# Trigonometry: Further Maths

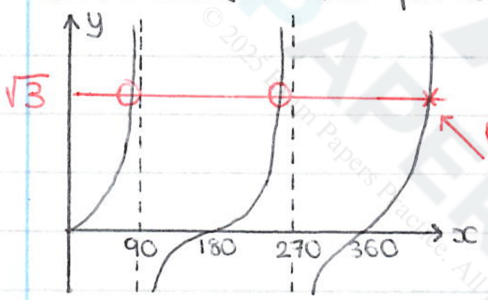
## Solving trigonometry equations

- A calculator only gives one answer for the solution to $\sin(x)$, $\cos(x)$ or $\tan(x)$. We can find more answers in a given interval using a graph.
- To solve a trigonometry equation within a given interval:
  1. Draw a graph of the trigonometric function with the given range as the $x$-axis & the value for the $y$-axis.
  2. Draw a horizontal line through the graph from the point of a solution to the equation.
  3. Where the graph crosses the horizontal line are solutions. Circle the solutions.

  → $\sin(x)$ and $\cos(x)$ have lines of symmetry. This can be used to find the other solutions in relation to a known solution.

  → $\tan(x)$ graphs also have lines of symmetry which can be used. You can also just $\pm 180°$ to get other solutions.

eg 1 "Solve $\tan(x) = \sqrt{3}$ for $0° \leq x \leq 360°$."

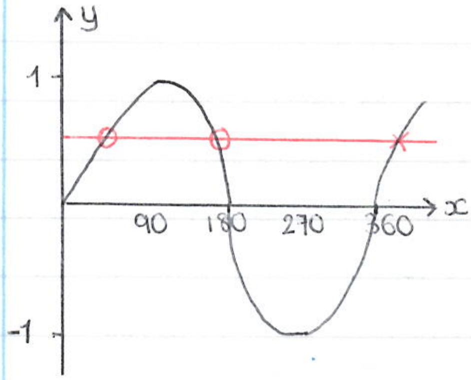



$x = \tan^{-1}(\sqrt{3}) = 60°$

$60° + 180° = 240°$

solutions $= 60°, 240°$

eg 2 "Solve $2\sin(x) = 1$ for $0° \leq x \leq 360°$."




$2\sin(x) = 1$ $(\div 2)$

$\sin(x) = 0.5$

$x = \sin^{-1}(0.5) = 30°$

$180° - 30° = 150°$ (due to symmetry)

solutions $= 30°, 150°$

## Trigonometry Identities

→ $\tan x \equiv \dfrac{\sin x}{\cos x}$

- This identity is normally used in equations with:

  → $\sin$ & $\cos$ where you can divide $\sin$ by $\cos$ (e.g $5\sin x = \cos x$).

  → A $\tan$ together with a $\sin$ or $\cos$ (eg $3\sin x - \tan x = 0$).

e.g. "Solve $5\sin x = \cos x$, for $0° \leq x \leq 360°$."

$5\sin x = \cos x$ $(\div \cos x)$

$\frac{5\sin x}{\cos x} = 1$

(substitute in $\tan x$)

$5\tan x = 1$ $(\div 5)$

$\tan x = 0.2$

$x = \tan^{-1}(0.2) = 11.30...^\circ$

$11.3^\circ + 180^\circ = 191.3^\circ$

$x = 11.3^\circ$ or $191.3^\circ$ (1 dp)

→ $\sin^2 x + \cos^2 x \equiv 1$ $\Rightarrow \sin^2 x = 1 - \cos^2 x$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

- This identity is normally used in equations to get rid of a $\sin^2 x$ or $\cos^2 x$.

eg a) "Show that $2\cos^2\theta \equiv 2 - 2\sin^2\theta$." (Proof)

$LHS = 2\cos^2\theta$

$= 2(1-\sin^2\theta)$

$= 2 - 2\sin^2\theta = RHS$

b) "Hence, solve $2\cos^2\theta - \sin\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$."

$2\cos^2\theta - \sin\theta = 1$

$2 - 2\sin^2\theta - \sin\theta = 1$ $(-1)$

$2 - 2\sin^2\theta - \sin\theta - 1 = 0$

$-2\sin^2\theta - \sin\theta + 1 = 0$ (let $y = \sin\theta$)

$-2y^2 - y + 1 = 0$

$(-2y+1)(y+1) = 0$

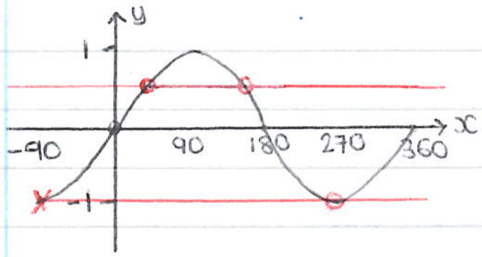
$-2y + 1 = 0$ | $y + 1 = 0$

$-2y = -1$ | $y = -1$

$y = 0.5$ OR $\therefore \sin\theta = -1$

$\therefore \sin\theta = 0.5$ | $\theta = \sin^{-1}(-1) = -90^\circ$

$\theta = \sin^{-1}(0.5) = 30^\circ$




$\theta = 30^\circ, 150^\circ, 270^\circ$

→ Proof using identities examples

eg 1 "Prove $\tan\theta\cos\theta \equiv \sin\theta$."

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $(\times \cos\theta)$

$\tan\theta\cos\theta = \sin\theta$

eg 2 "Show that $2 - 2\cos^2 x \equiv 2\sin^2 x$."

$\sin^2 x + \cos^2 x = 1$ $(\times 2)$

$2\sin^2 x + 2\cos^2 x = 2$ $(-2\cos^2 x)$

$2\sin^2 x = 2 - 2\cos^2 x$

eg 3 "Prove $\sin^2 x - \cos^2 x \equiv 1 - 2\cos^2 x$."

$\sin^2 x - \cos^2 x = (1 - \cos^2 x) - \cos^2 x$ (substitute in $\sin^2 x = 1 - \cos^2 x$)

$= 1 - 2\cos^2 x$

eg 4 "Prove that $\sin\theta - \sin\theta\cos^2\theta \equiv \sin^3\theta$."

$LHS = \sin\theta - \sin\theta\cos^2\theta$

$= \sin\theta - \sin\theta(1 - \sin^2\theta)$

$= \sin\theta - \sin\theta + \sin^3\theta$

$= \sin^3\theta = RHS$

eg 5 "Given that $4\sin x + \cos x = 0$, show that $\tan x = \frac{-1}{4}$."

$4\sin x + \cos x = 0$

$4\sin x = -\cos x$

$4(\cos x)(\tan x) = -\cos x$ $(\div 4)$

$\cos x \tan x = \dfrac{-\cos x}{4}$ $(\div \cos x)$

$\tan x = \dfrac{-1}{4}$

eg6 "Show that $5\sin^2 x + 5\sin x + 4\cos^2 x \equiv \sin^2 x + 5\sin x + 4$."

$LHS = 5\sin^2 x + 5\sin x + 4\cos^2 x$

$= 5\sin^2 x + 5\sin x + 4(1 - \sin^2 x)$

$= 5\sin^2 x + 5\sin x + 4 - 4\sin^2 x$

$= \sin^2 x + 5\sin x + 4 = RHS$