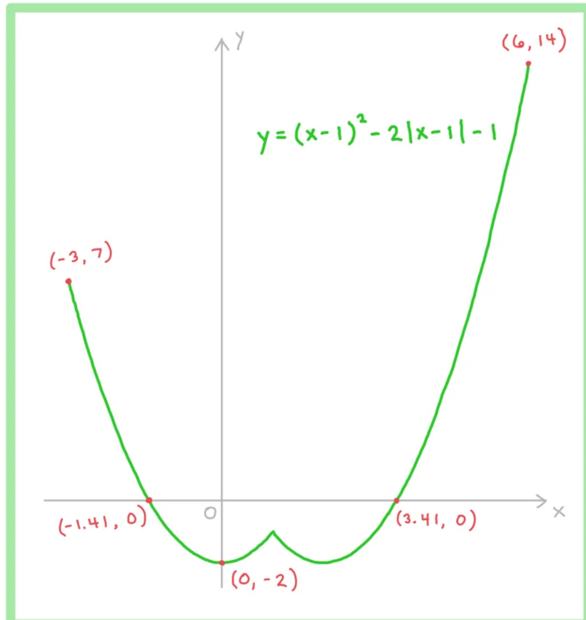
Question 1

(a) Sketch the graph of $y = (x-1)^2 - 2|x-1| - 1$, for $-3 \le x \le 6$. [3]

(b) Hence, solve the equation $y = (x-1)^2 - 2|x-1| - 1 = 0$. [2]

a) Use your GDC to help you sketch the function:



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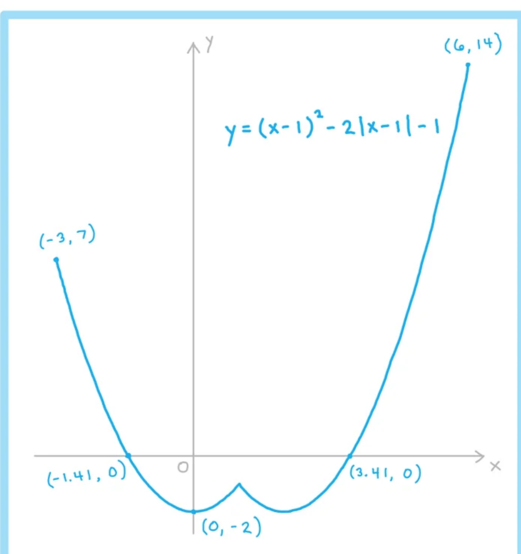


b) The solutions are the x-coordinates of the x-axis intercepts.

$$\boxed{x = -1.41 \text{ or } 3.41 \text{ (3 s.f.)}}$$

Question 2

Given that

$$f(x) = \ln x, \qquad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$

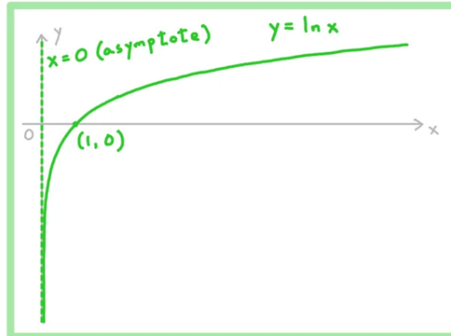
(ii) $y = |f(x)|$

(iii) $y = -f(x-3)$.

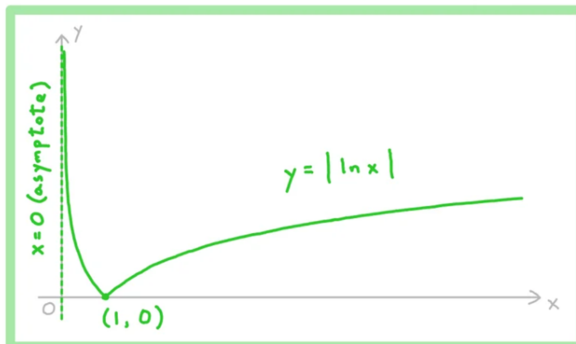
On each diagram, show the x-intercepts along with any asymptotes, including their equations.

