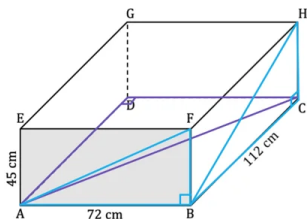


Geometry of 3D Shapes

Mark Schemes

Question 1

The diagram below shows a cuboid measuring $45\text{ cm} \times 72\text{ cm} \times 112\text{ cm}$.



- (a) (i) Calculate the distance from A to F.
 (ii) Calculate the distance from B to H.
 (iii) Calculate the distance from A to C.

(b) Calculate the distance from B to G.

a) Notice the right-angled triangles.

$$\begin{aligned} \text{i) } AF &= \sqrt{AB^2 + BF^2} \\ AF &= \sqrt{72^2 + 45^2} \end{aligned}$$

$$AF \approx 84.9\text{ cm}$$

$$\begin{aligned} \text{ii) } BH &= \sqrt{BC^2 + CH^2} \\ BH &= \sqrt{112^2 + 45^2} \end{aligned}$$

$$BH \approx 121\text{ cm}$$

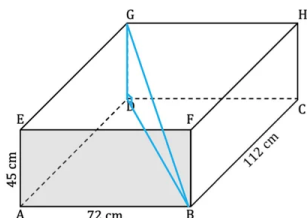
[3]

$$\begin{aligned} \text{iii) } AC &= \sqrt{AD^2 + DC^2} \\ AC &= \sqrt{72^2 + 112^2} \end{aligned}$$

[2]

$$AC \approx 133\text{ cm}$$

The diagram below shows a cuboid measuring $45\text{ cm} \times 72\text{ cm} \times 112\text{ cm}$.



- (a) (i) Calculate the distance from A to F.
 (ii) Calculate the distance from B to H.
 (iii) Calculate the distance from A to C.

$$AC \approx 133\text{ cm}$$

(b) Calculate the distance from B to G.

b) Notice the right-angled triangle BDG.

$$BG = \sqrt{BD^2 + DG^2}$$

$$BD = AC = 133\text{ cm}$$

$$DG = AE = 45$$

$$BG = \sqrt{133^2 + 45^2}$$

$$BG \approx 141\text{ cm}$$

[3]

[2]

Question 2

A nickel earring in the shape of a sphere has a radius of 4mm.

- (a) Find the volume of the earring, expressing your answer in the form of $a \times 10^k$, where $1 \leq a \leq 10$ and k is an integer.

[3]

The nickel earring is to be melted down and reshaped to form a cylinder with a height of 16mm.

- (b) Find the radius of the cylinder.

[2]

a) Volume of a sphere

$$V = \frac{4}{3} \pi r^3$$

(in formula booklet)

$$V = \frac{4}{3} \pi (4)^3$$

$$V = \frac{256}{3} \pi = 268.08257\dots$$

$$V = 2.68 \times 10^2 \text{ mm}^3 \text{ (3sf)}$$

A nickel earring in the shape of a sphere has a radius of 4mm.

- (a) Find the volume of the earring, expressing your answer in the form of $a \times 10^k$, where $1 \leq a \leq 10$ and k is an integer.

$$V = \frac{256}{3} \pi$$

[3]

The nickel earring is to be melted down and reshaped to form a cylinder with a height of 16mm.

- (b) Find the radius of the cylinder.

[2]

b) Volume of a cylinder

$$V = \pi r^2 h$$

(in formula booklet)

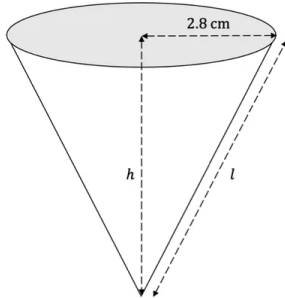
$$\frac{256}{3} \pi = \pi r^2 (16)$$

$$r = \sqrt{\frac{16}{3}} = 2.3094\dots$$

$$r = 2.31 \text{ mm (3sf)}$$

Question 3

A waffle ice cream cone forms a right circular cone that has a volume of 120 cm^3 and a radius of 2.8 cm .



(a) Find the height, h , of the cone.

