IB Maths: AA HL

Further Integration

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	5. Calculus
Topic	5.4 Further Integration
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Further Integration

Question 1

(a) Find the indefinite integral for

$$\int \sin x \ \mathrm{d}x$$

[1 mark]

(b) Find the exact value for

$$\int_1^4 \frac{1}{x} \, \mathrm{d}x$$

[3 marks]

(c) Find the indefinite integral for

$$\int 7e^{7x} \, \mathrm{d}x$$

[2 marks]

Question 2

(a) Integrate

$$\int \cos 2x \ \mathrm{d}x$$

[2 marks]

(b) Find the definite integral

$$\int_0^2 (3x-1)^3 \ \mathrm{d}x$$

[4 marks]

(c) Find an expression for y given that

$$\frac{dy}{dx} = e^{5x}$$

[2 marks]

Question 3

Using a suitable substitution, show that

$$\int_1^2 \frac{x}{x+4}\,dx = 1 + 4\ln\frac{5}{6}$$

[7 marks]

Question 4

Given that $\cos 2\theta \equiv 2\cos^2\theta - 1$, find the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta\,d\theta$$

[6 marks]

Question 5

(a) Given that $f(x) = 2x^3 + 4x$, find $f'(x)$.

[2 marks]

(b) Hence, or otherwise, find

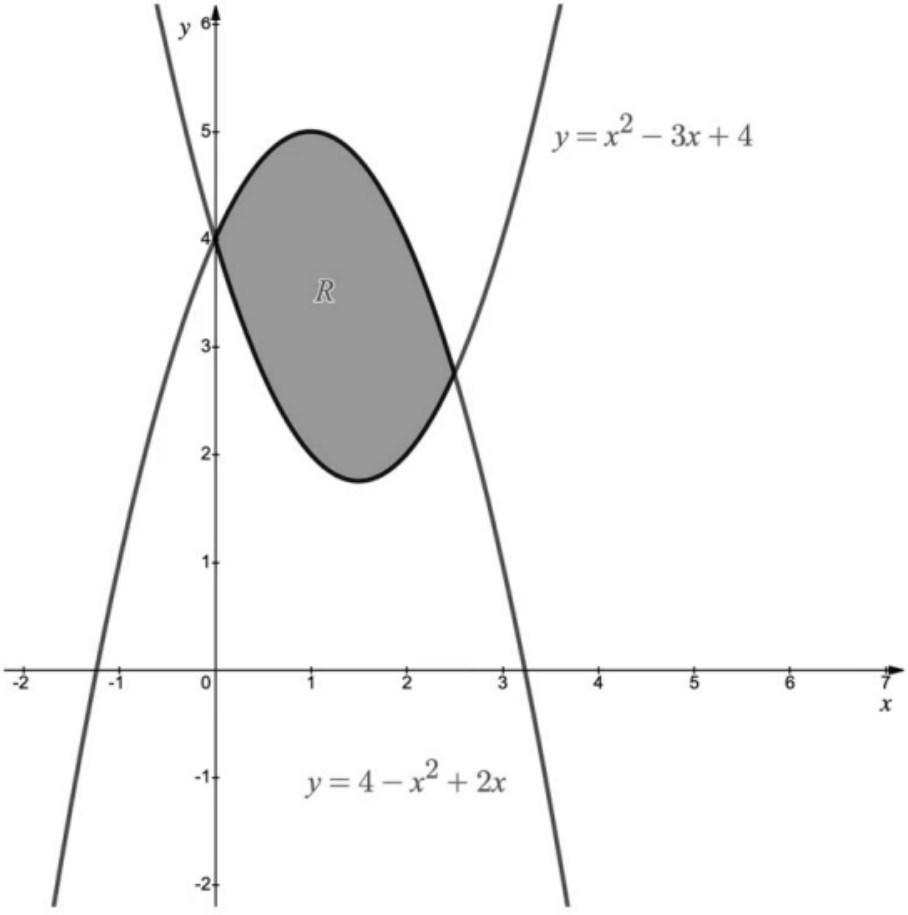
$$\int \frac{3x^2 + 2}{2x^3 + 4x} \, dx$$

[4 marks]

Question 6

The diagram below shows a sketch of the curves with equations

$$y = x^2 - 3x + 4 \quad \text{and} \quad y = 4 - x^2 + 2x$$



(a) Find the x-coordinates of the intersections of the two graphs.

