IB Maths: AA HL

Further Complex Numbers

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	1. Number & Algebra
Topic	1.9 Further Complex Numbers
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Further Complex Numbers

Question 1

Consider $w = \frac{z_1}{z_2}$, where $z_1 = 2 + 2\sqrt{3}i$ and $z_2 = 2 + 2i$.

a)

Express w in the form w =a +bi.

[3 marks]

b)

Write the complex numbers z_1 and z_2 in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta < \pi$.

[4 marks]

c)

Express w in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta < \pi$

[3 marks]

Question 2

Solve the equation $z^3 = 27i$, giving your answers in the form $a + bi$.

[6 marks]

Question 3

$$let\ z_1 = 6\ cis\ (\tfrac{\pi}{6})\ \textbf{and}\ z_2 = 3\sqrt{2e^{i(\frac{\pi}{4})}}$$

a)

Giving your answers in the form *rcisθ*, find

(i)

$$z_1 z_2$$

(ii)

$$\frac{z_1}{z_2}$$

[4 marks]

b)

Write $z_1\ and\ z_2\ \ in\ the\ form\ a\ +\ bi.$

[2 marks]

c)

Find $z_1 + z_2,\ giving\ your\ answer\ in\ the\ form\ a\ \ +\ bi.$

[2 marks]

Question 3d

It is given that z_1^* *and* z_2^* *are the complex conjugates of* z_1 *and* z_2 *respectively.*

d)

Find z_1^* *and* z_2^*, giving your answer in the form a +bi.

[2 marks]

Question 4a

let $z_1 = 2cis\left(\frac{\pi}{3}\right)$ *and* $z_2 = 2 + 2i.$

a)

Express

(i)

z_1 *in the form* $a + bi$

(ii)

z_2 in the form r cisθ

[2 marks]

b)

Find $w_1 = z_1 + z_2$, giving your answer in the form a +bi.

[2 marks]

c)

Find $w_1 = z_1 z_2$, *giving your answer in the form* $r\,cis\theta$.

[3 marks]

d)

Sketch w_1 *and* w_2 *on a single Argand diagram.*

[2 marks]

Question 5

It is given that that $z_1 = 2e^{i\left(\frac{\pi}{3}\right)}$ *and* $z_2 = 3cis\left(\frac{n\pi}{12}\right)$, $n \epsilon Z^+$.

a)

Find the value of $z_1\, z_2$ *for* $n = 3$.

[3 marks]

b)

Find the least value of n such that $z_1\, z_2\, \epsilon R^+$.

[3 marks]

Question 6

Consider the complex number $w = \frac{z_1}{z_2}$ where $z_1 = 3 - \sqrt{3}i$ and $z_2 = 2cis\left(\frac{2\pi}{3}\right)$

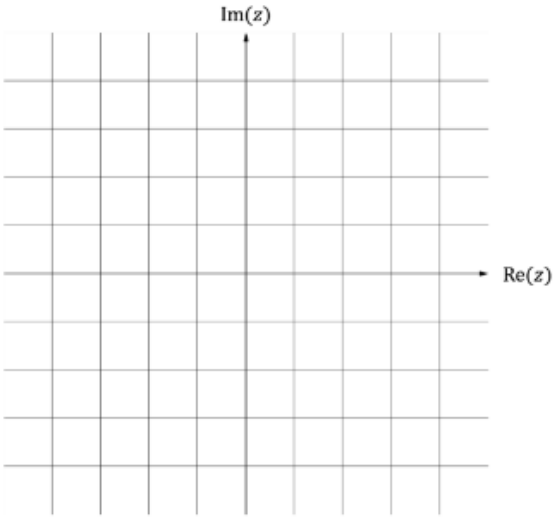
a)

Express w in the form $r\,cis\theta$

[5 marks]

b)

Sketch z_1, z_2 *and* w *on the Argand diagram below.*



[3 marks]

c)

Find the smallest positive integer value of n such that w^n is a real number.

[2 marks]

Question 7

Consider the complex number $z = -1 + \sqrt{3}i$.

(a)

Express z in the form $r\,cis\,\theta,\ where\ r > 0\ and\ -\pi < \theta \leq \pi$.

[4 marks]

(b)

Find the three roots of the equation $z^3 = -1 + \sqrt{3}i$. expressing your answers in the form $r\,cis\theta,\ where\ r > 0\ and\ -\pi < \theta \leq \pi$.

[4 marks]

Question 8

Consider the equation $z^4 - 1 = 15,\ where\ z \epsilon C$.

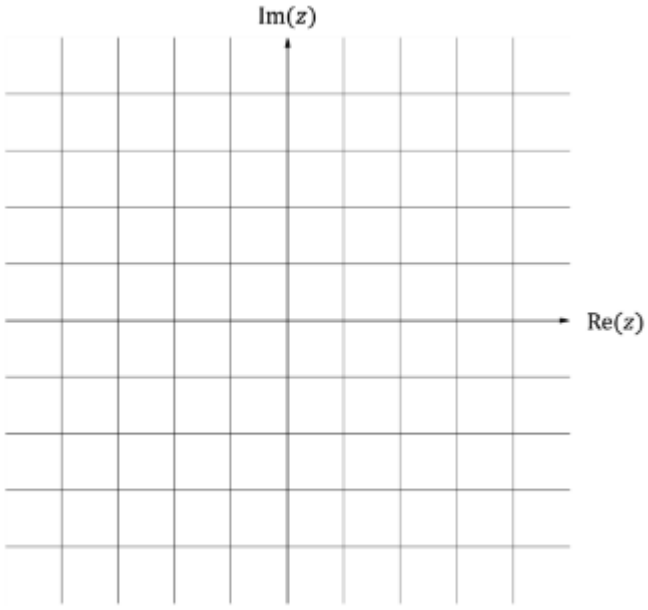
(a)

Find the four distinct roots of the equation, giving your answers in the form $a + bi$, where $a,b \in R$.

[4 marks]

(b)

Represent the roots found in part (a) on the Argand diagram below.



[2 marks]

(c)

Find the area of the polygon whose vertices are represented by the four roots on the Argand diagram.

[2 marks]

Question 9

Consider the complex numbers $w = 3\left(cos\frac{\pi}{3}\right) - isin - \frac{\pi}{3}$ $and\ z = 3 - \sqrt{3i}.$

(a)

Write $w\ and\ z\ in\ the\ form\ r\ cis\ \theta,\ where\ r > 0\ and\ -\pi < \theta \leq \pi.$

[4 marks]

(b)

Find the modulus and argument of zw.

[2 marks]

(c)

Write down the value of zw.

[2 marks]

Question 10

$let\ z = 12 + 16i,\ where\ a, b \in R.$

a)

Verify that 4+2i and −4−2i are the second roots of z.

[2 marks]

b)

Hence, or otherwise, find two distinct roots of the equation $w^2 + 4w + (1 - 4i) = 0$, where w ∈C. Give your answer in the form a +bi, where a,b ∈R.

[4 marks]

Question 11

The complex numbers $w_1 = 3$, and $w_2 = 2 - 2i$ are roots of the cubic equation

$w^3 + pw^2 + qw + r = 0$, where p, q, r ∈R.

a)

Write down the third root, w_3, of the equation.

[1 marks]

b)

Find the values of p, qand r.

[4 marks]

c)

Express $w_1,\ w_2\ and\ w_3$ in the form $r\ cis\theta$.

[4 marks]