IB Maths: AA HL

Further Modelling with Functions

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	2. Functions
Topic	2.6 Further Modelling with Functions
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Further Modelling with Functions

Question 1

The fare of a taxi ride starts at $2.50 and increases by $4.15 per km, for the first 5.5 km. After 5.5 km the rate charged per km decreases by $0.50.

(a) Write down a piecewise function that models the fare of the taxi ride and show that the function is continuous when the taxi ride reaches 5.5 km.

[4 marks]

(b) Calculate the distance of a taxi ride that costs $15.

[2 marks]

On a particular ride, a taxi driver accidentally takes a wrong turn 2.8 km into the ride that extends the journey by 2.1 km. Since the driver cannot switch off the fare calculator in the taxi without losing the details of the ride he decides to calculate the fair price for his customer, by removing the cost from the extra distance travelled. The total length of the journey, including the 2.1 km detour, was 9.4 km.

(c) Calculate the proper fare the customer should pay, having appropriately discounted the distance of the detour.

[3 marks]

Question 2

The average temperature, T °C, of a city in winter every day can be modelled by the function

$$T(t) = a\sin(bt + k\pi) + d, \qquad 0 \le t < 24,$$

where t is the time, in hours after midnight, a and d are constants, and b is measured in radians.

(a) Find the value of b. Give your answer in terms of π.

[2 marks]

The average daily temperature range in the city is 6°C. The average maximum daily temperature is 2 °C and the average minimum daily temperature is at 3:00 am.

(b) Find the values of a, k and d.

[3 marks]

(c) Sketch the graph of T and label any intersections with the axes, local maximums and local minimums.

[4 marks]

Question 3

The daily distance covered by an animal depends on the weather, and so varies each month. The following model represents the average daily distance, D km, covered by the animal each month.

$$D(t) = a\cos(bt + k) + d\,, \qquad 0 \le t < 12,$$

Where t is the time, in months after the beginning of the year, a and d are constants, and b is measured in degrees.

(a) Find the value of b.

[2 marks]

The range of D is 22 km, the maximum daily distance covered is 29 km and the minimum daily distance covered occurs when $t = 6.5$.

(b) Find the values of a, k and d.

[3 marks]

(c) Sketch the graph of D and label any intersections with the axes, local maximums and local minimums.

[4 marks]

(d) Given that the animal covers the most distance in summer, state which hemisphere the animal is likely to live in.

[1 mark]

Question 4

The following piecewise function models the depth of a pool, in metres.

$$d(x) = \begin{cases} \frac{1}{2}(x+a)^2 + b, & 0 \le x \le 2 \\ -2, & 2 < x \le 10 \\ \frac{1}{4}x + c, & 10 < x \le d \end{cases}$$

where x represents the horizontal distance in metres from the deep end and a, b and c are constants. The depth at both ends of the pool is 0 m.

(a) Find the values of a, b and c such that f is a continuous function.

[5 marks]

(b) Find the value of d.

[1 mark]

(c) Write down the maximum depth of the pool and the length of the pool.

[1 mark]

Question 5

A basketball streaming site offers three membership options depending on the number of matches the member wants to watch per week. However, if the member does want to watch more matches in a given week they will be charged per match. The membership options are summarized in the table below.

Membership	Matches/Week	Extra charge/Match	Weekly Cost
Casual	7	$0.75	$6.50
Baller	14	$0.50	$8.50
Premium	Unlimited	-	$11.50

(a) The total weekly cost, $\$C$, for the casual and baller memberships can be modelled as a piecewise linear function, where m is the number of matches watched in a given week. Determine the models for each type of membership.

[4 marks]

(b) Find the total weekly cost for each membership if in a given week a member wants to watch 20 games.

[3 marks]

Question 6

The depth, D m, of an underwater sound wave can be modelled by the function

$$D(t) = 18 - 3.4\sin(0.523t)$$

where t is the elapsed time, in seconds, since the first sound wave was detected by the sensor.

(a) Find the minimum and maximum depths of the sound waves as they pass the sensor.

[3 marks]

(b) Find the first time after 12 seconds at which the depth of the wave reaches 18.2 m.

[3 marks]

Question 7

The income tax rates for a country are shown in the table below.

Income, $\$x$	Income tax rate, $y\%$
$0 < x \leq 22\,000$	0
$22\,000 < x \leq 64\,000$	18
$64\,000 < x \leq 130\,000$	22
$x > 130\,000$	29

(a) Calculate the amount of tax payable on the first $65 000 of income.

[2 marks]

(b) Calculate the income of someone who has $11 520 of income tax payable.

[3 marks]

John is paid an annual salary, before tax, of $75 000. He works 10 months of the year and then he decides to take the rest of the year off.

(c) Calculate the amount of tax payable for John.

[3 marks]

Question 8

A Ferris wheel rotates at a constant speed and the height of a particular seat above the ground is modelled by the function

$$H(t) = a\sin(bt - c) + d, \qquad 0 \le t \le 48$$

where H is the height of the seat above the ground, in metres, and t is the elapsed time, in seconds, since the start of the ride.

