Question 1

In this question, give all answers in the form $a \times 10^k$, where $1 \le a \le 10$ and $k \in \mathbb{Z}$, correct to three significant figures.

The height of a regulation basketball is 2286 mm. Assuming the surface of the basketball is a sphere:

(a) Calculate the circumference of the basketball. [2]

(b) Calculate the surface area of the basketball. [3]

(c) Calculate the volume of the basketball. [3]

a) Circle circumference formula

$C = 2\pi r$ (in formula booklet)

$r = \frac{1}{2}$ height

$r = \frac{1}{2}(2286)$

$r = 1143$

Sub r into formula.

$C = 2\pi(1143)$

$C = 7181.68...$

$C = 7180$ (3sf)

$\boxed{C = 7.18 \times 10^3 \text{ mm}}$

In this question, give all answers in the form $a \times 10^k$, where $1 \le a \le 10$ and $k \in \mathbb{Z}$, correct to three significant figures.

The height of a regulation basketball is 2286 mm. Assuming the surface of the basketball is a sphere:

(a) Calculate the circumference of the basketball. [2]

(b) Calculate the surface area of the basketball. [3]

(c) Calculate the volume of the basketball. [3]

b) Surface area of a sphere formula

$A = 4\pi r^2$ (in formula booklet)

$r = 1143$

Sub r into formula.

$A = 4\pi(1143)^2$

$A = 16\ 417\ 322.32...$

$A = 16\ 400\ 000$ (3sf)

$\boxed{A = 1.64 \times 10^7 \text{ mm}^2}$

In this question, give all answers in the form $a \times 10^k$, where $1 \le a \le 10$ and $k \in \mathbb{Z}$, correct to three significant figures.

The height of a regulation basketball is 2286 mm. Assuming the surface of the basketball is a sphere:

(a) Calculate the circumference of the basketball.

[2]

(b) Calculate the surface area of the basketball.

[3]

(c) Calculate the volume of the basketball.

[3]

c) Volume of a sphere formula

$V = \frac{4}{3}\pi r^3$ (in formula booklet)

$r = 1143$

Sub r into formula.

$V = \frac{4}{3}\pi(1143)^3$

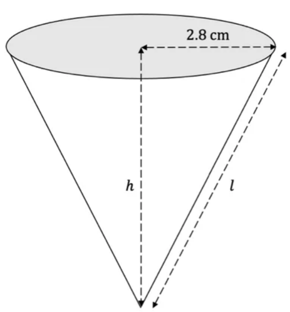
$V = 6\ 254\ 999\ 804.97$

$V = 6\ 250\ 000\ 000$ (3sf)

$\boxed{V = 6.25 \times 10^9 \text{ mm}^3}$

Question 2

A waffle ice cream cone forms a right circular cone that has a volume of 120 cm^3 and a radius of 2.8 cm.



(a) Find the height, h, of the cone.

[2]

(b) Find the slant height, l, of the cone.

[2]

(c) Calculate the curved surface area of the cone.

[2]

a) Volume of a right circular cone

$V = \frac{1}{3}\pi r^2 h$ (in formula booklet)

$V = 120$ $\quad r = 2.8$

Sub V and r into formula and rearrange for h.

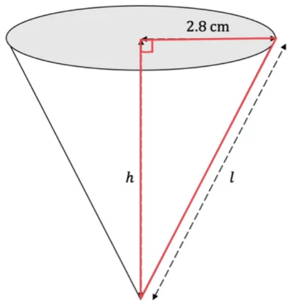
$120 = \frac{1}{3}\pi(2.8)^2 h$

$h = \frac{120}{\frac{1}{3}\pi(2.8)^2}$

$h = 14.616...$

$\boxed{h = 14.6 \text{ cm} \ (3sf)}$

A waffle ice cream cone forms a right circular cone that has a volume of 120 cm^3 and a radius of 2.8 cm.



(a) Find the height, h, of the cone.

