Question 1

A 'lucky dip' bag contains seven bars of chocolate and 5 packets of sweets. Suraya selects two items at random without replacing them.

The probability distribution table for the discrete random variable X, "the number of packets of sweets selected", is shown below.

X	0	1	2
$P(X = x)$	$\frac{21}{66}$	$\frac{7k}{66}$	$\frac{2k}{66}$

(a) Find the value of k.

[3]

(b) Find $E(X)$.

[2]

(c) Find $E(X^2)$.

[2]

(d) Find $Var(X)$.

[3]

a) Probabilities will sum to 1.

$\frac{21}{66} + \frac{7k}{66} + \frac{2k}{66}$

$\boxed{k = 5}$

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[3]

(b) Find $E(X)$.

[2]

(c) Find $E(X^2)$.

[2]

(d) Find $Var(X)$.

[3]

b) $E(X) = \sum x P(X = x)$ (in formula booklet)

$E(X) = (0)\left(\frac{21}{66}\right) + (1)\left(\frac{35}{66}\right) + (2)\left(\frac{10}{66}\right)$

$\boxed{E(X) = \frac{5}{6} = 0.833 \text{ (3 s.f.)}}$

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[3]

(b) Find $\mathrm{E}(X)$.

[2]

(c) Find $\mathrm{E}(X^2)$.

[2]

(d) Find $\mathrm{Var}(X)$.

[3]

c) $E(X^2) = \sum x^2 P(X = x)$ (not in formula booklet)

$E(X^2) = (0)^2\left(\frac{21}{66}\right) + (1)^2\left(\frac{35}{66}\right) + (2)^2\left(\frac{10}{66}\right)$

$\boxed{E(X^2) = \frac{25}{22} = 1.14 \ (3 \text{ s.f.})}$

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[3]

(b) Find $\mathrm{E}(X)$. $\boxed{E(X) = \frac{5}{6} = 0.833 \ (3 \text{ s.f.})}$

[2]

(c) Find $\mathrm{E}(X^2)$. $\boxed{E(X^2) = \frac{25}{22} = 1.14 \ (3 \text{ s.f.})}$

[2]

(d) Find $\mathrm{Var}(X)$.

[3]

d) $Var(X) = E(X - \mu)^2 = E(X)^2 - [E(X)]^2$ (in formula booklet)

$Var(X) = \frac{25}{22} - \left(\frac{5}{6}\right)^2$

$\boxed{Var(X) = \frac{175}{396} = 0.442 \ (3 \text{ s.f.})}$

Question 2

A population of grasshoppers is being studied. It is found that the length of an adult grasshopper, in cm, has PDF

$$f(x) = \begin{cases} kx^2(6-x), & 0 \le x \le 6 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of k.

[4]

(b) Sketch the probability density function.

[2]

(c) Find the probability that a grasshopper picked at random is less than 4 cm in length.

[2]

a) The area under the curve is equal to 1.

$k\int_0^6 x^2(6-x)\,dx = 1$ solve with GDC

$$\boxed{k = \frac{1}{108}}$$

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(a) Find the value of k.

$$\boxed{k = \frac{1}{108}}$$

[4]

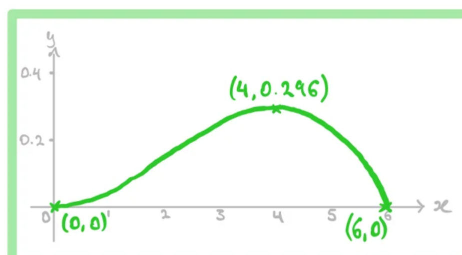
(b) Sketch the probability density function.

[2]

(c) Find the probability that a grasshopper picked at random is less than 4 cm in length.

[2]

b) Sketch $f(x)$ on your GDC.



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(a) Find the value of k.

[4]

(b) Sketch the probability density function.

[2]

(c) Find the probability that a grasshopper picked at random is less than 4 cm in length.

[2]

c) $P(X<4) = \frac{1}{108}\int_0^4 x^2(6-x)\,dx$

$\boxed{P(X<4) = \frac{16}{27} = 0.593 \text{ (3 s.f.)}}$

Question 3

A game is played with two fair spinners. Each spinner is divided into three sections numbered 1, 2 and 3. A player's score is obtained by spinning both spinners simultaneously and adding together the numbers that they land on.

