Gravitational Fields

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Course	HL IB Physics
Section	Fields
Topic	Gravitational Fields
Difficulty	Medium

To be used by all students preparing for HL IB Physics
Students of other boards may also find this useful

1

The correct answer is **D** because:

- The question is asking about the weight of the module on Earth and **not** on Jupiter
- Weight = mass × gravitational field strength on Earth
- Weight = 4000 × 10 = 40 000 N

A is incorrect as this is the mass of the module and **not** the weight of the module on Earth

B is incorrect as the question is asking about the weight of the object on Earth and **not** on Jupiter, so the mass should be multiplied by the gravitational field strength on Earth and not by that on Jupiter

C is incorrect as the weight of the module on Earth is its mass × gravitational field strength on Earth. It is not mass × the increase in gravitational field strength on Jupiter

This question is easy, but you need to read it carefully. It is asking about the weight of the object on Earth and **not** on Jupiter. Remember that you can assume *g* on earth is 10 N kg^{-1} for paper 1.

2

The correct answer is **B** because:

- The gravitational field strength on Earth is defined by the equation:
  - $g_{earth} = \frac{Gm}{r^2}$
- For the new planet:
  - mass = 3*m* and radius = $\frac{1}{3}r$
- Substituting these values into the equation gives:
  - $g_{planet} = \frac{G \times 3m}{\left(\frac{1}{3}r\right)^2} = \frac{3Gm}{\frac{1}{9}r^2} = \frac{3Gm \times 9}{r^2} = \frac{27Gm}{r^2}$
- Therefore:
  - $g_{planet} = 27 \times g_{earth} = 27 \times 10$ N kg^{-1}
  - $g_{planet} = 270$ N kg^{-1}

A is incorrect as this is the gravitational field strength on Earth and not on the new planet

C is incorrect as the radius of the new planet is $\frac{1}{3}$ of the radius of Earth.

When substituted into the formula $g = \frac{Gm}{r^2}$ then $\frac{1}{3}$ r must be **squared**, so g is 9 × 3 times bigger on the new planet than on Earth and not just 3 × 3 times bigger, as $3^2 = 9$

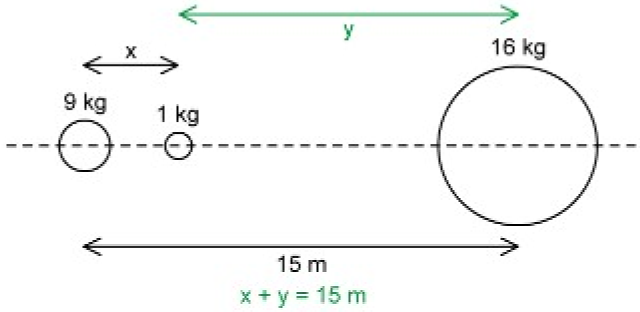
D is incorrect as the radius of the new planet is $\frac{1}{3}$ of the radius of Earth. 3^2 = 9 and not 6, so g is 9 × 3 times bigger on the new planet than on Earth and not just 6 × 3 times bigger

This question requires you to use the known equation for gravitational field strength and substitute the correct values in carefully. Show your workings clearly so that you do not make a mistake. For non-calculator questions, it is valid to assume that $g = 10\ N\ kg^{-1}$.

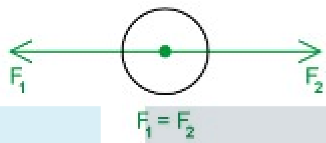
3

The correct answer is **A** because:

- The equation for the gravitational force F between two masses m_1 and m_2 is:
  - $F = \frac{Gm_1m_2}{r^2}$
- Therefore, the force between m_1 = 1 kg and m_2 = 9 kg if the distance between them is x is given by $F_1 = \frac{9G}{x^2}$
- Additionally, the force between m_1 = 1 kg and m_2 = 16 kg if the distance between them is is given by $F_2 = \frac{16G}{y^2}$
- Therefore $x + y = 15$ as shown below:



- If the 1 kg mass experiences zero resultant force, then $F_1 = F_2$:



- Therefore, equating the force equations gives:
    - $\frac{9G}{x^2} = \frac{16G}{y^2}$
    - $\frac{16}{9} = \frac{y^2}{x^2}$
    - $\frac{y}{x} = \frac{4}{3}$ and so $y = \frac{4}{3}x$
- Using the fact that x + y = 15, then substituting in $y = \frac{4}{3}x$ gives:
    - $x + \frac{4}{3}x = 15$
    - $\frac{7}{3}x = 15$
    - $7x = 45$
    - $x = \frac{45}{7} = 6.4$ m

