Question 1

Differentiate $\frac{5x^7}{\sin 2x}$ with respect to x.

[4]

Quotient Rule $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Let $u = 5x^7$ $\quad v = \sin 2x$ chain rule

$\frac{du}{dx} = 35x^6$ $\quad \frac{dv}{dx} = 2\cos 2x$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{(\sin 2x)35x^6 - 5x^7(2\cos 2x)}{(\sin 2x)^2}$$

$$\boxed{\frac{d\left(\frac{5x^7}{\sin 2x}\right)}{dx} = \frac{35x^6\sin 2x - 10x^7\cos 2x}{\sin^2 2x}}$$

Question 2

Find $\frac{dy}{dx}$ for each of the following:

(a) $y = \underbrace{\cos(x^2 - 3x + 7)}_{1} + \underbrace{\sin(e^x)}_{2}$ [4]

(b) $y = \ln(2x^3)$ [3]

(c) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ [3]

a) Chain rule 1 $\quad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Let $y = \cos u$ $\quad u = x^2 - 3x + 7$

$\frac{dy}{du} = -\sin u$ $\quad \frac{du}{dx} = 2x - 3$

$\frac{dy}{dx} = (-\sin u)(2x-3)$

sub in $u = x^2 - 3x + 7$

$\frac{dy}{dx} = (-\sin(x^2 - 3x + 7))(2x - 3)$

Chain rule 2

Let $y = \sin u$ $\quad u = e^x$

$\frac{dy}{du} = \cos u$ $\quad \frac{du}{dx} = e^x$

$\frac{dy}{dx} = (\cos u)e^x$

sub in $u = e^x$, $\frac{dy}{dx} = (\cos(e^x))e^x$

$$\boxed{\frac{dy}{dx} = -(2x-3)\sin(x^2 - 3x + 7) + e^x\cos(e^x)}$$

Find $\frac{dy}{dx}$ for each of the following:

(a) $y = \cos(x^2 - 3x + 7) + \sin(e^x)$ [4]

(b) $y = \ln(2x^3)$ [3]

(c) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ [3]

b) Method 1: Chain Rule $\qquad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$y = \ln u \qquad u = 2x^3$

$\frac{dy}{du} = \frac{1}{u} \qquad \frac{du}{dx} = 6x^2$

$\frac{dy}{dx} = \frac{1}{u}(6x^2)$

sub $u = 2x^3$

$\frac{dy}{dx} = \frac{1}{2x^3}(6x^2) = \boxed{\frac{3}{x}}$

OR

Method 2: Simplify using log laws

$y = \ln(2x^3) = \ln 2 + \ln x^3 = \ln 2 + 3\ln x$

$\frac{dy}{dx} = \boxed{\frac{3}{x}}$

Find $\frac{dy}{dx}$ for each of the following:

(a) $y = \cos(x^2 - 3x + 7) + \sin(e^x)$ [4]

(b) $y = \ln(2x^3)$ [3]

(c) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ [3]

c) $y = x^{1/2} + x^{-1/2}$

$\boxed{\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}}$

Question 3

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(4\cos x - 3\sin x)e^{3x-5}$ [3]

(b) $(x^3 - 4x^2 + 7)\ln x$ [3]

(c) $\sin\left(x^{\frac{1}{3}} + x^{-\frac{4}{5}} + \pi\right)$ [3]

a) Product rule $\qquad \frac{dy}{dx} = u\frac{dv}{du} + v\frac{du}{dv}$

Let $u = 4\cos x - 3\sin x \qquad v = e^{3x-5}$

$\frac{du}{dx} = -4\sin x - 3\cos x \qquad \frac{dv}{dx} = 3e^{3x-5}$

$$\frac{d(uv)}{dx} = (4\cos x - 3\sin x)3e^{3x-5} + e^{3x-5}(-4\sin x - 3\cos x)$$

$$= e^{3x-5}(12\cos x - 9\sin x - 4\sin x - 3\cos x)$$

$$= \boxed{e^{3x-5}(9\cos x - 13\sin x)}$$

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(4\cos x - 3\sin x)e^{3x-5}$ [3]

