

IB Maths: AA HL

Functions Toolkit

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	2. Functions
Topic	2.3 Functions Toolkit
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Functions Toolkit

Question 1

The functions f and g are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$.

[2 marks]

(b) Given that $(g \circ f)(a) = 27$, find the value of a .

[2 marks]

(c) Show that $(f \circ g)(x) = 2x + 6$.

[2 marks]

(d) Given that $(f \circ g)(b) = 44$, find the value of b .

[2 marks]

Question 2

The functions $f(x)$ and $g(x)$ are defined as follows

$$\begin{array}{ll} f(x) = x^2 & x \in \mathbb{R} \\ g(x) = 4x - 3 & x \in \mathbb{R} \end{array}$$

(a) Write down the range of $f(x)$.

[1 mark]

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

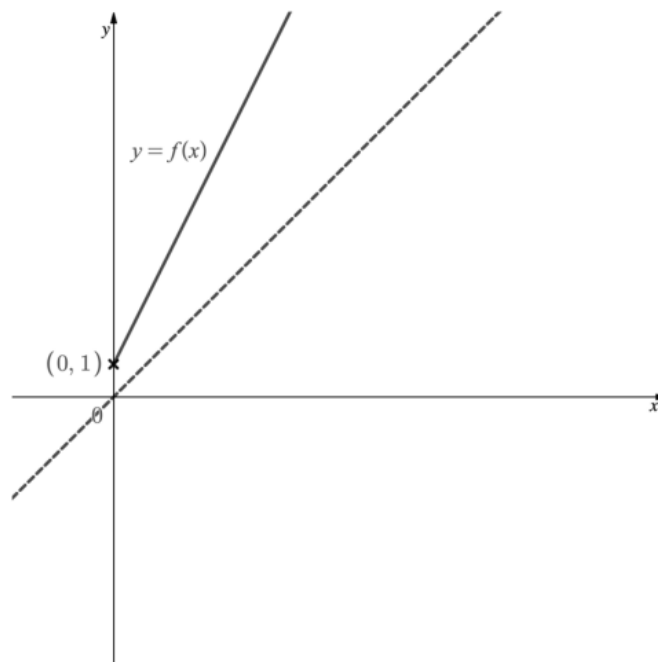
[4 marks]

(c) Solve the equation $f(x) = g(x)$.

[2 marks]

Question 3

The graph of $y = f(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $f(x)$.

(ii) Given that the point $(1, 1)$ lies on the dotted line, write down the equation of the line.

[3 marks]

(b) On the diagram above sketch the graph of $y = f^{-1}(x)$.

[2 marks]

Question 4

The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \quad x \in \mathbb{R}, x \neq 0$$

(a) Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

[2 marks]

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2 marks]

(c) The domain of $f(x)$ is changed to $x > 0$.

Find an expression for $f^{-1}(x)$ and state its domain and range.

[4 marks]

Question 5

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = \frac{1}{2}(4x - 3) \quad x \in \mathbb{R}$$

$$g(x) = 0.5x + 0.75 \quad x \in \mathbb{R}$$

(a) Find
(i) $fg(x)$
(ii) $gf(x)$

[3 marks]

(b) Write down $f^{-1}(x)$ and state its domain and range.

[3 marks]

Question 6

A function is defined by $f(x) = 54x - 13$, $-2 < x < 20$.

(a) Find the value of $f\left(\frac{5}{2}\right)$.

[1 mark]

(b) Write down the range of $f(x)$.

[2 marks]

(c) Find the inverse function $f^{-1}(x)$.

[2 marks]

(d) Write down the range of the inverse function.

[1 mark]

Question 7

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \leq x \leq 3$.

(a) Find

(i) $f(2)$

(ii) x when $f(x) = 15$.

[2 marks]

(b) Find the range of $f(x)$.