B is incorrect as $x + y = 15$ and not $15 + x = y$

C is incorrect as $\frac{16}{9} = \frac{y^2}{x^2}$ and not $\frac{16}{9} = \frac{y}{x}$

D is incorrect as,

$\frac{7}{3}x = 15$ when rearranged becomes $x = \frac{15 \times 3}{7} = \frac{45}{7}$ and not

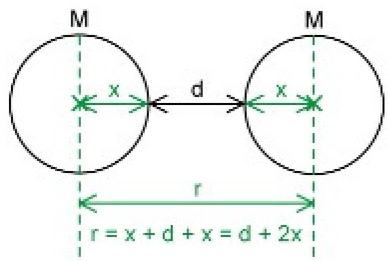
$x = \frac{15 \times 7}{3}$

Always annotate the diagrams to help visualise any missing distances so you can make the correct calculation. Simultaneous equations are a common occurrence in IB physics that you would have learnt from GCSE, an annotated diagram will also show you other equations such as the sum of distances.

4

The correct answer is **D** because:

- The equation for the gravitational force F between two masses m_1 and m_2 is:
  - $F = \frac{Gm_1m_2}{r^2}$
- r is the distance between the centre of masses, which means you need to consider the additional distance due to the radius of the spheres:



- Therefore:
  - $r = d + x + x = d + 2x$ where x is the radius of the sphere
- Substituting this equation for r into the force F equation gives:
  - $F = \frac{GM \times M}{r^2} = \frac{GM^2}{(d+2x)^2}$

- Rearranging the equation to obtain and expression for M is:
  - $M^2 = \frac{F(d+2x)^2}{G}$
  - So, $M = \sqrt{\frac{F(d+2x)^2}{G}}$

A is incorrect as $F = \frac{GM^2}{r^2}$ and not $F = \frac{GM}{r^2}$ (which is the equation for g) so when rearranged the final equation must be square rooted to obtain the correct expression for M

B is incorrect as the gravitational force acts from the centre of each mass and not just from the surface of each mass, so the radii of both masses need to be considered. F must be multiplied by d the distance between the masses + $2x$ the distance from the surface of each mass to the centre. This gives $d + 2x$

C is incorrect as in the equation for gravitational force $F = \frac{Gm_1m_2}{r^2}$ where r is squared and not just r this means the top line of the fraction should be $F \times (d + 2x)^2$ and not $F \times (d + 2x)$

You should remember that the distance between bodies in gravitational fields needs to be between the **centre of mass** of each body. This is an important consideration for the distance r.

Especially in questions with additional context, like a satellite orbiting the Earth, you are sometimes given only the distance between the Earth's **surface** and the body. This distance is **not** the distance you should substitute into equations for gravitational force and / or field strength: you would need to include the **radius** of the entire planet (i.e., from the surface to its centre) as well! Only then would the distance between the two bodies (the Earth and the satellite) be between their centres of mass.

5

The correct answer is **D** because:

- The gravitational force on the satellite from the Earth is a centripetal force
- Centripetal force is defined by the equation:
  - $F = m\omega^2 r$
- The gravitational is defined by Newton's law of gravitation:
  - $F = \frac{GMm}{r^2}$
- Therefore, equating both forces gives:
  - $m\omega^2 r = \frac{GMm}{r^2}$
  - Where m is the mass of the satellite, M is the mass of the body being orbited (the Earth) and r is the orbital radius
- Rearranging for r gives:
  - $\omega^2 r = \frac{GM}{r^2}$
  - $r^3 = \frac{GM}{\omega^2}$
  - $r = \sqrt[3]{\frac{GM}{\omega^2}} = \left(\frac{GM}{\omega^2}\right)^{\frac{1}{3}}$
- Therefore, the circumference of the satellite's orbit is:
  - $2\pi \times r = 2\pi\left(\frac{GM}{\omega^2}\right)^{\frac{1}{3}}$

A is incorrect as this is the expression for the radius of the satellites orbit and not the circumference. The answer for r must be multiplied by 2π as circumference is $2\pi r$

B is incorrect as ω^2 is not equal to $\frac{GMm}{r^2}$ the centripetal force $F = m\omega^2 r = \frac{GMm}{r^2}$

C is incorrect as the radius of the Earth's orbit is $\left(\frac{GM}{\omega^2}\right)^{\frac{1}{3}}$ and not just $\left(\frac{GM}{\omega^2}\right)$ because the equation is r^3 so needs to be cube-rooted.

This question requires very careful substitution and rearranging of the equations for gravitational and centripetal forces. Remembering also that the satellite remains in orbit because of the centripetal force caused by gravity is key to this question, and is very common in this topic! You will see many circular motion equations appearing in this topic too, so make sure to brush up on them.

