Question 1

Consider $w = \frac{z_1}{z_2}$, where $z_1 = 2 + 2\sqrt{3}i$ and $z_2 = 2 + 2i$.

(a) Express w in the form $w = a + bi$. [2]

(b) Write the complex numbers z_1 and z_2 in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta < \pi$. [2]

(c) Express w in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta < \pi$. [2]

(a) You can use your GDC to help you with this but be careful with your answers as they should be left as exact values where possible

$$w = \frac{2+2\sqrt{3}i}{2+2i} \times \frac{2-2i}{2-2i}$$

Multiply by conjugate of denominator to cancel i

$$= \frac{4+4\sqrt{3}i-4i+4\sqrt{3}}{4+4i-4i+4}$$

$$= \frac{4(1+\sqrt{3})+4(\sqrt{3}-1)i}{8}$$

$$\boxed{w = \frac{(1+\sqrt{3})}{2} + \frac{(\sqrt{3}-1)}{2}i}$$

Consider $w = \frac{z_1}{z_2}$, where $z_1 = 2 + 2\sqrt{3}i$ and $z_2 = 2 + 2i$.

(a) Express w in the form $w = a + bi$. [2]

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(c) Express w in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta < \pi$. [2]

(b) Find the modulus and argument for z_1 and z_2

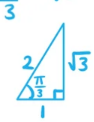
You can use the GDC to help you find the modulus and argument

$|z_1| = 4$

$$\theta_1 = \tan^{-1}\left(\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right) = \frac{\pi}{3}$$

Trig exact values

$\tan\frac{\pi}{3} = \sqrt{3}$



$$\boxed{z_1 = 4e^{i\left(\frac{\pi}{3}\right)}}$$

$$|z_2| = \sqrt{2^2+2^2} = 2\sqrt{2}$$

$$\theta_2 = \tan^{-1}\left(\frac{\cancel{2}}{\cancel{2}}\right) = \frac{\pi}{4}$$

Trig exact values

$\tan\frac{\pi}{4} = 1$

$$\boxed{z_2 = 2\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}}$$

Consider $w = \frac{z_1}{z_2}$, where $z_1 = 2 + 2\sqrt{3}i$ and $z_2 = 2 + 2i$.

(a) Express w in the form $w = a + bi$.

[2]

(b) Write the complex numbers z_1 and z_2 in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta < \pi$.

$\boxed{z_1 = 4e^{i\left(\frac{\pi}{3}\right)}}$

[2]

(c) Express w in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta < \pi$.

$\boxed{z_2 = 2\sqrt{2}\, e^{i\left(\frac{\pi}{4}\right)}}$

[2]

(c) $w = \dfrac{4e^{i\frac{\pi}{3}}}{2\sqrt{2}\, e^{i\frac{\pi}{4}}}$

$= \dfrac{2}{\sqrt{2}} e^{i\left(\frac{\pi}{3} - \frac{\pi}{4}\right)}$

$= \dfrac{\cancel{2}\sqrt{2}}{\cancel{2}} e^{i\left(\frac{\pi}{12}\right)}$

$\boxed{w = \sqrt{2}\, e^{i\left(\frac{\pi}{12}\right)}}$

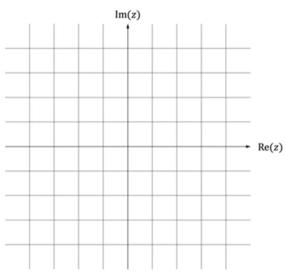
Question 2

Consider the equation $z^4 - 1 = 15$, where $z \in \mathbb{C}$.

(a) Find the four distinct roots of the equation, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

[4]

(b) Represent the roots found in part (a) on the Argand diagram below.



[2]

(c) Find the area of the polygon whose vertices are represented by the four roots on the Argand diagram.

[3]

(a) $z^4 - 1 = 15 \Rightarrow z^4 = 16$

Define a new complex number, w

Let $z^2 = w \Rightarrow w^2 = 16$

$w = \pm 4$

Substitute $w = \pm 4$ back into $z^2 = w$

$z^2 = 4$ $\qquad$ $z^2 = -4$

$z = \pm 2$ $\qquad$ $z = \pm 2i$

$\boxed{z_1 = 2 \quad z_2 = -2 \quad z_3 = 2i \quad z_4 = -2i}$

Consider the equation $z^4 - 1 = 15$, where $z \in \mathbb{C}$.