(a) Complete the table below for the probability distribution of the game.

Score, X	2	3	4	5	6
$P(X=x)$	1/9	2/9	3/9	2/9	1/9

[2]

(b) Find the expected score, $E(X)$.

[2]

Jian Wei wants to award prizes such that a player receives \$3 for the score that they achieve.

(c) Find the expected prize money for the game.

[2]

a) Min score is 2 (1+1) and max score is 6 (3+3) and there are 9 possible outcomes.

score = 2 $\rightarrow (1+1) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

score = 3 $\rightarrow (1+2)$ and $(2+1) = 2\left(\frac{1}{3}\right)^2 = \frac{2}{9}$

score = 4 $\rightarrow (2+2), (1+3)$ and $(3+1) = 3\left(\frac{1}{3}\right)^2 = \frac{3}{9}$

score = 5 $\rightarrow (2+3)$ and $(3+2) = 2\left(\frac{1}{3}\right)^2 = \frac{2}{9}$

score = 6 $\rightarrow (3+3) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

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Score, X	2	3	4	5	6
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[2]

(b) Find the expected score, $E(X)$.

[2]

Jian Wei wants to award prizes such that a player receives \$3 for the score that they achieve.

(c) Find the expected prize money for the game.

[2]

b) $E(X) = \sum x P(X=x)$ (in formula booklet)

$E(X) = (2)\left(\frac{1}{9}\right) + (3)\left(\frac{2}{9}\right) + (4)\left(\frac{3}{9}\right) + (5)\left(\frac{2}{9}\right) + (6)\left(\frac{1}{9}\right)$

$\boxed{E(X) = 4}$

A game is played with two fair spinners. Each spinner is divided into three sections numbered 1, 2 and 3. A player's score is obtained by spinning both spinners simultaneously and adding together the numbers that they land on.

(a) Complete the table below for the probability distribution of the game.

Score, X					
$P(X = x)$					

[2]

(b) Find the expected score, $E(X)$.

$\boxed{E(X) = 4}$

[2]

Jian Wei wants to award prizes such that a player receives \$3 for the score that they achieve.

(c) Find the expected prize money for the game.

[2]

c) $\boxed{\text{expected prize money} = 3 \times 4 = \$12}$

Question 4

A continuous random variable has a probability distribution function

$$f(x) = \begin{cases} \frac{3}{4}(-x^2 + 2x), & 0 \le x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that the mean of the random variable is equal to 1.

[4]

(b) Find the variance of the random variable.

[6]

(c) Hence, find the standard deviation of the random variable, leaving your answer in the form $\frac{\sqrt{a}}{b}$.

[3]

a) $E(X) = \mu = \int_{-\infty}^{\infty} x f(x)$ (in formula booklet)

$E(X) = \frac{3}{4}\int_0^2 x(-x^2 + 2x)\,dx = \frac{3}{4}\int_0^2 -x^3 + 2x^2\,dx$

$E(X) = \frac{3}{4}\left[-\frac{x^4}{4} + \frac{2x^3}{3}\right]_0^2 = \frac{3}{4}\left(-4 + \frac{16}{3}\right) = \frac{3}{4} \times \frac{4}{3}$

$\boxed{E(X) = 1}$

A continuous random variable has a probability distribution function

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(a) Show that the mean of the random variable is equal to 1.

[4]

(b) Find the variance of the random variable.

[6]

(c) Hence, find the standard deviation of the random variable, leaving your answer in the form $\frac{\sqrt{a}}{b}$.

[3]

b) $Var(X) = E(X - \mu)^2 = E(X)^2 - [E(X)]^2$ (in formula booklet)

$E(X^2) = \sum x^2 P(X = x)$ (not in formula booklet)

$E(X)^2 = \frac{3}{4}\int_0^2 x^2(-x^2 + 2x)\,dx = \frac{3}{4}\int_0^2 -x^4 + 2x^3\,dx$

$E(X)^2 = \frac{3}{4}\left[-\frac{x^5}{5} + \frac{x^4}{2}\right]_0^2 = \frac{3}{4}\left(-\frac{32}{5} + 8\right) = \frac{3}{\cancel{4}} \times \frac{\cancel{8}^{2}}{5} = \frac{6}{5}$

$\boxed{Var(X) = \frac{6}{5} - 1^2 = \frac{1}{5}}$

A continuous random variable has a probability distribution function

$$f(x) = \begin{cases} \frac{3}{4}(-x^2 + 2x), & 0 \le x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that the mean of the random variable is equal to 1.