6

The correct answer is **C** because:

- The gravitational field strength on Earth is defined by the equation:
  - $g_{earth} = \frac{Gm}{r^2}$
- For the new planet:
  - Mass = $2m$ and Radius = $3r$
- Substituting this into the gravitational field strength equation gives:
  - $g_{planet} = \frac{G \times 2m}{(3r)^2} = \frac{2Gm}{9r^2}$
- Therefore:
  - $g_{planet} = \frac{2}{9} g_{earth}$

A is incorrect as the mass of the planet is 2 × mass of earth and not 3 × m. The radius of the planet is 3 × radius of earth and not 2 × r.

B is incorrect as the 3 and *r* both need to be squared, so the final fraction should be and not as the formula is g = and not g =

D is incorrect as the fraction has been manipulated incorrectly to produce 2 × 9 as the numerator
This question requires you to use the known equation for gravitational field strength and substitute the correct values in carefully. Show your workings clearly so that you do not make a mistake and make sure that **all** values are squared that need to be!

7

The correct answer is **A** because:

- Newton's law of gravitational states that the force between two planets is:
  - $F = \frac{GMm}{r^2}$ where *r* is the separation between two masses *M* and *m*
- During separation *R* the force is:
  - $F = \frac{GMm}{R^2}$
- During separation 4*R* the force is:
  - $F = \frac{GMm}{(4R)^2} = \frac{GMm}{16R^2}$
- This means *F* for a separation of 4*R* is:
  - $\frac{F}{16}$ (i.e. 16 times less)

B is incorrect as the radius is 4 × *R*, so it is now 4 times further away. According to the inverse square law the force will be $\frac{1}{4^2}$ smaller and not 4 times larger

C is incorrect as the inverse square law states that the force will be reduced by $\frac{1}{4^2}$ and not just $\frac{1}{4}$

D is incorrect as the distance between the planets has changed, so the force between them will also change

The inverse square law relationship appears many times in this topic, remember to always consider it's impact.

8

The correct answer is **B** because:

- Gravitational field strength is a **vector** quantity
- In the second situation, *P* is now between two gravitational fields acting in **opposite** directions (towards P and towards Q)
- The overall magnitude of the gravitational field strength at *x* will be less than when *x* is in just one gravitational field

- Therefore, the correct answer is between *g* and zero
- This can be seen from the diagram below:

A is incorrect as the distances between *PX* and *QX* are not the same, so the gravitational field strengths from the two masses do not cancel each other out to give zero. PX is a lot smaller than QX

C & **D** are incorrect as gravitational field strength is a vector, so the gravitational field strength of *P* acts in one direction and in *Q* acts in the other, so the overall magnitude of the gravitational field strength at *X* is less and not more than between *P* and *X*.

This question requires you to recognise that gravitational field strength is a vector, so acting in one direction, it is positive and in the other it is negative. This means that it has a **direction** and this must be taken into consideration.

9

The correct answer is **C** because:

- Since the spacecraft is travelling with its motors shut down, this means it is decelerating due to the gravitational field strength of Earth
- Newton's second law states: $F = ma = mg$
- So, the acceleration of the rocket can be calculated using the equation:

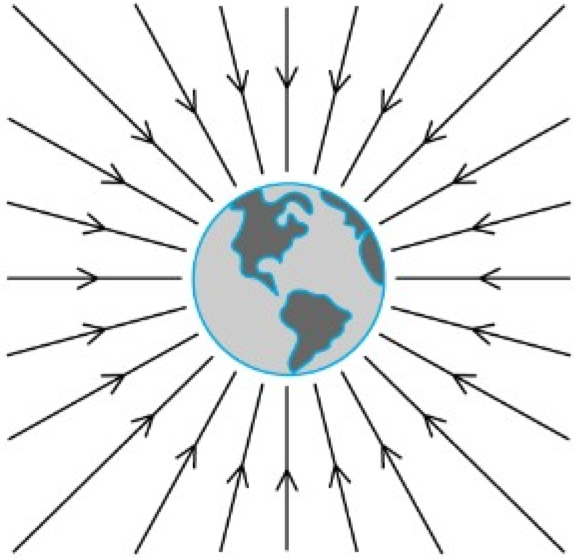
- a = change in velocity / change in time = $\frac{v-u}{t}$ where v is the final velocity and u is the initial velocity
- To use this formula km s^{-1} must be converted to m s^{-1}
  - There are 1000 m in 1 km
    - 2 km s^{-1} = 5200 m s^{-1}
    - 7 km s^{-1} = 5700 m s^{-1}
- Therefore:
  - Gravitational field strength, $g = \frac{v-u}{t} = \frac{5200-5700}{1000}$ = -0.5 N kg^{-1}
- In standard form, this is -5×10^{-1} N kg^{-1}
- The acceleration is negative because the space shuttle is moving away from the Earth and the gravitational field strength is acting towards the Earth, therefore it is decelerating due to the gravitational field pulling it back

