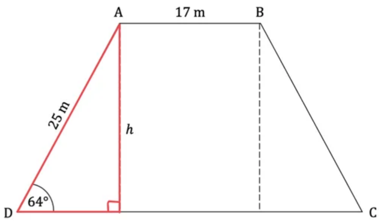
Question 1

ABCD is an isosceles trapezoid where $AB = 17$ m and $AD = BC = 25$ m, as shown in the diagram below.



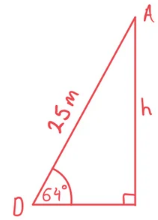
(a) Find the height, h, of the trapezoid.

[2]

(b) Find the area of the trapezoid.

[4]

a) Notice the right-angled triangle.



We have

$\theta = 64°$ opp $= h$ hyp $= 25$ m

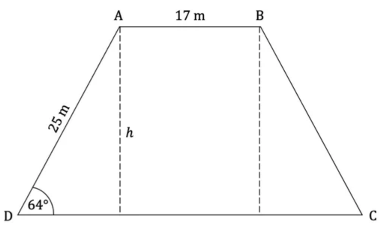
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ from SOH CAH TOA

$\therefore \sin 64° = \frac{h}{25}$

$h = \sin 64° \times 25$

$\boxed{h = 22.5 \text{ m}}$

ABCD is an isosceles trapezoid where $AB = 17$ m and $AD = BC = 25$ m, as shown in the diagram below.



(a) Find the height, h, of the trapezoid. $\boxed{h = 22.5 \text{ m}}$

[2]

(b) Find the area of the trapezoid.

[4]

b) Base of trapezoid, DC, is equal to AB + 2 (base of right-angled triangle).

$DC = 17 + 2\sqrt{25^2 - 22.5^2}$ (pythagoras)

Area of a trapezoid formula

$A = \frac{1}{2}(a+b)h$ (in formula booklet)

a and b are parallel sides, h is the height

$a = AB = 17$ $b = DC = 17 + 2\sqrt{25^2 - 22.5^2}$ $h = 22.5$

Sub a, b and h into formula.

$A = \frac{1}{2}\left(17 + 17 + 2\sqrt{25^2 - 22.5^2}\right) \times 22.5$

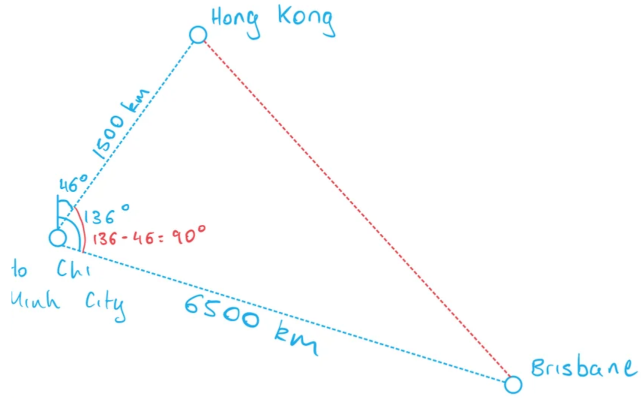
$\boxed{A \approx 628 \text{ m}^2}$

Question 2

The distance between Ho Chi Minh City and Hong Kong is known to be 1500 km. The bearing of Hong Kong from Ho Chi Minh City is 046°. Another city, Brisbane, is 6500 km from Ho Chi Minh City on a bearing of 136°. Calculate the distance between Hong Kong and Brisbane.

[3]

Draw a diagram



The three cities form a right-angled triangle.

$$\text{distance} = \sqrt{(1500)^2 + (6500)^2}$$

$$\text{distance} = \sqrt{44\ 500\ 000}$$

$$\boxed{\text{distance} = 6670 \text{ km} \quad (3sf)}$$

Question 3

Point A has coordinates $(4, -6)$ and point B has coordinates $(8, 6)$.

(a) Calculate the distance of the line segment AB.

