## Question 1

The functions $f$ and $g$ are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$. [2]

(b) Given that $(g \circ f)(a) = 27$, find the value of $a$. [2]

(c) Show that $(f \circ g)(x) = 2x + 6$. [2]

(d) Given that $(f \circ g)(b) = 44$, find the value of $b$. [2]

a) Sub $(4x - 10)$ into $g(x)$.

$$(g \circ f)(x) = \frac{(4x - 10) + 8}{2}$$

$$(g \circ f)(x) = \frac{4x - 2}{2}$$

$$\boxed{(g \circ f)(x) = 2x - 1}$$

The functions $f$ and $g$ are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$. $\boxed{(g \circ f)(x) = 2x - 1}$ [2]

(b) Given that $(g \circ f)(a) = 27$, find the value of $a$. [2]

(c) Show that $(f \circ g)(x) = 2x + 6$. [2]

(d) Given that $(f \circ g)(b) = 44$, find the value of $b$. [2]

b) Set $(g \circ f)(x) = 27$ and solve for $x$.

$$2x - 1 = 27$$

$$x = 14$$

$$\boxed{\therefore a = 14}$$

The functions $f$ and $g$ are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$. [2]

(b) Given that $(g \circ f)(a) = 27$, find the value of $a$. [2]

(c) Show that $(f \circ g)(x) = 2x + 6$. [2]

(d) Given that $(f \circ g)(b) = 44$, find the value of $b$. [2]

c) Sub $\frac{x+8}{2}$ into $f(x)$.

$(f \circ g)(x) = \cancel{4}^{2}\left(\frac{x+8}{\cancel{2}}\right) - 10$

$(f \circ g)(x) = 2(x + 8) - 10$

$(f \circ g)(x) = 2x + 16 - 10$

$\boxed{(f \circ g)(x) = 2x + 6}$

The functions $f$ and $g$ are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$. [2]

(b) Given that $(g \circ f)(a) = 27$, find the value of $a$. [2]

(c) Show that $(f \circ g)(x) = 2x + 6$. $\boxed{(f \circ g)(x) = 2x + 6}$ [2]

(d) Given that $(f \circ g)(b) = 44$, find the value of $b$. [2]

d) Set $(f \circ g)(x) = 44$ and solve for $x$.

$2x + 6 = 44$

$x = 19$

$\boxed{\therefore b = 19}$

EXAM PAPERS PRACTICE

## Question 2

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 \qquad x \in R$$

$$g(x) = 4x - 3 \qquad x \in R$$

(a) Write down the range of $f(x)$. [1]

a) $x^2 \geqslant 0$ ALWAYS POSITIVE

RANGE = OUTPUT VALUES

$\boxed{f(x) \geqslant 0}$

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

[4]

(c) Solve the equation $f(x) = g(x)$. [2]

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 \qquad x \in R$$

$$g(x) = 4x - 3 \qquad x \in R$$

(a) Write down the range of $f(x)$. [1]

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

COMPOSITE FUNCTIONS

[4]

(c) Solve the equation $f(x) = g(x)$. [2]

b)

i) $f \circ g(x)$

SUBSTITUTE $g(x)$ INTO $f(x)$

$(4x-3)^2$

$\boxed{f \circ g(x) = 16x^2 - 24x + 9}$

ii) $g \circ f(x)$

SUBSTITUTE $f(x)$ INTO $g(x)$

$4(x^2) - 3$

$\boxed{g \circ f(x) = 4x^2 - 3}$

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 \qquad x \in R$$

$$g(x) = 4x - 3 \qquad x \in R$$

(a) Write down the range of $f(x)$.

[1]

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

[4]

(c) Solve the equation $f(x) = g(x)$.

