EXAM PAPERS PRACTICE

IB Maths: AI HL

Further Differentiation

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Topic Questions

Course	IB Maths
Section	5. Calculus
Topic	5.2 Further Differentiation
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AI HL

Board: IB Maths

Topic: Further Differentiation

Question 1

Differentiate $\dfrac{5x^7}{\sin 2x}$ with respect to x.

[4 marks]

Question 2

Find $\dfrac{dy}{dx}$ for each of the following:

(a) $y = \cos(x^2 - 3x + 7) + \sin(e^x)$

[4 marks]

(b) $y = \ln(2x^3)$

[3 marks]

(c) $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

[3 marks]

Question 3

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(4\cos x - 3\sin x)e^{3x-5}$

[3 marks]

(b) $(x^3 - 4x^2 + 7)\ln x$

[3 marks]

(c) $\sin\left(x^{\frac{1}{3}} + x^{-\frac{4}{5}} + \pi\right)$

[3 marks]

Question 4

A curve has the equation $y = \mathrm{e}^{-3x} + \ln x,\ x > 0$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

(b) Hence find the gradient of the normal to the curve at the point $(1,\ \mathrm{e}^{-3})$, giving your answer correct to 3 decimal places.

[2 marks]

Question 5

Consider the curve with equation $y = e^{3x^2 + 5x - 2}$.

(a) Find $\frac{dy}{dx}$.

[1 mark]

(b) Hence find the equation of the tangent to the curve at the point $(-2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3 marks]

Question 6

Let $f(x) = \frac{g(x)}{h(x)}$, where $g(2) = 4$, $h(2) = -1$, $g'(2) = 0$ and $h'(2) = 2$.

Find the equation of the tangent of f at $x = 2$.

[6 marks]

Question 7

A curve has the equation $y = x^3 - 12x + 7$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

[3 marks]

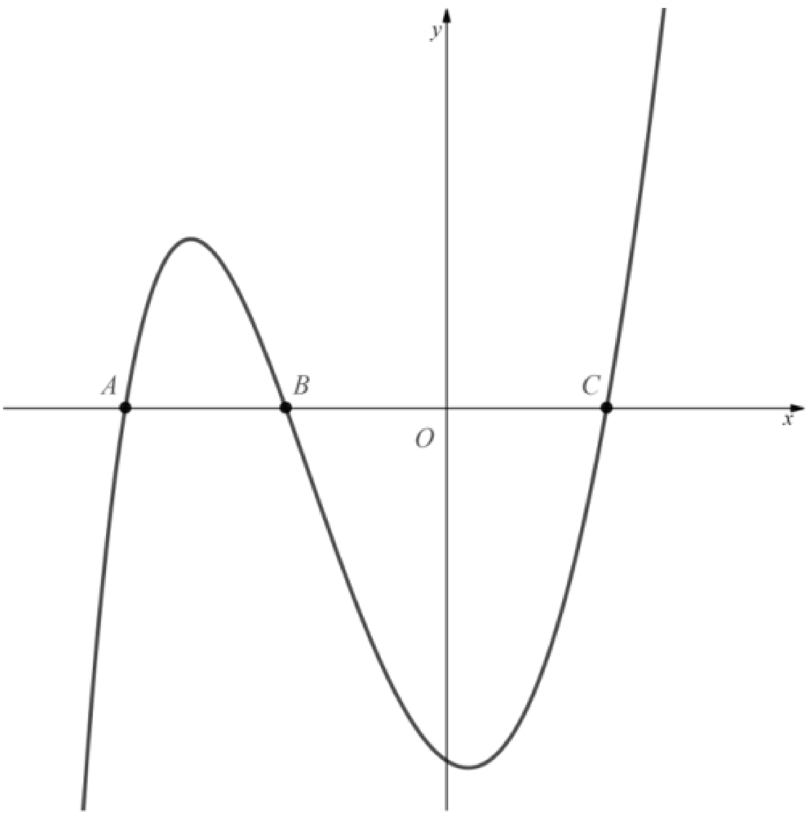
(b) Determine the coordinates of the local minimum of the curve.

[3 marks]

Question 8

The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = (x^2 - 1)\ln(x+3), \quad x > -3$$



Points A, B and C are the three places where the graph intercepts the x-axis.

(a) Find $\mathrm{f}'(x)$.

[4 marks]

(b) Show that the coordinates of point A are $(-2, 0)$.

[2 marks]

(c) Find the equation of the tangent to the curve at point A.

[3 marks]

Question 9

Let $f(x) = x^2 e^x$.

(a) Find $f'(x)$.

[3 marks]

(b) Find $f''(x)$.

[3 marks]

(c) Determine the ranges of x-values for which the graph of f is

(i) concave-up

(ii) concave-down

giving all boundary values for the ranges as exact values.

[4 marks]

(d) Hence find the exact x coordinates of the points of inflection for the graph of f. Be sure to show that any points identified are indeed points of inflection.

[2 marks]

Question 10

Let $f(x) = 2e^{2cosx}$, where $-\pi \leq x \leq \pi$.

(a) Find the number of points containing a horizontal tangent.

[1 mark]

(b) Show algebraically that the gradient of the tangent at $x = \frac{\pi}{2}$ is -4.

[4 marks]

(c) State the gradient of the tangent at $x = \frac{3\pi}{2}$.

[1 mark]

It can be found that as the function, f, undergoes a transformation $f(kx)$, the number of stationary points found between $-\pi \leq x \leq \pi$ increases.

(d) Find the number of stationary points on f after a transformation of $f(2x)$ and hence, state the general rule representing the number of stationary points in terms of k where $k \in Z^+$.

[3 marks]

Question 11

Let $f(x) = \sin x$ and $g(x) = sin^2 x$, for $0 \leq x \leq 2\pi$.

Solve $f'(x) = g'(x)$.

[5 marks]

Question 12

(a) Use the quotient rule to show that the derivative of $\tan x$ is $\frac{1}{\cos^2 x}$.

[3 marks]

Consider the function f defined by $f(x) = x \tan x$, $-\frac{3\pi}{2} \leq x \leq \frac{3\pi}{2}$.

(b) Find $f'(x)$.

[2 marks]

(c) Show that

$$f''(x) = \frac{2}{\cos^2 x}(1 + x \tan x)$$

[5 marks]

(d) Using your answers to parts (b) and (c), determine the x-coordinates of any

(i) local minima or maxima

(ii) points of inflection

on the curve $y = f(x)$.

[5 marks]

Question 13

An international mission has landed a rover on the planet Mars. After landing, the rover deploys a small drone on the surface of the planet, then rolls away to a distance of 6 metres in order to observe the drone as it lifts off into the air. Once the rover has finished moving away, the drone ascends vertically into the air at a constant speed of 2 metres per second.

Let D be the distance, in metres, between the rover and the drone at time t seconds.

Let h be the height, in metres, of the drone above the ground at time t seconds. The entire area where the rover and drone are situated may be assumed to be perfectly horizontal.

a)
Show that

$$D = \sqrt{h^2 + 36}$$

[2 marks]

b)
(i)
Explain why $\frac{\mathrm{d}h}{\mathrm{d}t} = 2$.

(ii)
Hence use implicit differentiation to show that

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{2h}{\sqrt{h^2 + 36}}$$

[5 marks]

c)
Find

(i)
the rate at which the distance between the rover and the drone is increasing at the moment when the drone is 8 metres above the ground.

(ii)
the height of the drone above the ground at the moment when the distance between the rover and the drone is increasing at a rate of $1\ \text{ms}^{-1}$.

[4 marks]