[2]

(b) Find the equation of the line connecting points A and B.
Give your answer in the form $y = mx + c$.

[2]

(c) (i) Find the midpoint of [AB].

(ii) Find the equation of the perpendicular bisector to the line segment AB.
Give your answer in the form $y = mx + c$.

[4]

a) Distance between two points formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \text{(in formula booklet)}$$

$A(4, -6) \qquad B(8, 6)$

Sub A and B into formula.

$$d = \sqrt{(4-8)^2 + (-6-6)^2}$$

$$\boxed{d \approx 12.6 \text{ units}}$$

Point A has coordinates $(4, -6)$ and point B has coordinates $(8, 6)$.

(a) Calculate the distance of the line segment AB. [2]

(b) Find the equation of the line connecting points A and B.
Give your answer in the form $y = mx + c$. [2]

(c) (i) Find the midpoint of [AB].

(ii) Find the equation of the perpendicular bisector to the line segment AB.
Give your answer in the form $y = mx + c$. [4]

b) Gradient formula

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (in formula booklet)

$A(4, -6)$ $B(8, 6)$

Sub A and B into formula.

$m = \frac{6 - (-6)}{8 - 4}$ $\therefore m = 3$

Sub A and m into $y - y_1 = m(x - x_1)$

$y - (-6) = 3(x - 4)$ expand both sides

$y + 6 = 3x - 12$ -6

$\boxed{y = 3x - 18}$

Point A has coordinates $(4, -6)$ and point B has coordinates $(8, 6)$.

(a) Calculate the distance of the line segment AB. [2]

(b) Find the equation of the line connecting points A and B.
Give your answer in the form $y = mx + c$. $\boxed{y = 3x - 18}$ [2]

(c) (i) Find the midpoint of [AB].

(ii) Find the equation of the perpendicular bisector to the line segment AB.
Give your answer in the form $y = mx + c$. [4]

c) i) Midpoint formula

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (in formula booklet)

$A(4, -6)$ $B(8, 6)$

Sub A and B into formula.

Midpoint $= \left(\frac{4 + 8}{2}, \frac{-6 + 6}{2}\right)$

$\boxed{\text{Midpoint} = (6, 0)}$

ii) Perpendicular gradients

$m_{\perp AB} = -\frac{1}{m_{AB}}$

$m_{AB} = 3$ $\therefore m_{\perp AB} = -\frac{1}{3}$

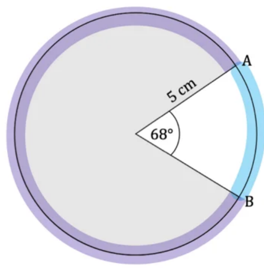
Sub the midpoint and $m_{\perp AB}$ into $y - y_1 = m(x - x_1)$.

$y - 0 = -\frac{1}{3}(x - 6)$ expand RHS

$\boxed{y = -\frac{1}{3}x + 2}$

Question 4

The diagram below shows a circle with a 68° sector cut from it. The radius of the circle is 5 cm.



(a) Find the length of

(i) the minor arc AB

(ii) the major arc AB.

[3]

(b) Find the area of the shaded region.

[3]

a) Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

i) Minor arc AB

$\theta = 68$ $r = 5$

Sub θ and r into formula.

$l = \frac{68}{360} \times 2\pi(5)$

$\boxed{l = 5.93 \text{ cm}}$

ii) Major arc AB

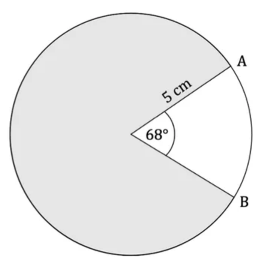
$\theta = 360 - 68$ $r = 5$
$= 292$

Sub θ and r into formula.

$l = \frac{292}{360} \times 2\pi(5)$

$\boxed{l = 25.5 \text{ cm}}$

The diagram below shows a circle with a 68° sector cut from it. The radius of the circle is 5 cm.



