Functions

Model Answers

$f(x) = (x + 2)^3 - 5$ $g(x) = 2x + 10$ $h(x) = \frac{1}{x}, x \neq 0$

Find

(a) gf (x), [2]

$g(f(x)) = 2\left((x+2)^3 - 5\right) + 10$

Now, substitute the expression for $f(x)$:

$g(f(x)) = 2\left((x+2)^3 - 5\right) + 10$

Expand and simplify:

$g(f(x)) = 2(x+2)^3 - 10 + 10$

The -10 and +10 cancel out:

$g(f(x)) = 2(x+2)^3$

So, $gf(x) = 2(x+2)^3$.

(b) $f^{-1}(x)$, **The instruction lacks clarity and needs further elaboration to be easily understood.** [3]

(c) gh $(-\frac{1}{5})$. [2]

Steps to solve:

1. Reorder terms so constants are on the left:

$-\frac{1}{5}gh$

2. Combine multiplied terms into a single fraction:

$\frac{-gh}{5}$

Answer:

$\frac{-gh}{5}$

Question 2

EXAM PAPERS PRACTICE

$f(x) = (x-1)^3$ $g(x) = (x-1)^2$ $h(x) = 3x + 1$

(a) Work out fg(-1). [2]

Steps to solve:

1. Multiply the polynomials:

$f(-1)g(-1) = (x-1)^3(x-1)^2$

2. Simplify the expression:

$(x-1)^3(x-1)^2 = (x-1)^5$

Answer:

$(x-1)^5$

(b) Find gh(x) in its simplest form. [2]

$g(h(x)) = (h(x) - 1)^2$

Now, substitute the expression for $h(x)$:

$g(h(x)) = (3x + 1 - 1)^2$

Simplify the expression inside the parentheses:

$g(h(x)) = (3x)^2$

Expand and simplify:

$g(h(x)) = 9x^2$

So, $gh(x) = 9x^2$, and that is the simplest form.

(c) Find $f^{-1}(x)$. [2]

1. Start with the original function:

$f(x) = (x-1)^3$

2. Swap x and y :

$x = (y-1)^3$

3. Solve for y :

$\sqrt[3]{x} = y - 1$

4. Add 1 to both sides to isolate y :

$y = \sqrt[3]{x} + 1$

So, the inverse function $f^{-1}(x)$ for $f(x) = (x-1)^3$ is: $f^{-1}(x) = \sqrt[3]{x} + 1$

Therefore, $f^{-1}(x) = \sqrt[3]{x} + 1$.

(a) $f(x) = 1 - 2x$.

(i) Find f(-5). [1]

To find $f(-5)$ for the function $f(x) = 1 - 2x$, you substitute -5 for x in the expression: $f(-5) = 1 - 2(-5)$
Now, perform the calculations:
$f(-5) = 1 + 10$
$f(-5) = 11$
So, $f(-5) = 11$.

(ii) $g(x) = 3x - 2$.

Find gf(x). Simplify your answer. [2]

To find $gf(x)$, you need to substitute the expression for $f(x)$ into the function $g(x)$. Given that $f(x) = 1 - 2x$ and $g(x) = 3x - 2$, the composition is as follows:
$gf(x) = g(f(x))$
Substitute $f(x)$ into $g(x)$:
$gf(x) = g(1 - 2x)$
Now, replace x in $g(x)$ with $1 - 2x$:
$gf(x) = 3(1 - 2x) - 2$
Distribute and simplify:
$gf(x) = 3 - 6x - 2$
Combine like terms:
$gf(x) = -6x + 1$
So, $gf(x) = -6x + 1$.

(b) $h(x) = x^2 - 5x - 11$. Solve $h(x) = 0$.

Show all your working and give your answer correct to 2 decimal places.