(b) $(x^3 - 4x^2 + 7)\ln x$ [3]

(c) $\sin\left(x^{\frac{1}{3}} + x^{-\frac{4}{5}} + \pi\right)$ [3]

b) Let $u = x^3 - 4x^2 + 7 \qquad v = \ln x \qquad \frac{dy}{dx} = u\frac{dv}{du} + v\frac{du}{dv}$

$\frac{du}{dx} = 3x^2 - 8x \qquad \frac{dv}{dx} = \frac{1}{x}$

$$\frac{d(uv)}{dx} = \frac{(x^3 - 4x^2 + 7)}{x} + (\ln x)(3x^2 - 8x)$$

$$= \boxed{x^2 - 4x + \frac{7}{x} + (\ln x)(3x^2 - 8x)}$$

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(4\cos x - 3\sin x)e^{3x-5}$ [3]

(b) $(x^3 - 4x^2 + 7)\ln x$ [3]

(c) $\sin\left(x^{\frac{1}{3}} + x^{-\frac{4}{5}} + \pi\right)$ [3]

c) Here, we must use the chain rule

The derivative is

$$\boxed{\left(\frac{1}{3}x^{-2/3} - \frac{4}{5}x^{-9/5}\right)\cos\left(x^{1/3} + x^{-4/5} + \pi\right)}$$

Question 4

A curve has the equation $y = e^{-3x} + \ln x,\ x > 0$.

(a) Find $\frac{dy}{dx}$.

[2]

(b) Hence find the gradient of the normal to the curve at the point $(1,\ e^{-3})$, giving your answer correct to 3 decimal places.

[2]

a) Here, we must use the chain rule

$$\boxed{\frac{dy}{dx} = -3e^{-3x} + \frac{1}{x}}$$

A curve has the equation $y = e^{-3x} + \ln x,\ x > 0$.

(a) Find $\frac{dy}{dx}$.

$$\boxed{\frac{dy}{dx} = -3e^{-3x} + \frac{1}{x}}$$

[2]

(b) Hence find the gradient of the normal to the curve at the point $(1,\ e^{-3})$, giving your answer correct to 3 decimal places.

[2]

b) $M_{normal} = \frac{-1}{\frac{dy}{dx}}$

$$M_{normal} = \frac{-1}{-3e^{-3(1)} + \frac{1}{(1)}} = \frac{-1}{1-3e^{-3}} = -1.1755...$$

$$\boxed{M_{normal} = -1.176 \quad (3dp)}$$

Question 5

Consider the curve with equation $y = e^{3x^2 + 5x - 2}$.

(a) Find $\frac{dy}{dx}$.

[1]

(b) Hence find the equation of the tangent to the curve at the point $(-2,\ 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3]

a) Here, we must use the chain rule

$$\boxed{\frac{dy}{dx} = (6x+5)e^{3x^2+5x-2}}$$

Consider the curve with equation $y = e^{3x^2 + 5x - 2}$.

(a) Find $\frac{dy}{dx}$.

$$\boxed{\frac{dy}{dx} = (6x+5)e^{3x^2+5x-2}}$$

[1]

(b) Hence find the equation of the tangent to the curve at the point $(-2,\ 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3]

b) sub $x = -2$ into $\frac{dy}{dx}$

$$\frac{dy}{dx} = (6(-2)+5)e^{3(-2)^2+5(-2)-2} = -7e^0 = -7$$

$y - y_1 = m(x - x_1) \rightarrow m = -7$, pt $\underset{(x_1,\ y_1)}{(-2,\ 1)}$

$y - (1) = (-7)(x - (-2))$

$$\boxed{y + 7x + 13 = 0}$$

Question 6

Let $f(x) = \frac{g(x)}{h(x)}$, where $g(2) = 4$, $h(2) = -1$, $g'(2) = 0$ and $h'(2) = 2$.

