IB Maths: AI HL

Further Complex Numbers

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AI HL Topic Questions

Course	IB Maths
Section	1. Number & Algebra
Topic	1.6 Further Complex Numbers
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AI HL

Board: IB Maths

Topic: Further Complex Numbers

Question 1

Consider $w = \frac{z_1}{z_2}$, where $z_1 = 2 + 2\sqrt{3}\mathrm{i}$ and $z_2 = 2 + 2\mathrm{i}$.

(a)
Express w in the form $w = a + b\mathrm{i}$.

[2 marks]

(b)
Write the complex numbers z_1 and z_2 in the form $re^{\mathrm{i}\theta}$, $r \geq 0$, $-\pi < \theta < \pi$.

[2 marks]

(c)
Express w in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta < \pi$.

[2 marks]

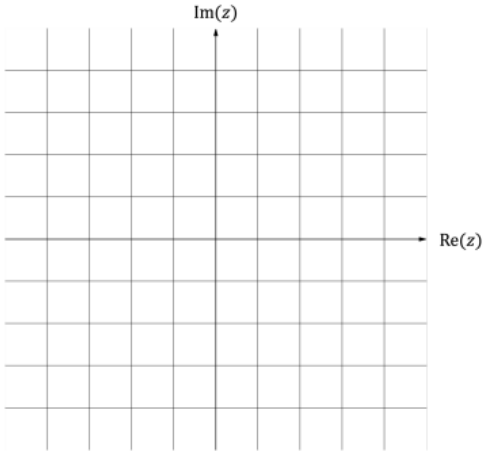
Question 2

Consider the equation $z^4 - 1 = 15$, where $z \in \mathbb{C}$.

(a)
Find the four distinct roots of the equation, giving your answers in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

[4 marks]

(b)
Represent the roots found in part (a) on the Argand diagram below.



[2 marks]

(c)
Find the area of the polygon whose vertices are represented by the four roots on the Argand diagram.

[2 marks]

Question 3

Let $z_1 = 6cis(\frac{\pi}{6})$ and $z_2 = 3\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$.

a)
Giving your answers in the form $rcis\theta$, find

(i)
$z_1 z_2$

(ii)
$\frac{z_1}{z_2}$.

[4 marks]

b)
Write z_1 and z_2 in the form $a+b\text{i}$.

[2 marks]

c)
Find z_1+z_2, giving your answer in the form $a+b\text{i}$.

[2 marks]

It is given that z_1^* and z_2^* are the complex conjugates of z_1 and z_2 respectively.

d)
Find $z_1^*+z_2^*$, giving your answer in the form $a+b\text{i}$.

[2 marks]

Question 4

Let $z_1=2cis(\frac{\pi}{3})$ and $z_2=2+2\text{i}$.

a)
Express

(i)
z_1 in the form $a+b\text{i}$

(ii)
z_2 in the form $r\,cis\theta$

[2 marks]

b)
Find $w_1=z_1+z_2$, giving your answer in the form $a+b\text{i}$.

[2 marks]

c)
Find $w_2 = z_1 z_2$, giving your answer in the form $r\,cis\theta$.

[3 marks]

d)
Sketch w_1 and w_2 on a single Argand diagram.

[2 marks]

Question 5

It is given that that $z_1 = 2e^{i\left(\frac{\pi}{3}\right)}$ and $z_2 = 3cis\left(\frac{n\pi}{12}\right)$, $n \in \mathbb{Z}^+$.

a)
Find the value of $z_1 z_2$ for $n = 3$.

[3 marks]

b)
Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$.

[3 marks]

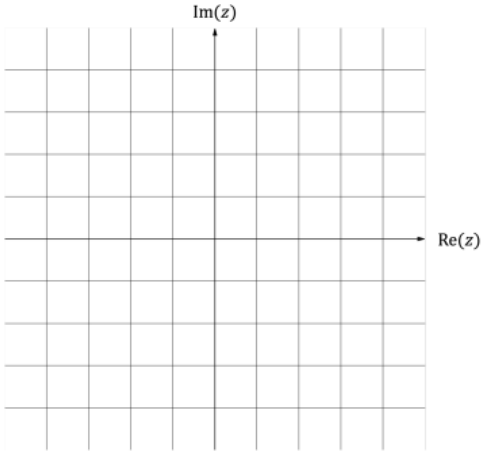
Question 6

Consider the complex number $w = \frac{z_1}{z_2}$ where $z_1 = 3 - \sqrt{3}i$ and $z_2 = 2cis\left(\frac{2\pi}{3}\right)$.

a)
Express w in the form $r\,cis\theta$.

[5 marks]

b)
Sketch z_1, z_2 and w on the Argand diagram below.



[3 marks]

c)
Find the smallest positive integer value of n such that w^n is a real number.

[2 marks]

Question 7

Consider the complex numbers $w=3\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$ and $z=3-\sqrt{3}i$.

(a)
Write w and z in the form $r\,\text{cis}\,\theta$, where $r>0$ and $-\pi<\theta\leq\pi$.

[4 marks]

(b)
Find the modulus and argument of zw.

[2 marks]

(c)
Write down the value of zw.

[2 marks]

Question 8

Write $5\cos(2t+3)+4\cos(2t+5)$ in the form $A\cos(2t+B)$ where $A>0,\ -\pi<B<\pi$.

[5 marks]