[2]

(b) Find the slant height, l , of the cone.

[2]

(c) Calculate the curved surface area of the cone.

[2]

a) Volume of a right circular cone

$$V = \frac{1}{3} \pi r^2 h \quad (\text{in formula booklet})$$

$$V = 120 \quad r = 2.8$$

Sub V and r into formula and rearrange for h .

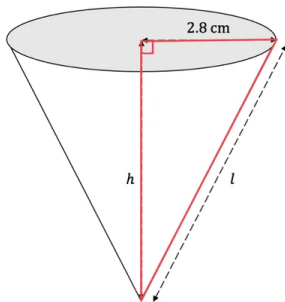
$$120 = \frac{1}{3} \pi (2.8)^2 h$$

$$h = \frac{120}{\frac{1}{3} \pi (2.8)^2}$$

$$h = 14.616\dots$$

$$h = 14.6 \text{ cm (3sf)}$$

A waffle ice cream cone forms a right circular cone that has a volume of 120 cm^3 and a radius of 2.8 cm .



(a) Find the height, h , of the cone.

$$h = 14.6 \text{ cm}$$

[2]

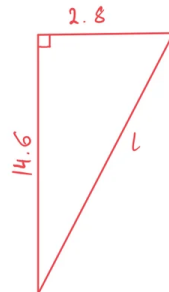
(b) Find the slant height, l , of the cone.

[2]

(c) Calculate the curved surface area of the cone.

[2]

b) Notice the right-angled triangle.



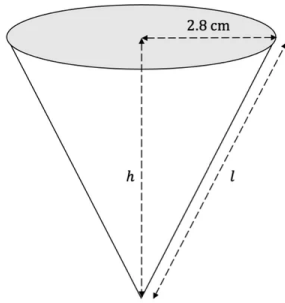
$$l = \sqrt{(14.6)^2 + (2.8)^2} \quad (\text{pythagoras})$$

$$l = \sqrt{221}$$

$$l = 14.866\dots$$

$$l = 14.9 \text{ cm (3sf)}$$

A waffle ice cream cone forms a **right circular cone** that has a volume of 120 cm^3 and a radius of 2.8 cm .



(a) Find the height, h , of the cone.

[2]

(b) Find the slant height, l , of the cone.

$l = 14.9 \text{ cm}$ (3sf)

[2]

(c) Calculate the curved surface area of the cone.

[2]

c) Curved surface area of a cone formula

$A = \pi r l$ (in formula booklet)

$r = 2.8$ $l = 14.9$

Sub r and l into formula.

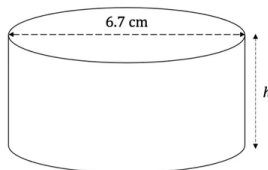
$A = \pi (2.8)(14.9)$

$A = 131.067\dots$

$A = 131 \text{ cm}^2$ (3sf)

Question 4

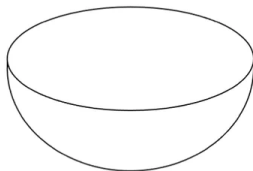
A baking container has the shape of a **cylinder**, as shown in the diagram below. The **diameter of the baking container is 6.7 cm** and its **volume, V , is 80 cm^3** .



(a) Find the **height, h** , of the baking container.

[2]

A bowl full of cake batter has the shape of a hemisphere, as shown in the diagram below. The cake batter is poured into the baking container and fills a quarter of the container.



(b) Find the radius, r , of the bowl.

[4]

a) Volume of a cylinder formula

$V = \pi r^2 h$ (in formula booklet)

$V = 80$ $r = \frac{6.7}{2} = 3.35$

Sub in V and r into formula and rearrange for h .

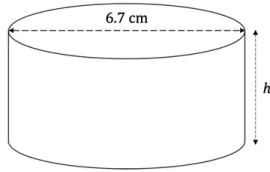
$80 = \pi (3.35)^2 h$

$h = \frac{80}{\pi (3.35)^2}$

$h = 2.269\dots$

$h = 2.27 \text{ cm}$ (3sf)

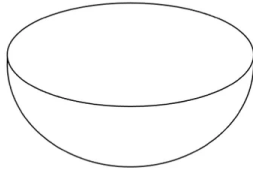
A baking container has the shape of a cylinder, as shown in the diagram below. The diameter of the baking container is 6.7 cm and its volume, V , is 80 cm^3 .