Find the equation of the tangent of f at $x = 2$.

[6]

$f(2) = \frac{g(2)}{h(2)}$

$f(2) = -4$ point : $(2, -4)$

Quotient rule

$y = \frac{u}{v} \longrightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (in formula booklet)

$u = g(2) = 4$ $v = h(2) = -1$

$\frac{du}{dx} = g'(2) = 0$ $\frac{dv}{dx} = h'(2) = 2$

$f'(2) = \frac{\cancel{(-1)(0)} - (4)(2)}{(-1)^2}$

$f'(2) = -8$

Sub $(2, -4)$ and $m = -8$ into $y - y_1 = m(x - x_1)$.

$y + 4 = -8(x - 2)$

$y = -8x + 16 - 4$

$\boxed{y = -8x + 12}$

Question 7

A curve has the equation $y = x^3 - 12x + 7$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

[3]

(b) Determine the coordinates of the local minimum of the curve.

[3]

a) Differenciate once

$\frac{dy}{dx} = \boxed{3x^2 - 12}$

... and differenciate again!

$\frac{d^2y}{dx^2} = \boxed{6x}$

A curve has the equation $y = x^3 - 12x + 7$.

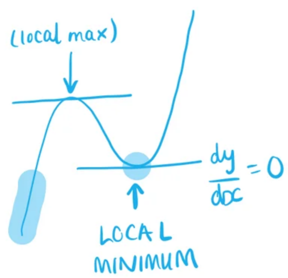
(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

$\frac{dy}{dx} = 3x^2 - 12$ $\qquad$ $\frac{d^2y}{dx^2} = 6x$

[3]

(b) Determine the coordinates of the local minimum of the curve.

[3]



b) Find x at the stationary points

$\frac{dy}{dx} = 3x^2 - 12 = 0$

$x^2 = 4$

$x = \pm 2 \rightarrow$ Two values since $\frac{dy}{dx} = 0$ at local min & local max.

Classify stationary points

At $x = 2$ $\quad \frac{d^2y}{dx^2} = 6(2) = 12 > 0 \quad \therefore$ local min

At $x = -2$ $\quad \frac{d^2y}{dx^2} = 6(-2) = -12 < 0 \quad \therefore$ local max

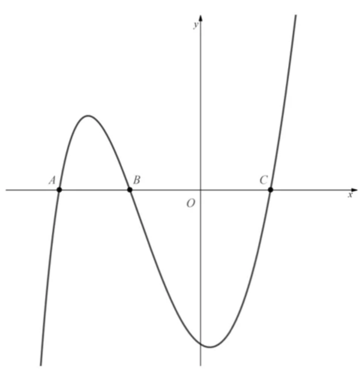
$y = 2^3 - 12(2) + 7 = -9$

LOCAL MINIMUM: $\boxed{(2, -9)}$

Question 8

The diagram below shows part of the graph of $y = f(x)$, where $f(x)$ is the function defined by

$$f(x) = (x^2 - 1)\ln(x + 3), \quad x > -3$$



Points A, B and C are the three places where the graph intercepts the x-axis.

(a) Find $f'(x)$.

[4]

(b) Show that the coordinates of point A are $(-2, 0)$.

[2]

(c) Find the equation of the tangent to the curve at point A.