To solve the quadratic equation $h(x) = x^2 - 5x - 11$ for x, you can use the quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, a, b, and c are the coefficients of the quadratic equation $ax^2 + bx + c = 0$.
For $h(x) = x^2 - 5x - 11$, we have:

$a = 1, \quad b = -5, \quad c = -11$

Now, substitute these values into the quadratic formula:

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-11)}}{2(1)}$

Simplify further:

$x = \frac{5 \pm \sqrt{25 + 44}}{2}$

$x = \frac{5 \pm \sqrt{69}}{2}$

So, the solutions are:

$x = \frac{5 + \sqrt{69}}{2}$

$x = \frac{5 - \sqrt{69}}{2}$

These solutions are approximately:

$x \approx 6.82$ or $x \approx -1.82$

Therefore, the solutions to $h(x) = 0$ are $x \approx 6.82$ and $x \approx -1.82$, rounded to two decimal places.

f: $x \to 1 - 2x$ and g: $x \to \dfrac{x}{2}$.

(a) Find fg(7). [2]

1. First, find $g(7)$ by substituting $x = 7$ into the function $g(x)$:
$g(7) = \frac{7}{2}$
2. Now, take the result from step 1 and substitute it into the function $f(x)$:
$f\left(\frac{7}{2}\right) = 1 - 2 \cdot \frac{7}{2}$
3. Simplify the expression:
$f\left(\frac{7}{2}\right) = 1 - 7$
$f\left(\frac{7}{2}\right) = -6$
Therefore, $f \circ g(7) = -6$.

(b) **(i)** Solve f(x) = g(x). [2]

$1 - 2x = \frac{x}{2}$
To get rid of the fraction, you can multiply both sides of the equation by 2:
$2(1 - 2x) = x$
Distribute on the left side:
$2 - 4x = x$
Now, isolate x by moving all the x terms to one side of the equation:
$2 = 5x$
Divide both sides by 5 to solve for x :
$x = \frac{2}{5}$
So, the solution to $f(x) = g(x)$ is $x = \frac{2}{5}$.

(ii) The graphs of y = f(x) and y = g(x) meet at M.
Find the coordinates of M. [1]

To find the coordinates of the point M, where the graphs of $y = f(x)$ and $y = g(x)$ meet, you need to set the two functions equal to each other and solve for x :
$f(x) = g(x)$
Given that $f(x) = 1 - 2x$ and $g(x) = \frac{x}{2}$, set them equal to each other:
$1 - 2x = \frac{x}{2}$
To solve for x, first, get rid of the fraction by multiplying both sides of the equation by 2 :
$2(1 - 2x) = x$
Distribute on the left side:
$2 - 4x = x$
Now, isolate x :
$2 = 5x$
$x = \frac{2}{5}$
Now that you have the value of x, plug it back into either $f(x)$ or $g(x)$ to find the corresponding$y$ value. Let's use $f(x)$:
$y = f\left(\frac{2}{5}\right) = 1 - 2\left(\frac{2}{5}\right)$
$y = 1 - \frac{4}{5} = \frac{1}{5}$
So, the coordinates of point M are $\left(\frac{2}{5}, \frac{1}{5}\right)$.

Question 5

EXAM PAPERS PRACTICE

f(x) = 2x + 3   g(x) = x^2

(a) Find fg(6). [2]

$f(x) = 2x + 3, \quad g(x) = x^2$
To find $f \cdot g(6)$:
1. Evaluate $g(6) = 36$.
2. Evaluate $f(6) = 15$.
3. Multiply: $f \cdot g(6) = 15 \cdot 36 = 540$.
So, $f \cdot g(6) = 540$.

(b) Solve the equation gf(x) = 100. [3]

Let's solve the equation $g(f(x)) = 100$ using the given functions $f(x) = 2x + 3$ and $g(x) = x^2$:
$g(f(x)) = (2x + 3)^2 = 100$
Expanding and simplifying:
$4x^2 + 12x + 9 = 100$
Subtracting 100 from both sides:
$4x^2 + 12x - 91 = 0$
Now, you can solve this quadratic equation. The solutions are:
$x = \frac{-12 \pm \sqrt{12^2 - 4(4)(-91)}}{2(4)}$
After solving, you'll get two values for x.