A is incorrect as the velocities of the spacecraft must be converted into m s^{-1} to use the equations of motion to calculate acceleration and not leave the velocities in km s^{-1}

B is incorrect as this is the initial velocity 5.7 minus final velocity 5.2. The equation states that $a = \frac{v-u}{t} = \frac{\text{final} - \text{initial}}{t}$, not the other way around

D is incorrect as this is the strength of the Earth's gravitational field on Earth's surface. Whilst the spacecraft is affected by the Earth's gravitational field, it is only affected by a **fraction** of it since it is moving away where the field strength is getting weaker

Remembering to use the equation $a = \frac{v-u}{t}$ is key to answering this question correctly. Also recognising that the gravitational field strength will be negative because the object is moving away from the Earth is important to success in this question. The Earth's gravitational field strength acts towards the centre of the Earth. So, it acts in the opposite direction to a rocket moving away from the Earth.



10

The correct answer is **B** because:

- The acceleration a of a mass m is given by $a = \frac{F}{m}$ where F is the resultant force on the mass
- The resultant force on a mass in a gravitational field is given by $F = mg$ (because the gravitational field strength $g = \frac{F}{m}$)
- Therefore, $a = \frac{F}{m} = \frac{mg}{m} = g$
- Therefore, statement **B** is incorrect

This question is vital to help you understand that the acceleration of any freefalling body in a gravitational field is independent of its mass: it only depends on the strength of the field. This is one of the major discoveries of physics, famously investigated by Galileo Galilei and verified by astronauts on the moon, who observed a hammer and a feather falling to the surface of the moon at exactly the same rate (in the absence of air resistance!)

11

The correct answer is **D** because:

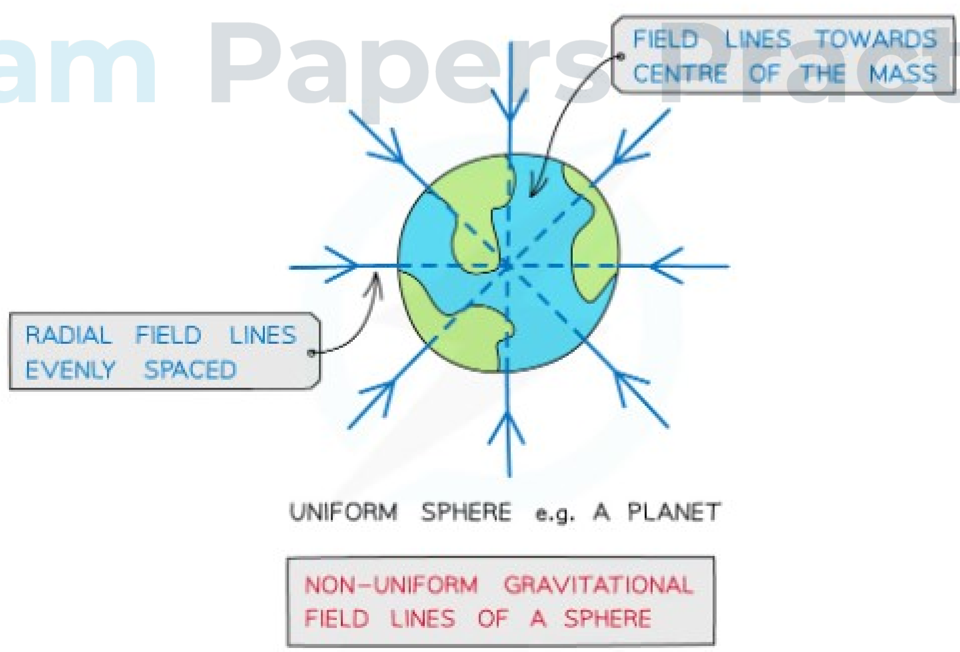
- The gravitational field around *M* is radial
- The path of the test mass *m* is parallel to a gravitational field line, and is in the same direction
    - Therefore, the gravitational field does (positive) work on the test mass
    - This is because work is defined as the product of force and distance moved in the same direction as the force

A is incorrect as positive work is done by the gravitational field, since the gravitational force attracts *m* to *M*. In other words, the motion of *m* is in the same direction as the field line, therefore, energy is transferred to *m*

B is incorrect as negative work would be done only if the test mass moved in the **opposite direction** to the field line, or in other words, if the test mass moved in the opposite direction to the gravitational force

