IB Maths: AA HL

Further Functions & Graphs

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	2. Functions
Topic	2.9 Further Functions & Graphs
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Further Functions & Graphs

Question 1

a)
Sketch the graph of $y=(x-1)^2-2|x-1|-1$, for $-3\leq x\leq 6$.

[3 marks]

b)
Hence, solve the equation $y=(x-1)^2-2|x-1|-1=0$.

[2 marks]

Question 2

Given that

$$f(x)=\ln x, \qquad x>0$$

sketch on separate axes the graphs of

(i)
$y=f(x)$

(ii)
$y=|f(x)|$

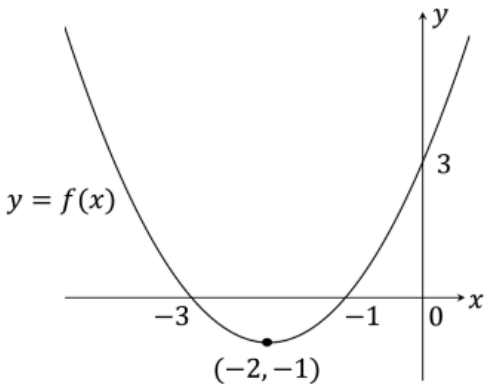
(iii)
$y=-f(x-3)$

On each diagram, show the x-intercepts along with any asymptotes, including their equations.

[7 marks]

Question 3

The graph of $y = f(x)$ is given below.



On separate axes, draw the graphs of

a)
$|f(x)|$

[3 marks]

b)
$[f(x)]^2$

[3 marks]

Question 4

a)

Sketch the curve $y = \frac{3}{x+4}$ and line $y = 4 - x$ on the same axes, clearly indicating any x- and y- intercepts and any asymptotes.

[3 marks]

b)
Consider the equation

$$4 - x = \left| \frac{3}{x+4} \right|$$

(i)
Explain why the cases $x < -4$, $x = -4$ and $x > -4$ must be considered separately in attempting to solve the equation.

(ii)
Hence find the exact solutions to the equation.

[5 marks]

Question 5

Consider the function f defined by $f(x) = 3x^2 \arcsin x$, $-1 \leq x \leq 1$.

a)
Sketch the graph of $y = f(x)$.

[3 marks]

b)
State the range of f.

[2 marks]

c)
Solve the inequality $|3x^2 \arcsin x| > 1$.

[3 marks]

Question 6

Consider the function f defined by $f(x)=\sqrt{9-x}$, where f has the largest possible valid domain.

a)
(i)
Sketch the graph of $y=f(x)$, labelling the x- and y-intercepts.

(ii)
State the domain and range of f.

[4 marks]

b)
(i)
On the same set of axes, sketch the graph of the function $f(|x|)$, labelling the x- and y-intercepts.

(ii)
State the domain and range of the function $f(|x|)$.

[4 marks]

Question 7

Let $f(x)=\frac{7-9x}{cx-12}$, $x\neq\frac{12}{c}$, where c is a non-zero constant.

The line $x=4$ is a vertical asymptote to the graph of $y=f(x)$.

a)
(i)
Find the value of c.

(ii)
State the equation of the horizontal asymptote to the graph of $y=f(x)$.

[4 marks]

b)
The line $y=k$, where $k\in\mathbb{R}$, intersects the graph of $y=|f(x)|$ at exactly one point. Find the possible values of k.

[3 marks]

Question 8

Let $f(x) = 2x^3 - 2x$, for $x \in \mathbb{R}$.

(a)
(i)
Sketch the graph of $y = |f(x)|$.

(ii)
State the transformation of the graph $y = f(x)$ to $y = |f(x)|$ for $f(x) < 0$.

[3 marks]

(b)
(i)
Sketch the graph of $y = f(|x|)$.

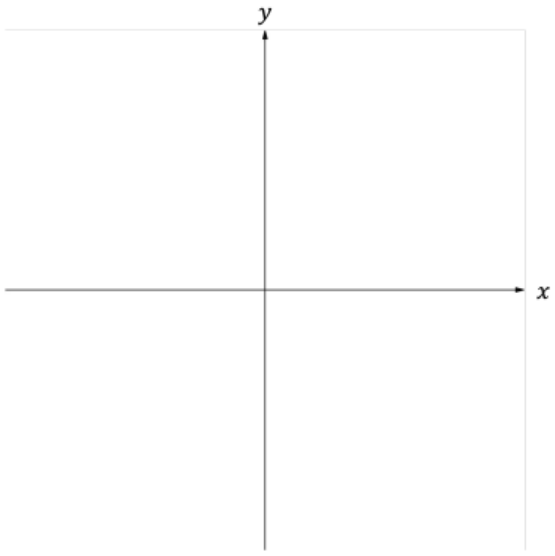
(ii) State the transformation of the graph $y = f(x)$ to $y = f(|x|)$ for $x < 0$.

[3 marks]

Question 9

Let $f(x) = x(x-2)$.

(a)
Sketch the graph of $y = f(x)$on the coordinate axes below. Be sure to label anywhere the graph intersects the coordinate axes and any extrema.



[3 marks]

(b)

On the same axes, sketch the graph of the reciprocal $y = \frac{1}{f(x)}$. Be sure to label anywhere the graph intersects the coordinate axes and any extrema.

[3 marks]

(c)

Find the equation of the horizontal and vertical asymptotes of the graph of $y = f(x)$.

[2 marks]