[2]

c)

$x^2 = 4x - 3$

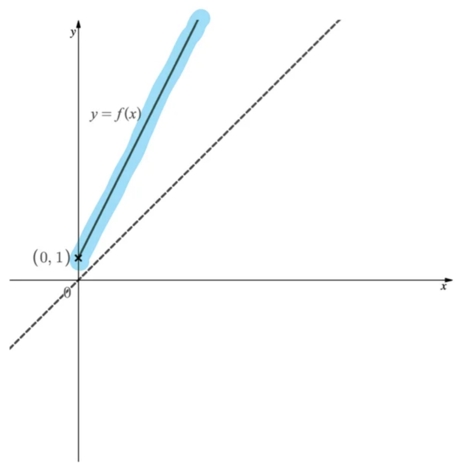
$x^2 - 4x + 3 = 0$ FACTORISE OR USE CALCULATOR

$(x-1)(x-3) = 0$

$\boxed{x = 1 \qquad x = 3}$

## Question 3

The graph of $y = \mathrm{f}(x)$ is shown below.




(a) (i) Use the graph to write down the domain and range of $\mathrm{f}(x)$.

(ii) Given that the point $(1, 1)$ lies on the dotted line, write down the equation of the line.

[3]

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

[2]

a)

i) DOMAIN INPUT VALUES

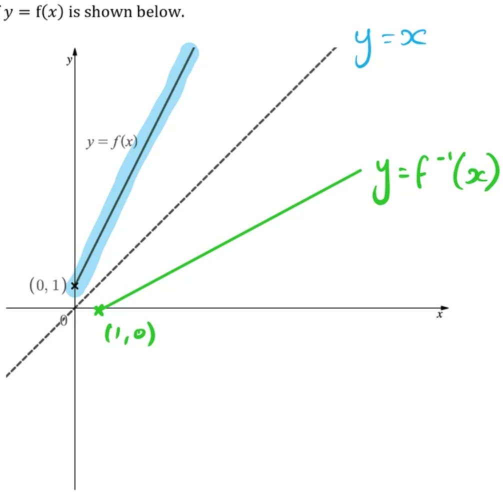
$\boxed{x \geq 0 \qquad x \in R}$

RANGE OUTPUT VALUES

$\boxed{f(x) \geq 1}$

ii) $\boxed{y = x}$

The graph of $y = \mathrm{f}(x)$ is shown below.




b) INVERSE FUNCTION

REFLECTION IN $y = x$

(a) (i) Use the graph to write down the domain and range of $\mathrm{f}(x)$.

(ii) Given that the point $(1, 1)$ lies on the dotted line, write down the equation of the line.

[3]

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

[2]

## Question 4

The function $\mathrm{f}(x)$ is defined as

$$\mathrm{f}: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

(a) Show that $\mathrm{f}(x)$ can be written in the form

$$\mathrm{f}: x \mapsto 1 + \frac{1}{x^2}$$

[2]

a) $f(x) = \frac{x^2}{x^2} + \frac{1}{x^2}$

$= \boxed{1 + \frac{1}{x^2}}$

(b) Explain why the inverse of $\mathrm{f}(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2]

(c) The domain of $\mathrm{f}(x)$ is changed to $x > 0$.
Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain and range.

[4]

The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

(a) Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

[2]

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2]

(c) The domain of $f(x)$ is changed to $x > 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

[4]

b)

THE FUNCTION $f(x)$ IS MANY TO ONE SO INVERSE (ONE TO MANY) DOES NOT EXIST

FUNCTIONS MUST BE ONE TO ONE FOR INVERSE TO EXIST SO WE CAN CHANGE DOMAIN TO

$x > 0$ OR $x < 0$

The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

(a) Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

[2]

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2]

(c) The domain of $f(x)$ is changed to $x > 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

[4]

c) LET $y = f(x)$

$$y = 1 + \frac{1}{x^2}$$

REARRANGE

$$y - 1 = \frac{1}{x^2}$$

$$x^2(y-1) = 1$$

$$x^2 = \frac{1}{y-1}$$

$$x = \sqrt{\frac{1}{y-1}} \quad \text{OR} \quad x = \frac{1}{\sqrt{y-1}}$$

$$f^{-1}(x) = \sqrt{\frac{1}{x-1}} \quad \text{OR} \quad \frac{1}{\sqrt{x-1}}$$

DOMAIN $x > 1$ RANGE $f(x) > 0$

## Question 5

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = \frac{1}{2}(4x - 3) \qquad x \in \mathbb{R}$$

$$g(x) = 0.5x + 0.75 \qquad x \in \mathbb{R}$$

(a) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

[3]

(b) Write down $f^{-1}(x)$ and state its domain and range.