$\boxed{h = 14.6 \text{ cm}}$

[2]

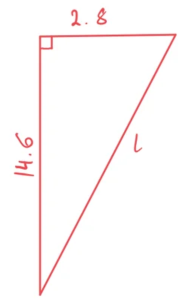
(b) Find the slant height, l, of the cone.

[2]

(c) Calculate the curved surface area of the cone.

[2]

b) Notice the right-angled triangle.



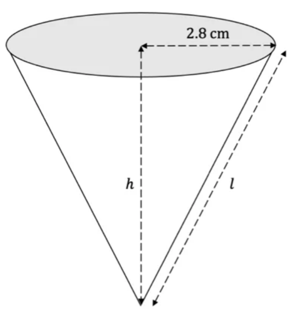
$l = \sqrt{(14.6)^2 + (2.8)^2}$ (pythagoras)

$l = \sqrt{221}$

$l = 14.866\ldots$

$\boxed{l = 14.9 \text{ cm } (3sf)}$

A waffle ice cream cone forms a right circular cone that has a volume of 120 cm^3 and a radius of 2.8 cm.



(a) Find the height, h, of the cone.

[2]

(b) Find the slant height, l, of the cone.

$\boxed{l = 14.9 \text{ cm } \quad (3sf)}$

[2]

(c) Calculate the curved surface area of the cone.

[2]

c) Curved surface area of a cone formula

$A = \pi r l$ (in formula booklet)

$r = 2.8 \qquad l = 14.9$

Sub r and l into formula.

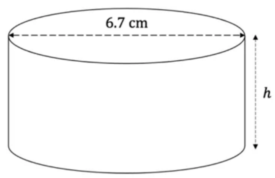
$A = \pi (2.8)(14.9)$

$A = 131.067\ldots$

$\boxed{A = 131 \text{ cm}^2 \quad (3sf)}$

Question 3

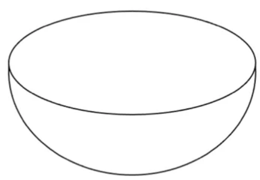
A baking container has the shape of a cylinder, as shown in the diagram below. The diameter of the baking container is 6.7 cm and its volume, V, is 80 cm^3.



(a) Find the height, h, of the baking container.

[2]

A bowl full of cake batter has the shape of a hemisphere, as shown in the diagram below. The cake batter is poured into the baking container and fills a quarter of the container.



(b) Find the radius, r, of the bowl.

[4]

a) Volume of a cylinder formula

$V = \pi r^2 h$ (in formula booklet)

$V = 80$ $\quad r = \frac{6.7}{2} = 3.35$

Sub in V and r into formula and rearrange for h.

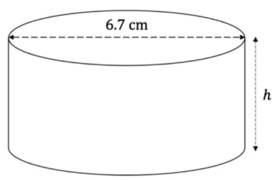
$80 = \pi(3.35)^2 h$

$h = \frac{80}{\pi(3.35)^2}$

$h = 2.269...$

$\boxed{h = 2.27 \text{ cm } (3sf)}$

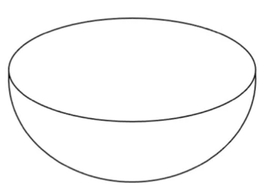
A baking container has the shape of a cylinder, as shown in the diagram below. The diameter of the baking container is 6.7 cm and its volume, V, is 80 cm^3.



(a) Find the height, h, of the baking container.

[2]

A bowl full of cake batter has the shape of a hemisphere, as shown in the diagram below. The cake batter is poured into the baking container and fills a quarter of the container.



(b) Find the radius, r, of the bowl.

[4]

b) Volume of a hemisphere formula

$V = \frac{2}{3}\pi r^3$ $\quad \left(\frac{V_{sphere}}{2}\right)$

NB the volume of a hemisphere is half the volume of a sphere with the same radius.

$V = 80 \times \frac{1}{4} = 20$

Sub V into formula and rearrange for r.