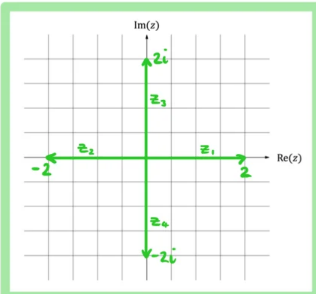
(a) Find the four distinct roots of the equation, giving your answers in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

$\boxed{z_1 = 2 \quad z_2 = -2 \quad z_3 = 2i \quad z_4 = -2i}$

[4]

(b) Represent the roots found in part (a) on the Argand diagram below.

(b)



[2]

(c) Find the area of the polygon whose vertices are represented by the four roots on the Argand diagram.

[3]

Consider the equation $z^4 - 1 = 15$, where $z \in \mathbb{C}$.

(a) Find the four distinct roots of the equation, giving your answers in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

[4]

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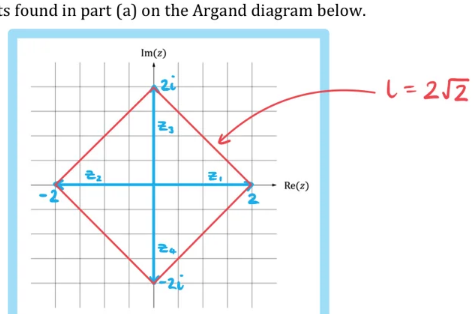


[2]

(c) Find the area of the polygon whose vertices are represented by the four roots on the Argand diagram.

[3]

(c) The shape formed is a square

Length, $l = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

Area, $A = (2\sqrt{2})^2$

$\boxed{A = 8 \text{ units}^2}$

Question 3

Let $z_1 = 6\text{cis}\left(\frac{\pi}{6}\right)$ and $z_2 = 3\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$.

(a) Giving your answers in the form $r\text{cis}\theta$, find

(i) $z_1 z_2$

(ii) $\frac{z_1}{z_2}$.

[4]

(b) Write z_1 and z_2 in the form $a + bi$.

[2]

(c) Find $z_1 + z_2$, giving your answer in the form $a + bi$.

[2]

It is given that z_1^* and z_2^* are the complex conjugates of z_1 and z_2 respectively.

(d) Find $z_1^* + z_2^*$, giving your answer in the form $a + bi$.

[2]

(a) (i) $z_1 z_2 = 6e^{i\left(\frac{\pi}{6}\right)} \times 3\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$

$= 18\sqrt{2}e^{i\left(\frac{\pi}{6}+\frac{\pi}{4}\right)}$

$= 18\sqrt{2}e^{i\left(\frac{5\pi}{12}\right)}$

$$\boxed{z_1 z_2 = 18\sqrt{2}\,\text{cis}\left(\frac{5\pi}{12}\right)}$$

(ii) $\frac{z_1}{z_2} = \frac{6e^{i\left(\frac{\pi}{6}\right)}}{3\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}}$

$= \frac{2}{\sqrt{2}}e^{i\left(\frac{\pi}{6}-\frac{\pi}{4}\right)}$

$= \sqrt{2}e^{i\left(-\frac{\pi}{12}\right)}$

$$\boxed{\frac{z_1}{z_2} = \sqrt{2}\,\text{cis}\left(-\frac{\pi}{12}\right)}$$

Let $z_1 = 6\text{cis}\left(\frac{\pi}{6}\right)$ and $z_2 = 3\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$.

(a) Giving your answers in the form $r\text{cis}\theta$, find

(i) $z_1 z_2$

(ii) $\frac{z_1}{z_2}$.

[4]

(b) Write z_1 and z_2 in the form $a + bi$.

[2]

(c) Find $z_1 + z_2$, giving your answer in the form $a + bi$.

[2]

It is given that z_1^* and z_2^* are the complex conjugates of z_1 and z_2 respectively.

(d) Find $z_1^* + z_2^*$, giving your answer in the form $a + bi$.

