Question 1

Complete the table.

Degrees	Radians	sin	cos	tan
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$120°$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
$270°$	$\frac{3\pi}{2}$	-1	0	

[5]

Notes

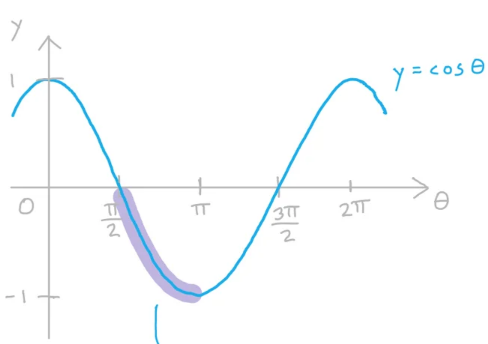
① $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

② The tangent of 270° (like the tangent of every multiple of 90°) is undefined. Sometimes '∞' or '$\pm\infty$' is used to indicate this.

Question 2

Given that $\sin\theta = \frac{3}{5}$, where $\frac{\pi}{2} < \theta < \pi$, find the possible values of $\cos\theta$ and $\tan\theta$.

[3]



$\cos^2\theta + \sin^2\theta = 1$ } Pythagorean identity

$\cos^2\theta + \left(\frac{3}{5}\right)^2 = \cos^2\theta + \frac{9}{25} = 1$

$\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$

$\cos\theta = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$

But for $\frac{\pi}{2} < \theta < \pi$, $\cos\theta$ is negative so

$$\boxed{\cos\theta = -\frac{4}{5}}$$

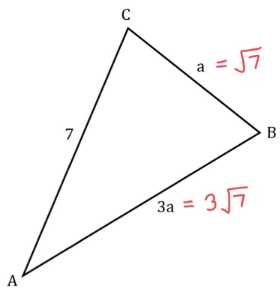
$\tan\theta = \frac{\sin\theta}{\cos\theta}$ } Identity for $\tan\theta$

$\tan\theta = \frac{3/5}{-4/5} = \frac{3}{5} \times \left(-\frac{5}{4}\right)$

$$\boxed{\tan\theta = -\frac{3}{4}}$$

Question 3

The following triangle shows triangle ABC, with $AB = 3a$, $BC = a$ and $AC = 7$.



Given that $\cos A\hat{B}C = \frac{1}{2}$, find the area of the triangle. Give your answer in the form $\frac{p\sqrt{3}}{r}$, where $p, q \in \mathbb{R}$.

[7]

$c^2 = a^2 + b^2 - 2ab\cos C$ } Cosine rule

$\cos^2\theta + \sin^2\theta = 1$ } Pythagorean identity

$\text{Area} = \frac{1}{2}ab\sin C$ } area of a triangle

Use cosine rule to find value of a

$7^2 = (a)^2 + (3a)^2 - 2(a)(3a)\left(\frac{1}{2}\right)$

$49 = a^2 + 9a^2 - 3a^2$

$7a^2 = 49 \Rightarrow a^2 = 7 \Rightarrow a = \sqrt{7}$

Use identity to find $\sin A\hat{B}C$

$\sin A\hat{B}C = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Now use formula to find area of triangle

$\text{Area} = \frac{1}{2}(\sqrt{7})(3\sqrt{7})\left(\frac{\sqrt{3}}{2}\right)$

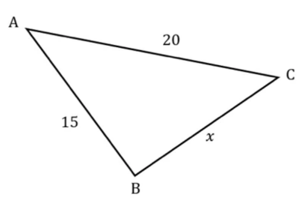
$= \frac{1}{2}(21)\left(\frac{\sqrt{3}}{2}\right) = \frac{21\sqrt{3}}{4}$

$\boxed{\text{Area} = \frac{21\sqrt{3}}{4} \text{ units}^2}$

$p = 21$
$r = 4$

Question 4

The following triangle shows triangle ABC, with $AB = 15$, $AC = 20$, $BC = x$.