(a) Find the length of

(i) the minor arc AB

(ii) the major arc AB.

[3]

(b) Find the area of the shaded region.

[3]

b) Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

$\theta = 292$ $r = 5$

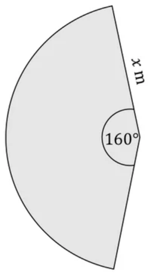
Sub θ and r into formula.

$A = \frac{292}{360} \times \pi(5)^2$

$\boxed{A \approx 63.7 \text{ cm}^2}$

Question 5

A lawn sprinkler sprays water over a lawn covering an arc of 160° with a maximum spray distance of x m as shown in the diagram below. The lawn sprinkler waters 20 m^2 of the lawn.



(a) Calculate the value of x.

[4]

(b) Calculate the length of the outer arc.

[3]

a) Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

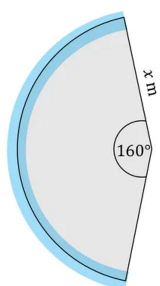
$A = 20$ $\theta = 292$ $r = x$

Sub A and θ into formula and rearrange for x.

$20 = \frac{292}{360} \times \pi x^2$

$\boxed{x \approx 3.78 \text{ m}}$

A lawn sprinkler sprays water over a lawn covering an arc of 160° with a maximum spray distance of x m as shown in the diagram below. The lawn sprinkler waters 20 m^2 of the lawn.



(a) Calculate the value of x. $\boxed{x \approx 3.78 \text{ m}}$

[4]

(b) Calculate the length of the outer arc.

[3]

b) Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

$\theta = 160$ $r = 3.78$

Sub θ and r into formula.

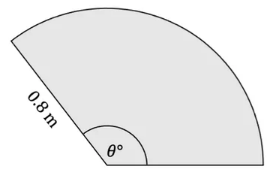
$l = \frac{160}{360} \times 2\pi(3.78)$

$\boxed{l \approx 10.6 \text{ m}}$

For more help, please visit www.exampaperspractice.co.uk

Question 6

A windscreen wiper blade is 0.8 m long. When in motion the blade moves through an arc of $\theta°$ and wipes an area of $\frac{4}{15}\pi$ m^2.



(a) Calculate the value of θ. [4]

(b) Calculate the length travelled by the outer edge of the blade. [3]

a) Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

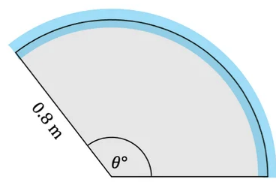
$A = \frac{4}{15}\pi \quad r = 0.8$

Sub A and r into formula and rearrange for θ.

$\frac{4}{15}\pi = \frac{\theta}{360} \times \pi(0.8)^2$

$\boxed{\theta = 150°}$

A windscreen wiper blade is 0.8 m long. When in motion the blade moves through an arc of $\theta°$ and wipes an area of $\frac{4}{15}\pi$ m^2.



(a) Calculate the value of θ. $\boxed{\theta = 150°}$ [4]

(b) Calculate the length travelled by the outer edge of the blade. [3]

b) Arc length formula

$L = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

$\theta = 150 \quad r = 0.8$

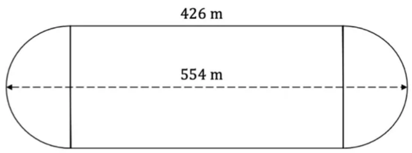
Sub θ and r into formula.

$L = \frac{150}{360} \times 2\pi(0.8)$

$\boxed{L \approx 2.09 \text{ m}}$

Question 7

The diagram below shows a dirt racetrack where the straights are 426 m long and the longest distance from one end of the track to the other is 554m.



(a) Find the total distance around the racetrack.

[3]

(b) Find the total area enclosed by the racetrack.