$20 = \frac{2}{3}\pi r^3$

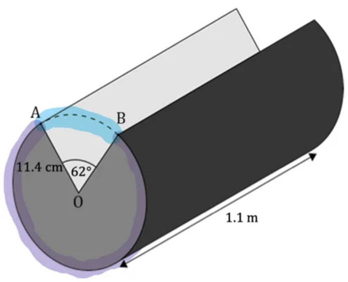
$r = \sqrt[3]{\frac{20}{\frac{2}{3}\pi}}$

$r = 2.1215...$

$\boxed{r = 2.12 \text{ cm } (3sf)}$

Question 4

Hamish is building a tree hut using cylindrical logs of length 1.1m and radius 11.4cm.
A wedge is cut from the logs as shown.



(a) Find the length, in cm, of the

(i) minor arc AB

(ii) major arc AB.

[3]

(b) Find the area of the empty sector OAB.

[2]

(c) Find the volume of each log. Give your answer in cm^3.

[3]

a) Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

i) $\theta = 62$ $r = 11.4$

Sub θ and r into formula.

$l = \frac{62}{360} \times 2\pi(11.4)$

$l = 12.3359...$

$\boxed{l = 12.3 \text{ cm (3sf)}}$

ii) $\theta = 360 - 62 = 298$ $r = 11.4$

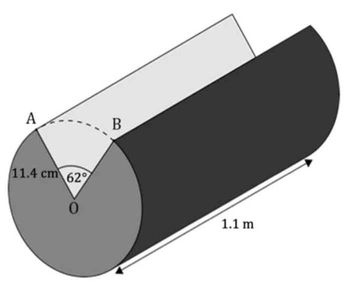
Sub θ and r into formula.

$l = \frac{298}{360} \times 2\pi(11.4)$

$l = 59.2923...$

$\boxed{l = 59.3 \text{ cm (3sf)}}$

Hamish is building a tree hut using cylindrical logs of length 1.1m and radius 11.4cm.
A wedge is cut from the logs as shown.



(a) Find the length, in cm, of the

(i) minor arc AB

(ii) major arc AB.

[3]

(b) Find the area of the empty sector OAB.

[2]

(c) Find the volume of each log. Give your answer in cm^3.

[3]

b) Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

$\theta = 62$ $r = 11.4$

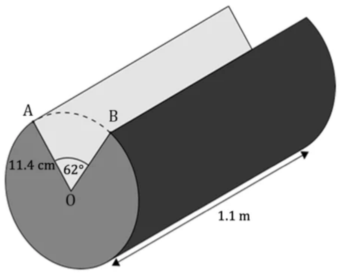
Sub θ and r into formula.

$A = \frac{62}{360} \times \pi(11.4)^2$

$A = 70.3151...$

$\boxed{A = 70.3 \text{ cm}^2 \text{ (3sf)}}$

Hamish is building a tree hut using cylindrical logs of length 1.1m and radius 11.4cm.
A wedge is cut from the logs as shown.



(a) Find the length, in cm, of the

(i) minor arc AB

(ii) major arc AB.

[3]

(b) Find the area of the empty sector OAB.

[2]

(c) Find the volume of each log. Give your answer in cm³.

[3]

c) Volume (V) = Cross-sectional area (A) × length (l)

Cross-sectional area is the major sector OAB.

$\therefore V = \underbrace{\frac{\theta}{360} \times \pi r^2}_{\text{sector area}} \times l$

$\theta = 298 \quad r = 11.4 \quad l = 110 \quad (1.1\text{ m} = 110\text{ cm})$

Sub θ, r and l into formula.

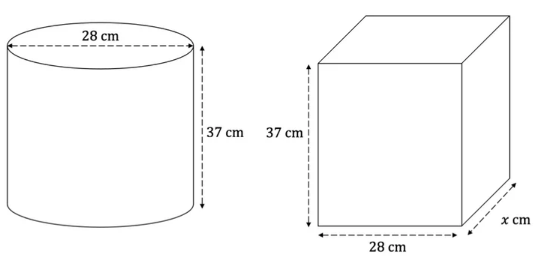
$V = \frac{298}{360} \times \pi (11.4)^2 \times 110$

$V = 37\,176.2879\ldots$

$\boxed{V = 37\,200\text{ cm}^3 \text{ (3sf)}}$

Question 5

Vivian has two containers. The first container is in the shape of a cylinder with diameter 28 cm and height 37 cm. The second container is in the shape of a cuboid with width 28 cm, height 37 cm and length x cm.