(a) Given that $\cos B\hat{A}C = \frac{2}{3}$, find the value of $\sin B\hat{A}C$. [3]

(b) Find the exact area of triangle ABC. [3]

(c) By finding the value of x, show that triangle ABC is isosceles. [3]

a) $\cos^2\theta + \sin^2\theta = 1$ } Pythagorean identity

Use identity to find $\sin B\hat{A}C$

$\left(\frac{2}{3}\right)^2 + \sin^2 B\hat{A}C = 1$

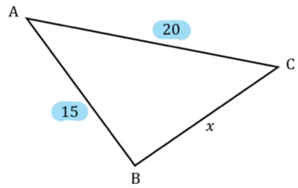
$\sin^2 B\hat{A}C + \frac{4}{9} = 1$

$\sin^2 B\hat{A}C = 1 - \frac{4}{9} = \frac{5}{9}$

$\sin B\hat{A}C = \sqrt{\frac{5}{9}}$

$\boxed{\sin B\hat{A}C = \frac{\sqrt{5}}{3}}$

The following triangle shows triangle ABC, with AB = 15, AC = 20, BC = x.



(a) Given that $\cos B\hat{A}C = \frac{2}{3}$, find the value of $\sin B\hat{A}C$.

$\boxed{\sin B\hat{A}C = \frac{\sqrt{5}}{3}}$

[3]

(b) Find the exact area of triangle ABC.

[3]

(c) By finding the value of x, show that triangle ABC is isosceles.

[3]

b) $\text{Area} = \frac{1}{2}ab\sin C$ } area of a triangle

Use formula to find area of triangle

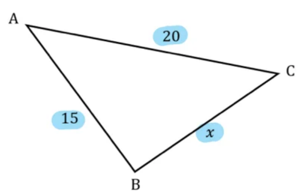
$\text{Area} = \frac{1}{2}(15)(20)\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{1}{2}(300)\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{300\sqrt{5}}{6} = 50\sqrt{5}$

$\boxed{\text{Area} = 50\sqrt{5} \text{ units}^2}$

The following triangle shows triangle ABC, with AB = 15, AC = 20, BC = x.



(a) Given that $\cos B\hat{A}C = \frac{2}{3}$, find the value of $\sin B\hat{A}C$.

[3]

(b) Find the exact area of triangle ABC.

[3]

(c) By finding the value of x, show that triangle ABC is isosceles.

[3]

c) $c^2 = a^2 + b^2 - 2ab\cos C$ } Cosine rule

Use cosine rule to find value of x

$x^2 = (15)^2 + (20)^2 - 2(15)(20)\left(\frac{2}{3}\right)$

$= 225 + 400 - 600\left(\frac{2}{3}\right)$

$= 225 + 400 - 400 = 225$

$\implies x = \sqrt{225} = 15$

$x = 15$, so $AB = BC = 15$.

Two sides are equal, therefore triangle ABC is isosceles.

Question 5

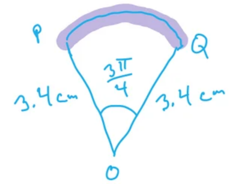
A sector of a circle, OPQ, is such that it has radius 3.4 cm and the angle at its centre, O, is $\frac{3\pi}{4}$ radians.

(i) Find the length of the arc PQ.

(ii) Find the area of the sector OPQ.

[4]

This is the arc PQ



A sketch can be helpful, but isn't needed to get the marks

$\left.\begin{array}{l}\underline{\text{Arc length}}:\ l = r\theta \\ \underline{\text{Area of sector}}:\ A = \frac{1}{2}r^2\theta\end{array}\right\}$ θ must be in radians!

(i) $l = r\theta = (3.4)\left(\frac{3\pi}{4}\right) = \boxed{\frac{51\pi}{20}\text{ cm}}$

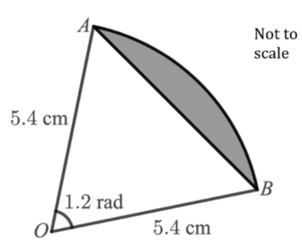
$(\approx 8.0\text{ cm})$

(ii) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(3.4)^2\left(\frac{3\pi}{4}\right) = \boxed{\frac{867\pi}{200}\text{ cm}^2}$

$(\approx 13.6\text{ cm}^2)$

Question 6

The diagram below shows the sector of a circle OAB.