[3]

a) Product rule $\qquad \frac{dy}{dx} = u\frac{dv}{du} + v\frac{du}{dv}$

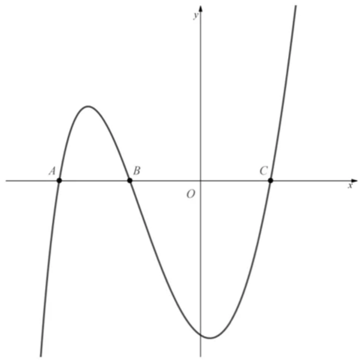
Let $u = x^2 - 1$ $\qquad v = \ln(x+3)$

$\frac{du}{dx} = 2x$ $\qquad \frac{dv}{dx} = \frac{1}{x+3}$

$\boxed{f'(x) = \frac{(x^2-1)}{x+3} + (\ln(x+3))2x}$

The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = (x^2 - 1)\ln(x + 3)\,, \quad x > -3$$



Points A, B and C are the three places where the graph intercepts the x-axis.

(a) Find $\mathrm{f}'(x)$.

[4]

(b) Show that the coordinates of point A are $(-2, 0)$.

[2]

(c) Find the equation of the tangent to the curve at point A.

[3]

b) At A, the curve intersects the x axis, so

$$y = f(x) = 0$$

$$(x^2-1)\ln(x+3) = 0$$

$$(x+1)(x-1)\underbrace{\ln(x+3)}_{} = 0$$

$$\ln 1 = 0$$

$$x + 3 = 1$$

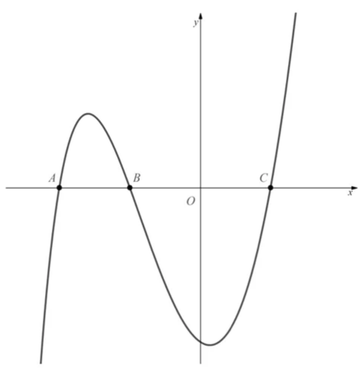
$x = -1, 1$ $\qquad$ $x = -2$

A is the most negative point of intersection,

$\therefore$ $\boxed{A(-2, 0)}$

The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = (x^2 - 1)\ln(x + 3)\,, \quad x > -3$$



Points A, B and C are the three places where the graph intercepts the x-axis.

(a) Find $\mathrm{f}'(x)$.

$$f'(x) = \frac{x^2-1}{x+3} + 2x\ln(x+3)$$

[4]

(b) Show that the coordinates of point A are $(-2, 0)$.

[2]

(c) Find the equation of the tangent to the curve at point A.

[3]

c) $$y - y_1 = m(x - x_1)$$

When $x = -2$

$$m = f'(x) = \frac{(-2)^2 - 1}{(-2)+3} + 2(-2)\ln(-2+3)$$

$$= -4\ln 1 + 3 = 3$$

Sub in $x_1 = -2$, $y_1 = 0$ and $m = 3$

$$y - (0) = 3(x - (-2))$$

$$\boxed{y = 3x + 6}$$

Question 9

Let $f(x) = x^2 e^x$.

(a) Find $f'(x)$.

[3]

(b) Find $f''(x)$.

[3]

(c) Determine the ranges of x-values for which the graph of f is

(i) concave-up

(ii) concave-down

giving all boundary values for the ranges as exact values.

[4]

(d) Hence find the exact x coordinates of the points of inflection for the graph of f. Be sure to show that any points identified are indeed points of inflection.

[2]

a) Product rule (in formula booklet)

$y = uv \longrightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x^2$ $\quad$ $v = e^x$

$\frac{du}{dx} = 2x$ $\quad$ $\frac{dv}{dx} = e^x$

$$\boxed{f'(x) = x^2 e^x + 2xe^x}$$

Let $f(x) = x^2 e^x$.

(a) Find $f'(x)$.

[3]

(b) Find $f''(x)$.

$$\boxed{f'(x) = x^2 e^x + 2xe^x}$$

[3]

(c) Determine the ranges of x-values for which the graph of f is

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[4]

(d) Hence find the exact x coordinates of the points of inflection for the graph of f. Be sure to show that any points identified are indeed points of inflection.