[7]

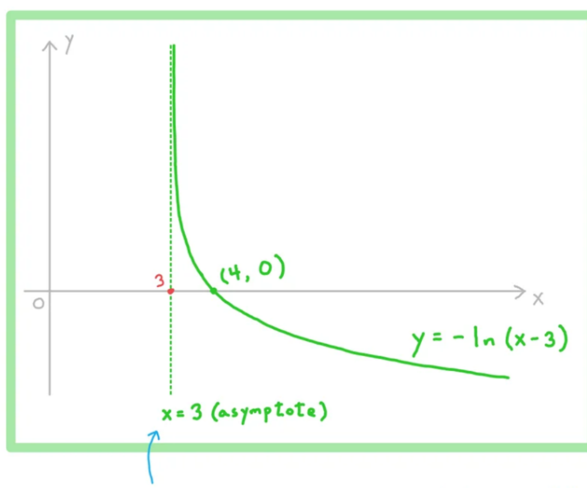
(i) You should be familiar with the graph of ln x !



(ii) Where f(x) < 0, reflect in the x-axis :



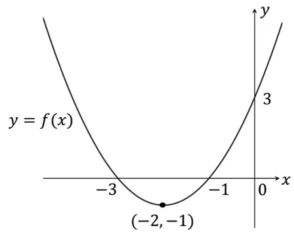
(iii) Translate f(x) 3 units to the right, then reflect in the x-axis :



Don't forget to translate asymptote as well!

Question 3

The graph of $y = f(x)$ is given below.

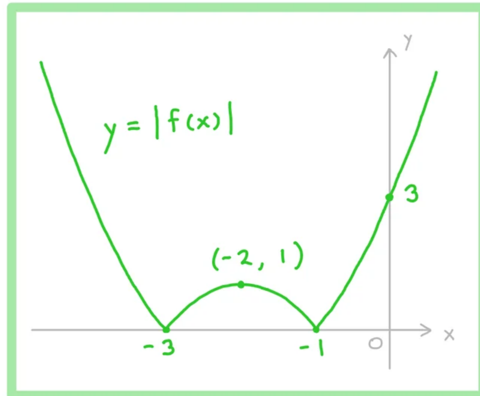


On separate axes, draw the graphs of

(a) $|f(x)|$ [3]

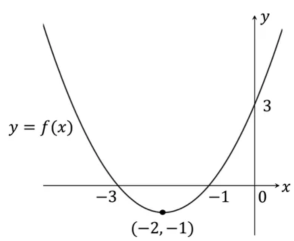
(b) $[f(x)]^2$. [3]

a) Where $f(x) < 0$, reflect in the x-axis:



Note the sharp 'points' where the curve meets the x-axis — compare part (b).

The graph of $y = f(x)$ is given below.

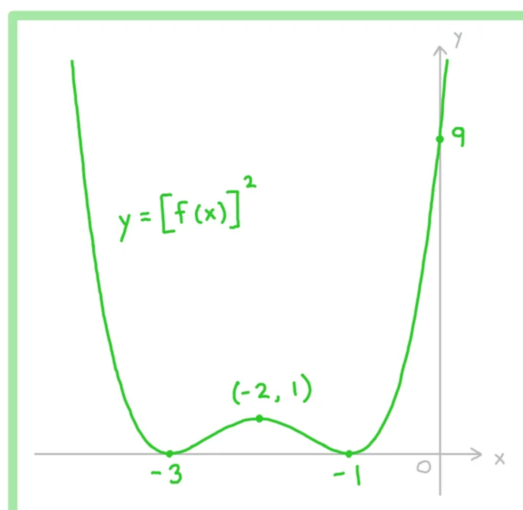


On separate axes, draw the graphs of

(a) $|f(x)|$ [3]

(b) $[f(x)]^2$. [3]

b) x-coordinates stay the same, y-coordinates are all squared (this means none of the new y-coordinates will be negative):



Note that curve is smooth and rounded where it meets the x-axis — compare part (a).

Question 4

(a) Sketch the curve $y = \frac{3}{x+4}$ and line $y = 4 - x$ on the same axes, clearly indicating any x- and y- intercepts and any asymptotes.