(a) (i) Find the area of the sector OAB, giving your answer to 3 significant figures.

(ii) Find the area of the triangle OAB, giving your answer to 3 significant figures.

(iii) Find the area of the shaded segment, giving your answer to 3 significant figures.

[5]

(b) (i) Find the length of the arc AB.

(ii) Find the perimeter of the sector OAB.

[3]

$\left.\begin{array}{l}\underline{\text{Arc length}}:\ l = r\theta \\ \underline{\text{Area of sector}}:\ A = \frac{1}{2}r^2\theta\end{array}\right\}$ θ must be in radians!

$\text{Area} = \frac{1}{2}ab\sin\theta$

a) (i) area $= \frac{1}{2}(5.4)^2(1.2) = 17.496\text{ cm}^2$

$\boxed{= 17.5\text{ cm}^2\ (3\text{ s.f.})}$

(ii) area $= \frac{1}{2}(5.4)(5.4)\sin(1.2) = 13.589\ldots\text{ cm}^2$

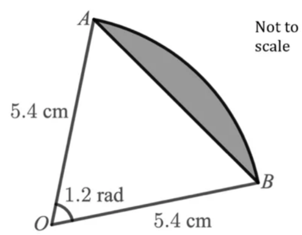
$\boxed{= 13.6\text{ cm}^2\ (3\text{ s.f.})}$

(iii) area $= 17.496 - 13.589\ldots$

$= 3.906\ldots\text{ cm}^2$

$\boxed{= 3.91\text{ cm}^2\ (3\text{ s.f.})}$

The diagram below shows the sector of a circle OAB.



(a) (i) Find the area of the sector OAB, giving your answer to 3 significant figures.

(ii) Find the area of the triangle OAB, giving your answer to 3 significant figures.

(iii) Find the area of the shaded segment, giving your answer to 3 significant figures.

[5]

(b) (i) Find the length of the arc AB.

(ii) Find the perimeter of the sector OAB.

[3]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

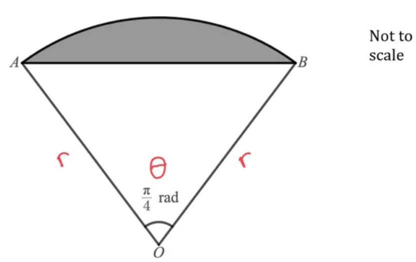
b) (i) $l = (5.4)(1.2) = \boxed{6.48 \text{ cm}}$

(ii) Perimeter $= 5.4 + 5.4 + 6.48$

$\boxed{= 17.28 \text{ cm}}$

Question 7

The canopy of a parachute and the outermost connecting cords form a sector of a circle as shown in the diagram below, with the parachutist modelled as a particle at point O.



The area of the sector OAB is $\frac{81\pi}{200}$ m^2.
Find the length of one of the connecting cords on the parachute.

[3]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

area $= \frac{81\pi}{200} = \frac{1}{2}r^2\left(\frac{\pi}{4}\right)$

$\frac{\cancel{\pi}}{8}r^2 = \frac{81\cancel{\pi}}{200}$

$\frac{1}{8}r^2 = \frac{81}{200}$

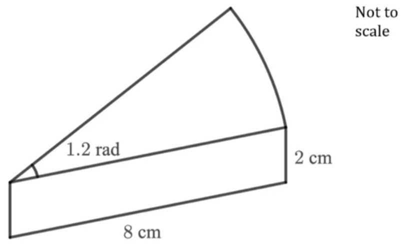
$r^2 = \frac{81}{25}$

radius $= \sqrt{\frac{81}{25}} = \frac{9}{5}$ m

length of connecting cord is $\frac{9}{5}$ m (1.8 m)

Question 8

A plastic puzzle piece is in the form of a prism with a cross-section that is the sector of a circle, as shown in the diagram below. The radius of the sector is 8 cm, and the angle at the centre is 1.2 radians.
The height of the puzzle piece is 2 cm.



(i) Work out the area of the cross-section.

(ii) Hence, or otherwise, work out the volume of the puzzle piece.

