclosed by the graph of f, the graph of g, the x-axis, and the line $x = k$, where $k \in \mathbb{Z}$.



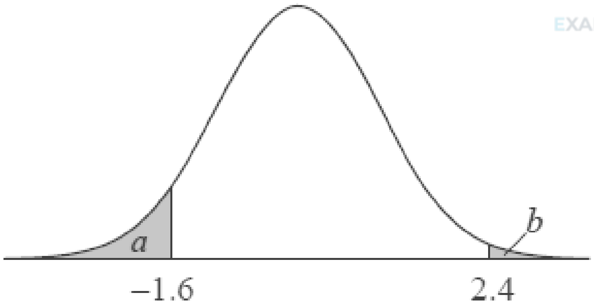
The area of the shaded region can be written as $\ln(p) + 8$, where $p \in \mathbb{Z}$.

a. Find the coordinates of B. [5]

b. Find the value of k and the value of p. [10]

19M.1.SL.TZ1.S_9

A random variable Z is normally distributed with mean 0 and standard deviation 1. It is known that $P(z < -1.6) = a$ and $P(z > 2.4) = b$. This is shown in the following diagram.



A second random variable X is normally distributed with mean m and standard deviation s.

It is known that $P(x < 1) = a$.

a. Find $P(-1.6 < z < 2.4)$. Write your answer in terms of a and b. [2]

b.

Given that $z > -1.6$, find the probability that $z < 2.4$. Write your answer in terms of a and b.

[4]

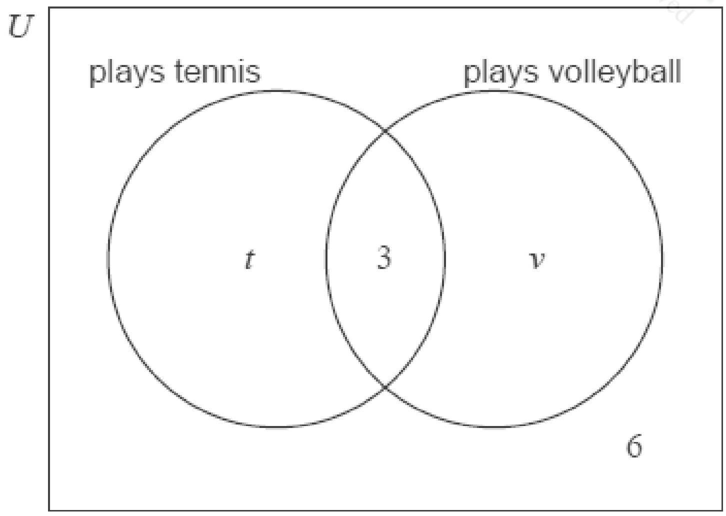
c. Write down the standardized value for $x = 1$. [1]

d. It is also known that $P(x > 2) = b$. Find s. [6]

20N.1.SL.TZ0.S_1

In a class of 30 students, 19 play tennis, 3 play both tennis and volleyball, and 6 do not play either sport.

The following Venn diagram shows the events "plays tennis" and "plays volleyball". The values t and v represent numbers of students.



a.i. Find the value of t. [2]

a.ii. Find the value of v. [2]

Find the probability that a randomly selected student from the class plays tennis or volleyball, but not both.

[2]

21N.1.SL.TZ0.5

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

The function g is defined for all $x \in \mathbb{R}$ where $gx = x^2 - 3x$ and $hx = fgx$.

a. Write down the value of $f'(4)$. [1]

b. Find $f(4)$. [1]

c. Find $h(4)$. [2]

d. Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

21N.1.SL.TZ0.7

A particle P moves along the x-axis. The velocity of P is $v \text{ m s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \le t \le 3$. When $t = 0$, P is at the origin O.

a.i. Find the value of t when P reaches its maximum velocity. [2]

a.ii. Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [5]

b.

Sketch a graph of v against t, clearly showing any points of intersection with the axes.

[4]

c. Find the total distance travelled by P. [5]

21N.1.SL.TZ0.8

Consider the function $f x = a^x$ where $x,\ a \in \mathbb{R}$ and $x > 0,\ a > 1$.

The graph of f contains the point $\frac{2}{3},\ 4$.

Consider the arithmetic sequence $\log_8 27$, $\log_8 p$, $\log_8 q$, $\log_8 125$, where $p > 1$ and $q > 1$.

a. Show that $a = 8$. [2]

b. Write down an expression for $f^{-1} x$. [1]

c. Find the value of $f^{-1} \sqrt{32}$. [3]

d.i.