[2]

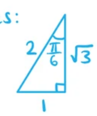
(b) $z_1 = x + yi$

Express $\tan\theta$ in terms of x and y

$\tan\frac{\pi}{6} = \left(\frac{y}{x}\right)$

Exact trig values:

$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$



$\frac{k}{k\sqrt{3}} = \frac{y}{x}$

$\frac{y}{x}$ is a multiple of $\frac{1}{\sqrt{3}}$

Find the modulus, 6 in terms of x and y

$6 = \sqrt{x^2 + y^2}$

Substitute the expressions for x and y in terms of k into the equation for the modulus

$6 = \sqrt{(k\sqrt{3})^2 + k^2}$

$6 = \sqrt{4k^2}$

$k = 3 \quad \Rightarrow \quad y = 3, \; x = 3\sqrt{3}$

$$\boxed{z_1 = 3\sqrt{3} + 3i}$$

z_2 will be the complex conjugate of z_1

$$\boxed{z_2 = 3 + 3i}$$

Let $z_1 = 6\text{cis}\left(\frac{\pi}{6}\right)$ and $z_2 = 3\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$.

(a) Giving your answers in the form $r\text{cis}\theta$, find

(i) $z_1 z_2$

(ii) $\frac{z_1}{z_2}$.

Exact trig values:

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\sin\frac{\pi}{6} = \frac{1}{2}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

[4]

(b) Write z_1 and z_2 in the form $a + bi$.

[2]

(c) Find $z_1 + z_2$, giving your answer in the form $a + bi$.

[2]

It is given that z_1^* and z_2^* are the complex conjugates of z_1 and z_2 respectively.

(d) Find $z_1^* + z_2^*$, giving your answer in the form $a + bi$.

[2]

(c) $z_1 + z_2 = 6\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) + 3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= 6\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) + 3\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$

$= 3\sqrt{3} + 3i + 3 + 3i$

$\boxed{z_1 + z_2 = 3(1+\sqrt{3}) + 6i}$

Let $z_1 = 6\text{cis}\left(\frac{\pi}{6}\right)$ and $z_2 = 3\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$.

(a) Giving your answers in the form $r\text{cis}\theta$, find

(i) $z_1 z_2$

(ii) $\frac{z_1}{z_2}$.

[4]

(b) Write z_1 and z_2 in the form $a + bi$.

$\boxed{z_1 = 3\sqrt{3} + 3i}$ $\boxed{z_2 = 3 + 3i}$

[2]

(c) Find $z_1 + z_2$, giving your answer in the form $a + bi$.

[2]

It is given that z_1^* and z_2^* are the complex conjugates of z_1 and z_2 respectively.

(d) Find $z_1^* + z_2^*$, giving your answer in the form $a + bi$.

[2]

(d) $z_1^* = 3\sqrt{3} - 3i$ $z_2^* = 3 - 3i$

$z_1^* + z_2^* = (3\sqrt{3} - 3i) + (3 - 3i)$

$= 3\sqrt{3} + 3 - 3i - 3i$

$\boxed{z_1^* + z_2^* = 3(\sqrt{3}+1) - 6i}$

Question 4

Let $z_1 = 2\text{cis}\left(\frac{\pi}{3}\right)$ and $z_2 = 2 + 2i$.

(a) Express

(i) z_1 in the form $a + bi$

(ii) z_2 in the form $r\text{cis}\theta$.

[2]

(b) Find $w_1 = z_1 + z_2$, giving your answer in the form $a + bi$.

[2]

(c) Find $w_2 = z_1 z_2$, giving your answer in the form $r\text{cis}\theta$.

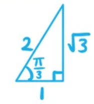
[3]

(d) Sketch w_1 and w_2 on a single Argand diagram.

[2]

(a) (i) $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ ← Trig exact values

$\cos\frac{\pi}{3} = \frac{1}{2}$



$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

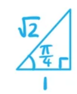
$= 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$\boxed{z_1 = 1 + \sqrt{3}i}$

(ii) $|z_2| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$\theta_2 = \tan^{-1}\left(\frac{2}{2}\right) = \frac{\pi}{4}$ ← Trig exact values

$\tan\left(\frac{\pi}{4}\right) = 1$



$\boxed{z_2 = 2\sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)}$

Let $z_1 = 2\text{cis}\left(\frac{\pi}{3}\right)$ and $z_2 = 2 + 2i$.

(a) Express

(i) z_1 in the form $a + bi$

(ii) z_2 in the form $r\text{cis}\theta$.