[4]

a) Circle circumference formula

$C = 2\pi r$ (in formula booklet)

Radius of semicircles

$r = \frac{554 - 426}{2}$ $\therefore r = 64\text{ m}$

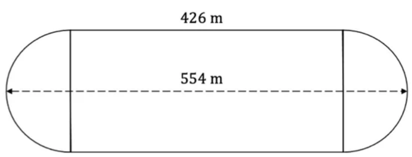
Total distance

$d = 2(426) + 2\pi(64)^*$

$\boxed{d \approx 1250\text{ m}}$

*N.B 2 semicircles make a full circle.

The diagram below shows a dirt racetrack where the straights are 426 m long and the longest distance from one end of the track to the other is 554m.



(a) Find the total distance around the racetrack.

$\boxed{r = 64\text{ m}}$

[3]

(b) Find the total area enclosed by the racetrack.

[4]

b) Total area = rectangle + 2 semicircles

The height of the rectangle is equal to the diameter of the semicircles.

$r = 64$ $\therefore$ height $= 2(64)$
$= 128\text{ m}$

Circle area formula

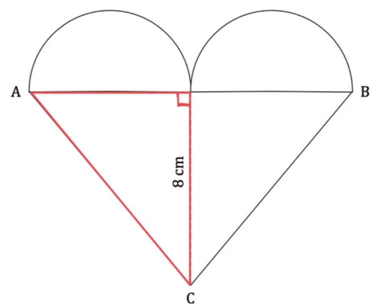
$A = \pi r^2$ (in formula booklet)

Total area $= (426)(128) + \pi(64)^2$

$\boxed{\text{Total area} \approx 67\,400\text{ m}^2}$

Question 8

The diagram below shows a cookie cutter in the shape of a heart constructed from a triangle and two identical semi circles. The height of the triangle is 8 cm and its base AB is 13.34 cm.



(a) Find the length of the line AC.

[2]

(b) Calculate the total area of the heart.

[4]

Bob makes some cookie dough and rolls it out on his kitchen bench. The cookie dough covers 1314 cm².

(c) Find the number of **full** cookies Bob can cut from the dough.

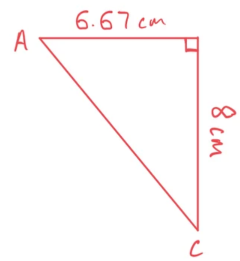
[2]

a) Notice the right-angled triangle.

triangle base $= \frac{AB}{2}$

triangle base $= \frac{13.34}{2}$

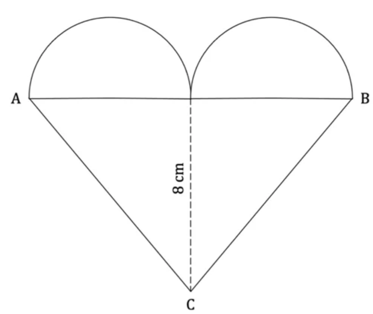
triangle base $= 6.67$



$\therefore AC = \sqrt{8^2 + 6.67^2}$

$\boxed{AC \approx 10.4 \text{ cm}}$

The diagram below shows a cookie cutter in the shape of a heart constructed from a triangle and two identical semi circles. The height of the triangle is 8 cm and its base AB is 13.34 cm.



(a) Find the length of the line AC.

[2]

(b) Calculate the total area of the heart.

[4]

Bob makes some cookie dough and rolls it out on his kitchen bench. The cookie dough covers 1314 cm².

(c) Find the number of **full** cookies Bob can cut from the dough.

[2]

b) Total area (A) = triangle + 2 semicircles*

$\therefore A = \frac{1}{2}bh + \pi r^2$

Semicircle radius $(r) = \frac{AB}{4}$

$r = \frac{13.34}{4}$

$b = 13.34 \quad h = 8 \quad r = 3.335$

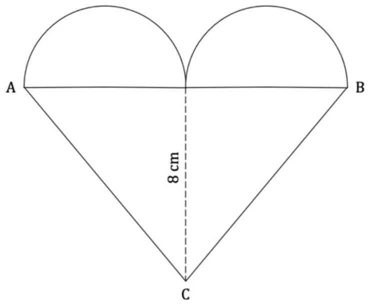
Sub b, h and r into formula

$A = \frac{1}{2}(13.34)(8) + \pi(3.335)^2$

$\boxed{A \approx 88.3 \text{ cm}^2}$

*N.B 2 semicircles make a full circle.