The ride starts from the lowest point on the Ferris wheel and takes a total of 48 seconds.

(a) Find the value of b.

[2 marks]

The seat reaches a minimum height of 12 m and a maximum height of 42 m.

(b) Find the values of a, c and d.

[3 marks]

Passengers on the Ferris wheel have the best view when their seat is above 25 m.

(c) Calculate the number of seconds for which the passengers have the best view.

[3 marks]

Question 9

A company producing small boats sells 60 boats per month for a sale price of $1200, with each boat costing $700 to produce. Reliable market research suggests that for each increase (or decrease) of the sale price by $50 the company will sell 10 units less (or more).

(a) Given that N is the number of boats the company can sell per month with a sale price of $$x$, show that $N(x) = -\frac{1}{5}x + 300$.

[2 marks]

(b) Given that P is the total monthly profit the company makes from selling the boats for a sale price of $$x$, show that $P(x) = -\frac{1}{5}x^2 + 440x - 210\,000$.

[3 marks]

(c) Find the number of boats the company must produce to maximise monthly profit, given that the maximum monthly production is

(i) 75 boats

(ii) 115 boats.

[3 marks]

(d) Write down two intervals of x for which the company makes a loss and state an economic reason why for each interval.

[1 mark]

Question 10

For a particular type of coffee, a typical mug contains 100 mg of caffeine. The half-life of the amount of caffeine in the bloodstream is 2.5 hours.

Assuming the 100 mg of caffeine from a mug of coffee is absorbed immediately after drinking it, the amount of caffeine, C mg, left in the bloodstream t hours after consumption can be modelled by the equation

$$C = Ae^{-kt}$$

where A and k are positive constants.

a)
Write down the value of A.

[2 marks]

(b)
Find the value of k, giving your answer to 2 significant figures.

[2 marks]

(c)
Find the amount of caffeine in the bloodstream after 6 hours.

[3 marks]

A consumer wishes to cut down their caffeine intake and so makes a drink using half the amount of coffee in a mug.

d)
Find the amount of caffeine in the bloodstream for this consumer after 6 hours and state an assumption you have made in finding your answer.

[3 marks]

Question 11

A rhino is raised in a zoo and his height, h metres, is modelled by the logistic function

$$h(t) = \frac{L}{1 + 1.9e^{-027t}}, \qquad t \geq 0,$$

where t is the number of years since his birth. The rhino's height reaches a limit of 1.82 m as he ages.

(a)
State the value of L.

[2 marks]

(b)
Find the rhino's height on his 12th birthday.

[2 marks]

The rhino's nth birthday is the first birthday in which he is double the value of $h(0)$.

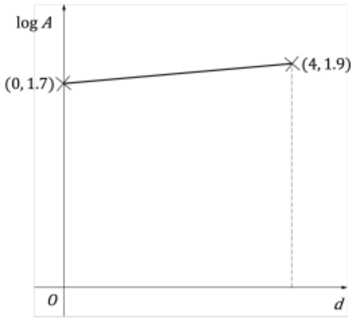
(c)
Find the value of n.

[3 marks]

Question 12

The surface of a large pond is partially covered by algae. A limnologist (a scientist who studies freshwater systems) monitors the area, A m^2, of the pond covered by algae, d days, after first discovering its presence.

The limnologist plots a graph of $\log A$ against d, and after 4 days the graph is a straight line passing through the points (0, 1.7) and (4, 1.9).



The limnologist believes the area of the pond covered by algae can be modelled by the equation $A = A_0 b^d$.

a)
Find the value of A_0, giving your answer to two significant figures, and explain its meaning in the context of the algae covering the lake.

[2 marks]

b)
i)
Find the gradient of the straight line.
ii)
Hence find the value of b correct to 2 significant figures

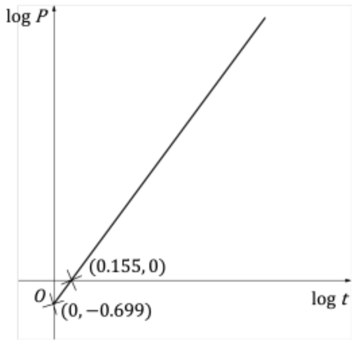
[2 marks]

c)
Using the rounded values for A_0 and b in the model predict the area of the pond covered by algae after 20 days.

[2 marks]

Question 13

In a production process the amount of a pollutant, P ppm (parts per million), in the surrounding air seconds after the process began, is monitored. A chemist produces the graph below of the first 10 seconds of the process.



The graph passes through the points $(0,-0,7)$ and $(1,4,0)$. The chemist suggests that a model of the form $P=at^b$, where a and b are constants, can be used to predict the amount of pollutant in the air.

a)
Find the gradient of the graph.

[2 marks]

b)
Find the equation of the straight line.

[1 mark]

c)
Find the values of a and b.

[3 marks]

The process stops after a maximum running time of 20 seconds.

d)
Find the maximum amount of the pollutant produced during one occurrence of the production process. State your answer to 2 significant figures.

[2 marks]