IB Maths: AA HL

Further Probability Distributions

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	4. Statistics & Probability
Topic	4.7 Further Probability Distributions
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Further Probability Distributions

Question 1

A 'lucky dip' bag contains seven bars of chocolate and 5 packets of sweets. Suraya selects two items at random without replacing them.

The probability distribution table for the discrete random variable X, "the number of packets of sweets selected", is shown below.

X	0	1	2
P($X=x$)	$\dfrac{21}{66}$	$\dfrac{7k}{66}$	$\dfrac{2k}{66}$

a)
Find the value of k.

[3 marks]

b)
Find $\mathrm{E}(X)$.

[2 marks]

c)
Find $\mathrm{E}(X^2)$.

[2 marks]

d)
Find $\mathrm{Var}(X)$.

[3 marks]

Question 2

A population of grasshoppers is being studied. It is found that the length of an adult grasshopper, in cm, has PDF

$$f(x) = \begin{cases} kx^2(6-x), & 0 \le x \le 6 \\ 0, & \text{otherwise.} \end{cases}$$

a)
Find the value of k.

[4 marks]

b)
Sketch the probability density function.

[2 marks]

c)
Find the probability that a grasshopper picked at random is less than 4 cm in length.

[2 marks]

Question 3

A game is played with two fair spinners. Each spinner is divided into three sections numbered 1, 2 and 3. A player's score is obtained by spinning both spinners simultaneously and adding together the numbers that they land on.

a)
Complete the table below for the probability distribution of the game.

Score, X					
$P(X=x)$					

[2 marks]

b)
Find the expected score, $E(X)$.

[2 marks]

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Jian Wei wants to award prizes such that a player receives $3 for the score that they achieve.

c)
Find the expected prize money for the game.

[2 marks]

Question 4

A continuous random variable has a probability distribution function

$$f(x) = \begin{cases} \dfrac{3}{4}(-x^2 + 2x), & 0 \le x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

a)
Show that the mean of the random variable is equal to 1.

[4 marks]

b)
Find the variance of the random variable.

[6 marks]

c)
Hence, find the standard deviation of the random variable, leaving your answer in the form $\dfrac{\sqrt{a}}{b}$.

[3 marks]

Question 5

At a school probability fair, some students create a game using one complete suit from a standard pack of cards. A player must pay $1 to pick a card at random. If their card is a jack, queen or a king they will receive $1 back, if their card is an ace they will receive $5 otherwise if their card is an ordinary number card from 2 to 10, they will receive nothing.

a)
Show that the game is not fair.

[4 marks]

b)
Calculate

(i)
E(X^2)

(ii)
Var(X)

[4 marks]

The students want to make the game fair, so decide to give a prize to anyone who picks an ordinary number card.

c)
Calculate the value of the new prize for choosing an ordinary number card.

[2 marks]

Question 6

A discrete random variable B has probability distribution given by $B = ab(b+1)$, where $b = 5, 6, 7$.

a)
Find the value of a.

[3 marks]

b)
Complete the probability distribution table below.

B	5	6	7
P($B = b$)			

[2 marks]

c)
Find the mean of B.

[2 marks]

d)
Find the standard deviation of B.

[5 marks]

Question 7

A continuous random variable has the probability density function given by

$$f(x) = \begin{cases} tx^3 - \dfrac{x^2}{18} + \dfrac{7}{36}x, & 0 \le x < 6 \\ 0, & \text{otherwise.} \end{cases}$$

a)
Find the value of t.

[4 marks]

b)
Hence, find the values of

(i)
the mean

(ii)
the mode

(iii)
the median.

[8 marks]

Question 9

A random variable has $E(X) = 23$ and $Var(X) = 1.5$.

Find

(i)
$E(X - 6)$

(ii)
$E(-2X + 5)$

(iii)
$Var(X + 7)$

(iv)
$Var(3X - 3)$

[6 marks]

Question 9

Consider the function defined by

$$f(x) = \begin{cases} \dfrac{1}{10}x^2 & 0 \le x < 2 \\ \dfrac{82}{135} - \dfrac{14}{135}x & 2 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

where $f(x)$ is the probability density function of a continuous random variable.

a)
Sketch the graph of $f(x)$

[3 marks]

b)
Find the value of $E(X)$.

[4 marks]

c)
Find the value of $Var(X)$.

[4 marks]