[3]

a)

i) $f \circ g(x)$ SUBSTITUTE $g(x)$ INTO $f(x)$

$$\frac{1}{2}\left(4\left(\frac{1}{2}x + \frac{3}{4}\right) - 3\right)$$

$$\cancel{\frac{1}{2}}(\cancel{2}x \cancel{+3} \cancel{-3})$$

$$\boxed{f \circ g(x) = x}$$

ii) $g \circ f(x)$ SUBSTITUTE $f(x)$ INTO $g(x)$

$$\frac{1}{2}\left(\frac{1}{2}(4x - 3)\right) + \frac{3}{4}$$

$$\frac{1}{\cancel{4}}(\cancel{4}x - 3) + \frac{3}{4}$$

$$x - \cancel{\frac{3}{4}} + \cancel{\frac{3}{4}}$$

$$\boxed{g \circ f(x) = x}$$

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = \frac{1}{2}(4x - 3) \qquad x \in \mathbb{R}$$

$$g(x) = 0.5x + 0.75 \qquad x \in \mathbb{R}$$

(a) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

$$\boxed{f \circ g(x) = x \quad g \circ f(x) = x}$$

[3]

(b) Write down $f^{-1}(x)$ and state its domain and range.

[3]

b) IF $f \circ g(x) = g \circ f(x) = x$

$f(x)$ AND $g(x)$ ARE INVERSE FUNCTIONS OF EACHOTHER

$f^{-1}(x) = 0.5x + 0.75$

DOMAIN $x \in \mathbb{R}$

RANGE $x \in \mathbb{R}$

## Question 6

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20$.

(a) Find the value of $f\left(\frac{5}{2}\right)$. [1]

(b) Write down the range of $f(x)$. [2]

(c) Find the inverse function $f^{-1}(x)$. [2]

(d) Write down the range of the inverse function. [1]

a) Sub $x = \frac{5}{2}$ into $f(x)$.

$f\left(\frac{5}{2}\right) = 54\left(\frac{5}{2}\right) - 13$

$\boxed{f\left(\frac{5}{2}\right) = 122}$

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20$.

(a) Find the value of $f\left(\frac{5}{2}\right)$. [1]

(b) Write down the range of $f(x)$. [2]

(c) Find the inverse function $f^{-1}(x)$. [2]

(d) Write down the range of the inverse function. [1]

b) Use the domain of $f(x)$ to find its range.

$f(-2) = 54(-2) - 13$

$f(-2) = -121$

$f(20) = 54(20) - 13$

$f(20) = 1067$

$\boxed{\text{Range is } \{y \mid -121 < y < 1067\}}$

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20$.

(a) Find the value of $f\left(\frac{5}{2}\right)$. [1]

(b) Write down the range of $f(x)$. [2]

(c) Find the inverse function $f^{-1}(x)$. [2]

(d) Write down the range of the inverse function. [1]

c) Set $y = f(x)$.

$y = 54x - 13$

Swap $x$ and $y$.

$x = 54y - 13$

Rearrange into $y = mx + c$.

$54y - 13 = x$

$54y = x + 13$

$y = \frac{x + 13}{54}$

$y = \frac{1}{54}x + \frac{13}{54}$

$$\boxed{f^{-1}(x) = \frac{1}{54}x + \frac{13}{54}}$$

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20$.

(a) Find the value of $f\left(\frac{5}{2}\right)$. [1]

(b) Write down the range of $f(x)$. [2]

(c) Find the inverse function $f^{-1}(x)$. [2]

(d) Write down the range of the inverse function. [1]

d) The domain of $f(x)$ is the range of $f^{-1}(x)$.

$$\boxed{\text{Range is } \{y \mid -2 < y < 20\}}$$

## Question 7

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \le x \le 3$.