The diagram below shows a cookie cutter in the shape of a heart constructed from a triangle and two identical semi circles. The height of the triangle is 8 cm and its base AB is 13.34 cm.



(a) Find the length of the line AC.

[2]

(b) Calculate the total area of the heart.

$\boxed{A \approx 88.3\,cm^2}$

[4]

Bob makes some cookie dough and rolls it out on his kitchen bench. The cookie dough covers 1314 cm^2.

(c) Find the number of **full** cookies Bob can cut from the dough.

[2]

c) Number of cookies = $\frac{\text{dough area}}{\text{heart area}}$

Number of cookies = $\frac{1314}{88.3}$

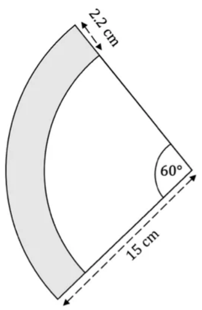
Number of cookies = 14.9

$\boxed{\therefore 14 \text{ full cookies}}$

N.B Cookie dough can be reformed so no need to account for the irregular shape.

Question 9

The diagram below shows a slice of pizza that forms a sector of a circle with an arc of 60° and radius of 15 cm. The width of the crust is 2.2 cm.



(a) Find the perimeter of the slice of pizza.

[3]

(b) Find the area of the crust.

[3]

a) Perimeter = arc + 2 (radius)

Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

$\theta = 60$ $r = 15$

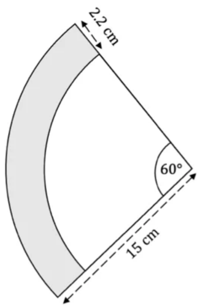
Sub θ and r into formula.

$l = \frac{60}{360} \times 2\pi(15)$

Perimeter $= \frac{60}{360} \times 2\pi(15) + 2(15)$

$\boxed{\text{Perimeter} \approx 45.7\,cm}$

The diagram below shows a slice of pizza that forms a sector of a circle with an arc of 60° and radius of 15 cm. The width of the crust is 2.2 cm.



(a) Find the perimeter of the slice of pizza.

[3]

(b) Find the area of the crust.

[3]

b) Crust area (A_c) = Pizza area (A_p) - Toppings area (A_T)

Toppings radius (r_T) = Pizza radius (r_p) - crust width

$\therefore r_T = 15 - 2.2$
$= 12.8\text{ cm}$

Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

$\theta = 60 \quad r_p = 15 \quad r_T = 12.8$

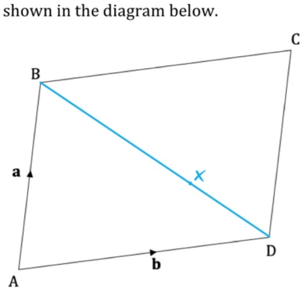
Sub θ, r_p and r_T into formula to find A_c.

$A_c = \frac{60}{360} \times \pi(15)^2 - \frac{60}{360} \times \pi(12.8)^2$

$\boxed{A_c \approx 32\text{ cm}^2}$

Question 10

A parallelogram ABCD is shown in the diagram below.



$\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$.

A new line is added to the diagram connecting B to D.

A point X lies $\frac{2}{3}$ of the way along $\overrightarrow{BD}$.

(a) Express $\overrightarrow{CX}$ in terms of **a** and **b**.

[4]

A new point Y lies on the line CD such that AXY is a straight line.

(b) Express $\overrightarrow{AY}$ in terms of **a** and **b**.