[3]

(b) Consider the equation

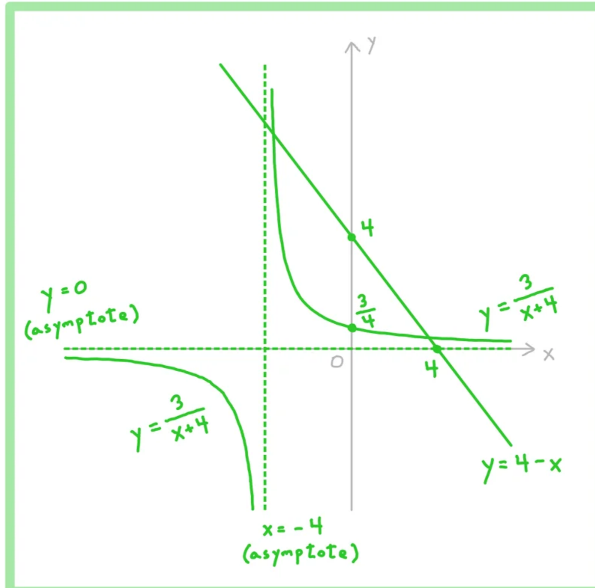
$$4 - x = \left|\frac{3}{x+4}\right|$$

(i) Explain why the cases $x < -4$, $x = -4$ and $x > -4$ must be considered separately in attempting to solve the equation.

(ii) Hence find the exact solutions to the equation.

[5]

a)



(a) Sketch the curve $y = \frac{3}{x+4}$ and line $y = 4 - x$ on the same axes, clearly indicating any x- and y- intercepts and any asymptotes.

[3]

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$$4 - x = \left|\frac{3}{x+4}\right|$$

(i) Explain why the cases $x < -4$, $x = -4$ and $x > -4$ must be considered separately in attempting to solve the equation.

(ii) Hence find the exact solutions to the equation.

[5]

b) (i)

When $x = -4$, $\frac{3}{x+4}$ is undefined so there can be no solutions.

When $x > -4$, $\frac{3}{x+4} > 0$ and so $\left|\frac{3}{x+4}\right| = \frac{3}{x+4}$

When $x < -4$, $\frac{3}{x+4} < 0$ and so $\left|\frac{3}{x+4}\right| = -\frac{3}{x+4}$

(ii) When $x > -4$

$4 - x = \left|\frac{3}{x+4}\right| \Rightarrow 4 - x = \frac{3}{x+4}$

$\Rightarrow (4-x)(x+4) = 3 \Rightarrow 16 - x^2 = 3$

$\Rightarrow x^2 = 13 \Rightarrow x = \pm\sqrt{13}$

Note that both these solutions are greater than -4.

When $x < -4$

$4 - x = \left|\frac{3}{x+4}\right| \Rightarrow 4 - x = -\frac{3}{x+4}$

$\Rightarrow (4-x)(x+4) = -3 \Rightarrow 16 - x^2 = -3$

$\Rightarrow x^2 = 19 \Rightarrow x = \pm\sqrt{19}$

But $x < -4$ must also be true, so $x = -\sqrt{19}$

$$\boxed{x = -\sqrt{19},\ -\sqrt{13},\ \sqrt{13}}$$

Question 5

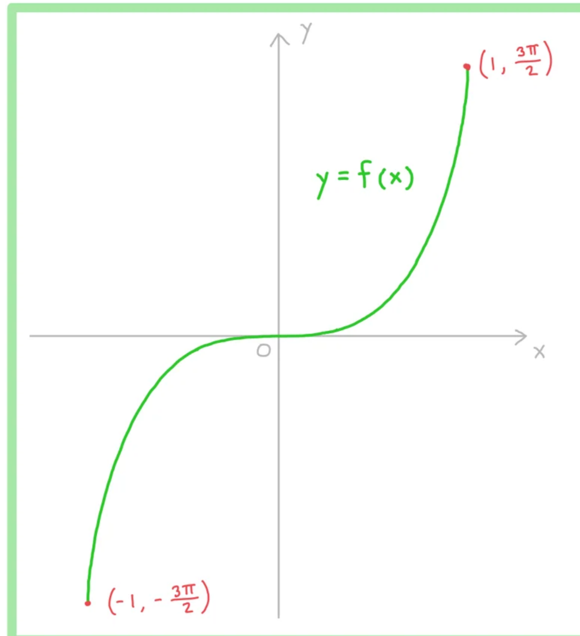
Consider the function f defined by $f(x) = 3x^2 \arcsin x, \ -1 \le x \le 1$.