Show that $27,\ p,\ q$ and 125 are four consecutive terms in a geometric sequence.

[4]

d.ii. Find the value of p and the value of q. [5]

19M.1.SL.TZ2.S_1

The following table shows the probability distribution of a discrete random variable X.

X	0	1	2	3
$P(X=x)$	$\frac{3}{13}$	$\frac{1}{13}$	$\frac{4}{13}$	k

a. Find the value of k. [3]

b. Find $E(X)$. [3]

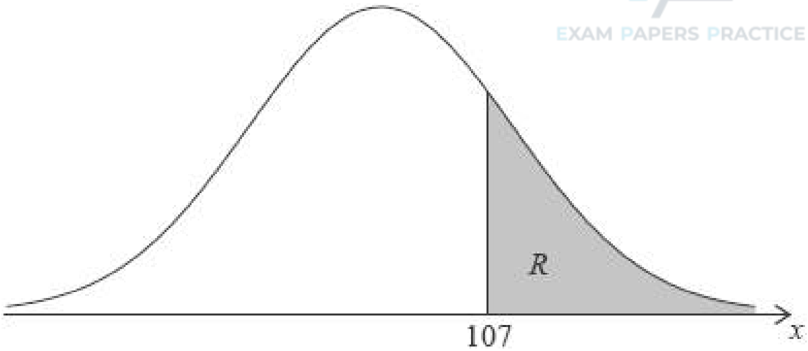
18M.1.SL.TZ1.S_7

Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in R$ where $h(x) = (f \circ g)(x)$.

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.

17M.1.SL.TZ2.S_3

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

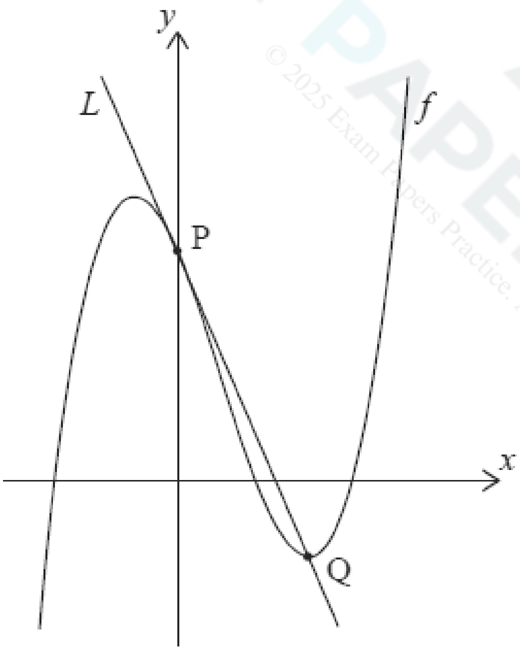
a. Write down $\mathrm{P}(X > 107)$. [1]

b. Find $\mathrm{P}(100 < X < 107)$. [3]

c. Find $\mathrm{P}(93 < X < 107)$. [2]

18N.1.SL.TZ0.S_10

Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram.



The graph of f crosses the y-axis at the point P. The line L is tangent to the graph of f at P.

b.i. Find $f'(x)$. [2]

b.ii. Hence, find the equation of L in terms of a. [4]

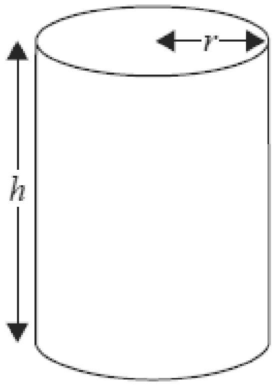
c. The graph of f has a local minimum at the point Q. The line L passes through Q.

Find the value of a. [8]

18M.1.SL.TZ2.S_9

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of 20π cm^3.

diagram not to scale



The material for the base and top of the can costs 10 cents per cm^2 and the material for the curved side costs 8 cents per cm^2. The total cost of the material, in cents, is *C*.

a. Express h in terms of *r*. [2]