(a) Calculate the surface area of the first container.

[3]

Both containers have the same surface area.

(b) Find the value of x.

[4]

a) Surface area of a cylinder formula

*$A = \underbrace{2\pi r h}_{\text{curved surface area}} + \underbrace{2\pi r^2}_{2 \times \text{circular ends}}$

$r = \frac{28}{2} = 14 \qquad h = 37$

Sub r and h into formula.

$A = 2\pi(14)(37) + 2\pi(14)^2$

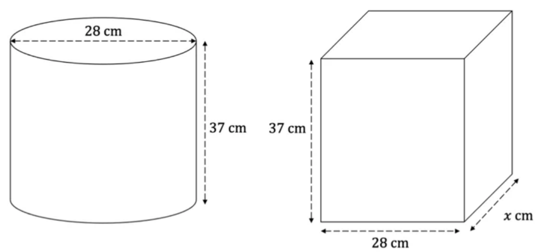
$A = 1428\pi$

$A = 4486.19\ldots$

$\boxed{A = 4490\text{ cm}^2 \text{ (3sf)}}$

* Curved surface area and circle area formula are in the formula booklet.

Vivian has two containers. The first container is in the shape of a cylinder with diameter 28 cm and height 37 cm. The second container is in the shape of a cuboid with width 28 cm, height 37 cm and length x cm.



(a) Calculate the surface area of the first container.

$\boxed{A = 4490 \text{ cm}^2 \ (3sf)}$

[3]

Both containers have the same surface area.

(b) Find the value of x.

[4]

b) Surface area of a cuboid formula

$A = 2lw + 2lh + 2wh$

$A = 4490 \quad w = 28 \quad h = 37 \quad l = x$

Sub A, w and h into formula and solve for x on your GDC.

$4490 = 2x(28) + 2x(37) + 2(28)(37)$

$\boxed{x = 18.6 \text{ cm}}$

Question 6

A stone is in the shape of a sphere with radius 1.84 m.

(a) Calculate the volume of the stone.

[2]

The stone is cooled and its volume decreases by 1%.

(b) Calculate the radius of the stone following this decrease.

[3]

a) Volume of a sphere formula

$V = \frac{4}{3}\pi r^3$ (in formula booklet)

$r = 1.84$

Sub r into formula.

$V = \frac{4}{3}\pi(1.84)^3$

$V = 26.094...$

$\boxed{V = 26.1 \text{ m}^3 \ (3sf)}$

A stone is in the shape of a sphere with radius 1.84 m.

(a) Calculate the volume of the stone. [2]

$\boxed{V = 26.1\ m^3\ (3sf)}$

The stone is cooled and its volume decreases by 1%.

(b) Calculate the radius of the stone following this decrease. [3]

b) Let V_c = cooled volume and r_c = cooled radius.

Method 1

$$0.99 = \frac{V_c}{V} = \frac{\cancel{\tfrac{4}{3}}\cancel{\pi} r_c^3}{\cancel{\tfrac{4}{3}}\cancel{\pi}(1.84)^3}$$

$$r_c = \sqrt[3]{0.99} \times 1.84$$

$$r_c = 1.8338...$$

$$\boxed{r_c = 1.83\ m\ \ (3sf)}$$

Method 2

$$V_c = 26.1 \times 0.99 \approx 25.8\ m^3$$

$$25.8 = \frac{4}{3}\pi r_c^3$$

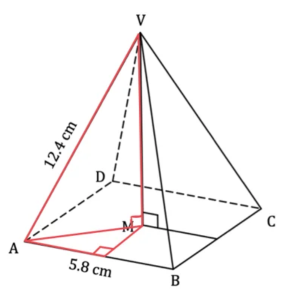
$$r_c = \sqrt[3]{\frac{25.8}{\frac{4}{3}\pi}}$$

$$r_c = 1.8338...$$

$$\boxed{r_c = 1.83\ m\ \ (3sf)}$$

Question 7

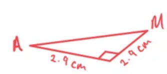
A right pyramid has square base ABCD and apex V. The sides of the square base are 5.8 cm and the sloping edges are 12.4 cm.