$\boxed{z_1 = 1 + \sqrt{3}i}$

[2]

(b) Find $w_1 = z_1 + z_2$, giving your answer in the form $a + bi$.

[2]

(c) Find $w_2 = z_1 z_2$, giving your answer in the form $r\text{cis}\theta$.

[3]

(d) Sketch w_1 and w_2 on a single Argand diagram.

[2]

(b) $w_1 = (1 + \sqrt{3}i) + (2 + 2i)$

$\boxed{w_1 = 3 + (2 + \sqrt{3})i}$

EXAM PAPERS PRACTICE

Let $z_1 = 2\text{cis}\left(\frac{\pi}{3}\right)$ and $z_2 = 2 + 2i$.

(a) Express

(i) z_1 in the form $a + bi$

(ii) z_2 in the form $r\text{cis}\theta$.

$$\boxed{z_2 = 2\sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)}$$

[2]

(b) Find $w_1 = z_1 + z_2$, giving your answer in the form $a + bi$.

[2]

(c) Find $w_2 = z_1 z_2$, giving your answer in the form $r\text{cis}\theta$.

[3]

(d) Sketch w_1 and w_2 on a single Argand diagram.

[2]

(c) $\omega_2 = \left(2\,\text{cis}\left(\frac{\pi}{3}\right)\right)\left(2\sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)\right)$

$= 2e^{i\left(\frac{\pi}{3}\right)} \times 2\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$

$= 4\sqrt{2}e^{i\left(\frac{\pi}{3}+\frac{\pi}{4}\right)}$

$$\boxed{\omega_2 = 4\sqrt{2}\,\text{cis}\left(\frac{7\pi}{12}\right)}$$

Let $z_1 = 2\text{cis}\left(\frac{\pi}{3}\right)$ and $z_2 = 2 + 2i$.

(a) Express

(i) z_1 in the form $a + bi$

(ii) z_2 in the form $r\text{cis}\theta$.

[2]

(b) Find $w_1 = z_1 + z_2$, giving your answer in the form $a + bi$.

$$\boxed{\omega_1 = 3 + \left(2+\sqrt{3}\right)i}$$

[2]

(c) Find $w_2 = z_1 z_2$, giving your answer in the form $r\text{cis}\theta$.

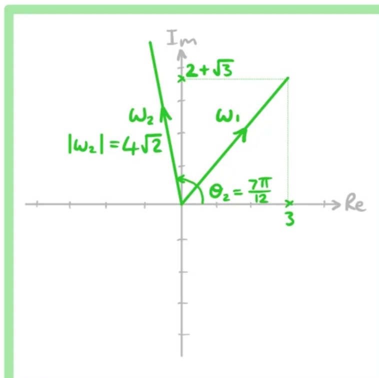
$$\boxed{\omega_2 = 4\sqrt{2}\,\text{cis}\left(\frac{7\pi}{12}\right)}$$

[3]

(d) Sketch w_1 and w_2 on a single Argand diagram.

[2]

(d)



Question 5

It is given that that $z_1 = 2e^{i\left(\frac{\pi}{3}\right)}$ and $z_2 = 3\,\mathrm{cis}\left(\frac{n\pi}{12}\right)$, $n \in \mathbb{Z}^+$.

(a) Find the value of $z_1 z_2$ for $n = 3$. [3]

(b) Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$. [3]

(a)

$$z_1 z_2 = 2e^{i\left(\frac{\pi}{3}\right)} \times 3e^{i\left(\frac{3\pi}{12}\right)}$$

$$= 6e^{i\left(\frac{\pi}{3} + \frac{3\pi}{12}\right)}$$

$$= 6e^{i\left(\frac{7\pi}{12}\right)}$$

$$\boxed{z_1 z_2 = 6\,\mathrm{cis}\left(\frac{7\pi}{12}\right)}$$

It is given that that $z_1 = 2e^{i\left(\frac{\pi}{3}\right)}$ and $z_2 = 3\,\mathrm{cis}\left(\frac{n\pi}{12}\right)$, $n \in \mathbb{Z}^+$.

