## Question 1

Show that

(i) $\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$

(ii) $\tan(\theta - \pi) = \tan\theta$

(iii) $\sin\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}(\sin\theta - \cos\theta)$.

[6]

i) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ (in formula booklet)

$$\cos\left(\theta + \frac{\pi}{2}\right) = \cos\theta\underbrace{\cos\frac{\pi}{2}}_{=0} - \sin\theta\underbrace{\sin\frac{\pi}{2}}_{=1}$$

$$\boxed{\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta}$$

ii) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ (in formula booklet)

$$\tan(\theta - \pi) = \frac{\tan\theta - \tan\pi}{1 + \tan\theta\tan\pi} \qquad \tan\pi = 0$$

$$\tan(\theta - \pi) = \frac{\tan\theta - 0}{1 + 0}$$

$$\boxed{\tan(\theta - \pi) = \tan\theta}$$

iii) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ (in formula booklet)

$$\sin\left(\theta - \frac{\pi}{4}\right) = \sin\theta\underbrace{\cos\frac{\pi}{4}}_{=\frac{1}{\sqrt{2}}} - \cos\theta\underbrace{\sin\frac{\pi}{4}}_{=\frac{1}{\sqrt{2}}}$$

$$\sin\left(\theta - \frac{\pi}{4}\right) = \sin\theta\left(\frac{1}{\sqrt{2}}\right) - \cos\theta\left(\frac{1}{\sqrt{2}}\right)$$

$$\boxed{\sin\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}(\sin\theta - \cos\theta)}$$

## Question 2

Let $f(x) = \tan(x + \pi)\sin\left(x + \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$.

By using the compound angle formulae, express $f(x)$ in terms of $\sin x$.

[4]

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ (in formula booklet)

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ (in formula booklet)

$$\tan(x+\pi)\sin\left(x+\frac{\pi}{2}\right) = \left(\frac{\tan x + \overbrace{\tan\pi}^{=0}}{1 - \tan x\underbrace{\tan\pi}_{=0}}\right)\left(\sin x\underbrace{\cos\frac{\pi}{2}}_{=0} + \cos x\underbrace{\sin\frac{\pi}{2}}_{=1}\right)$$

$$\tan(x+\pi)\sin\left(x+\frac{\pi}{2}\right) = \left(\frac{\tan x + 0}{1 - 0}\right)(0 + \cos x(1))$$

$$\tan(x+\pi)\sin\left(x+\frac{\pi}{2}\right) = \underbrace{\tan x}_{=\frac{\sin x}{\cos x}}\cos x = \frac{\sin x}{\cancel{\cos x}}\cdot\cancel{\cos x}$$

$$\boxed{\tan(x+\pi)\sin\left(x+\frac{\pi}{2}\right) = \sin x}$$

## Question 3

Consider the equation $\cos(x - 45) = 2\sin x$ in the interval $0 \le x \le 360°$.

Find an exact value for $\tan x$.

[5]

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ (in formula booklet)

$\cos(x - 45) = 2\sin x$

$\cos x \underbrace{\cos 45}_{=\frac{1}{\sqrt{2}}} + \sin x \underbrace{\sin 45}_{=\frac{1}{\sqrt{2}}} = 2\sin x$

$\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = 2\sin x$

$\frac{1}{\sqrt{2}}\cos x = \left(\frac{2\sqrt{2} - 1}{\sqrt{2}}\right)\sin x$

$\frac{1}{\sqrt{2}} \div \left(\frac{2\sqrt{2} - 1}{\sqrt{2}}\right) = \frac{\sin x}{\cos x} = \tan x$

$\boxed{\tan x = \frac{1}{2\sqrt{2} - 1}}$

## Question 4

(a) Express $\cos 4\theta$ in terms of $\cos 2\theta$.

[1]

(b) Hence, show that $\cos 4\theta = 8\cos^2\theta\,(\cos^2\theta - 1) + 1$.

[5]

a) $\cos 2\theta = 2\cos^2\theta - 1$ (in formula booklet)

$\cos 4\theta = \cos 2(2\theta)$

$\boxed{\cos 4\theta = 2\cos^2 2\theta - 1}$

(a) Express $\cos 4\theta$ in terms of $\cos 2\theta$.

$\boxed{\cos 4\theta = 2\cos^2 2\theta - 1}$

[1]

(b) Hence, show that $\cos 4\theta = 8\cos^2\theta\,(\cos^2\theta - 1) + 1$.