(a) Calculate the length of VM. [3]

a) Notice the right-angled triangles.

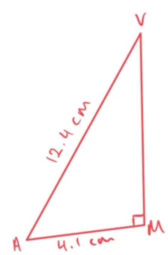
First, we need to find AM.



$$AM = \sqrt{2.9^2 + 2.9^2} \quad \left(\frac{5.8}{2} = 2.9\right)$$

$$AM = \sqrt{16.82} \quad (AM^2 = 16.82)$$

Use AM to find VM.



$$VM = \sqrt{12.4^2 - 16.82}$$

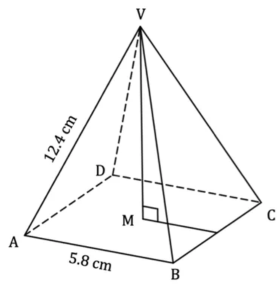
$$VM = \sqrt{136.94}$$

$$VM = 11.7021...$$

$$\boxed{VM = 11.7\ cm\ \ (3sf)}$$

(b) Calculate the volume of the pyramid. [2]

A right pyramid has square base ABCD and apex V. The sides of the square base are 5.8 cm and the sloping edges are 12.4 cm.



(a) Calculate the length of VM.

$\boxed{VM = 11.7 \text{ cm } (3sf)}$

[3]

(b) Calculate the volume of the pyramid.

[2]

b) Volume of a right pyramid formula

$V = \frac{1}{3} A h$ (in formula booklet)

where A is the area of the base.

$A = 5.8^2$ $h = 11.7$

Sub A and h into formula.

$V = \frac{1}{3}(5.8^2)(11.7)$

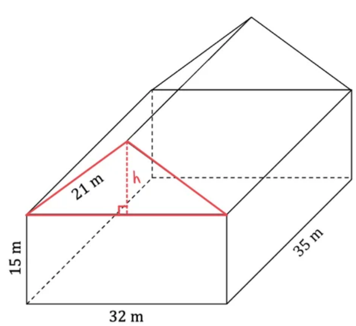
$V = 131.196$

$\boxed{V = 131 \text{ cm}^3 \ (3sf)}$

Question 8

A storage warehouse consists of a cuboid measuring 15 m × 32 m × 35 m and a roof in the shape of an isosceles triangular prism with side lengths of 21 m, as shown in the diagram. The total exterior surface of the storage warehouse is to be painted.

Find the area to be painted. Give your answer to the nearest m^2.



[8]

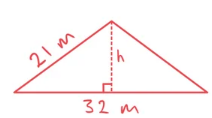
Area = 2 (warehouse ends) + 2 (warehouse sides) + 2 (roof slanted sides) + 2 (roof ends)

Roof ends are isosceles triangles.

Area of a triangle formula

$A = \frac{1}{2} b h$ (in formula booklet)

b is the base, h is the perpendicular height



$b = 32$ $h = \sqrt{21^2 - 16^2}$ (pythagoras)

Sub b and h into formula and sum all the surfaces.

$A = 2(15 \times 32) + 2(15 \times 35) + 2(35 \times 21) + 2\left(\frac{1}{2} \times 32 \times \sqrt{21^2 - 16^2}\right)$

$A = 3915.247...$

$\boxed{A = 3915 \text{ m}^2 \ (\text{nearest m}^2)}$

Question 9

Two planes, A and B, are coming into land at Sharp airport. The locations of the planes and Sharp airport can be described by coordinates on an x, y, z axes, where x and y represent the distance east and north of Sharp airport respectively and z represents the altitude of the planes. Plane A has coordinates $(11, 14, 4)$, plane B has coordinates $(4, 17, 3)$ and Sharp airport has coordinates $(0, 0, 0)$. All distances are in km.