C is incorrect as gravitational equipotential lines are circles of constant radius from the centre of mass of *M*. The line AB is parallel to the gravitational field lines around *M*

A radial field around a planet consists of field lines that radiate out of the centre, as shown in the image below



12

The correct answer is **C** because:

- The work done is perpendicular to equipotential lines
  - In other words, in the direction of the electric field caused by *Q*
- The electric potential difference ΔV_e is greater from P along path X
  - Path X starts at the outermost equipotential and finishes at the innermost equipotential
  - This potential gap is greater than path Y, which only crosses a potential difference to the middle equipotential
- Therefore, more work is done moving the test charge from P along path X towards *Q*

Remember that **no work is done** by electric field when a test charge moves **along** an equipotential surface. This is because the change in potential, $\Delta V_e = 0$. Hence, the work done, $W = q\Delta V_e = 0$.

13

The incorrect statement is **D** because:

- The gravitational potential $V_g = -\frac{GM}{r}$
- For constant radial distance *r*, the value of the potential V_g is proportional to mass *M*
  - Imagine an equipotential surface corresponding to –400 V, with some radius *r* from the centre of mass
  - The radius of this equipotential surface can only be varied by varying the mass of *M*, not its diameter

A is correct as equipotentials are lines or surfaces corresponding to constant gravitational potential. For a spherical mass *M*, points of constant gravitational potential would be joined by a sphere around *M*

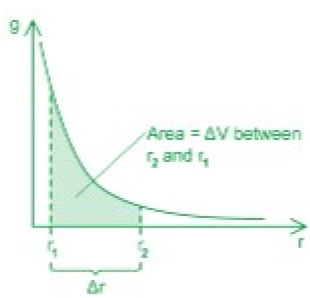
B is correct as the gravitational field strength g is proportional to the **gradient** of the potential, $\frac{\Delta V}{\Delta r}$. Since g decreases with distance by the inverse square law, or $g \propto \frac{1}{r^2}$, then equal changes in potential ΔV should happen over greater distances Δr further away from M. This corresponds to a shallower potential gradient

C is correct as work is only done by the gravitational field if a test mass moves parallel to the direction of gravitational force - i.e., along a gravitational **field line**. Equipotential surfaces are **perpendicular** to field lines; hence, no work is done by the gravitational field along an equipotential surface

14

The correct answer is **D** because:

- The gravitational field strength $g = -\frac{GM}{r^2}$
  - This has the units of N kg^{-1}
- The area under a gravitational field strength–distance curve has the units of field strength multiplied by the units of distance
  - Therefore (N kg^{-1}) × (m) = N m kg^{-1} = J kg^{-1}
- Therefore, the area represents a change in gravitational potential between two points
- This is shown on the diagram below:



A is incorrect as there is a point between the Earth and the Moon at which the resultant gravitational field strength is zero, but the potential is non-zero. This is represented as maximum point on a potential–distance graph

B is incorrect as the gravitational field (hence, the gravitational force) is always attractive – directed toward the centre of mass. Gravitational potential is the work done per unit of mass to bring a test mass from infinity. Since zero gravitational potential is defined at infinity, everywhere else is at negative potential (i.e., work must be done on a test mass to move it to infinity, against the attractive gravitational field)

C is incorrect as

the gradient of the gravitational potential is related to the gravitational field strength $g = -\frac{\Delta V}{\Delta r}$ Therefore, the gradient of the potential $\frac{\Delta V}{\Delta r} \propto \frac{1}{r^2}$

Therefore, the gradient of the gravitational potential is inversely proportional to the square of the radial distance *r* from some massive body

15

The correct answer is **B** because:

- The escape velocity v_{esc} of a spherical mass *M* is given by $\sqrt{\frac{2GM}{r}}$ where *r* is the radius of the mass
- Therefore:
  - The escape velocity from Europa is given by $\sqrt{\frac{2Gm_E}{r_E}}$
  - The escape velocity from Jupiter is given by $\sqrt{\frac{2Gm_J}{r_J}}$
- The ratio $\frac{escape\ velocity\ of\ Europa}{escape\ velocity\ of\ Jupiter}$ is then
  - $\sqrt{\frac{2Gm_E}{r_E}} \div \sqrt{\frac{2Gm_J}{r_J}} = \sqrt{\frac{2Gm_E}{r_E}} \times \sqrt{\frac{r_J}{2Gm_J}}$

- This simplifies to:
  - $\sqrt{\frac{2Gm_E r_J}{r_E \times (2Gm_J)}} = \sqrt{\frac{m_E r_J}{m_J r_E}}$
  - Therefore, the correct answer is **B**