[5]

b) $\cos 2\theta = 2\cos^2\theta - 1$ (in formula booklet)

$\cos 4\theta = 2\cos^2 2\theta - 1 = 2(\cos 2\theta)(\cos 2\theta) - 1$

$= 2(2\cos^2\theta - 1)(2\cos^2\theta - 1) - 1$

$= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1 = 8\cos^4\theta - 8\cos^2\theta + 1$

$\boxed{\cos 4\theta = 8\cos^2\theta\,(\cos^2\theta - 1) + 1}$

## Question 5

Given that $\tan A = \frac{\sqrt{3}}{2}$, solve the equation $\tan(A + x) = \frac{4}{5}$ in the interval $0 \le x \le 360°$.

[6]

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ (in formula booklet)

$\tan(A + x) = \dfrac{\tan A + \tan x}{1 - \tan A \tan x} = \dfrac{4}{5}$ $\qquad \tan A = \dfrac{\sqrt{3}}{2}$

$$\frac{\frac{\sqrt{3}}{2} + \tan x}{1 - \frac{\sqrt{3}}{2}\tan x} = \frac{4}{5}$$

$5\left(\dfrac{\sqrt{3}}{2} + \tan x\right) = 4\left(1 - \dfrac{\sqrt{3}}{2}\tan x\right)$

$\dfrac{5\sqrt{3}}{2} + 5\tan x = 4 - \dfrac{\cancel{4}^{2}\sqrt{3}}{\cancel{2}}\tan x$

$5\tan x + 2\sqrt{3}\tan x = 4 - \dfrac{5\sqrt{3}}{2}$

$\tan x = \dfrac{4 - \frac{5\sqrt{3}}{2}}{5 + 2\sqrt{3}}$ $\qquad$ solve with GDC

$x = 177.76\ldots,\ 357.76\ldots$

$\boxed{x = 178°,\ 358° \ (3\text{ s.f.})}$ $\qquad$ ignore $x = -2.233\ldots$ since $0 \le x \le 360$

EXAM PAPERS PRACTICE

## Question 6

Prove that $\cos 3x \equiv 4\cos^3 x - 3\cos x$.

[6]

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ (in formula booklet)

$\cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$

$= \overbrace{(\cos^2 x - \sin^2 x)}^{= \cos 2x} \cos x - \overbrace{(2\sin x \cos x)}^{= \sin 2x} \sin x$

$= \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x$

$= \cos^3 x - 3\sin^2 x \cos x = \cos^3 x - 3\cos x(1 - \cos^2 x)$

$= \cos^3 x - 3\cos x + 3\cos^3 x$

$\boxed{\cos 3x \equiv 4\cos^3 x - 3\cos x}$

## Question 7

Solve the equation $\sin 2x - \cos 2x = \frac{\sin x + \cos x}{2} - 1$ for the interval $-\pi < x < 0$.

[7]

$\sin 2x - \cos 2x = \frac{\sin x + \cos x}{2} - 1$

$\overbrace{2\sin x \cos x}^{= \sin 2x} - \overbrace{(1 - 2\sin^2 x)}^{= \cos 2x} = \frac{\sin x + \cos x}{2} - 1$

$2\sin x \cos x + 2\sin^2 x = \frac{\sin x + \cos x}{2}$

$4\sin x \cos x + 4\sin^2 x = \sin x + \cos x$

$4\sin x \cos x - \cos x + 4\sin^2 x - \sin x = 0$

$\cos x(4\sin x - 1) + \sin x(4\sin x - 1) = 0$

$(\cos x + \sin x)(4\sin x - 1) = 0$

$\therefore \cos x + \sin x = 0 \quad \cos x = -\sin x \quad \tan x = -1$

$\therefore 4\sin x - 1 = 0 \quad \sin x = \frac{1}{4}$ $x$ outside interval

$\boxed{x = -\frac{\pi}{4}}$

## Question 8

(a) Show that $1 - \cos 2x = 2 - 2\cos^2 x$ [2]

(b) Show that $\frac{1}{\cos 2x} - \tan 2x = \frac{\cos x - \sin x}{\cos x + \sin x}$ [5]

a) Make the LHS match the RHS.

$1 - \cos 2x = 1 - \overbrace{(2\cos^2 x - 1)}^{= \cos 2x}$

$\boxed{1 - \cos 2x = 2 - 2\cos^2 x}$

(a) Show that $1 - \cos 2x = 2 - 2\cos^2 x$ [2]

$\boxed{1 - \cos 2x = 2 - 2\cos^2 x}$

(b) Show that $\frac{1}{\cos 2x} - \tan 2x = \frac{\cos x - \sin x}{\cos x + \sin x}$ [5]

b) Make the LHS match the RHS.