[2 marks]

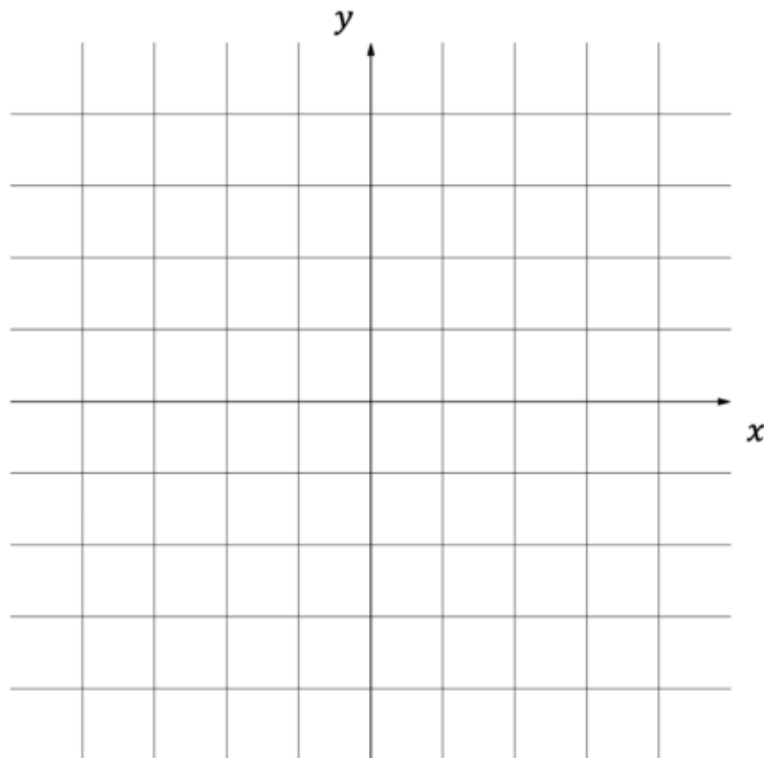
(c) Write down the domain of the inverse function.

[3 marks]

Question 8

Consider the function $g(x) = \sqrt{4 - x}$.

(a) Sketch the graph of the function $g(x)$, labelling the x and y intercepts.



[3 marks]

(b) Find

(i) $g(-5)$

(ii) x when $g(x) = \frac{1}{2}$.

[2 marks]

(c) Find

(i) the maximum possible domain of the function $g(x)$

(ii) the range of the function $g(x)$ that corresponds to the domain found in part (c) (i).

[2 marks]

Question 9

The functions f and g are defined for $x \in R$ by $f(x) = 3x^2 + 10x + 7$ and $g(x) = x + d$, where $d \in R$.

(a) Find the range of f .

[2 marks]

(b) Given that $(g \circ f)(x)$ is always positive for all x , determine the set of possible values for d .

[4 marks]

Question 10

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

- (i) the value of a
- (ii) the range of f .

[2 marks]

(b) For the graph of f , find the equations of all the asymptotes.

[1 mark]

(c) Find $f^{-1}(x)$.

[2 marks]

(d) For the graph of f^{-1} , find the equation of

- (i) the horizontal asymptote
- (ii) the vertical asymptote.

[2 marks]

Question 11

Determine, for each of the following functions, whether they are even, odd or neither:

(i) $f(x) = \frac{1}{x^2} + 2$

(ii) $g(x) = x^3 - 3x$

(iii) $h(x) = x^2 + 2x - 5.$

[5 marks]

Question 12

Prove that the sum of two odd functions is also an odd function.

[5 marks]

Question 13

Let $f(x) = \frac{\pi^2}{x}$, where $x \neq 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function.

[2 marks]

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \neq p, x \in \mathbb{R}$.

(b) Find the value of p .

[1 mark]

(c) Show that $g(x)$ is a self-inverse function.

[3 marks]

Question 14

Consider the function f defined by $f(x) = 2x^3 + 3x^2 - 36x + 7$, $x \in \mathbb{R}$.

(a) Sketch the graph of f . Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima.

[2 marks]

Let the function g be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7$, $x \leq p$.

(b) Given that g has an inverse:

- (i) Find the largest possible value of p
- (ii) Find the domain of g^{-1} for the value of p identified in part (b)(i)
- (iii) Find the value of $g^{-1}(0)$.

[3 marks]

Let the function h be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7$, $x \geq q$.

(c) Given that h has an inverse:

- (i) Find the smallest possible value of q
- (ii) Find the domain of h^{-1} for the value of q identified in part (c)(i)
- (iii) Find the value of $h^{-1}(0)$.

[3 marks]