[3]

(i) $\text{Area} = \frac{1}{2}(8)^2(1.2) = \boxed{38.4\ \text{cm}^2}$

(ii) $\text{Volume} = 38.4 \times 2 = \boxed{76.8\ \text{cm}^3}$

Volume of Prism = Area of Cross-section × Height

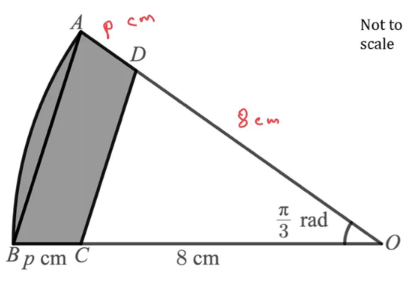
Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

θ must be in radians!

Question 9

The circle sector OAB is shown in the diagram below.
The angle at the centre is $\frac{\pi}{3}$ radians, and the line segments OC and BC have lengths of 8 cm and p cm respectively.
Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



(a) Show that the area of the sector OAB is $\frac{\pi}{6}(p+8)^2$ cm^2.

[2]

(b) Show that the area of the triangle OCD is $16\sqrt{3}$ cm^2.

[2]

(c) Given that the area of the shaded shape $ABCD$ is $\left(\frac{50\pi}{3} - 16\sqrt{3}\right)$ cm^2, find the value of p.

[4]

Arc length : $l = r\theta$

Area of sector : $A = \frac{1}{2}r^2\theta$

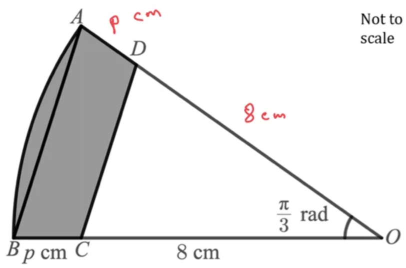
θ must be in radians!

a) radius of sector $OAB = (p+8)$ cm

$\text{Area} = \frac{1}{2}(p+8)^2\left(\frac{\pi}{3}\right)$

$= \frac{\pi}{6}(p+8)^2$ cm^2

The circle sector OAB is shown in the diagram below.
The angle at the centre is $\frac{\pi}{3}$ radians, and the line segments OC and BC have lengths of 8 cm and p cm respectively.
Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



(a) Show that the area of the sector OAB is $\frac{\pi}{6}(p+8)^2$ cm^2.

[2]

(b) Show that the area of the triangle OCD is $16\sqrt{3}$ cm^2.

[2]

(c) Given that the area of the shaded shape $ABCD$ is $\left(\frac{50\pi}{3} - 16\sqrt{3}\right)$ cm^2, find the value of p.

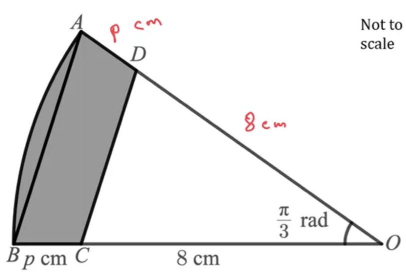
[4]

Area $= \frac{1}{2}ab\sin\theta$

b)

$$\text{Area} = \frac{1}{2}(8)(8)\sin\left(\frac{\pi}{3}\right)$$
$$= 32\sin\left(\frac{\pi}{3}\right)$$
$$= 32\left(\frac{\sqrt{3}}{2}\right)$$
$$= 16\sqrt{3} \text{ cm}^2$$

The circle sector OAB is shown in the diagram below.
The angle at the centre is $\frac{\pi}{3}$ radians, and the line segments OC and BC have lengths of 8 cm and p cm respectively.
Additionally, CD is parallel to AB, so that $AD = BC$ and $OD = OC$.



(a) Show that the area of the sector OAB is $\frac{\pi}{6}(p+8)^2$ cm^2.

[2]

(b) Show that the area of the triangle OCD is $16\sqrt{3}$ cm^2.

[2]

(c) Given that the area of the shaded shape $ABCD$ is $\left(\frac{50\pi}{3} - 16\sqrt{3}\right)$ cm^2, find the value of p.