[2]

b) Product rule (in formula booklet)

$y = uv \longrightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$f'(x) = x^2 e^x + 2xe^x$

$f'(x) = e^x(x^2 + 2x)$

$u = e^x$ $\quad$ $v = x^2 + 2x$

$\frac{du}{dx} = e^x$ $\quad$ $\frac{dv}{dx} = 2x + 2$

$f''(x) = e^x(2x + 2) + e^x(x^2 + 2x)$

$$\boxed{f''(x) = e^x(x^2 + 4x + 2)}$$

Let $f(x) = x^2e^x$.

(a) Find $f'(x)$.

[3]

(b) Find $f''(x)$.

$f''(x) = e^x(x^2 + 4x + 2)$

[3]

(c) Determine the ranges of x-values for which the graph of f is

(i) concave-up

(ii) concave-down

giving all boundary values for the ranges as exact values.

[4]

(d) Hence find the exact x coordinates of the points of inflection for the graph of f. Be sure to show that any points identified are indeed points of inflection.

[2]

c) $e^x > 0$ for all x values, so the sign $f''(x)$ depends on $x^2 + 4x + 2$

$x^2 + 4x + 2 = 0$ $\quad$ $x = -2 \pm \sqrt{2}$

Test sign of $f''(x)$ on either side

$f''(-2) = -0.270670... < 0$

$f''(2) = 2 > 0$

i) concave up for $x < -2 - \sqrt{2}$ and $x > -2 + \sqrt{2}$

ii) concave down for $-2 - \sqrt{2} < x < -2 + \sqrt{2}$

Let $f(x) = x^2e^x$.

(a) Find $f'(x)$.

[3]

(b) Find $f''(x)$.

$f''(x) = e^x(x^2 + 4x + 2)$

[3]

(c) Determine the ranges of x-values for which the graph of f is

(i) concave-up

(ii) concave-down

giving all boundary values for the ranges as exact values.

[4]

(d) Hence find the exact x coordinates of the points of inflection for the graph of f. Be sure to show that any points identified are indeed points of inflection.

[2]

d) Points of inflection occur when $f''(x) = 0$ and concavity changes.

$0 = e^x(x^2 + 4x + 2)$

$0 = x^2 + 4x + 2$

$x = -2 \pm \sqrt{2}$ $\quad$ part (c)

$x = -2 \pm \sqrt{2}$ are inflection points because the concavity changes - from up to down and then down to up.

Question 10

Let $f(x) = 2e^{2cosx}$, where $-\pi \leq x \leq \pi$.

(a) Find the number of points containing a horizontal tangent.

[1]

(b) Show algebraically that the gradient of the tangent at $x = \frac{\pi}{2}$ is -4.

[4]

(c) State the gradient of the tangent at $x = \frac{3\pi}{2}$.

[1]

It can be found that as the function, f, undergoes a transformation $f(kx)$, the number of stationary points found between $-\pi \leq x \leq \pi$ increases.

(d) Find the number of stationary points on f after a transformation of $f(2x)$ and hence, state the general rule representing the number of stationary points in terms of k where $k \in Z^+$.

[3]

a) Graph $f(x)$ on your GDC and count the number of points the gradient is 0 in the given domain.

$\boxed{\text{3 points}}$

Let $f(x) = 2e^{2cosx}$, where $-\pi \leq x \leq \pi$.

(a) Find the number of points containing a horizontal tangent.

[1]

(b) Show algebraically that the gradient of the tangent at $x = \frac{\pi}{2}$ is -4.

[4]

(c) State the gradient of the tangent at $x = \frac{3\pi}{2}$.

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(d) Find the number of stationary points on f after a transformation of $f(2x)$ and hence, state the general rule representing the number of stationary points in terms of k where $k \in Z^+$.

[3]

b) Chain rule

$y = g(u)$, where $u = f(x)$

$\frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}$ (in formula booklet)

$u = 2\cos x$ $\qquad$ $y = 2e^u$

$\frac{du}{dx} = -2\sin x$ $\qquad$ $\frac{dy}{du} = 2e^u$

$f'(x) = 2e^{2\cos x} \times -2\sin x$

$f'(x) = -4\sin x \, e^{2\cos x}$

Sub $x = \pi/2$ into $f'(x)$.