(a) Find the height, h , of the baking container.

[2]

A bowl full of cake batter has the shape of a hemisphere, as shown in the diagram below. The cake batter is poured into the baking container and fills a quarter of the container.



(b) Find the radius, r , of the bowl.

[4]

b) Volume of a hemisphere formula

$$V = \frac{2}{3} \pi r^3 \quad \left(\frac{V_{\text{sphere}}}{2} \right)$$

NB the volume of a hemisphere is half the volume of a sphere with the same radius.

$$V = 80 \times \frac{1}{4} = 20$$

Sub V into formula and rearrange for r .

$$20 = \frac{2}{3} \pi r^3$$

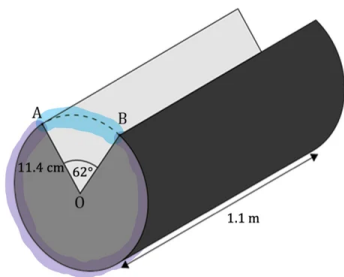
$$r = \sqrt[3]{\frac{20}{\frac{2}{3}\pi}}$$

$$r = 2.1215\dots$$

$$r = 2.12 \text{ cm (3sf)}$$

Question 5

Hamish is building a tree hut using cylindrical logs of length 1.1m and radius 11.4cm. A wedge is cut from the logs as shown.



(a) Find the length, in cm, of the

(i) minor arc AB

(ii) major arc AB.

(b) Find the area of the empty sector OAB.

(c) Find the volume of each log. Give your answer in cm^3 .

[3]

[2]

[3]

a) Arc length formula

$$l = \frac{\theta}{360} \times 2\pi r \quad (\text{in formula booklet})$$

$$\text{i) } \theta = 62 \quad r = 11.4$$

Sub θ and r into formula.

$$l = \frac{62}{360} \times 2\pi (11.4)$$

$$l = 12.3359\dots$$

$$l = 12.3 \text{ cm (3sf)}$$

$$\text{ii) } \theta = 360 - 62 = 298 \quad r = 11.4$$

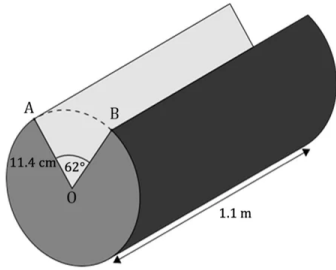
Sub θ and r into formula.

$$l = \frac{298}{360} \times 2\pi (11.4)$$

$$l = 59.2923\dots$$

$$l = 59.3 \text{ cm (3sf)}$$

Hamish is building a tree hut using cylindrical logs of length 1.1m and radius 11.4cm. A wedge is cut from the logs as shown.



(a) Find the length, in cm, of the

- (i) minor arc AB
- (ii) major arc AB.

(b) Find the area of the empty sector OAB.

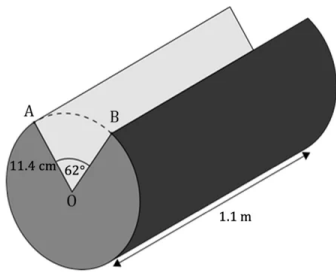
(c) Find the volume of each log. Give your answer in cm^3 .

[3]

[2]

[3]

Hamish is building a tree hut using cylindrical logs of length 1.1m and radius 11.4cm. A wedge is cut from the logs as shown.



(a) Find the length, in cm, of the

- (i) minor arc AB
- (ii) major arc AB.

(b) Find the area of the empty sector OAB.

(c) Find the volume of each log. Give your answer in cm^3 .