(a) Find

(i) $f(2)$

(ii) $x$ when $f(x) = 15$.

[2]

(b) Find the range of $f(x)$.

[3]

(c) Write down the domain of the inverse function.

[1]

a)i) Sub $x = 2$ into $f(x)$.

$f(2) = -6(2) - 3$

$\boxed{f(2) = -15}$

ii) Set $f(x) = 15$ and rearrange for $x$.

$f(x) = 15$

$-6x - 3 = 15 \quad \Big) + 3$

$-6x = 18 \quad \Big) \div(-6)$

$\boxed{x = -3}$

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \le x \le 3$.

(a) Find

(i) $f(2)$

(ii) $x$ when $f(x) = 15$.

[2]

(b) Find the range of $f(x)$.

[3]

(c) Write down the domain of the inverse function.

[1]

b) Use the domain of $f(x)$ to find its range.

$f(-5) = -6(-5) - 3$

$f(-5) = 27$

$f(3) = -6(3) - 3$

$f(3) = -21$

$\boxed{\text{Range is } \{y \mid -21 \le y \le 27\}}$

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \leq x \leq 3$.

(a) Find

(i) $f(2)$

(ii) $x$ when $f(x) = 15$.

[2]

(b) Find the range of $f(x)$.

$\boxed{\text{Range is } \{y \mid -21 \leq y \leq 27\}}$

[3]

(c) Write down the domain of the inverse function.

[1]

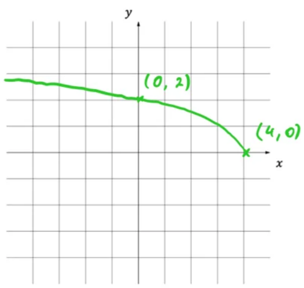
c) The range of $f(x)$ is the domain of $f^{-1}(x)$.

$\boxed{\text{Domain is } \{x \mid -21 \leq x \leq 27\}}$

## Question 8

Consider the function $g(x) = \sqrt{4-x}$.

(a) Sketch the graph of the function $g(x)$, labelling the $x$ and $y$ intercepts.




[3]

a) $x$-intercept is when $g(x) = 0$.

$\boxed{x\text{-intercept is at } (4, 0)}$

$y$-intercept is when $x = 0$.

$\boxed{y\text{-intercept is at } (0, 2)}$

Graph $g(x)$ on your GDC to find its shape.

(b) Find

(i) $g(-5)$

(ii) $x$ when $g(x) = \frac{1}{2}$.

[2]

(c) Find

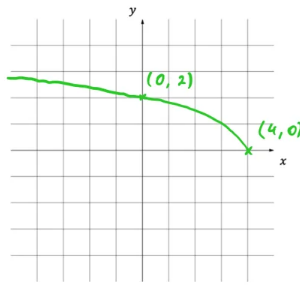
(i) the maximum possible domain of the function $g(x)$

(ii) the range of the function $g(x)$ that corresponds to the domain found in part (c) (i).

[2]

Consider the function $g(x) = \sqrt{4-x}$.

(a) Sketch the graph of the function $g(x)$, labelling the $x$ and $y$ intercepts.




[3]

(b) Find

(i) $g(-5)$

(ii) $x$ when $g(x) = \frac{1}{2}$.

[2]

(c) Find

(i) the maximum possible domain of the function $g(x)$

(ii) the range of the function $g(x)$ that corresponds to the domain found in part (c) (i).

[2]

b)i) Sub $x = -5$ into $g(x)$.

$g(-5) = \sqrt{4-(-5)}$

$g(-5) = \sqrt{9}$

$\boxed{g(-5) = 3}$

ii) Set $g(x) = \frac{1}{2}$ and rearrange for $x$.

$g(x) = \frac{1}{2}$

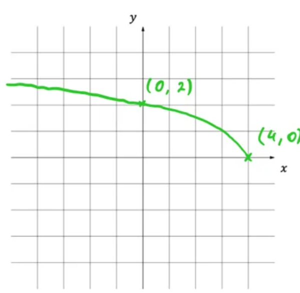
$\sqrt{4-x} = \frac{1}{2}$

$4 - x = \frac{1}{4}$

$\boxed{x = 3.75}$

Consider the function $g(x) = \sqrt{4-x}$.