$f'(\pi/2) = -4\sin(\pi/2)e^{2\cos(\pi/2)}$

$f'(\pi/2) = -4(1)e^{2(0)}$

$\boxed{f'(\pi/2) = -4}$

Let $f(x) = 2e^{2\cos x}$, where $-\pi \leq x \leq \pi$.

(a) Find the number of points containing a horizontal tangent.

[1]

(b) Show algebraically that the gradient of the tangent at $x = \frac{\pi}{2}$ is -4.

[4]

(c) State the gradient of the tangent at $x = \frac{3\pi}{2}$.

[1]

It can be found that as the function, f, undergoes a transformation $f(kx)$, the number of stationary points found between $-\pi \leq x \leq \pi$ increases.

(d) Find the number of stationary points on f after a transformation of $f(2x)$ and hence, state the general rule representing the number of stationary points in terms of k where $k \in Z^+$.

[3]

c) $f'(x) = -4\sin x\, e^{2\cos x}$

Sub $x = 3\pi/2$ into $f'(x)$.

$f'(3\pi/2) = -4\sin(3\pi/2)e^{2\cos(3\pi/2)}$

$f'(3\pi/2) = -4(-1)e^{2(0)}$

$\boxed{f'(3\pi/2) = 4}$

Let $f(x) = 2e^{2\cos x}$, where $-\pi \leq x \leq \pi$.

(a) Find the number of points containing a horizontal tangent.

$\boxed{3 \text{ points}}$

[1]

(b) Show algebraically that the gradient of the tangent at $x = \frac{\pi}{2}$ is -4.

[4]

(c) State the gradient of the tangent at $x = \frac{3\pi}{2}$.

[1]

It can be found that as the function, f, undergoes a transformation $f(kx)$, the number of stationary points found between $-\pi \leq x \leq \pi$ increases.

(d) Find the number of stationary points on f after a transformation of $f(2x)$ and hence, state the general rule representing the number of stationary points in terms of k where $k \in Z^+$.

[3]

d) $f(x)$ has 3 stationary points in the given domain $(-\pi, 0, \pi)$.

$f(2x)$ has 5 stationary points in the given domain $(-\pi, -\pi/2, 0, \pi/2, \pi)$

$\boxed{\therefore \text{rule}: 2k+1}$

Question 11

Let $f(x) = \sin x$ and $g(x) = sin^2 x$, for $0 \le x \le 2\pi$.

Solve $f'(x) = g'(x)$.

[5]

Derivative of $\sin x$ (in formula booklet)

$f(x) = \sin x \longrightarrow f'(x) = \cos x$

Chain rule

$g(x) = \sin^2 x$

$g(x) = (\sin x)^2 \longrightarrow g'(x) = 2\sin x \cos x$

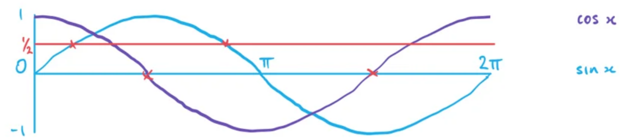
$f'(x) = g'(x)$

when $\cos x = 0$, $f'(x) = g'(x) = 0$.

$\cos x = 2\sin x \cos x$

$1 = 2\sin x$

$\sin x = 1/2$



$$\boxed{\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}}$$

Question 12

(a) Use the quotient rule to show that the derivative of $\tan x$ is $\frac{1}{\cos^2 x}$.

[3]

Consider the function f defined by $f(x) = x\tan x$, $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$.

(b) Find $f'(x)$.

[2]

(c) Show that

$$f''(x) = \frac{2}{\cos^2 x}(1 + x\tan x)$$

[5]

(d) Using your answers to parts (b) and (c), determine the x-coordinates of any

(i) local minima or maxima

(ii) points of inflection

on the curve $y = f(x)$.