(a) Sketch the graph of $y = f(x)$. [3]

(b) State the range of f. [2]

(c) Solve the inequality $|3x^2 \arcsin x| > 1$. [3]

a) Use your GDC to help you sketch the function:



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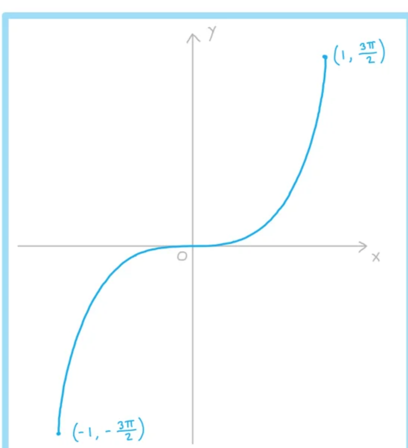


b) You can work this out:

$f(-1) = 3(-1)^2 \arcsin(-1) = 3\left(-\frac{\pi}{2}\right) = -\frac{3\pi}{2}$

$f(1) = 3(1)^2 \arcsin(1) = 3\left(\frac{\pi}{2}\right) = \frac{3\pi}{2}$

Or else get the values from the graph on your GDC.

$$\boxed{-\frac{3\pi}{2} \le f(x) \le \frac{3\pi}{2}}$$

Consider the function f defined by $f(x) = 3x^2 \arcsin x,\ -1 \le x \le 1$.

(a) Sketch the graph of $y = f(x)$.

[3]

(b) State the range of f.

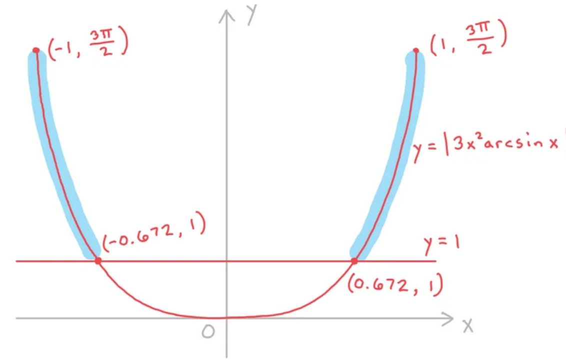
[2]

(c) Solve the inequality $|3x^2 \arcsin x| > 1$.

[3]

c) Graph $y = |3x^2 \arcsin x|$ and $y = 1$ on your GDC.

Then find x-coordinates of the points of intersection and identify the regions of the graph that satisfy the inequality.



$\boxed{-1 \le x < -0.672 \ (3\text{ s.f.}) \quad \text{or} \quad 0.672 < x \le 1 \ (3\text{ s.f.})}$

Question 6

Consider the function f defined by $f(x) = \sqrt{9-x}$, where f has the largest possible valid domain.

(a) (i) Sketch the graph of $y = f(x)$, labelling the x- and y-intercepts.

(ii) State the domain and range of f.

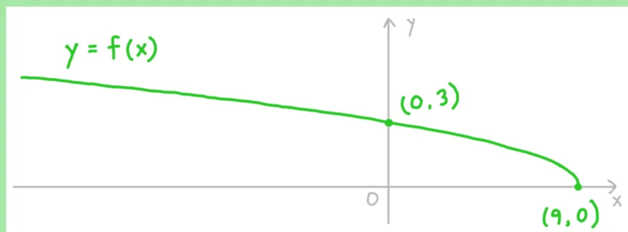
[4]

(b) (i) On the same set of axes, sketch the graph of the function $f(|x|)$, labelling the x- and y-intercepts.

(ii) State the domain and range of the function $f(|x|)$.

[4]

a) (i) Use your GDC to help you sketch the function:



(ii) Domain: $x \le 9$

Range: $f(x) \ge 0$

Consider the function f defined by $f(x) = \sqrt{9 - x}$, where f has the largest possible valid domain.