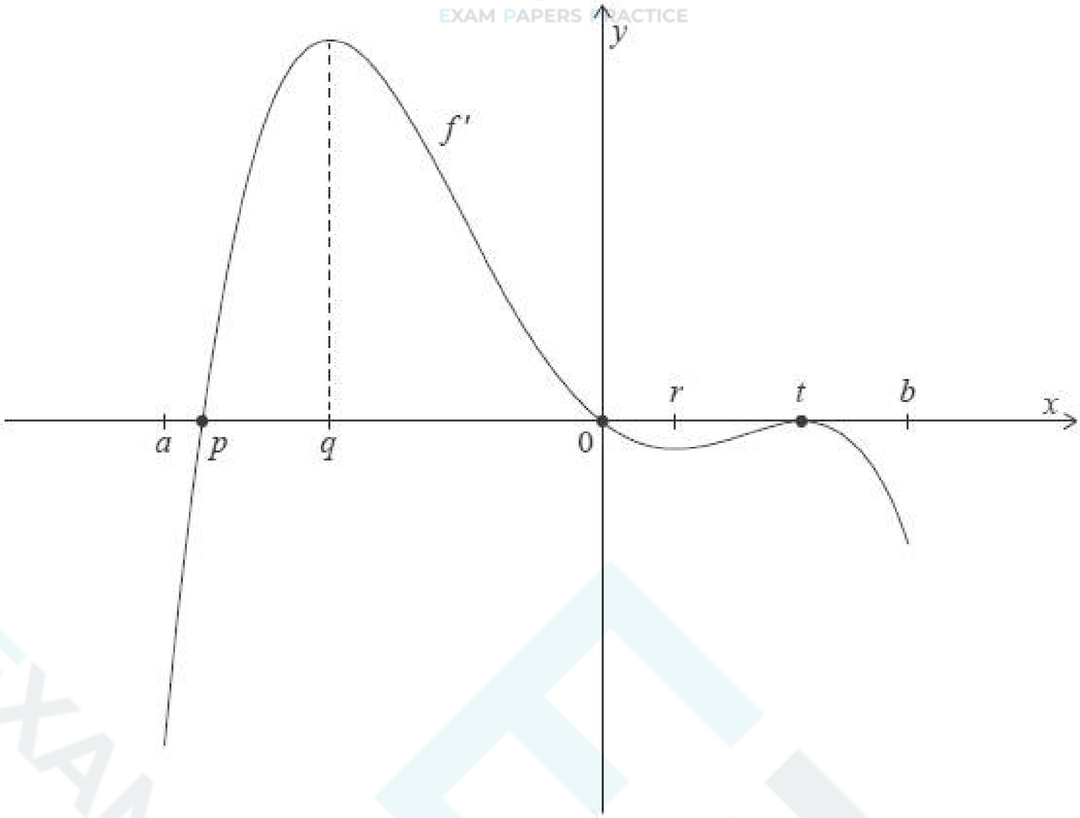
b. Show that $C = 20\pi r^2 + \frac{320\pi}{r}$. [4]

c. Given that there is a minimum value for *C*, find this minimum value in terms of π.

[9]

21N.1.SL.TZ0.9

Consider a function f with domain $a < x < b$. The following diagram shows the graph of f', the derivative of f.



The graph of f', the derivative of f, has x-intercepts at $x = p$, $x = 0$ and $x = t$. There are local maximum points at $x = q$ and $x = t$ and a local minimum point at $x = r$.

a. Find all the values of x where the graph of f is increasing. Justify your answer. [2]

b. Find the value of x where the graph of f has a local maximum. [1]

c.i. Find the value of x where the graph of f has a local minimum. Justify your answer. [2]

c.ii.

Find the values of x where the graph of f has points of inflexion. Justify your answer. [3]

d.

The total area of the region enclosed by the graph of f', the derivative of f, and the x-axis is 20.

Given that $fp + ft = 4$, find the value of $f0$. [6]

16N.1.SL.TZ0.S_10

Let $f(x) = \cos x$.

Let $g(x) = x^k$, where $k \in Z^+$.

Let $k = 21$ and $h(x) = \left(f^{(19)}(x) \times g^{(19)}(x)\right)$.

a. (i) Find the first four derivatives of $f(x)$. (ii) Find $f^{(19)}(x)$. [4]

b. (i) Find the first three derivatives of $g(x)$.

(ii) Given that $g^{(19)}(x) = \frac{k!}{(k-p)!}(x^{k-19})$, find p. [5]

c. (i) Find $h'(x)$. (ii) Hence, show that $h'(\pi) = \frac{-21!}{2}\pi^2$. [7]

17N.1.SL.TZ0.S_7

Consider $f(x) = \log k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$.