(a) Find the value of $z_1 z_2$ for $n = 3$. [3]

(b) Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$. [3]

(b) $z_1 z_2 = 6e^{i\left(\frac{\pi}{3} + \frac{n\pi}{12}\right)}$

$$= 6e^{i\left(\frac{(4+n)\pi}{12}\right)}$$

For $z_1 z_2$ to be $\mathbb{R}^+$, it must lie on the positive, real axis and θ must therefore be a multiple of 2π

The smallest value for n will be when $\theta = 2\pi$

$$\frac{(4+n)\cancel{\pi i}}{12} = 2\cancel{\pi i}$$

$$4+n = 24$$

$$\boxed{n = 20}$$

Question 6

Consider the complex number $w = \frac{z_1}{z_2}$ where $z_1 = 3 - \sqrt{3}i$ and $z_2 = 2\text{cis}\left(\frac{2\pi}{3}\right)$.

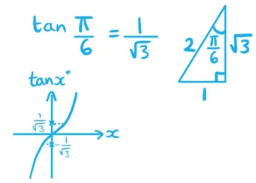
(a) Express w in the form $r\text{cis}\theta$. [5]

(a) Rewrite z_1 in the form of $r\text{cis}\theta$

$|z_1| = \sqrt{3^2 + (-\sqrt{3})^2} = 2\sqrt{3}$

$\theta_1 = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ Trig exact values

$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$



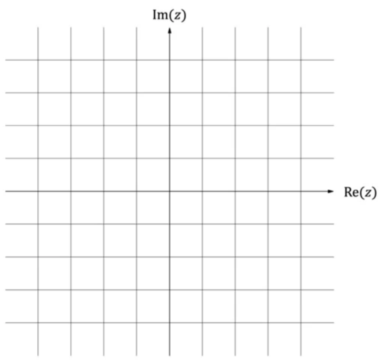
$= -\frac{\pi}{6}$

$w = \frac{2\sqrt{3}e^{i\left(-\frac{\pi}{6}\right)}}{2e^{i\left(\frac{2\pi}{3}\right)}}$

$= \frac{2\sqrt{3}}{2}e^{i\left(-\frac{\pi}{6} - \frac{2\pi}{3}\right)}$

$\boxed{w = \sqrt{3}\,\text{cis}\left(-\frac{5\pi}{6}\right)}$

(b) Sketch z_1, z_2 and w on the Argand diagram below.



[3]

(c) Find the smallest positive integer value of n such that w^n is a real number.

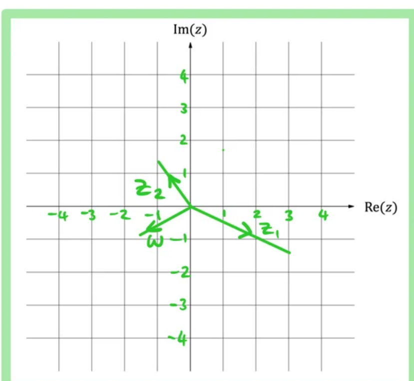
[2]

Consider the complex number $w = \frac{z_1}{z_2}$ where $z_1 = 3 - \sqrt{3}i$ and $z_2 = 2\text{cis}\left(\frac{2\pi}{3}\right)$.

(a) Express w in the form $r\text{cis}\theta$. [5]

(b) Sketch z_1, z_2 and w on the Argand diagram below.

(b)



[3]

(c) Find the smallest positive integer value of n such that w^n is a real number.

[2]

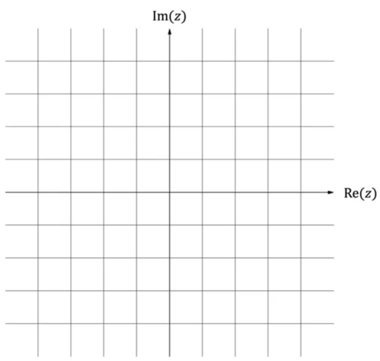
Consider the complex number $w = \frac{z_1}{z_2}$ where $z_1 = 3 - \sqrt{3}i$ and $z_2 = 2\text{cis}\left(\frac{2\pi}{3}\right)$.

(a) Express w in the form $r\,\text{cis}\theta$.

$$\boxed{w = \sqrt{3}\ \text{cis}\left(-\frac{5\pi}{6}\right)}$$

[5]

(b) Sketch z_1, z_2 and w on the Argand diagram below.