(a) (i) Sketch the graph of $y = f(x)$, labelling the x- and y-intercepts.

(ii) State the domain and range of f.

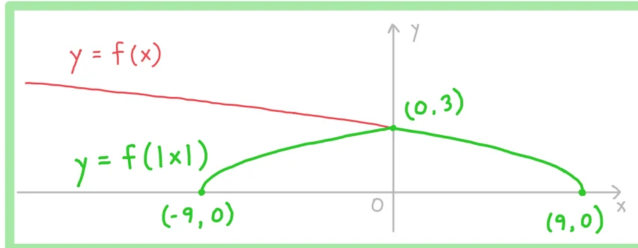
[4]

(b) (i) On the same set of axes, sketch the graph of the function $f(|x|)$, labelling the x- and y-intercepts.

(ii) State the domain and range of the function $f(|x|)$.

[4]

b) (i) Use your GDC to help you sketch the function:



Note how the part of f(x) where $x \geq 0$ has been reflected in the y-axis.

(ii) Domain: $-9 \leq x \leq 9$

Range: $0 \leq f(x) \leq 3$

Question 7

Let $f(x) = \frac{7-9x}{cx-12}$, $x \neq \frac{12}{c}$, where c is a non-zero constant.

The line $x = 4$ is a vertical asymptote to the graph of $y = f(x)$.

(a) (i) Find the value of c.

(ii) State the equation of the horizontal asymptote to the graph of $y = f(x)$.

[4]

(b) The line $y = k$, where $k \in \mathbb{R}$, intersects the graph of $y = |f(x)|$ at exactly one point. Find the possible values of k.

[3]

a) (i) f(x) will have a vertical asymptote for any x values that make its denominator zero.

$x = 4$ is a vertical asymptote, therefore:

$c(4) - 12 = 0 \Rightarrow 4c = 12 \Rightarrow \boxed{c = 3}$

(ii) Take limit as $x \to \pm\infty$ to identify horizontal asymptotes:

$$\lim_{x \to \pm\infty} \left(\frac{7-9x}{3x-12} \right) = \lim_{x \to \pm\infty} \left(\frac{\frac{7}{x} - 9}{3 - \frac{12}{x}} \right) = \frac{0-9}{3-0} = -3$$

$\boxed{y = -3 \text{ is a horizontal asymptote}}$

Let $f(x) = \frac{7-9x}{cx-12}$, $x \neq \frac{12}{c}$, where c is a non-zero constant.

The line $x = 4$ is a vertical asymptote to the graph of $y = f(x)$.

(a) (i) Find the value of c.

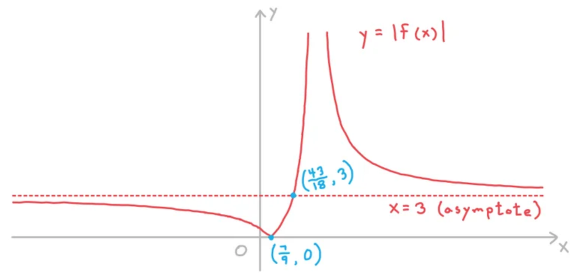
(ii) State the equation of the horizontal asymptote to the graph of $y = f(x)$.

[4]

(b) The line $y = k$, where $k \in \mathbb{R}$, intersects the graph of $y = |f(x)|$ at exactly one point. Find the possible values of k.

[3]

b) Graph the function on your GDC to help you visualise the situation:



If $k = 0$ the line intersects at $(\frac{7}{9}, 0)$ only.

If $k = 3$ the line intersects at $(\frac{43}{18}, 3)$ only.

If $k < 0$ the line doesn't intersect at all.

If $0 < k < 3$ or $k > 3$ the intersects in two places.

$\boxed{k = 0 \text{ or } k = 3}$

Question 8

Let $f(x) = 2x^3 - 2x$, for $x \in \mathbb{R}$.

(a) (i) Sketch the graph of $y = |f(x)|$.

(ii) State the transformation of the graph $y = f(x)$ to $y = |f(x)|$ for $f(x) < 0$.