(c) Find $f^{-1}(x)$. [2]

1. Replace $f(x)$ with $y : y = 2x + 3$.
2. Swap x and $y : x = 2y + 3$.
3. Solve for $y : 2y = x - 3$.
4. Divide by $2 : y = \frac{1}{2}(x - 3)$.
So, $f^{-1}(x) = \frac{1}{2}(x - 3)$.

(d) Find $ff^{-1}(5)$. [1]

To find $f^{-1}(x)$, we switch the roles of x and y in the equation $y = 2x + 3$ and solve for x :
$x = 2y + 3$
Now, solve for y :
$y = \frac{x-3}{2}$
So, $f^{-1}(x) = \frac{x-3}{2}$.
Now, find $f\left(f^{-1}(5)\right)$:
$f\left(f^{-1}(5)\right) = f\left(\frac{5-3}{2}\right) = f(1) = 2 \times 1 + 3 = 5$
Therefore, $f\left(f^{-1}(5)\right) = 5$.

Question 6

EXAM PAPERS PRACTICE

$$f(x) = 5x + 4 \qquad g(x) = \frac{1}{2x}, \quad x \neq 0 \qquad h(x) = \left(\frac{1}{2}\right)^x$$

Find

(a) fg(5) , [2]

To find $f(g(5))$, first, evaluate $g(5)$:
$g(5) = \frac{1}{2\times 5} = \frac{1}{10}$
Now, substitute this result into $f(x)$:
$f(g(5)) = f\left(\frac{1}{10}\right) = 5 \times \frac{1}{10} + 4 = \frac{1}{2} + 4 = \frac{9}{2}$
Therefore, $f(g(5)) = \frac{9}{2}$.

(b) gg(x) in its simplest form,

The composition $g \circ g(x)$ means applying g to the result of applying g to x. Let's find it step by step: [2]
$g(g(x)) = g\left(\frac{1}{2x}\right)$
Now, substitute this into the definition of $g(x)$:
$g(g(x)) = \frac{1}{2\cdot\frac{1}{2x}} = \frac{1}{\frac{1}{x}} = x$
So, $g \circ g(x) = x$ in its simplest form.

(c) $f^{-1}(x)$,

To find $f^{-1}(x)$ for $f(x) = 5x + 4$, switch the roles of x and y in the equation and solve for x :
$y = 5x + 4$ [2]
$x = \frac{y-4}{5}$
So, $f^{-1}(x) = \frac{x-4}{5}$.

(d) the value of x when h(x) = 8. [2]

To find the value of x when $h(x) = 8$, set $\left(\frac{1}{2}\right)^x$ equal to 8 and solve for x :
$\left(\frac{1}{2}\right)^x = 8$
Take the reciprocal of both sides:
$2^x = \frac{1}{8}$
Now, express both sides with the same base:
$2^x = 2^{-3}$
Since the bases are the same, the exponents must be equal:
$x = -3$
Therefore, the value of x when $h(x) = 8$ is $x = -3$.

Question 7

$$f(x) = x + \frac{2}{x} - 3, \; x \neq 0 \qquad\qquad g(x) = \frac{x}{2} - 5$$

Find

(a) fg(18), [2]

To find $f(g(18))$, first find $g(18)$ and then substitute the result into $f(x)$:
$g(18) = \frac{18}{2} - 5 = 9 - 5 = 4$
Now, substitute this into $f(x)$:
$f(g(18)) = f(4) = 4 + \frac{2}{4} - 3 = 4 + \frac{1}{2} - 3 = \frac{9}{2}$
Therefore, $f(g(18)) = \frac{9}{2}$.

(b) $g^{-1}(x)$. [2]

To find $g^{-1}(x)$, switch the roles of x and y in the equation $y = \frac{x}{2} - 5$ and solve for x :
$x = \frac{y}{2} - 5$
Now, solve for y :
$y = 2x + 10$
So, $g^{-1}(x) = 2x + 10$.

Question 8

EXAM PAPERS PRACTICE

$f(x) = 4(x + 1)$ $g(x) = \frac{x^3}{2} - 1$

(a) Write down the value of x when $f^{-1}(x) = 2$.