[4]

c) area of shape ABCD = area of sector OAB − area of triangle OCD

$$= \frac{\pi}{6}(p+8)^2 - 16\sqrt{3}$$

Therefore

$$\frac{\pi}{6}(p+8)^2 - \cancel{16\sqrt{3}} = \frac{50\pi}{3} - \cancel{16\sqrt{3}}$$
$$\frac{\cancel{\pi}}{6}(p+8)^2 = \frac{50\cancel{\pi}}{3}$$
$$\frac{1}{6}(p+8)^2 = \frac{50}{3}$$
$$(p+8)^2 = 100$$
$$p+8 = \pm 10$$
$$p = 2 \text{ or } -18$$

But p can't be negative

because it is the length of a line segment!

$$\boxed{\text{So } p = 2 \text{ cm}}$$

Question 10

Solve the equation $2\sin x = \frac{1}{\sin x}$ for $0° \le x \le 360°$.

[5]

$2\sin x = \frac{1}{\sin x}$

$2\sin^2 x = 1$

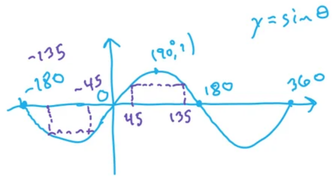
$\sin^2 x = \frac{1}{2} \implies \sin x = \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$

If $\sin x = \frac{1}{\sqrt{2}}$

$x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$ principal solution

$180 - 45 = 135$ use symmetry or CAST to find other solution

So $x = 135°$ is another solution

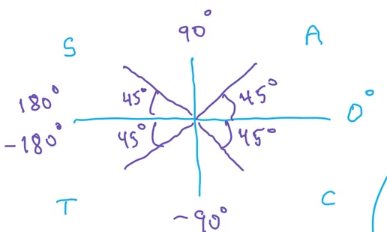


If $\sin x = -\frac{1}{\sqrt{2}}$

$x = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -45°$ principal solution

$-180 + 45 = -135$ use symmetry or CAST to find other solution

So $x = -135°$ is another solution



Use fact that sine function repeats every 360° to find solutions in the interval $0° \le x \le 360°$

$-45 + 360 = 315$ $\quad -135 + 360 = 225$

So $x = 225°$ and $x = 315°$ are also solutions

The solutions are

$\boxed{x = 45°, 135°, 225°, 315°}$

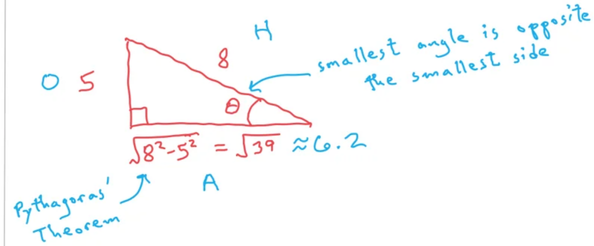
Question 11

A right-angled triangle has hypotenuse 8 cm. One of its other sides is 5 cm.

Find exact values for $\sin\theta$, $\cos\theta$ and $\tan\theta$, where θ is the smallest angle in the triangle.

[6]

Be sure to use surd form for the third side!



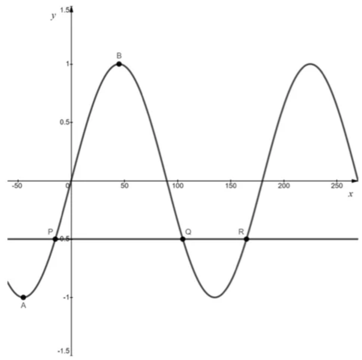
$\sin\theta = \frac{5}{8}$ SOH

$\cos\theta = \frac{\sqrt{39}}{8}$ CAH

$\tan\theta = \frac{5}{\sqrt{39}}$ TOA

Question 12

The graph below shows the curve with equation $y = \sin 2x$ in the interval $-60° \leq x \leq 270°$.



(a) Point A has coordinates $(-45°, -1)$ and is the minimum point closest to the origin. Point B is the maximum point closest to the origin. State the coordinates of B.

[1]

(b) A straight line with equation $y = -\frac{1}{2}$ meets the graph of $y = \sin 2x$ at the three points P, Q and R, as shown in the diagram.