[3]

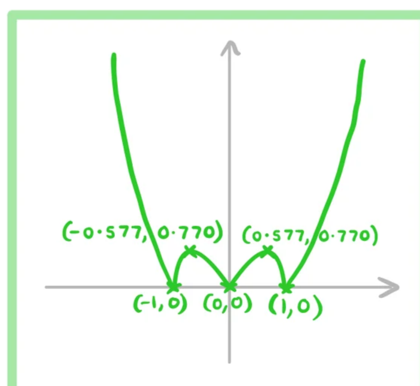
(b) (i) Sketch the graph of $y = f(|x|)$.

(ii) State the transformation of the graph $y = f(x)$ to $y = f(|x|)$ for $x < 0$.

[3]

(a)

(i)



(ii) $\boxed{\text{Reflection in the } x\text{-axis}}$

EXAM PAPERS PRACTICE

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[3]

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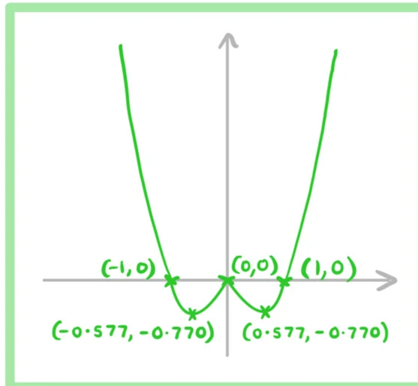
(ii) State the transformation of the graph $y = f(x)$ to $y = f(|x|)$ for $x < 0$.

[3]

(b)

(i)



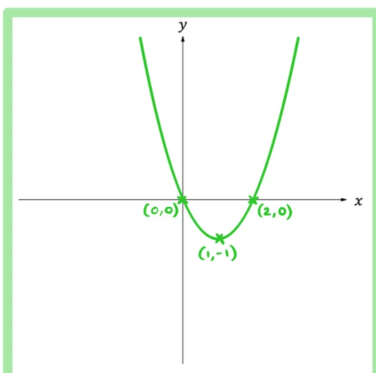
(ii) Reflection in the x-axis

Question 9

Let $f(x) = x(x-2)$.

(a) Sketch the graph of $y = f(x)$ on the coordinate axes below. Be sure to label anywhere the graph intersects the coordinate axes and any local maxima or minima.

(a)



[3]

(b) On the same axes, sketch the graph of the reciprocal $y = \frac{1}{f(x)}$. Be sure to label anywhere the graph intersects the coordinate axes and any local maxima or minima.

[3]

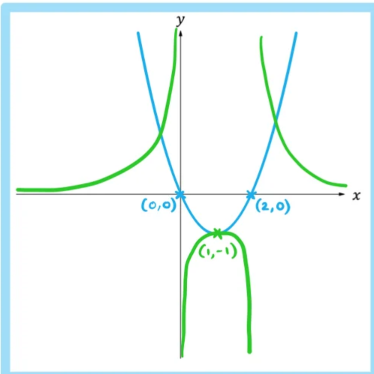
(c) Find the equation of the horizontal and vertical asymptotes of the graph of $y = \frac{1}{f(x)}$.

[2]

Let $f(x) = x(x-2)$.

(a) Sketch the graph of $y = f(x)$ on the coordinate axes below. Be sure to label anywhere the graph intersects the coordinate axes and any local maxima or minima.

(b)



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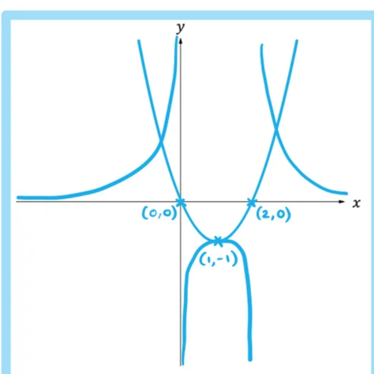

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[3]

(c) Find the equation of the horizontal and vertical asymptotes of the graph of $y = \frac{1}{f(x)}$.

[2]

(c) You can use your GDC to help you find the asymptotes

Vertical asymptotes: $x = 0$ $x = 2$

Horizontal asymptotes: $y = 0$