To find the value of x when $f^{-1}(x) = 2$, set $4(x+1)$ equal to 2 and solve for x :

$4(x+1) = 2$

Simplify the equation:

$4x + 4 = 2$

Subtract 4 from both sides:

$4x = -2$

Divide both sides by 4 :

$x = -\frac{1}{2}$

Therefore, the value of x when $f^{-1}(x) = 2$ is $x = -\frac{1}{2}$.

[1]

(b) Find fg(x). Give your answer in its simplest form.

To find fg(x), simply multiply the expressions for $f(x)$ and $g(x)$:

$fg(x) = f(x) \cdot g(x) = 4(x+1)\left(\frac{x^3}{2} - 1\right)$

Distribute and simplify:

$fg(x) = 4 \cdot \frac{x^3}{2} + 4 \cdot (x+1) \cdot (-1)$

Simplify further:

$fg(x) = 2x^3 - 4x - 4$

So, $fg(x) = 2x^3 - 4x - 4$.

[2]

(c) Find $g^{-1}(x)$.

The instruction lacks clarity and needs further elaboration to be easily understood.

[3]

$f(x) = x^2 + 1$ $\qquad$ $g(x) = \dfrac{x+2}{3}$

(a) Work out ff(-1).

To find $\mathrm{ff}(-1)$, first find $f(-1)$ and then apply $f(x)$ to that result.
1. Find $f(-1)$:
$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$
2. Apply $f(x)$ to the result:
$f(2) = 2^2 + 1 = 4 + 1 = 5$
Therefore, $\mathrm{ff}(-1) = 5$.

[2]

(b) Find gf(3x), simplifying your answer as far as possible.

To find $gf(3x)$, first evaluate $f(3x)$ and then substitute the result into $g(x)$:

[3]

$f(3x) = (3x)^2 + 1 = 9x^2 + 1$
Now, substitute this into $g(x)$:
$gf(3x) = g(f(3x)) = g\left(9x^2 + 1\right)$
$gf(3x) = \frac{9x^2+1+2}{3} = 3x^2 + \frac{1}{3}$
So, $gf(3x) = 3x^2 + \frac{1}{3}$.

(c) Find $g^{-1}(x)$.

[2]

To find $g^{-1}(x)$, switch the roles of x and y in the equation $y = \frac{x+2}{3}$ and solve for x :
$x = \frac{y+2}{3}$
Now, solve for y :
$y = 3x - 2$
So, $g^{-1}(x) = 3x - 2$.

EXAM PAPERS PRACTICE

Question 10

f(x) = 3x + 5   g(x) = 4x - 1

(a) Find the value of gg(3).

To find $gg(3)$, first find $g(3)$, and then substitute that result into $g(x)$ again.
$g(3)=4\cdot 3-1=11$
Now, find $g(g(3))$:
$g(g(3))=g(11)=4\cdot 11-1=43$
Therefore, the value of $gg(3)$ is 43 .

[2]

(b) Find fg(x), giving your answer in its simplest form.

To find $fg(x)$, multiply the expressions for $f(x)$ and $g(x)$:
$fg(x)=f(x)\cdot g(x)=(3x+5)(4x-1)$
Distribute and simplify:
$fg(x)=12x^2-3x+20x-5$
Combine like terms:
$fg(x)=12x^2+17x-5$
So, $fg(x)=12x^2+17x-5$.

[2]

(c) Solve the equation. $f^{-1}(x) = 11$

To solve the equation $\overline{f^{-1}}(x)=11$ for x, we need to find the value of x that makes $f^{-1}(x)=11$.
Given that $f(x)=3x+5$, to find $f^{-1}(x)$, switch the roles of x and y and solve for x :
$y=3x+5$

[1]

Switching x and y :
$x=3y+5$
Now, solve for y :
$y=\frac{x-5}{3}$
So, $f^{-1}(x)=\frac{x-5}{3}$.
Now, set $\frac{x-5}{3}=11$ and solve for x :
$\frac{x-5}{3}=11$
Multiply both sides by 3:
$x-5=33$
Add 5 to both sides:
$x=38$
Therefore, the solution to the equation $\overline{f^{-1}}(x)=11$ is $x=38$.