Given that point P has coordinates $(-15°, -\frac{1}{2})$, use graph symmetries to determine the coordinates of Q and R.

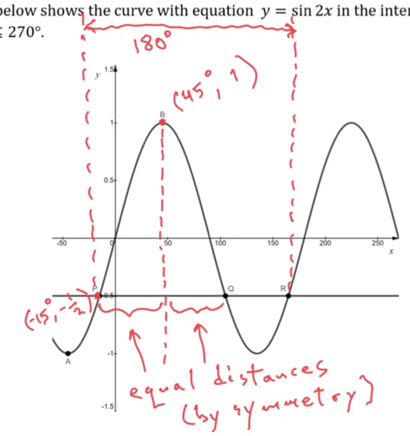
[2]

$\sin^{-1}(1) = 90°$ so $\sin(90°) = 1$

If $x = 45°$, $\sin(2x) = \sin(90°) = 1$ MAXIMUM

a) Point B has coordinates $(45°, 1)$

The graph below shows the curve with equation $y = \sin 2x$ in the interval $-60° \leq x \leq 270°$.



(a) Point A has coordinates $(-45°, -1)$ and is the minimum point closest to the origin. Point B is the maximum point closest to the origin. State the coordinates of B.

[1]

(b) A straight line with equation $y = -\frac{1}{2}$ meets the graph of $y = \sin 2x$ at the three points P, Q and R, as shown in the diagram.

Given that point P has coordinates $(-15°, -\frac{1}{2})$, use graph symmetries to determine the coordinates of Q and R.

[2]

$\sin 2x$ is a horizontal stretch of $\sin x$, with scale factor ½ (i.e., a 'squash'), around the y-axis. $\sin x$ repeats every 360°, so $\sin 2x$ repeats every 180°.

b) $45 - (-15) = 60$ } horizontal distance from point P to point B

$45 + 60 = 105$

Point Q has coordinates $(105°, -\frac{1}{2})$

$-15 + 180 = 165$

Point R has coordinates $(165°, -\frac{1}{2})$

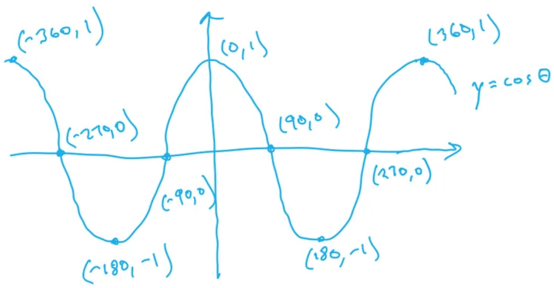
Note: Remember that you can use your GDC to check your answers on a question like this!

Question 13

(i) Sketch the graph of $y = \cos(\theta + 30°)$ in the interval $-180° \leq \theta \leq 360°$.

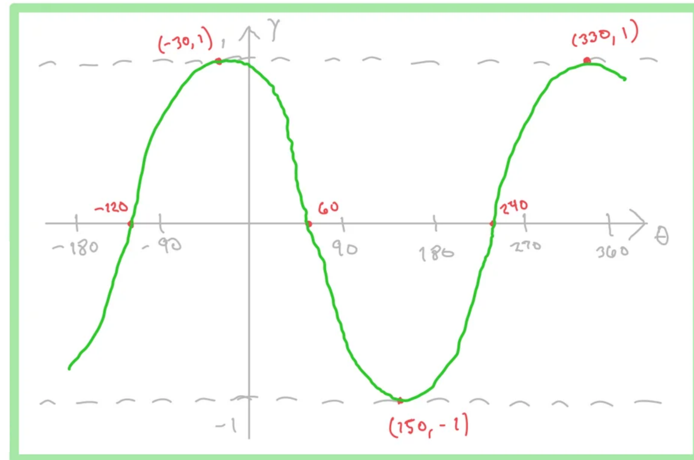
(ii) Write down all the values where $\cos(\theta + 30°) = 0$ in the given interval.

[4]



$y = \cos(\theta + 30°)$ is $y = \cos\theta$ translated $30°$ to the left

(i)



(ii) From the graph

$$\boxed{\theta = -120°, 60°, 240°}$$