(a) Sketch the graph of the function $g(x)$, labelling the $x$ and $y$ intercepts.




[3]

(b) Find

(i) $g(-5)$

(ii) $x$ when $g(x) = \frac{1}{2}$.

[2]

(c) Find

(i) the maximum possible domain of the function $g(x)$

(ii) the range of the function $g(x)$ that corresponds to the domain found in part (c) (i).

[2]

c)i) $g(x)$ is undefined for $x > 4$.

$\boxed{\text{Domain is } \{x \mid x \leq 4\}}$

ii) $g(x) = 0$ when $x = 4$.

$\boxed{\text{Range is } \{y \mid y \geq 0\}}$

## Question 9

The functions $f$ and $g$ are defined for $x \in R$ by $f(x) = 3x^2 + 10x + 7$ and $g(x) = x + d$, where $d \in R$.

(a) Find the range of $f$.

[2]

(b) Given that $(g \circ f)(x)$ is always positive for all $x$, determine the set of possible values for $d$.

[4]

a) The graph of $f$ is a parabola.

Axis of symmetry

$x = -\frac{b}{2a}$ (in formula booklet)

Sub $b = 10$ and $a = 3$ into formula.

$x = -\frac{\cancel{10}^5}{\cancel{2}(3)}$ $\therefore x = -\frac{5}{3}$

Sub $x = -\frac{5}{3}$ into $f(x)$.

$f(-\frac{5}{3}) = 3(-\frac{5}{3})^2 + 10(-\frac{5}{3}) + 7$

$f(-\frac{5}{3}) = 3(\frac{25}{9}) - \frac{50}{3} + 7$

$f(-\frac{5}{3}) = \frac{75}{9} - \frac{150}{9} + \frac{63}{9}$

$f(-\frac{5}{3}) = -\frac{\cancel{12}^4}{\cancel{9}_3} = -\frac{4}{3}$

$\boxed{\text{Range is } \{y \mid y \geq -\frac{4}{3}\}}$

The functions $f$ and $g$ are defined for $x \in R$ by $f(x) = 3x^2 + 10x + 7$ and $g(x) = x + d$, where $d \in R$.

(a) Find the range of $f$.

[2]

(b) Given that $(g \circ f)(x)$ is always positive for all $x$, determine the set of possible values for $d$.

[4]

b) $(g \circ f)(x) = 3x^2 + 10x + 7 + d$

$(g \circ f)(x)$ is a quadratic equation, with

$a = 3$ $b = 10$ $c = 7 + d$

Discriminant formula.

$\Delta = b^2 - 4ac$ (in formula booklet)

$\Delta = (10)^2 - 4(3)(7 + d)$

$\Delta = 100 - (84 + 12d)$

$\Delta = 16 - 12d$

$(g \circ f)(x)$ is positive for all $x$ when $\Delta < 0$.

$16 - 12d < 0$

$\boxed{d > \frac{4}{3}}$

## Question 10

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

(i) the value of $a$

(ii) the range of $f$. [2]

(b) For the graph of $f$, find the equations of all the asymptotes. [1]

(c) Find $f^{-1}(x)$. [2]

(d) For the graph of $f^{-1}$, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote. [2]

a) 'a' is the value of $x$ that makes the denominator equal zero.

i) $\boxed{a = -8}$

ii) As $x$ tends towards $\pm\infty$, $f(x)$ tends towards 2.

$\boxed{\text{Range: } \{f(x) : f(x) \in \mathbb{R} \mid f(x) \neq 2\}}$

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

(i) the value of $a$

(ii) the range of $f$. [2]

(b) For the graph of $f$, find the equations of all the asymptotes. [1]

(c) Find $f^{-1}(x)$. [2]

(d) For the graph of $f^{-1}$, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote. [2]

b) Vertical asymptote is the value of $x$ that makes the denominator equal zero.