[4]

(b) Find the variance of the random variable.

$\boxed{Var(X) = \frac{6}{5} - 1^2 = \frac{1}{5}}$

[6]

(c) Hence, find the standard deviation of the random variable, leaving your answer in the form $\frac{\sqrt{a}}{b}$.

[3]

c) Standard deviation, $\sigma = \sqrt{\text{variance}}$

$\boxed{\sigma = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}}$

Question 5

At a school probability fair, some students create a game using one complete suit from a standard pack of cards. A player must pay \$1 to pick a card at random. If their card is a jack, queen or a king they will receive \$1 back, if their card is an ace they will receive \$5 otherwise if their card is an ordinary number card from 2 to 10, they will receive nothing.

(a) Show that the game is not fair.

[4]

(b) Calculate

(i) $E(X^2)$

(ii) $Var(X)$

[4]

The students want to make the game fair, so decide to give a prize to anyone who picks an ordinary number card.

(c) Calculate the value of the new prize for choosing an ordinary number card.

[2]

a) Create a table of outcomes

X	number card	royal	ace
$P(X=x)$	$\frac{9}{13}$	$\frac{3}{13}$	$\frac{1}{13}$
Prize	\$0	\$1	\$5
Profit	-\$1	\$0	\$4

For a fair game the expected profit = 0

expected profit = $(-1)\left(\frac{9}{13}\right) + (0) + (4)\left(\frac{1}{13}\right) = \frac{-5}{13}$

$\boxed{\text{expected profit} = \frac{-5}{13} \neq 0, \therefore \text{the game is not fair}}$

Note : $E(X) = \frac{8}{13}$

At a school probability fair, some students create a game using one complete suit from a standard pack of cards. A player must pay $1 to pick a card at random. If their card is a jack, queen or a king they will receive $1 back, if their card is an ace they will receive $5 otherwise if their card is an ordinary number card from 2 to 10, they will receive nothing.

(a) Show that the game is not fair.

[4]

(b) Calculate

(i) $E(X^2)$

(ii) $\mathrm{Var}(X)$

[4]

The students want to make the game fair, so decide to give a prize to anyone who picks an ordinary number card.

(c) Calculate the value of the new prize for choosing an ordinary number card.

[2]

b) i) $E(X^2) = \sum x^2 P(X = x)$ (not in formula booklet)

$E(X^2) = 0 + \frac{3}{13}(1)^2 + \frac{1}{13}(5)^2$

$\boxed{E(X^2) = \frac{28}{13} = \$2.15 \text{ (3 s.f.)}}$

ii) $\mathrm{Var}(X) = E(X-\mu)^2 = E(X)^2 - [E(X)]^2$ (in formula booklet)

$\mathrm{Var}(X) = \frac{28}{13} - \left(\frac{8}{13}\right)^2$

$\boxed{\mathrm{Var}(X) = \frac{300}{169} = \$1.78 \text{ (3 s.f.)}}$

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[4]

(b) Calculate

(i) $E(X^2)$

(ii) $\mathrm{Var}(X)$

[4]

The students want to make the game fair, so decide to give a prize to anyone who picks an ordinary number card.

(c) Calculate the value of the new prize for choosing an ordinary number card.

[2]

c) Let x = prize for number card

$E(X) = 1 = \frac{9}{13}x + \frac{3}{13}(1) + \frac{1}{13}(5)$

$\boxed{x = \frac{5}{9} = \$0.56}$

Question 6

A discrete random variable B has probability distribution given by $B = ab(b+1)$, where $b = 5, 6, 7$.

(a) Find the value of a.

[3]

(b) Complete the probability distribution table below.

B	5	6	7
$P(B = b)$			

[2]

(c) Find the mean of B.

[2]

(d) Find the standard deviation of B.