[3]

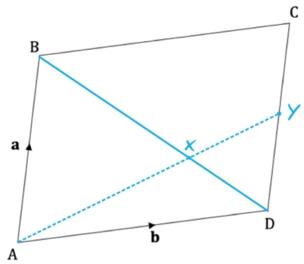
a) $\overrightarrow{CX} = \overrightarrow{CB} - \overrightarrow{BX}$

$\overrightarrow{BD} = b - a \quad \therefore \overrightarrow{BX} = \frac{2}{3}(b - a)$

$\overrightarrow{CX} = -b + \frac{2}{3}(b - a) = -b + \frac{2}{3}b - \frac{2}{3}a = -\frac{1}{3}b - \frac{2}{3}a$

$\boxed{\overrightarrow{CX} = -\frac{1}{3}(b + 2a)}$

A parallelogram ABCD is shown in the diagram below.



$\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$.

A new line is added to the diagram connecting B to D.

A point X lies $\frac{2}{3}$ of the way along $\overrightarrow{BD}$.

(a) Express $\overrightarrow{CX}$ in terms of **a** and **b**.

[4]

A new point Y lies on the line CD such that AXY is a straight line.

(b) Express $\overrightarrow{AY}$ in terms of **a** and **b**.

[3]

b) $\overrightarrow{AX}$ and $\overrightarrow{AY}$ are parallel, $\therefore$ the ratio of b:a must be the same

$\overrightarrow{AY} = \overrightarrow{AD} + \overrightarrow{DY} = b + ka$

$\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BX} = a + \frac{2}{3}(b - a) = \frac{2}{3}b + \frac{1}{3}a$

$(b : a = 2:1)$

$\boxed{\therefore \overrightarrow{AY} = b + \frac{1}{2}a}$

Question 11

Three points are located at A(0, 5), B(6, 4) and C(16, 8).

(a) (i) Find the magnitude of vector $\overrightarrow{AB}$.

(ii) Find the magnitude of vector $\overrightarrow{BC}$.

[3]

(b) Given that the angle $A\hat{B}C$ is a right angle, find the area of triangle ABC.

[2]

a) Vector magnitude is equal to the distance between the two points

i) $|\overrightarrow{AB}| = \left|\sqrt{(6-0)^2 + (4-5)^2}\right|$

$|\overrightarrow{AB}| = \sqrt{37} = 6.0827...$

$\boxed{|\overrightarrow{AB}| = \sqrt{37} = 6.08 \text{ units 3sf.}}$

ii) $|\overrightarrow{BC}| = \left|\sqrt{(16-6)^2 + (8-4)^2}\right|$

$|\overrightarrow{BC}| = \sqrt{116} = 2\sqrt{29} = 10.7703...$

$\boxed{|\overrightarrow{BC}| = \sqrt{116} = 10.8 \text{ units 3sf.}}$

Three points are located at A(0, 5), B(6, 4) and C(16, 8).

(a) (i) Find the magnitude of vector $\overrightarrow{AB}$.

(ii) Find the magnitude of vector $\overrightarrow{BC}$.

$\boxed{|\overrightarrow{AB}| = \sqrt{37} \quad |\overrightarrow{BC}| = \sqrt{116}}$

[3]

(b) Given that the angle $A\hat{B}C$ is a right angle, find the area of triangle ABC.

[2]

b) $\text{AREA} = \frac{1}{2}bh$

$A = \frac{1}{2}\sqrt{37} \times \sqrt{116} = 32.7566\ldots$

$\boxed{A = 32.8 \text{ units}^2 \text{ 3sf.}}$

Question 12

The points A, B, C and D have position vectors **a**, **b**, **c** and **d**, relative to the origin O.

The position vectors are given by

$$\mathbf{a} = 2i + 4j - k$$
$$\mathbf{b} = -ri + j + 2k$$
$$\mathbf{c} = 3i + sj$$
$$\mathbf{d} = 2i - 2j - tk$$

where r, s and t are constants.

(a) Given that $\overrightarrow{BA} = \overrightarrow{CD}$, find r, s and t

[5]

A fifth point, E, has position vector **e**, relative to the origin O.