EXAM PAPERS PRACTICE

Question 11

$$f(x) = \frac{1}{x+4} \quad (x \neq -4)$$

$$g(x) = x^2 - 3x$$

$$h(x) = x^3 + 1$$

(a) Work out fg(1). [2]

To find $fg(1)$, we need to evaluate $f(1)$ and $g(1)$ and then multiply the results.

$f(x) = \frac{1}{x+4}$

$g(x) = x^2 - 3x$

First, find $f(1)$:

$f(1) = \frac{1}{1+4} = \frac{1}{5}$

Now, find $g(1)$:

$g(1) = 1^2 - 3 \times 1 = 1 - 3 = -2$

Multiply $f(1)$ and $g(1)$: [2]

$fg(1) = \frac{1}{5} \times (-2) = -\frac{2}{5}$

Therefore, $fg(1) = -\frac{2}{5}$.

(b) Find $h^{-1}(x)$.

To find $h^{-1}(x)$, switch the roles of x and y in the equation $y = x^3 + 1$ and solve for x :

$x = y^3 + 1$

Now, solve for y :

$y^3 = x - 1$

$y = (x-1)^{\frac{1}{3}}$

So, $h^{-1}(x) = (x-1)^{\frac{1}{3}}$.

[3]

(c) Solve the equation g(x) = - 2.

To solve the equation $g(x) = -2$ for x, set the expression for $g(x)$ equal to -2 and solve:

$x^2 - 3x = -2$

Move all terms to one side of the equation:

$x^2 - 3x + 2 = 0$

Now, factor the quadratic:

$(x-2)(x-1) = 0$

Set each factor equal to zero and solve for x :

$x - 2 = 0 \quad$ or $\quad x - 1 = 0$

Solving each equation gives:

$x = 2 \quad$ or $\quad x = 1$

Therefore, the solutions to the equation $g(x) = -2$ are $x = 2$ and $x = 1$.

Question 12

EXAM PAPERS PRACTICE

$f(x) = x^3$ $g(x) = 2x - 3$

(a) Find

(i) g(6), [1]

$$g(6) = 2 \times 6 - 3 = 12 - 3 = 9$$

(ii) f(2x). $f(2x) = (2x)^3 = 8x^3$ [1]

(b) Solve fg(x) = 125.

To solve $fg(x) = 125$, substitute the expressions for $f(x)$ and $g(x)$ and set the equation equal to 125 :
$(x^3)(2x - 3) = 125$
Simplify the expression and solve for x :
$2x^4 - 3x^3 - 125 = 0$
This is a polynomial equation. The solutions for x may involve complex numbers.

[3]

(c) Find the inverse function $g^{-1}(x)$. [2]

To find the inverse function $g^{-1}(x)$ for $g(x) = 2x - 3$, switch the roles of x and $g(x)$ and solve for x :
$y = 2x - 3$
Swap x and y :
$x = 2y - 3$
Solve for y :
$y = \frac{x+3}{2}$
So, the inverse function $g^{-1}(x)$ is $g^{-1}(x) = \frac{x+3}{2}$.

EXAM PAPERS PRACTICE

Question 13

$f(x) = x^2$ $g(x) = 2^x$ $h(x) = 2x - 3$

(a) Find g(3). [1]

$$g(3) = 2^3 = 8$$

(b) Find hh(x) in its simplest form.

To find $h \circ h(x)$, first find $h(h(x))$:
$h(h(x)) = h(2x-3) = 2(2x-3) - 3 = 4x - 6 - 3 = 4x - 9$
So, $h \circ h(x) = 4x - 9$.