[3]

[2]

[3]

b) Sector area formula

$$A = \frac{\theta}{360} \times \pi r^2 \quad (\text{in formula booklet})$$

$$\theta = 62 \quad r = 11.4$$

Sub θ and r into formula.

$$A = \frac{62}{360} \times \pi (11.4)^2$$

$$A = 70.3151\dots$$

$$A = 70.3 \text{ cm}^2 \text{ (3sf)}$$

c) Volume (V) = Cross-sectional area (A) \times length (l)

Cross-sectional area is the major sector OAB.

$$\therefore V = \frac{\theta}{360} \times \pi r^2 \times l$$

sector area

$$\theta = 298 \quad r = 11.4 \quad l = 110 \quad (1.1 \text{ m} = 110 \text{ cm})$$

Sub θ , r and l into formula.

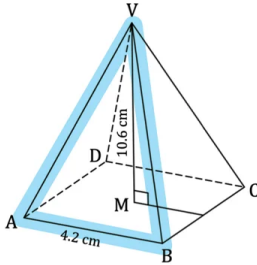
$$V = \frac{298}{360} \times \pi (11.4)^2 \times 110$$

$$V = 37176.2879\dots$$

$$V = 37200 \text{ cm}^3 \text{ (3sf)}$$

Question 6

In the diagram below ABCD is the square base of a right pyramid with vertex V. The centre of the base is M. The sides of the square base are 4.2 cm and the vertical height is 10.6 cm.



(a) Calculate the area of the triangle ABV.

[3]

(b) Calculate the length of AV.

[3]

(c) Find the size of the angle AV makes with the square base ABCD.

[3]

a) Area of a triangle

$$A = \frac{1}{2} bh$$

(in formula booklet)

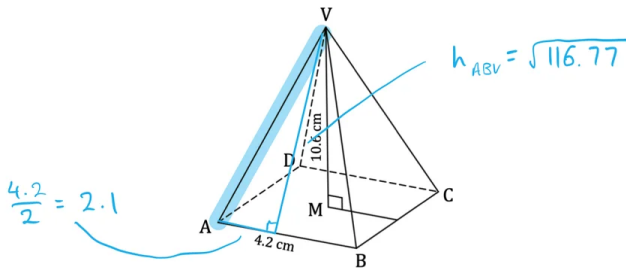
$$b_{ABV} = 4.2 \quad h_{ABV} = \sqrt{(2.1)^2 + (10.6)^2} = \sqrt{116.77}$$

$$A = \frac{1}{2} (4.2) (\sqrt{116.77})$$

$$A = 22.6926\dots$$

$$A = 22.7 \text{ cm}^2 \text{ (3sf)}$$

In the diagram below ABCD is the square base of a right pyramid with vertex V. The centre of the base is M. The sides of the square base are 4.2 cm and the vertical height is 10.6 cm.



(a) Calculate the area of the triangle ABV.

[3]

(b) Calculate the length of AV.

[3]

(c) Find the size of the angle AV makes with the square base ABCD.

[3]

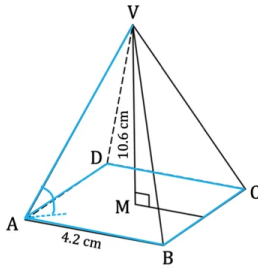
b) Find AV using pythagoras.

$$AV = \sqrt{(2.1)^2 + (\sqrt{116.77})^2}$$

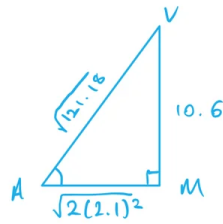
$$AV = \sqrt{121.18} = 11.008\dots$$

$$AV = 11.0 \text{ cm (3sf)}$$

In the diagram below ABCD is the square base of a right pyramid with vertex V. The centre of the base is M. The sides of the square base are 4.2 cm and the vertical height is 10.6 cm.



c) We have a right-angled triangle VAM.



$$\sin \hat{VAM} = \frac{10.6}{\sqrt{121.18}}$$

$$\hat{VAM} = \sin^{-1} \left(\frac{10.6}{\sqrt{121.18}} \right)$$

$$\hat{VAM} = 74.3^\circ \quad (3 \text{ s.f.})$$

(a) Calculate the area of the triangle ABV.

[3]

(b) Calculate the length of AV.

$$AV = \sqrt{121.18}$$

[3]

(c) Find the size of the angle AV makes with the square base ABCD.

[3]