[5]

a) $\sum B\,P(B=b) = 1$

$1 = 5a(5+1) + 6a(6+1) + 7a(7+1)$

$\boxed{a = \frac{1}{128}}$

A discrete random variable B has probability distribution given by $B = ab(b+1)$, where $b = 5, 6, 7$.

(a) Find the value of a.

$\boxed{a = \frac{1}{128}}$

[3]

(b) Complete the probability distribution table below.

B	5	6	7
$P(B = b)$	$\frac{15}{64}$	$\frac{21}{64}$	$\frac{7}{16}$

[2]

(c) Find the mean of B.

[2]

(d) Find the standard deviation of B.

[5]

b) $P(B=5) = \left(\frac{1}{128}\right)5(5+1) = \frac{30}{128} = \frac{15}{64}$

$P(B=6) = \left(\frac{1}{128}\right)6(6+1) = \frac{42}{128} = \frac{21}{64}$

$P(B=7) = \left(\frac{1}{128}\right)7(7+1) = \frac{56}{128} = \frac{7}{16}$

A discrete random variable B has probability distribution given by $B = ab(b+1)$, where $b = 5, 6, 7$.

(a) Find the value of a.

[3]

(b) Complete the probability distribution table below.

B	5	6	7
$P(B = b)$	$\frac{15}{64}$	$\frac{21}{64}$	$\frac{7}{16}$

[2]

(c) Find the mean of B.

[2]

(d) Find the standard deviation of B.

[5]

c) $E(X) = \sum x P(X = x)$ (in formula booklet)

$E(B) = (5)\left(\frac{15}{64}\right) + (6)\left(\frac{21}{64}\right) + (7)\left(\frac{7}{16}\right)$

$\boxed{E(B) = \frac{397}{64} = 6.20 \quad (3 \text{ s.f.})}$

A discrete random variable B has probability distribution given by $B = ab(b+1)$, where $b = 5, 6, 7$.

(a) Find the value of a.

[3]

(b) Complete the probability distribution table below.

B	5	6	7
$P(B = b)$	$\frac{15}{64}$	$\frac{21}{64}$	$\frac{7}{16}$

[2]

(c) Find the mean of B. $\boxed{E(B) = \frac{397}{64} = 6.20 \quad (3 \text{ s.f.})}$

[2]

(d) Find the standard deviation of B.

[5]

d) $E(X^2) = \sum x^2 P(X = x)$ (not in formula booklet)

$E(B^2) = (5^2)\left(\frac{15}{64}\right) + (6^2)\left(\frac{21}{64}\right) + (7^2)\left(\frac{7}{16}\right) = \frac{2503}{64}$

$Var(X) = E(X - \mu)^2 = E(X)^2 - [E(X)]^2$ (in formula booklet)

$\sigma_B = \sqrt{Var(B)} = \sqrt{E(B^2) - E(B)} = \sqrt{\frac{2503}{64} - \left(\frac{397}{64}\right)^2} = \frac{3\sqrt{287}}{64}$

$\boxed{\sigma_B = 0.7941\ldots = 0.794 \quad (3 \text{ s.f.})}$

Question 7

A continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} tx^3 - \frac{x^2}{18} + \frac{7}{36}x, & 0 \le x < 6 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of t.

[4]

(b) Hence, find the values of

(i) the mean

(ii) the mode

(iii) the median.

[8]

a) The area under the curve is equal to 1.

$$\int_0^6 \left(tx^3 - \frac{x^2}{18} + \frac{7}{36}x\right)dx = 1$$

$$\therefore \frac{t(6)^4}{4} - \frac{(6)^3}{54} + \frac{7}{72}(6)^2 = 1$$

$$\boxed{t = \frac{1}{216}}$$

A continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} tx^3 - \frac{x^2}{18} + \frac{7}{36}x, & 0 \le x < 6 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of t. $\boxed{t = \frac{1}{216}}$

[4]

(b) Hence, find the values of

(i) the mean

(ii) the mode

(iii) the median.

[8]

b) i) $E(X) = \mu = \int_{-\infty}^{\infty} x f(x)\,dx$ (in formula booklet)

$$E(X) = \mu = \int_0^6 x\left(\frac{1}{216}x^3 - \frac{1}{18}x^2 + \frac{7}{36}x\right)dx$$

$$\boxed{E(X) = \mu = \frac{16}{5} = 3.2}$$

ii) Mode $\longrightarrow$ Graph $f(x)$ and find its maximum.