$\frac{1}{\cos 2x} - \tan 2x = \frac{1}{\cos 2x} - \frac{\sin 2x}{\cos 2x}$

$= \frac{1 - \sin 2x}{\cos 2x} = \frac{\overbrace{\cos^2 x + \sin^2 x}^{=1} - \overbrace{2\sin x \cos x}^{= \sin 2x}}{\underbrace{\cos^2 x - \sin^2 x}_{= \cos 2x}}$

$= \frac{(\cos x - \sin x)^{\cancel{2}}}{(\cos x + \sin x)\cancel{(\cos x - \sin x)}}$

$\boxed{\frac{1}{\cos 2x} - \tan 2x = \frac{\cos x - \sin x}{\cos x + \sin x}}$

## Question 9

(a) Find the exact values for $\tan x$ given that $\tan^2 x + 4\tan x + 1 = 0$ [3]

(b) Hence, solve the equation $\frac{\tan x}{2\tan x + 1} = \tan 2x$ algebraically for the interval $0 \leq x \leq 2\pi$. [5]

a) $\tan^2 x + 4\tan x + 1 = 0$

Let $y = \tan x$ and use the quadratic formula.

$y^2 + 4y + 1 = 0$

$y = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-\cancel{4}^{2} \pm \cancel{2}\sqrt{3}}{\cancel{2}}$

$\boxed{\tan x = (\sqrt{3} - 2), \ (-\sqrt{3} - 2)}$

(a) Find the exact values for $\tan x$ given that $\tan^2 x + 4\tan x + 1 = 0$

$\tan x = (\sqrt{3}-2),\ (-\sqrt{3}-2)$

[3]

(b) Hence, solve the equation $\frac{\tan x}{2\tan x+1} = \tan 2x$ algebraically for the interval $0 \le x \le 2\pi$.

[5]

b) $\dfrac{\tan x}{2\tan x + 1} = \tan 2x$

$\dfrac{\tan x}{2\tan x + 1} = \dfrac{2\tan x}{1-\tan^2 x}$

cross multiply

$(\tan x)(1-\tan^2 x) = (2\tan x)(2\tan x + 1)$

$\tan x - \tan^3 x = 4\tan^2 x + 2\tan x$

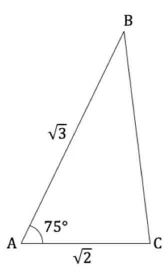
$\tan^3 x + 4\tan^2 x + \tan x = \tan x(\tan^2 x + 4\tan x + 1) = 0$

$\therefore \tan x = 0,\ (\sqrt{3}-2),\ (-\sqrt{3}-2)$

$\boxed{x = 0,\ \frac{7\pi}{12},\ \frac{11\pi}{12},\ \pi,\ \frac{19\pi}{24},\ \frac{23\pi}{24},\ 2\pi}$

## Question 10

The following diagram shows the triangle ABC where $AB = \sqrt{2}$, $AC = \sqrt{3}$ and $B\hat{A}C = 75°$.




(a) By writing $75°$ as $30° + 45°$ find the value of $\sin(75°)$.

[3]

(b) Find the area of the triangle, giving your answer in the form $\frac{a+\sqrt{b}}{c}$, where $a, b, c \in \mathbb{Z}$.

[4]

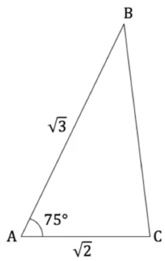
a) $\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$ (in formula booklet)

$\sin 75° = \sin(30° + 45°) = \underbrace{\sin 30°}_{=\frac{1}{2}}\underbrace{\cos 45°}_{=\frac{\sqrt{2}}{2}} + \underbrace{\cos 30°}_{=\frac{\sqrt{3}}{2}}\underbrace{\sin 45°}_{=\frac{\sqrt{2}}{2}}$

$\sin 75° = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} + \frac{\sqrt{3}\sqrt{2}}{4}$

$\boxed{\sin 75° = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}}$

The following diagram shows the triangle ABC where $AB = \sqrt{2}$, $AC = \sqrt{3}$ and $B\hat{A}C = 75°$.




(a) By writing 75° as 30° + 45° find the value of sin(75°).

$$\boxed{\sin 75° = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}}$$

[3]

(b) Find the area of the triangle, giving your answer in the form $\frac{a+\sqrt{b}}{c}$, where $a, b, c \in \mathbb{Z}$.

[4]

b) $A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2}(\sqrt{2})(\sqrt{3}) \sin 75° = \frac{1}{2}(\cancel{\sqrt{2}})(\sqrt{3})\left(\frac{1+\sqrt{3}}{2\cancel{\sqrt{2}}}\right)$

$A = \frac{\sqrt{3}}{2}\left(\frac{1+\sqrt{3}}{2}\right)$

$$\boxed{A = \frac{3+\sqrt{3}}{4} \text{ units}^2}$$