(b) Given that $\overrightarrow{AE} = 3\overrightarrow{CD}$, find the position vector of E.

[5]

(c) Find the unit vector that has the same direction as **e**.

[2]

a) $\overrightarrow{BA} = \overrightarrow{CD} \longrightarrow \therefore a - b = d - c$

$$\begin{pmatrix}2\\4\\-1\end{pmatrix} - \begin{pmatrix}-r\\1\\2\end{pmatrix} = \begin{pmatrix}2\\-2\\-t\end{pmatrix} - \begin{pmatrix}3\\s\\0\end{pmatrix}$$

$2 + r = -1 \longrightarrow \therefore r = -3$

$3 = -2 - s \longrightarrow \therefore s = -5$

$-3 = -t - 0 \longrightarrow \therefore t = 3$

The points A, B, C and D have position vectors **a**, **b**, **c** and **d**, relative to the origin O.

The position vectors are given by

$$\mathbf{a} = 2i + 4j - k$$
$$\mathbf{b} = -ri + j + 2k$$
$$\mathbf{c} = 3i + sj$$
$$\mathbf{d} = 2i - 2j - tk$$

where r, s and t are constants.

(a) Given that $\overrightarrow{BA} = \overrightarrow{CD}$, find r, s and t

$\boxed{r = -3, \quad s = -5, \quad t = 3}$

[5]

A fifth point, E, has position vector **e**, relative to the origin O.

(b) Given that $\overrightarrow{AE} = 3\overrightarrow{CD}$, find the position vector of E.

[5]

(c) Find the unit vector that has the same direction as **e**.

[2]

b) $\overrightarrow{AE} = 3\overrightarrow{CD} \longrightarrow \therefore e - a = 3(d - c)$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}2\\4\\-1\end{pmatrix} = 3\left[\begin{pmatrix}2\\-2\\-t\end{pmatrix} - \begin{pmatrix}3\\-5\\0\end{pmatrix}\right] = 3\begin{pmatrix}-1\\3\\-3\end{pmatrix} = \begin{pmatrix}-3\\9\\-9\end{pmatrix}$$

$x - 2 = -3 \longrightarrow \therefore x = -1$

$y - 4 = 9 \longrightarrow \therefore y = 13$

$z + 1 = -9 \longrightarrow \therefore z = -10$

$\boxed{\therefore e = -i + 13j - 10k}$

The points A, B, C and D have position vectors **a**, **b**, **c** and **d**, relative to the origin O.

The position vectors are given by

$$\mathbf{a} = 2i + 4j - k$$
$$\mathbf{b} = -ri + j + 2k$$
$$\mathbf{c} = 3i + sj$$
$$\mathbf{d} = 2i - 2j - tk$$

where r, s and t are constants.

(a) Given that $\overrightarrow{BA} = \overrightarrow{CD}$, find r, s and t [5]

A fifth point, E, has position vector **e**, relative to the origin O.

(b) Given that $\overrightarrow{AE} = 3\overrightarrow{CD}$, find the position vector of E. [5]

$$\boxed{\therefore e = -i + 13j - 10k}$$

(c) Find the unit vector that has the same direction as **e**. [2]

c) unit vector for a 3D vector (not in formula booklet)

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \rightarrow u = \frac{1}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$u = \frac{1}{\sqrt{(-1)^2 + (13)^2 + (-10)^2}} (-i + 13j - 10k)$$

$$u = \frac{1}{\sqrt{1 + 169 + 100}} (-i + 13j - 10k)$$

$$u = \frac{1}{\sqrt{270}} (-i + 13j - 10k)$$

$$u = \frac{1}{3\sqrt{30}} (-i + 13j - 10k)$$

$$\boxed{u = -\frac{1}{3\sqrt{30}} i + \frac{13}{3\sqrt{30}} j - \frac{10}{3\sqrt{30}} k}$$