$\boxed{\text{Mode occurs at } x = 2.5857\ldots = 2.59 \text{ (3 s.f.)}}$

iii) Median $\longrightarrow \int_0^m f(x)\,dx = 0.5$

$$0.5 = \int_0^m \left(\frac{1}{216}x^3 - \frac{1}{18}x^2 + \frac{7}{36}x\right)dx$$

$$\boxed{m = 3.1508\ldots}$$

Question 8

A random variable has $\mathrm{E}(X) = 23$ and $\mathrm{Var}(X) = 1.5$.

Find

(i) $\mathrm{E}(X - 6)$

(ii) $\mathrm{E}(-2X + 5)$

(iii) $\mathrm{Var}(X + 7)$

(iv) $\mathrm{Var}(3X - 3)$

[6]

$E(aX + b) = aE(X) - b$ (in formula booklet)

$Var(aX + b) = a^2 Var(X)$ (in formula booklet)

i) $\boxed{E(X - 6) = 23 - 6 = 17}$

ii) $\boxed{E(-2X + 5) = -2(23) + 5 = -41}$

iii) $\boxed{Var(X + 7) = 1.5}$

iv) $\boxed{Var(3X - 3) = 3^2(1.5) = 13.5}$

Question 9

Consider the function defined by

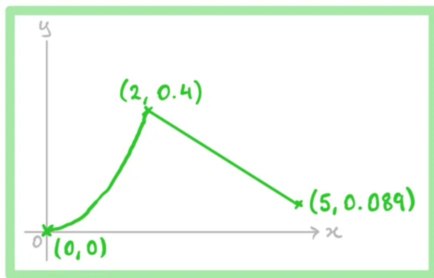
$$f(x) = \begin{cases} \frac{1}{10}x^2 & 0 \le x < 2 \\ \frac{82}{135} - \frac{14}{135}x & 2 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

where $f(x)$ is the probability density function of a continuous random variable.

(a) Sketch the graph of $f(x)$.

[3]

a) Graph $f(x)$ on your GDC.



(b) Find the value of $\mathrm{E}(X)$.

[4]

(c) Find the value of $\mathrm{Var}(X)$.

[4]

Consider the function defined by

$$f(x)=\begin{cases}\frac{1}{10}x^2 & 0\le x<2\\ \frac{82}{135}-\frac{14}{135}x & 2\le x\le 5\\ 0 & \text{otherwise}\end{cases}$$

where $f(x)$ is the probability density function of a continuous random variable.

(a) Sketch the graph of $f(x)$.

[3]

(b) Find the value of $E(X)$.

[4]

(c) Find the value of $Var(X)$.

[4]

b) $E(X)=\mu=\int_{-\infty}^{\infty}xf(x)\,dx$ (in formula booklet)

$E(X)=\int_0^2 \frac{1}{10}x^3\,dx+\int_2^5 x\left(\frac{82}{135}-\frac{14}{135}x\right)dx$

$\boxed{E(X)=\frac{41}{15}=2.73\ (3\text{ s.f.})}$

Consider the function defined by

$$f(x)=\begin{cases}\frac{1}{10}x^2 & 0\le x<2\\ \frac{82}{135}-\frac{14}{135}x & 2\le x\le 5\\ 0 & \text{otherwise}\end{cases}$$

where $f(x)$ is the probability density function of a continuous random variable.

(a) Sketch the graph of $f(x)$.

[3]

(b) Find the value of $E(X)$. $\boxed{E(X)=\frac{41}{15}=2.73\ (3\text{ s.f.})}$

[4]

(c) Find the value of $Var(X)$.

[4]

c) $Var(X)=E(X-\mu)^2=E(X)^2-[E(X)]^2$ (in formula booklet)

$E(X^2)=\int_0^2 \frac{1}{10}x^4\,dx+\int_2^5 x^2\left(\frac{82}{135}-\frac{14}{135}x\right)dx=\frac{427}{50}$

$\boxed{Var(X)=\frac{427}{50}-\left(\frac{41}{15}\right)^2=\frac{481}{450}=1.07\ (3\text{ s.f.})}$