(a) Determine which plane is farthest away from Sharp airport.

[2]

(b) Calculate the distance between plane A and plane B.

[3]

After an hour of flying, plane A has coordinates $(-8, 20, 5)$. Realizing the plane is low on fuel, the pilot decides to make an emergency landing at the closest airport. His two options are Sharp airport or Kit airport, located at $(-15, 1, 0)$.

(c) State which airport the pilot land the plane.

[3]

a) Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ (in formula booklet)

$A(11, 14, 4)$ $S(0, 0, 0)$

Sub A and S into formula.

$d_A = \sqrt{11^2 + 14^2 + 4^2}$

$d_A = \sqrt{333}$

$d_A = 18.2$ km (3sf)

$B(4, 17, 3)$ $S(0, 0, 0)$

Sub B and S into formula.

$d_B = \sqrt{4^2 + 17^2 + 3^2}$

$d_B = \sqrt{314}$

$d_B = 17.7$ km. (3sf)

∴ Plane A is farthest from Sharp airport.

N.B No need to sub S into formula as all values are zero.

Two planes, A and B, are coming into land at Sharp airport. The locations of the planes and Sharp airport can be described by coordinates on an x, y, z axes, where x and y represent the distance east and north of Sharp airport respectively and z represents the altitude of the planes. Plane A has coordinates $(11, 14, 4)$, plane B has coordinates $(4, 17, 3)$ and Sharp airport has coordinates $(0, 0, 0)$. All distances are in km.

(a) Determine which plane is farthest away from Sharp airport.

[2]

(b) Calculate the distance between plane A and plane B.

[3]

After an hour of flying, plane A has coordinates $(-8, 20, 5)$. Realizing the plane is low on fuel, the pilot decides to make an emergency landing at the closest airport. His two options are Sharp airport or Kit airport, located at $(-15, 1, 0)$.

(c) State which airport the pilot land the plane.

[3]

b) Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ (in formula booklet)

$A(11, 14, 4)$ $B(4, 17, 3)$

Sub A and B into formula.

$d = \sqrt{(11-4)^2 + (14-17)^2 + (4-3)^2}$

$d = \sqrt{59}$

$\boxed{d = 7.68 \text{ km} \quad (3sf)}$

Two planes, A and B, are coming into land at Sharp airport. The locations of the planes and Sharp airport can be described by coordinates on an x, y, z axes, where x and y represent the distance east and north of Sharp airport respectively and z represents the altitude of the planes. Plane A has coordinates $(11, 14, 4)$, plane B has coordinates $(4, 17, 3)$ and Sharp airport has coordinates $(0, 0, 0)$. All distances are in km.

(a) Determine which plane is farthest away from Sharp airport.

[2]

(b) Calculate the distance between plane A and plane B.

[3]

After an hour of flying, plane A has coordinates $(-8, 20, 5)$. Realizing the plane is low on fuel, the pilot decides to make an emergency landing at the closest airport. His two options are Sharp airport or Kit airport, located at $(-15, 1, 0)$.

(c) State which airport the pilot land the plane.

[3]

c) Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ (in formula booklet)

$A(-8, 20, 5)$ $S(0, 0, 0)$ $K(-15, 1, 0)$

Distance to Sharp airport

Sub A and S into formula.

$d_S = \sqrt{(-8)^2 + 20^2 + 5^2}$

$d_S = \sqrt{489}$

$d_S = 22.1$ km (3sf)

Distance to Kit airport

Sub A and K into formula.

$d_K = \sqrt{(-8 - (-15))^2 + (20 - 1)^2 + 5^2}$

$d_K = \sqrt{435}$

$d_K = 20.9$ km (3sf)

$\therefore$ The pilot should land at Kit airport.