

# IB Maths: AI HL

## Further Integration

### Topic Questions

**These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Topic Questions**

Course	IB Maths
Section	5. Calculus
Topic	5.4 Further Integration
Difficulty	Medium

**Level: IB Maths**

**Subject: IB Maths AI HL**

**Board: IB Maths**

**Topic: Further Integration**

### Question 1

(a) Find the indefinite integral for

$$\int \sin x \, dx$$

[1 mark]

(b) Show that the exact value of the definite integral

$$\int_1^4 \frac{1}{x} \, dx$$

is  $2 \ln 2$ .

[3 marks]

(c) Find the indefinite integral for

$$\int 7e^{7x} \, dx$$

[2 marks]

## Question 2

(a) Integrate

$$\int \cos 2x \, dx$$

[2 marks]

(b) Show that

$$\int (3x - 1)^3 \, dx = \frac{1}{12} (3x - 1)^4 + c$$

where  $c$  is a constant of integration.

[3 marks]

(c) Find an expression for  $y$  given that

$$\frac{dy}{dx} = e^{5x}$$

and that  $y = 1$  when  $x = 0$ .

[3 marks]

### Question 3

(a) Find the indefinite integral for

$$\int \left( \sqrt{x} + \frac{3}{\sqrt{x}} \right) dx$$

[3 marks]

(b) Find the indefinite integral for

$$\int \frac{x^{\frac{2}{3}} + x^{\frac{11}{6}}}{x^2} dx$$

[3 marks]

### Question 4

(a) Given that  $f(x) = 2x^3 + 4x$ , find  $f'(x)$ .

[2 marks]

(b) Hence, or otherwise, find

$$\int \frac{3x^2 + 2}{2x^3 + 4x} dx$$

[4 marks]

### Question 5

Consider the function  $f(x) = \ln(2x^2 + 1)$ .

(a) Find  $f'(x)$ .

[3 marks]

(b) Hence, find

$$\int \frac{x}{2x^2 + 1} dx$$

[3 marks]

### Question 6

Let  $f'(x) = x^2 \cos(x^3 + 1)$ .

Find  $f(x)$  given that  $f(-1) = 1$ .

[5 marks]

### Question 7

(a) Show that

$$\frac{\tan x}{\sin x \cos x} = \frac{1}{\cos^2 x}$$

[2 marks]

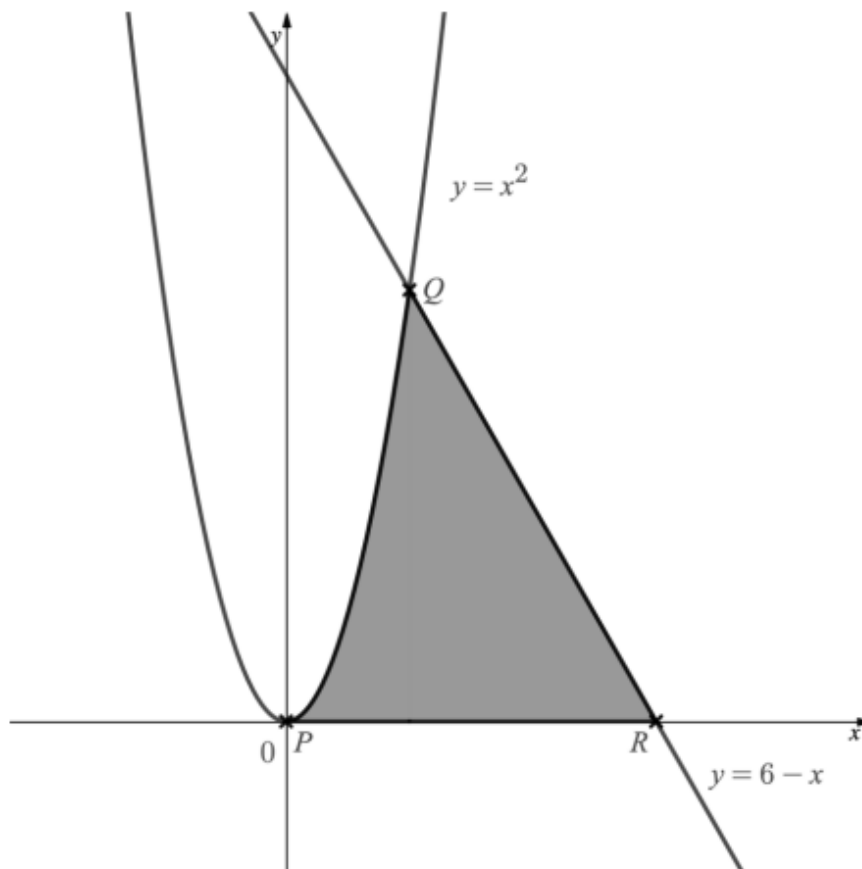
(b) Hence find

$$\int \frac{3 \tan x}{5 \sin x \cos x} dx$$

[3 marks]

### Question 8

The diagram below shows the graphs of the line  $y = 6 - x$  and the curve  $y = x^2$ .



Point  $P$  is the point of intersection of the curve  $y = x^2$  with the  $x$ -axis. Point  $Q$  is the point of intersection of the curve  $y = x^2$  with the line  $y = 6 - x$  for which  $x > 0$ . Point  $R$  is the point of intersection of the line  $y = 6 - x$  with the  $x$ -axis.

(a) Write down the  $x$ -coordinates of points  $P$ ,  $Q$  and  $R$ .