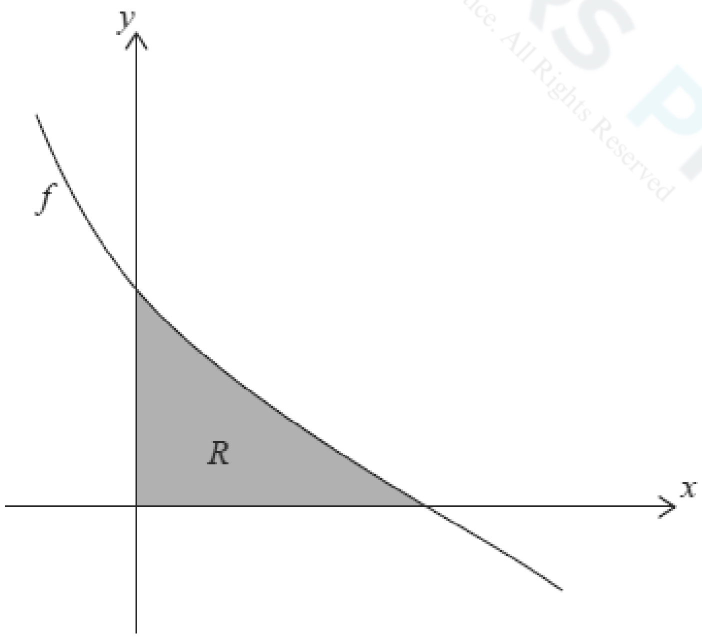
The equation $f(x) = 2$ has exactly one solution. Find the value of k.

17M.1.SL.TZ2.S_5

Let $f'(x) = \frac{3x^2}{(x^3+1)^5}$. Given that $f(0) = 1$, find $f(x)$.

18N.1.SL.TZ0.S_6

Let $f(x) = \frac{6-2x}{\sqrt{16+6x-x^2}}$. The following diagram shows part of the graph of f.



The region R is enclosed by the graph of f, the x-axis, and the y-axis. Find the area of R.

16N.1.SL.TZ0.S_6

Let $f'(x) = \sin^3(2x)\cos(2x)$. Find $f(x)$, given that $f\left(\frac{\pi}{4}\right) = 1$.

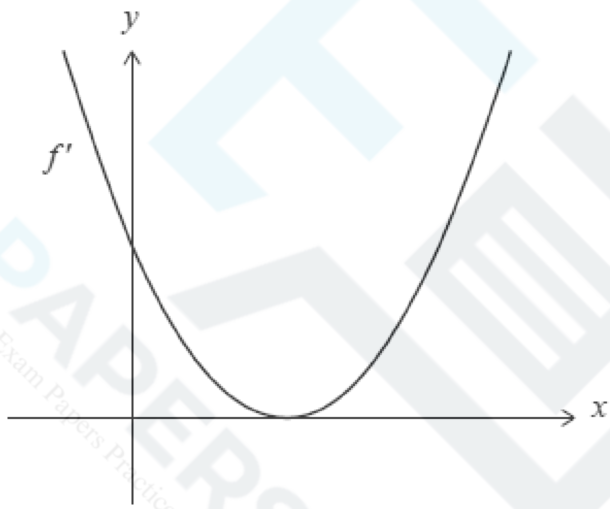
19M.1.SL.TZ1.S_5

The derivative of a function f is given by $f'(x) = 2e^{-3x}$. The graph of f passes through $\left(\frac{1}{3}, 5\right)$.

Find $f(x)$.

22M.1.SL.TZ1.7

A function, f, has its derivative given by $f'(x) = 3x^2 - 12x + p$, where $p \in \mathbb{R}$. The following diagram shows part of the graph of f'.



The graph of f' has an axis of symmetry $x = q$.

The vertex of the graph of f' lies on the x-axis.

The graph of f has a point of inflexion at $x = a$.

a. Find the value of q. [2]

b.i. Write down the value of the discriminant of f'. [1]

b.ii. Hence or otherwise, find the value of p. [3]

c. Find the value of the gradient of the graph of f' at $x = 0$. [3]

d.

Sketch the graph of f'', the second derivative of f. Indicate clearly the x-intercept and the y-intercept.

[2]

e.i. Write down the value of a. [1]

e.ii.

Find the values of x for which the graph of f is concave-down. Justify your answer.

[2]

22M.1.SL.TZ1.8

Consider the series $\ln x + p \ln x + \frac{1}{3}\ln x + \ldots$, where $x \in \mathbb{R}$, $x > 1$ and $p \in \mathbb{R}$, $p \neq 0$.

