

IB Maths: AA HL

Further Proof & Reasoning

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

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| Course | IB Maths |
| Section | 1. Number & Algebra |
| Topic | 1.5 Further Proof & Reasoning |
| Difficulty | Medium |

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Further Proof & Reasoning

Question 1

Prove that there is no x such that $\frac{2}{x-2} = x - 3$.

[5 marks]

Question 2

Using the method of proof by contradiction, prove that $\sqrt{7}$ is irrational.

[4 marks]

Question 3

Using mathematical induction, prove that $6^n - 1$ is divisible by 5 for $n \in \mathbb{Z}$, $n \geq 1$.

[4 marks]

Question 4

The n th triangular number is given by the formula $u_n = \frac{1}{2}n(n + 1)$.

(a)

Write down the first five triangular numbers.

[1 marks]

(b)

Prove by exhaustion that the first five triangular numbers are all factors of 180.

[2 marks]

Question 5

Determine, with appropriate reasoning, whether the following statements are true or false:

(i)

Given $n \in Z$ and n^2 is divisible by 4, then n is divisible by 4.

(ii)

Given $n \in Z$ then $n^2 - 1$ is a prime number.

(iii)

Given $n \in Z$ and n^2 is divisible by 3, then n is divisible by 3.

(iv)

Given an integer is a multiple 8 and 6 then it is a multiple of 48.

[8 marks]

Question 6

Prove that $x^2 - 3x + 3$ is positive for all real values of x .

[3 marks]

Question 7

(a)

Show that $(3n + 2)^2 - (n + 2)^2 = 8n^2 + 8n$, where $n \in Z$.

[2 marks]

(b)

Hence, or otherwise, prove that $(3n + 2)^2 - (n + 2)^2$ is a multiple of 8.

[2 marks]

Question 8

Prove that $(a - b)^2 - (a + b)^2 = -4ab$.

[3 marks]

Question 9

Prove that $(4x - 1)(2x + 3) - (2x + 1)^2 = 2(2x - 1)(x + 2)$.

[3 marks]

Question 10

Prove by mathematical induction $3^n \geq 1 + 2n$, given $n \geq 0$.

[6 marks]

Question 11

Prove by mathematical induction that if $y = \frac{1}{1-x}$ then $\frac{d^n y}{dx^n} = \frac{n!}{(1-x)^{n+1}}$.

[6 marks]

Question 12

Prove by induction that

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

for all values of n , $n \in \mathbb{Z}^+$.

[6 marks]

Question 13

Given $z = x + yi$

(i)

prove that $zz^* = |z|^2$,

(ii)

prove that, for $x \geq 0$, $\arg(z) + \arg(z^*) = 0$.

[4 marks]