[3]

(c) Find the smallest positive integer value of n such that w^n is a real number.

[2]

(c) $w = \sqrt{3}\,e^{i\left(\frac{-5\pi}{6}\right)}$

$w^n = \sqrt{3}^n e^{i\left(\frac{-5\pi n}{6}\right)}$

For w^n to be $\mathbb{R}$, it must lie on the real axis and θ must therefore be a multiple of π

The smallest value for n will be when $\theta = -k\pi$

$$-\frac{5\cancel{\pi}n}{6} = -k\cancel{\pi}$$

For k to be an integer value

$$\boxed{n = 6}$$

Question 7

Consider the complex numbers $w = 3\left(\cos\frac{\pi}{3} - \text{i}\sin\frac{\pi}{3}\right)$ and $z = 3 - \sqrt{3}\text{i}$.

(a) Write w and z in the form $r\,\text{cis}\,\theta$, where $r > 0$ and $-\pi < \theta \leq \pi$.

[4]

(b) Find the modulus and argument of zw.

[2]

(c) Write down the value of zw.

[2]

(a) $w = 3\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$

spot the negative!

$-\sin\theta = \sin(-\theta)$

$\cos\theta = \cos(-\theta)$

$$\boxed{w = 3\,\text{cis}\left(-\frac{\pi}{3}\right)}$$

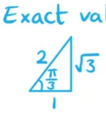
$|z| = \sqrt{3^2 + (-\sqrt{3})^2}$

$= 2\sqrt{3}$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$

$\theta = -\frac{\pi}{6}$

Exact values:



$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

$$\boxed{z = 2\sqrt{3}\,\text{cis}\left(\frac{-\pi}{6}\right)}$$

Consider the complex numbers $w = 3\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right)$ and $z = 3 - \sqrt{3}\mathrm{i}$.

(a) Write w and z in the form $r\operatorname{cis}\theta$, where $r > 0$ and $-\pi < \theta \leq \pi$. [4]

$\boxed{w = 3\operatorname{cis}\left(-\frac{\pi}{3}\right)}$ $\boxed{z = 2\sqrt{3}\operatorname{cis}\left(-\frac{\pi}{6}\right)}$

(b) Find the modulus and argument of zw. [2]

(c) Write down the value of zw. [2]

(b) $|zw| = 2\sqrt{3} \times 3$

$\boxed{|zw| = 6\sqrt{3}}$

$\arg(zw) = -\frac{\pi}{6} + -\frac{\pi}{3}$

$\boxed{\arg(zw) = -\frac{\pi}{2}}$

Consider the complex numbers $w = 3\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right)$ and $z = 3 - \sqrt{3}\mathrm{i}$.

(a) Write w and z in the form $r\operatorname{cis}\theta$, where $r > 0$ and $-\pi < \theta \leq \pi$. [4]

(b) Find the modulus and argument of zw. [2]

$\boxed{|zw| = 6\sqrt{3}}$ $\boxed{\arg(zw) = -\frac{\pi}{2}}$

(c) Write down the value of zw. [2]

(c) $zw = 6\sqrt{3}\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

$zw = 6\sqrt{3}(0 - i)$

$\boxed{zw = 6\sqrt{3}i}$

Question 8

Write $5\cos(2t+3) + 4\cos(2t+5)$ in the form $A\cos(2t+B)$ where $A > 0, -\pi < B \leq \pi$. [5]

The given expression can be written as the real part of a complex number

$\operatorname{Re}(5\operatorname{cis}(2t+3) + 4\operatorname{cis}(2t+5))$

Write in exponential form and simplify

$\operatorname{Re}\left(5e^{(2t+3)i} + 4e^{(2t+5)i}\right) = \operatorname{Re}\left(5e^{2ti}e^{3i} + 4e^{2ti}e^{5i}\right)$

$= \operatorname{Re}\left(e^{2ti}\underbrace{\left(5e^{3i} + 4e^{5i}\right)}_{\text{simplify on your GDC}}\right)$

$= \operatorname{Re}\left(e^{2ti}\left((4.93\ldots)e^{-2.45\ldots i}\right)\right)$

$= \operatorname{Re}\left((4.93\ldots)e^{(2t-2.45\ldots)i}\right)$

Re-write in polar form

$\boxed{4.93\cos(2t - 2.45)}$