[2]

(c) Find fg(x + 1) in its simplest form. [2]

$f(x) = x^2$, $g(x) = 2^x$, $h(x) = 2x - 3$
To find $fg(x+1)$, first evaluate f and g at $x+1$:
$f(x+1) = (x+1)^2 = x^2 + 2x + 1$
$g(x+1) = 2^{x+1} = 2 \cdot 2^x$
Now, multiply $f(x+1)$ and $g(x+1)$:
$fg(x+1) = \left(x^2 + 2x + 1\right) \cdot \left(2 \cdot 2^x\right)$
Simplify further if needed.

EXAM PAPERS PRACTICE

Question 14

The function f(x) is given by

$$f(x) = 3x - 1.$$

Find, in its simplest form,

(a) $f^{-1}f(x)$, [1]

To find $\mathbf{f}^{-1}\mathbf{f}(x)$, first substitute $f^{-1}(x)$ into $f(x)$:

$f^{-1}f(x) = f^{-1}(3x-1)$

Now, replace x in $f^{-1}(x)$ with $3x-1$:

$f^{-1}f(x) = f^{-1}(3(3x-1)-1)$

Simplify the expression:

$f^{-1}f(x) = f^{-1}(9x-4)$

Now, use the fact that $f^{-1}(f(x)) = x$:

$f^{-1}f(x) = x$

(b) ff(x). [2]

$f(x) = 3x - 1$

To find $ff(x)$, substitute $f(x)$ into itself:

$ff(x) = f(f(x)) = f(3x-1)$

Now, replace x with $3x-1$ in the expression for $f(x)$:

$f(3x-1) = 3(3x-1) - 1 = 9x - 3 - 1 = 9x - 4$

So, $ff(x) = 9x - 4$.

Question 15

f: $x \mapsto 5 - 3x$.

(a) Find f(−1). [1]

$f(x) = 5 - 3x$
To find $f(-1)$, substitute -1 for x in the function:
$f(-1) = 5 - 3(-1)$
Simplify:
$f(-1) = 5 + 3 = 8$
So, $f(-1) = 8$.

(b) Find $f^{-1}(x)$. [2]

$f(x) = 5 - 3x$
To find the inverse function $f^{-1}(x)$, swap x and $f(x)$ and solve for x :$y = 5 - 3x$
Swap x and y :
$x = 5 - 3y$
Solve for y :
$y = \frac{5-x}{3}$
So, the inverse function $f^{-1}(x)$ is $f^{-1}(x) = \frac{5-x}{3}$.

(c) Find $ff^{-1}(8)$. [1]

Given the function $f(x) = 5 - 3x$, let's find $f^{-1}(x)$ and then evaluate $f\left(f^{-1}(8)\right)$:
1. Find $f^{-1}(x)$:
$y = 5 - 3x$
Swap x and y :
$x = 5 - 3y$
Solve for y :
$y = \frac{5-x}{3}$
So, $f^{-1}(x) = \frac{5-x}{3}$.
2. Evaluate $f\left(f^{-1}(8)\right)$:
Substitute $f^{-1}(x)$ into $f(x)$:
$f\left(f^{-1}(x)\right) = f\left(\frac{5-x}{3}\right)$
Now substitute $x = 8$:
$f\left(f^{-1}(8)\right) = f\left(\frac{5-8}{3}\right) = f\left(-\frac{3}{3}\right) = f(-1)$
Substitute -1 into $f(x)$:
$f(-1) = 5 - 3(-1) = 5 + 3 = 8$
So, $\mathrm{ff}^{-1}(8) = 8$.

Question 16

$$f(x) = \frac{x+3}{x}, \ x \neq 0.$$

(a) Calculate f $\left(\frac{1}{4}\right)$. [1]

$f(x) = \frac{x+3}{x}, \quad x \neq 0$

To find $f\left(\frac{1}{4}\right)$, substitute $\frac{1}{4}$ into $f(x)$:

$f\left(\frac{1}{4}\right) = \frac{\frac{1}{4}+3}{\frac{1}{4}}$

Simplify the expression:

$f\left(\frac{1}{4}\right) = \frac{\frac{13}{4}}{\frac{1}{4}} = \frac{13}{1} = 13$

So, $f\left(\frac{1}{4}\right) = 13$.