Consider the case where the series is geometric.

Now consider the case where the series is arithmetic with common difference d.

a.i. Show that $p = \pm\frac{1}{\sqrt{3}}$. [2]

a.ii. Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x. [3]

b.i. Show that $p = \frac{2}{3}$. [3]

b.ii. Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$. [1]

b.iii. The sum of the first n terms of the series is $-3 \ln x$. Find the value of n. [6]

22M.1.SL.TZ1.9

Consider $f x = 4 \cos x 1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x$.

a.i. Expand and simplify $(1 - a)^3$ in ascending powers of a. [2]

a.ii.

By using a suitable substitution for a, show that $1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x = 8 \sin^6 x$.

[4]

b.i. Show that $\int_0^m f x \, \mathrm{d} x = \frac{32}{7}\sin^7 m$, where m is a positive real constant. [4]

b.ii. It is given that $\int_m^{\frac{\pi}{2}} f x \, \mathrm{d} x = \frac{127}{28}$, where $0 \leq m \leq \frac{\pi}{2}$. Find the value of m. [5]

22M.1.SL.TZ1.1

Consider the points $\mathrm{A}(-2, 20)$, $\mathrm{B}(4, 6)$ and $\mathrm{C}(-14, 12)$. The line L passes through the point A and is perpendicular to $[\mathrm{BC}]$.

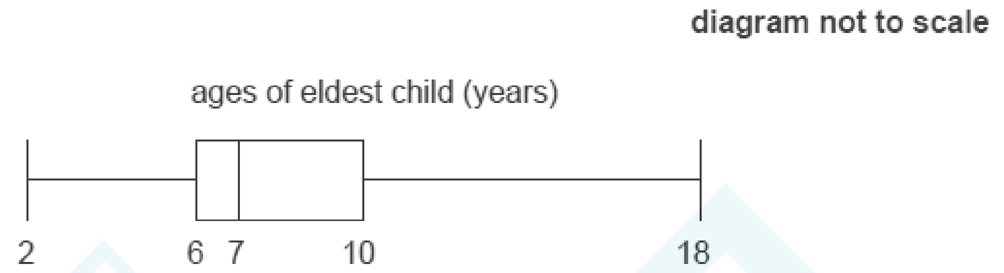
a. Find the equation of L. [3]

b. The line L passes through the point $(k, 2)$. Find the value of k. [2]

22M.1.SL.TZ1.3

A survey at a swimming pool is given to one adult in each family. The age of the adult, a years old, and of their eldest child, c years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.



The regression line of a on c is $a = \frac{7}{4}c + 20$. The regression line of c on a is $c = \frac{1}{2}a - 9$.

a. Find the largest value of c that would not be considered an outlier. [3]

b.i. One of the adults surveyed is 42 years old. Estimate the age of their eldest child. [2]

b.ii. Find the mean age of all the adults surveyed. [2]

22M.1.SL.TZ1.4

Consider the functions $f(x) = \sqrt{3}\sin x + \cos x$ where $0 \le x \le \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

a. Find $(f \circ g)(x)$. [2]

b. Solve the equation $(f \circ g)(x) = 2\cos 2x$ where $0 \le x \le \pi$. [5]

22M.1.SL.TZ1.5

Consider the curve with equation $y = (2x - 1)e^{kx}$, where $x \in \mathbb{R}$ and $k \in \mathbb{Q}$.

The tangent to the curve at the point where $x = 1$ is parallel to the line $y = 5e^{k}x$.

Find the value of k.

22M.1.SL.TZ1.6

Consider $f(x) = 4\sin x + 2.5$ and $gx = 4\sin x - \frac{3\pi}{2} + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

a. Describe these two transformations. [2]

b. The y-intercept of the graph of g is at $(0, \ r)$.

Given that $g(x) \geq 7$, find the smallest value of r. [5]

22M.1.SL.TZ2.1

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	-2	0	3	4
$f(x)$	8	4	0	-3
$g(x)$	-5	-2	4	0

a. Find $g(0)$. [1]

b. Find $(f \circ g)(0)$. [2]

c. Find the value of x such that $f(x) = 0$. [2]

17M.1.SL.TZ1.S_9

A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0, \ -6)$

a. Find the value of p. [3]

b. Find the value of a. [3]

c. The line $y = kx - 5$ is a tangent to the curve of f. Find the values of k. [8]