[5]

a) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

$\frac{d}{dx}(\tan x) = \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$

$\frac{d}{dx}(\tan x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$$\boxed{\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x}}$$

(a) Use the quotient rule to show that the derivative of $\tan x$ is $\frac{1}{\cos^2 x}$. [3]

$\boxed{\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x}}$

Consider the function f defined by $f(x) = x\tan x$, $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$.

(b) Find $f'(x)$. [2]

b) Product rule : $y = uv$ $\frac{dy}{dx} = u'v + uv'$

$f'(x) = (1)(\tan x) + (x)\left(\frac{1}{\cos^2 x}\right)$

$\boxed{f'(x) = \tan x + \frac{x}{\cos^2 x}}$

(c) Show that

$$f''(x) = \frac{2}{\cos^2 x}(1 + x\tan x)$$

[5]

(d) Using your answers to parts (b) and (c), determine the x-coordinates of any

(i) local minima or maxima

(ii) points of inflection

on the curve $y = f(x)$. [5]

(a) Use the quotient rule to show that the derivative of $\tan x$ is $\frac{1}{\cos^2 x}$. [3]

Consider the function f defined by $f(x) = x\tan x$, $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$.

(b) Find $f'(x)$. [2]

(c) Show that

$$f''(x) = \frac{2}{\cos^2 x}(1 + x\tan x)$$

[5]

c) Quotient rule : $y = \frac{u}{v}$ $\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$

$f''(x) = \frac{1}{\cos^2 x} + \frac{(1)(\cos^2 x) - (x)(-2\cos x \sin x)}{\cos^4 x}$

$f''(x) = \frac{1}{\cos^2 x} + \frac{1}{\cos^2 x} + \frac{2x\sin x}{\cos^2 x \cdot \cos x}$

$f''(x) = \frac{2}{\cos^2 x} + \frac{2}{\cos^2 x}\left(x \cdot \frac{\sin x}{\cos x}\right)$

$\boxed{f''(x) = \frac{2}{\cos^2 x}(1 + x\tan x)}$

(d) Using your answers to parts (b) and (c), determine the x-coordinates of any

(i) local minima or maxima

(ii) points of inflection

on the curve $y = f(x)$. [5]

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[5]

d) Solve $f'(x) = 0$ and $f''(x) = 0$ with GDC.

$f'(x) = 0 \quad x = 0$ (only solution)

$f''(x) = 0 \quad x = -2.798\ldots$ or $2.798\ldots$ (only solution in domain)

i) $f''(0) = \frac{2}{1}(1 + 0 \cdot 0) = 2$, $\therefore$ concave up when $x = 0$

$\boxed{\therefore \text{ local minima}}$

ii) Test for change in concavity

$f''(-\pi) = 2 \quad \therefore$ concave up

$f''(-2) = -38.9$ (3sf) $\quad \therefore$ concave down

$f''(2) = -38.9 \quad \therefore$ concave down

$f''(\pi) = 2 \quad \therefore$ concave up

$\boxed{\therefore \text{ inflection points at } x = \pm 2.80 \text{ (3sf)}}$

Question 13

An international mission has landed a rover on the planet Mars. After landing, the rover deploys a small drone on the surface of the planet, then rolls away to a distance of 6 metres in order to observe the drone as it lifts off into the air. Once the rover has finished moving away, the drone ascends vertically into the air at a constant speed of 2 metres per second.

Let D be the distance, in metres, between the rover and the drone at time t seconds. Let h be the height, in metres, of the drone above the ground at time t seconds. The entire area where the rover and drone are situated may be assumed to be perfectly horizontal.

(a) Show that

$$D = \sqrt{h^2 + 36}$$

[2]

(b) (i) Explain why $\frac{dh}{dt} = 2$.