[3 marks]

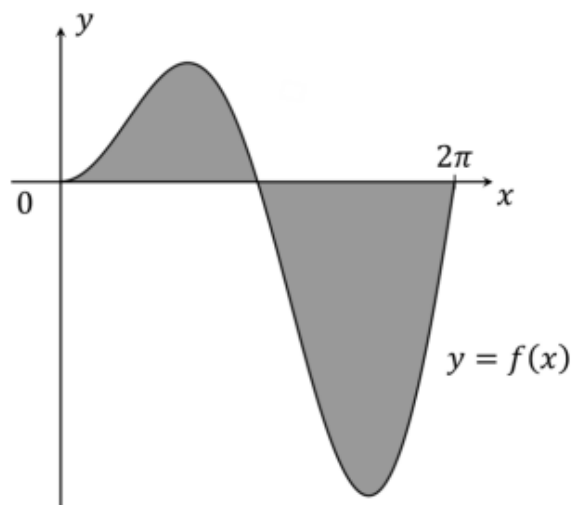
(b) Calculate the area of the shaded region.

[2 marks]

### Question 9

The diagram below shows the graph of the function  $f$  which is defined by

$$f(x) = x \sin x, \quad 0 \leq x \leq 2\pi$$



The shaded region in the diagram is the region enclosed by the  $x$ -axis and the graph of  $y = f(x)$ .

(a) Find the area of the part of the shaded region that lies above the  $x$ -axis.

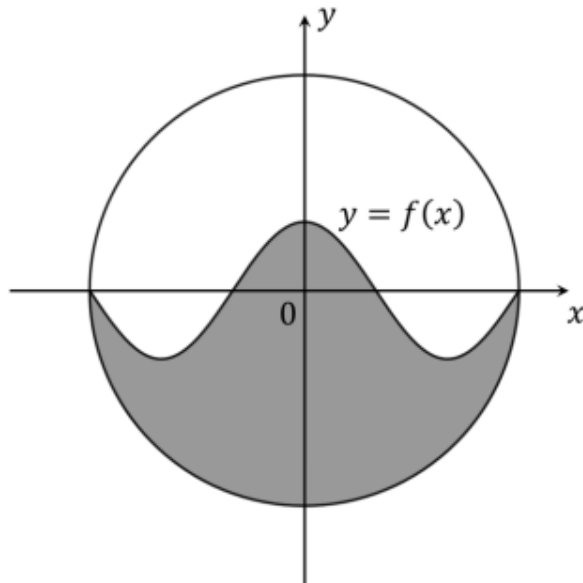
[4 marks]

(b) Find the area of the entire shaded region.

[3 marks]

### Question 10

The diagram below depicts the design for a new company logo. The logo is formed by a circle centred on the origin, which is divided into two regions by the curve  $y = f(x)$  where  $f$  is the function defined by  $f(x) = \cos \frac{3x}{2}$ ,  $-\pi \leq x \leq \pi$ . The points where the circle and the curve intersect lie on the  $x$ -axis, as shown.



The shaded region in the diagram is the region inside that circle that lies below the curve  $y = f(x)$ .

- (a) (i) Write down the radius of the circle that forms the outer border of the logo.  
 (ii) Hence determine the exact area of the shaded region.

[6 marks]

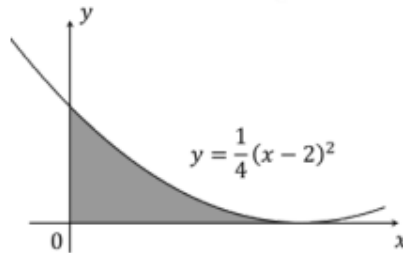
- (b) Find the percentage of the circular logo that is shaded.

[2 marks]



## Question 11

The following diagram shows a part of the graph of the curve  $y = \frac{1}{4}(x-2)^2$ . The shaded region is the region enclosed by the graph and the positive  $x$ - and  $y$ -axes.



(a)

(i)

Find the coordinates of the points where the graph intersects the coordinate axes.

(ii)

For the part of the curve that forms the boundary of the shaded region, show that  $x = 2 - 2\sqrt{y}$ .

[3 marks]

(b)

Find the area of the shaded region

(i)

by calculating it as an area between the curve and the  $x$ -axis.

(ii)

by calculating it as an area between the curve and the  $y$ -axis.

[6 marks]

(c)

Find the volume of the solid formed when the shaded region is rotated  $2\pi$  radians about the  $x$ -axis.

[5 marks]

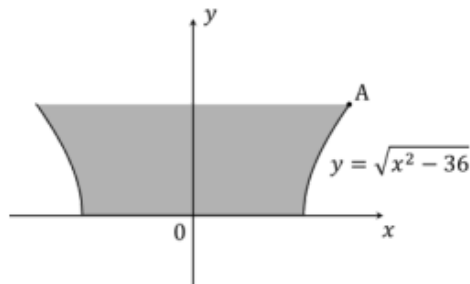
(d)

Find the volume of the solid formed when the shaded region is rotated  $2\pi$  radians about the  $y$ -axis.

[5 marks]

## Question 12

The diagram below shows the cross-section of a bowl that a company is planning to begin producing.



As indicated on the diagram, one of the sides of the bowl in the cross-section may be described by the curve  $y = \sqrt{x^2 - 36}$ , where units for  $x$  and  $y$  are centimetres. The cross-section is entirely symmetrical about the  $y$ -axis. The flat circular bottom of the bowl has a diameter of 12 cm, and the vertical depth of the bowl is 6 cm. For purposes of answering this question, the thickness of the bottom and sides of the bowl may be regarded as negligible.

(a)

Find the exact coordinates of the point marked **A** on the diagram.

[3 marks]

(b)

Show that the capacity of the bowl in  $\text{cm}^3$  is given by

$$\pi \int_0^b (y^2 + 36) dy$$

where  $b$  is a constant to be determined.

[4 marks]

(c)

Hence find the capacity of the bowl.

[2 marks]