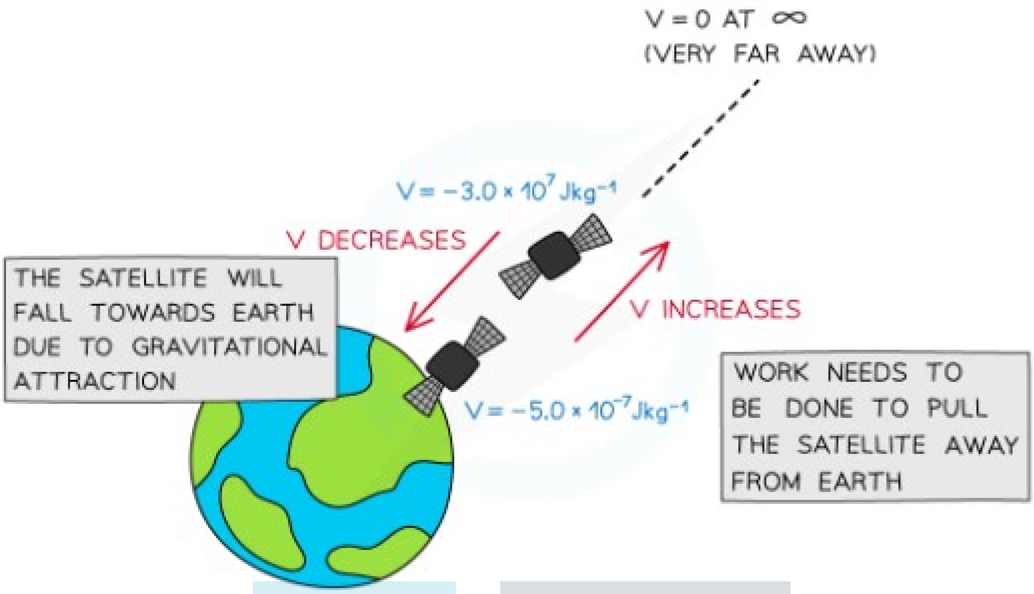
Make sure you can manipulate expressions algebraically, carefully keeping track of subscripts such that the appropriate quantities refer to the correct bodies as stated in the question.

16

The correct answer is **D** because:

- The change in gravitational potential is from –40 MJ kg^{-1} to –10 MJ kg^{-1}
  - Hence, the gravitational potential increases by 30 MJ kg^{-1} and gets closer to 0
  - Therefore, the satellite must be moving further away from the Earth
- Since the satellite is moving further away from Earth, it must be moving antiparallel to a field line
  - This is because field lines point toward the centre of mass of the Earth
  - This eliminates options A and C
- The work done W (or energy transferred) as a satellite of mass m moves across a gravitational potential difference ΔV_g is given by $W = m\Delta V_g$
  - The change in potential ΔV_g = 30 MJ kg^{-1} = 30 × 10^6 J kg^{-1}
  - Hence, the change in gravitational potential energy W = 2000 × (30 × 10^6) = 6 × 10^{10} = 60 GJ
- Therefore, the correct answer is **D**

You should be able to the direction of motion of any test mass in a gravitational field depending on the initial and final values of potential. Since gravitational potential is defined as **zero at infinity**, and negative everywhere else, if the final value of potential is **less negative** than the initial value - as it is in this question - the test mass must be moving **further away** from the source mass (i.e., Earth) towards infinity. Therefore, it is moving **antiparallel** to a gravitational field line, which always points towards the centre of the source mass.



17

The correct answer is **B** because:

- The international space station is in orbit around Earth; it therefore acts as a satellite
- The kinetic energy of a satellite of mass *m* moving with speed *v* is always positive, and is given by $E_K = \frac{1}{2}mv^2$
  - This eliminates options A and C
- The potential energy of a satellite moving in a gravitational field is always negative, and is given by $E_P = -\frac{GMm}{r}$ where *M* is the source mass in the field
- The total energy *E* of an orbiting satellite is negative, and is given by $E = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right)$
  - Therefore, $E_P = E - E_K$
  - Hence, $E > E_P$
- Therefore the correct answer is **B**
  - Because the line corresponding to the total energy *E* is always greater than the line corresponding to the potential energy E_P

Any orbiting satellite has a **total energy** equal to the **sum** of the kinetic and potential energy. Some important facts to remember are:

- The potential energy is **always negative** in a gravitational field
- The total energy is **always negative** for an orbiting body
- The total energy is **always greater** than the potential energy

18

The correct answer is **C** because:

- The total energy of the probe at launch $E_T = E_K + E_P$, where E_K and E_P are the probe's kinetic and gravitational potential energy respectively
  - Therefore, $E_T = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$ where M and R are the Earth's mass and radius respectively, and m is the mass of the probe
- The total energy of the probe at maximum height from the surface of the Earth $E_T = -\frac{GMm}{r}$
  - This is because the kinetic energy is zero at maximum height, as the probe momentarily comes to rest
  - Here, $r = R + h$, where h is the maximum height above the Earth's surface
- Since the velocity $v = \frac{v_{esc}}{2}$ at launch, then the total energy at launch becomes:
  - $E_T = \frac{1}{2}m\left(\frac{v_{esc}}{2}\right)^2 + \left(-\frac{GMm}{R}\right) = \frac{mv_{esc}^2}{8} - \frac{GMm}{R}$
- Using $v_{esc} = \sqrt{\frac{2GM}{r}}$
  - $E_T = \frac{m\left(\sqrt{\frac{2GM}{R}}\right)^2}{8} - \frac{GMm}{R} = \frac{2GMm}{8R} - \frac{GMm}{R}$
  - Hence, $E_T = \frac{2GMm}{8R} - \frac{8GMm}{8R} = -\frac{6GMm}{8R} = -\frac{3GMm}{4R}$

- Total energy of the probe is conserved, therefore the total energy is equal to the energy at the probe's maximum height. Therefore:
  - $-\frac{3GMm}{4R} = -\frac{GMm}{r}$
  - $\frac{3}{4R} = \frac{1}{r}$
  - Hence, $r = \frac{4R}{3}$
- Recalling that r is the distance from the Earth's centre of mass, such that $r = R + h$ where h is the height above Earth's surface, then:
  - $R + h = \frac{4R}{3}$
  - Hence, $h = \frac{4R}{3} - R = \frac{R}{3}$
- Therefore, the correct answer is **C**

There are a couple of super important things to remember when tackling questions about launching probes or satellites:

- The total energy E_T is always **conserved**. Hence, you can find an equation for the total energy at launch, which you can equate to **other equations later** in the probe's launch: the form may change, but the quantity does not
- Note the gravitational potential energy **at launch** is given in terms of the planet's radius R, whereas later in the probe's journey it is given in terms of a general distance r. This is because the launch begins at the Earth's surface, therefore the potential energy must be the value of the potential energy at this point
- Know the equation for the escape velocity, $v_{esc} = \sqrt{\frac{2GM}{r}}$. This is given in your data booklet, but being familiar with its form is very handy!

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The correct answer is **C** because:

- The escape velocity from any spherical mass, like the Sun, is given by $v_{esc} = \sqrt{\frac{2GM}{r}}$ where M is the mass of the Sun and r is its radius
- Hence, if the escape velocity is exactly equal to the speed of light c, then:
  - $c = \sqrt{\frac{2GM}{r}}$ which gives $c^2 = \frac{2GM}{r}$
- Therefore, the radius at which the escape velocity is exactly equal to the speed of light is given by $r = \frac{2GM}{c^2}$
- Hence,
  - $r = \frac{2 \times (6.67 \times 10^{-11}) \times (2 \times 10^{30})}{(3 \times 10^8)^2}$
- Approximating G, M and c to 1 significant figure allows an estimation of this radius to be computed as follows:
  - $r \approx \frac{2 \times (7 \times 10^{-11}) \times (2 \times 10^{30})}{(3 \times 10^8)^2} = \frac{28 \times 10^{19}}{9 \times 10^{16}}$
  - Hence, $r \approx 3 \times 10^3$ which is approximately 3 km
- Therefore, the correct answer is **C**

The special radius at which any spherical mass M has an escape velocity equal to the speed of light c is known as the **Schwarzschild Radius**. You do not need to know this for your examination, however; understanding how to use the equation for escape velocity, and estimating quantities by rounding to an appropriate number of significant figures, are useful techniques to practise.

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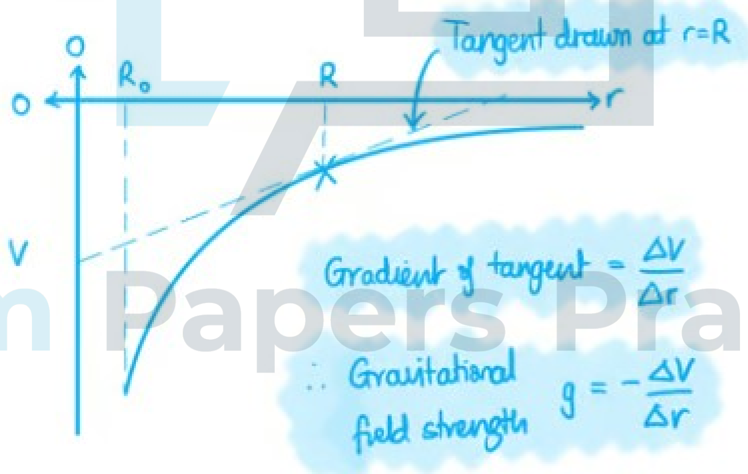
The correct answer is **D** because:

- The gravitational field strength $g = -\frac{\Delta V_g}{\Delta r}$
  - The quantity $\frac{\Delta V_g}{\Delta r}$ represents the gradient of a potential-distance graph, like the one in this question
  - Therefore, the gravitational field strength at the distance $r = R$ is determined by calculating the gradient of the potential at that point and multiplying it by –1
- Therefore, the correct answer is **D**

This question catches a lot of candidates out because many are in the (good!) habit of calculating the gradient of a graph **and then stopping there**. Though subtle, you must remember that the field strength is **proportional** to the gradient of a potential, but is **exactly equal** to the **negative** of the gradient. This is sketched out below:



A bit of (non-rigorous!) calculus also shows this nicely:

$$V = -\frac{GM}{r} = -GMr^{-1}$$

$$\frac{dV}{dr} = GMr^{-2} = \frac{GM}{r^2} = -g$$

So, $g \propto \frac{dV}{dr}$

$$g = -\frac{dV}{dr}$$

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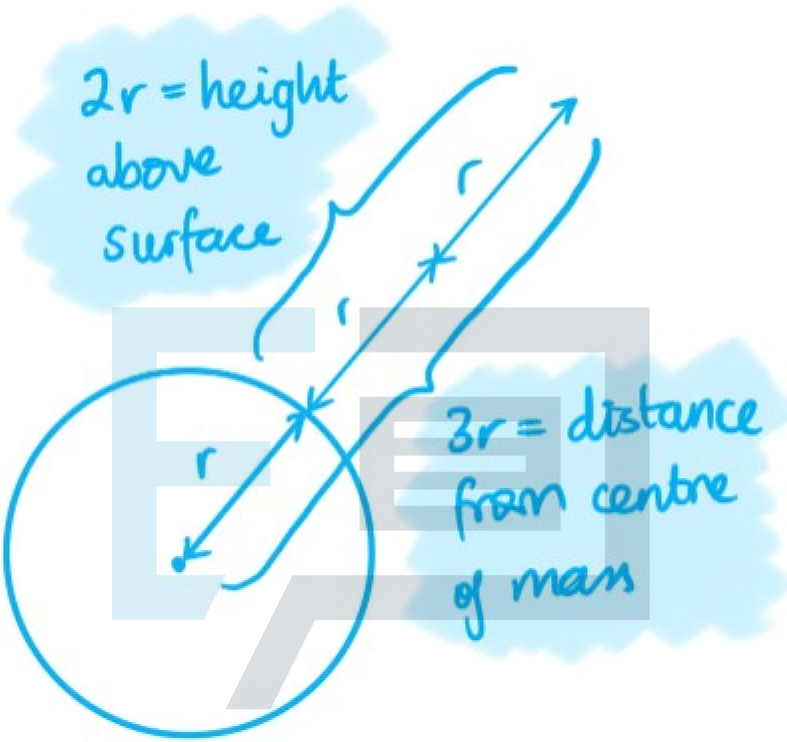
The correct answer is **C** because:

- The magnitude of the gravitational field strength is given by $g = \frac{GM}{r^2}$ for a spherical source of mass M at a distance r from its centre of mass
  - Hence, if the radius of Earth is given as r, then this equation gives the magnitude of the gravitational field strength at its surface
- At a height of $2r$ from the Earth's surface, the distance from Earth's centre is therefore $r + 2r = 3r$
  - Hence, the gravitational field strength at this height $g' = \frac{GM}{(3r)^2} = \frac{GM}{9r^2} = \frac{g}{9}$
- The magnitude of the gravitational potential is given by $V = -\frac{GM}{r}$ for a spherical source of mass M at a distance r from its centre of mass
  - Again, the height from the Earth's surface is $2r$
  - Therefore, from its centre of mass the total distance is $r + 2r = 3r$
  - The gravitational potential becomes $V' = -\frac{GM}{(3r)} = \frac{V}{3}$

- Therefore the correct answer is **C**

Drawing a quick sketch - as should be the case for any question without one! - will help you visualise the correct distances to use in equations for the Fields At Work topic. Check this out below:



As you can see, the correct distance to use in equations for gravitational field strength and potential at the surface of a planet is *r*, since this is determined from the object's centre of mass. This question, though seemingly simple at first, requires a bit of thought - and as you can see, a sketch to help! It's a commonly used exam trick: so be on the lookout for similar questions which give you distances from the surface of a planet or spherical mass.