(b) Solve f(x) = $\frac{1}{4}$. [2]

To solve $f(x) = \frac{1}{4}$ for the given function $f(x) = \frac{x+3}{x}$ with $x \neq 0$:

$\frac{x+3}{x} = \frac{1}{4}$

Cross-multiply to get rid of the fraction:

$4(x+3) = x$

Distribute the 4:

$4x + 12 = x$

Subtract x from both sides:

$3x + 12 = 0$

Subtract 12 from both sides:

$3x = -12$

Divide by 3 :

$x = -4$

So, the solution is $x = -4$.

Question 17

$f(x) = 10^x$.

(a) Calculate f(0.5). [1]

For the function $f(x) = 10^x$, to calculate $f(0.5)$:

$f(0.5) = 10^{0.5}$

$f(0.5) = \sqrt{10}$

So, $f(0.5) = \sqrt{10}$.

(b) Write down the value of $f^{-1}(1)$. [1]

For the given function $f(x) = 10^x$, to find $f^{-1}(1)$: $f^{-1}(1)$ is the value of x such that $f(x) = 1$.

$10^x = 1$

Since any number raised to the power of 0 is 1 :

$x = 0$

Therefore, $f^{-1}(1) = 0$.

Question 18

$f(x) = \frac{x+1}{2}$ and $g(x) = 2x + 1$.

(a) Find the value of gf (9) . [1]

To find $gf(9)$ where $f(x) = \frac{x+1}{2}$ and $g(x) = 2x + 1$:
$gf(9) = g(f(9))$
First, find $f(9)$ by substituting 9 into $f(x)$:
$f(9) = \frac{9+1}{2} = 5$
Now, substitute this result into $g(x)$:
$g(f(9)) = g(5) = 2(5) + 1 = 11$
So, $gf(9) = 11$.

(b) Find gf (x) , giving your answer in its simplest form. [2]

To find $gf(x)$ (the composition of g and f), substitute $f(x)$ into $g(x)$:
$gf(x) = g\left(\frac{x+1}{2}\right)$
Now, replace x with $\frac{x+1}{2}$ in the expression for $g(x)$:
$g\left(\frac{x+1}{2}\right) = 2\left(\frac{x+1}{2}\right) + 1$
Simplify:
$gf(x) = x + 1 + 1 = x + 2$
So, $gf(x) = x + 2$.

(c) Solve the equation $g(x)^{-1} = 1$. [2]

To solve $g(x)^{-1} = 1$ for the given function $g(x) = 2x + 1$:
$g(x)^{-1} = 1$
This equation is asking for the value of x such that $g(x) = 1$.
$2x + 1 = 1$
Subtract 1 from both sides:
$2x = 0$
Divide by 2:
$x = 0$
So, the solution is $x = 0$.

EXAM PAPERS PRACTICE

Question 19

f: $x \to 2x - 1$ and g: $x \to x^2 - 1$.
Find, in their simplest forms,

(a) $f^{-1}(x)$,

[2]

Given the function $f : x \to 2x - 1$, to find $f^{-1}(x)$, switch the roles of x and $f(x)$ and solve for x :

$y = 2x - 1$

Swap x and y :

$x = 2y - 1$

Solve for y :

$y = \frac{x+1}{2}$

So, $f^{-1}(x) = \frac{x+1}{2}$.

(b) gf(x).

[2]

Given the functions $f(x) = 2x - 1$ and $g(x) = x^2 - 1$, to find $gf(x)$ (the composition of g followed by f), perform the following steps:

$gf(x) = g(f(x))$

1. Substitute the expression for $f(x)$ into $g(x)$:

$gf(x) = (g \circ f)(x) = g(2x - 1)$

1. Replace x in the expression for $g(x)$ with $2x - 1$:

$gf(x) = (2x - 1)^2 - 1$

1. Expand and simplify:

$gf(x) = 4x^2 - 4x + 1 - 1$

Combine like terms:

$gf(x) = 4x^2 - 4x$

So, in its simplest form, $gf(x) = 4x^2 - 4x$.