[2 marks]

(b) Show that the area of the shaded region labelled R is given by

$$\int_0^{\frac{5}{2}} (5x - 2x^2)\, dx$$

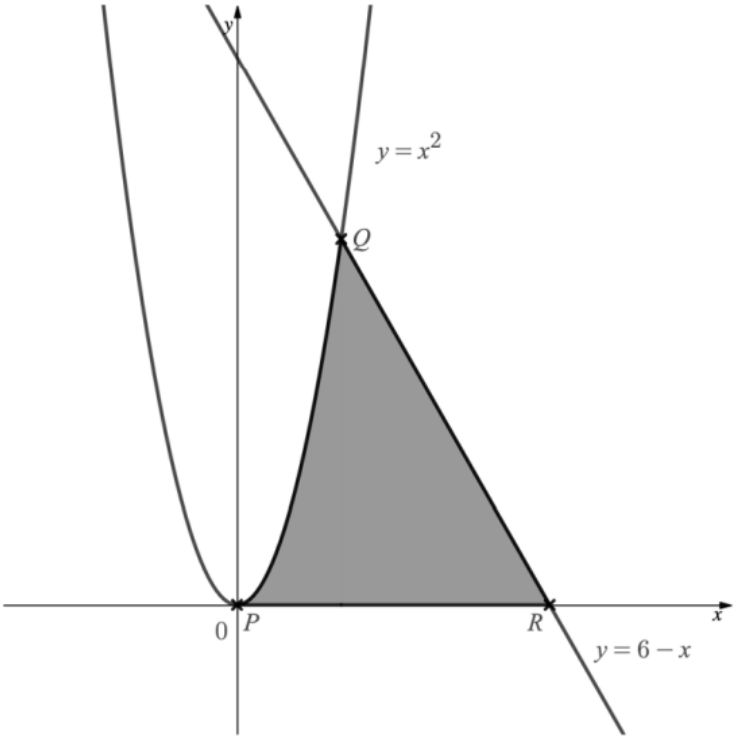
[2 marks]

(c) Find the area of the shaded region labelled R.

[2 marks]

Question 7

The diagram below shows the graphs of the line $y = 6 - x$ and the curve $y = x^2$.



Point P is the point of intersection of the curve $y = x^2$ with the x-axis. Point Q is the point of intersection of the curve $y = x^2$ with the line $y = 6 - x$ for which $x > 0$. Point R is the point of intersection of the line $y = 6 - x$ with the x-axis.

(a) Work out the x-coordinates of points P, Q and R.

[3 marks]

(b) Work out the area of the shaded region.

[4 marks]

Question 8

Consider the function $h(x)$ such that

$$\int_1^5 h(x)\,\mathrm{d}x = 2.$$

(a) Find

$$\int_5^1 h(x)\,\mathrm{d}x$$

[2 marks]

(b) Find

$$\int_1^5 \frac{h(x)+1}{2}\,\mathrm{d}x$$

[3 marks]

(c) Find

$$\int_1^5 (h(x)+2x)\,\mathrm{d}x$$

[3 marks]

Question 9

Consider the function $f(x) = \ln(2x^2 + 1)$.

(a) Find $f'(x)$.

[3 marks]

(b) Hence, find

$$\int \frac{x}{2x^2 + 1}\, dx$$

[3 marks]

Question 10

Let $f'(x) = x^2 \cos(x^3 + 1)$.

Find $f(x)$ given that $f(-1) = 1$.

[5 marks]