(ii) Hence use implicit differentiation to show that

$$\frac{dD}{dt} = \frac{2h}{\sqrt{h^2 + 36}}$$

[5]

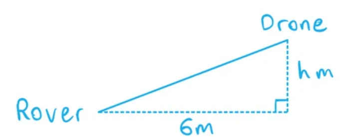
(c) Find

(i) the rate at which the distance between the rover and the drone is increasing at the moment when the drone is 8 metres above the ground.

(ii) the height of the drone above the ground at the moment when the distance between the rover and the drone is increasing at a rate of 1 ms^{-1}.

[4]

a) This is just Pythagoras.



$\boxed{D = \sqrt{h^2 + 6^2} = \sqrt{h^2 + 36}}$

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Let D be the distance, in metres, between the rover and the drone at time t seconds. Let h be the height, in metres, of the drone above the ground at time t seconds. The entire area where the rover and drone are situated may be assumed to be perfectly horizontal.

(a) Show that

$$D = \sqrt{h^2 + 36}$$

[2]

(b) (i) Explain why $\frac{\mathrm{d}h}{\mathrm{d}t} = 2$.

(ii) Hence use implicit differentiation to show that

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{2h}{\sqrt{h^2 + 36}}$$

[5]

(c) Find

(i) the rate at which the distance between the rover and the drone is increasing at the moment when the drone is 8 metres above the ground.

(ii) the height of the drone above the ground at the moment when the distance between the rover and the drone is increasing at a rate of 1 ms^{-1}.

[4]

b) i) $\boxed{\frac{dh}{dt} \text{ is the vertical speed in ms}^{-1}\text{, and the question says this is } 2\text{ms}^{-1}}$

ii) $D = \sqrt{h^2 + 36} = (h^2 + 36)^{1/2}$

$$\frac{dD}{dt} = \frac{1}{2}(h^2 + 36)^{-1/2}\left(2h\frac{dh}{dt}\right) = \frac{h}{\sqrt{h^2 + 36}}\frac{dh}{dt}$$

$$\boxed{\frac{dD}{dt} = \frac{2h}{\sqrt{h^2 + 36}}}$$

An international mission has landed a rover on the planet Mars. After landing, the rover deploys a small drone on the surface of the planet, then rolls away to a distance of 6 metres in order to observe the drone as it lifts off into the air. Once the rover has finished moving away, the drone ascends vertically into the air at a constant speed of 2 metres per second.

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[4]

c) i) sub $h = 8$ into $\frac{dD}{dt}$

$$\frac{dD}{dt} = \frac{2(8)}{\sqrt{(8)^2+36}} = \frac{16}{10}$$

$$\boxed{\frac{dD}{dt} = 1.6\ \text{ms}^{-1}}$$

ii) rearrange $\frac{dD}{dt}$ for h

$$\frac{dD}{dt} = \frac{2h}{\sqrt{h^2+36}} \longrightarrow \left(\frac{dD}{dt}\right)^2 = \frac{4h^2}{h^2+36}$$

$$\left(\frac{dD}{dt}\right)^2(h^2+36) = h^2\left(\frac{dD}{dt}\right)^2 + 36\left(\frac{dD}{dt}\right)^2 = 4h^2$$

$$4h^2 - h^2\left(\frac{dD}{dt}\right)^2 = 36\left(\frac{dD}{dt}\right)^2$$

$$h^2\left(4 - \left(\frac{dD}{dt}\right)^2\right) = 36\left(\frac{dD}{dt}\right)^2$$

$$h = \sqrt{\frac{36\left(\frac{dD}{dt}\right)^2}{4-\left(\frac{dD}{dt}\right)^2}}$$

sub in $\frac{dD}{dt} = 1$

$$h = \sqrt{\frac{36(1)^2}{4-(1)}} = \sqrt{12} = 2\sqrt{3}$$

$$\boxed{h = 3.464\ldots = 3.46\text{m (3 s.f.)}}$$