

IB Maths: AA HL

Further Limits (inc l'Hôpital's Rule)

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	5. Calculus
Topic	5.12 Further Limits (inc l'Hôpital's Rule)
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Further Limits (inc l'Hôpital's Rule)

Question 1

For each of the following limits,

(i) determine whether or not l'Hôpital's rule may be used to evaluate the limit, giving a reason for your answer; and

(ii) if l'Hôpital's rule may be used, then use the rule to evaluate the limit.

a)

$$\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + 2x}$$

[4 marks]

b)

$$\lim_{x \rightarrow 0} \frac{\cos x}{x^2 + 2x}$$

[2 marks]

c)

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sec x}{1 - \tan x}$$

[5 marks]

Question 2

Consider the following limit:

$$\lim_{x \rightarrow 0} \frac{-1 + \cos 2x}{x^2}$$

a)

Explain why it is appropriate to use l'Hôpital's rule to attempt to evaluate this limit.

[2 marks]

b)

Show that employing l'Hôpital's rule once leads to an indeterminate form when you attempt to evaluate the limit.

[2 marks]

c)

By employing l'Hôpital's rule a second time, show that the limit exists and find its value.

[2 marks]

Question 3

Consider the function f defined by

$$f(x) = \frac{7-3x}{12x+5}$$

a)

Use l'Hôpital's rule to evaluate $\lim_{x \rightarrow \infty} f(x)$.

[3 marks]

b)

Hence write down the equation(s) of any horizontal asymptotes on the graph of $y = f(x)$, giving a reason for your answer.

[2 marks]

c)

(i)

Show that $f(x)$ may be rewritten in the form

$$f(x) = \frac{\frac{7}{x} - 3}{12 + \frac{5}{x}}$$

(ii)

Hence show that $\lim_{x \rightarrow \infty} f(x)$ may also be evaluated without the use of l'Hôpital's rule.

[4 marks]

Question 4

a)

By substituting $-x$ into the Maclaurin series for e^x , determine the Maclaurin series for e^{-x} .

[2 marks]

Consider the limit

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{2x}$$

b)

Use Maclaurin series to evaluate the limit.

[5 marks]

c)

(i)

Show that it would also be appropriate to use l'Hôpital's rule to attempt to evaluate the limit.

(ii)

Evaluate the limit using l'Hôpital's rule, and confirm that this matches your answer in part (b).

[4 marks]

Question 5

a)

Find the Maclaurin series for $\cos 2x$.

[3 marks]

b)

Hence evaluate the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$$

[4 marks]

Question 6

Use an appropriate method to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$$

[5 marks]