$\boxed{x = -8}$

Horizontal asymptote is the value $f(x)$ tends towards as $x$ tends towards $\pm\infty$.

$\boxed{y = 2}$

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

(i) the value of $a$

(ii) the range of $f$.

[2]

(b) For the graph of $f$, find the equations of all the asymptotes.

[1]

(c) Find $f^{-1}(x)$.

[2]

(d) For the graph of $f^{-1}$, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote.

[2]

c) Let $y = f(x)$ and rearrange for $x$.

$$y = \frac{2x-5}{x+8}$$

$\times(x+8)$

$$xy + 8y = 2x - 5$$

$-2x - 8y$

$$xy - 2x = -5 - 8y$$

factorise

$$x(y-2) = -5 - 8y$$

$\div(y-2)$ and multiply by $\left(\frac{-1}{-1}\right)$ for a more appropriate form

$$x = \frac{-5-8y}{y-2} = \frac{8y+5}{2-y}$$

$$\boxed{f^{-1}(x) = \frac{8x+5}{2-x}}$$

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

(i) the value of $a$

(ii) the range of $f$.

[2]

(b) For the graph of $f$, find the equations of all the asymptotes.

[1]

(c) Find $f^{-1}(x)$.

[2]

(d) For the graph of $f^{-1}$, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote.

[2]

d) The asymptotes of $f^{-1}$ are the same as $f$, but with the $x$ and $y$ switched.

$\boxed{x = 2}$ $\boxed{y = -8}$

## Question 11

Determine, for each of the following functions, whether they are even, odd or neither:

(i) $f(x) = \frac{1}{x^2} + 2$

(ii) $g(x) = x^3 - 3x$

(iii) $h(x) = x^2 + 2x - 5$.

[5]

i) $f(-x) = \frac{1}{(-x)^2} + 2 = \frac{1}{x^2} + 2$

$\boxed{f(-x) = f(x) \quad \therefore f(x) \text{ is even}}$

ii) $g(-x) = (-x)^3 - 3(-x) = 3x - x^3 = -(x^3 - 3x)$

$\boxed{g(-x) = -g(x) \quad \therefore g(x) \text{ is odd}}$

iii) $h(-x) = (-x)^2 + 2(-x) - 5 = x^2 - 2x - 5$

$\boxed{h(x) \text{ is neither}}$

## Question 12

Prove that the sum of two odd functions is also an odd function.

[5]

Let $h(x) = f(x) + g(x)$, where $f$ and $g$ are odd

$h(-x) = f(-x) + g(-x) = -f(x) - g(x) = -(f(x) + g(x))$

$\therefore h(-x) = -h(x)$ for all $x$

## Question 13

Let $f(x) = \frac{\pi^2}{x}$, where $x \neq 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function.

[2]

a) Let $y = f(x)$ and rearrange for $x$.

$y = \frac{\pi^2}{x} \longrightarrow x = \frac{\pi^2}{y} \quad \therefore f^{-1}(x) = \frac{\pi^2}{x}$

$\boxed{\therefore f(x) \text{ is a self inverse function}}$

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \neq p, x \in \mathbb{R}$.

(b) Find the value of $p$.

[1]

(c) Show that $g(x)$ is a self-inverse function.

[4]

Let $f(x) = \frac{\pi^2}{x}$, where $x \neq 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function. [2]

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \neq p, x \in \mathbb{R}$.

(b) Find the value of $p$. [1]

(c) Show that $g(x)$ is a self-inverse function. [4]

b) The value of $p$ is equal to the value of $x$ that makes the denominator equal zero.

$\boxed{p = -\frac{1}{5}}$

Let $f(x) = \frac{\pi^2}{x}$, where $x \neq 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function. [2]

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \neq p, x \in \mathbb{R}$.

(b) Find the value of $p$. [1]

(c) Show that $g(x)$ is a self-inverse function. [4]

c) Let $y = g(x)$ and rearrange for $x$.

$y = \frac{-x-2}{5x+1}$

$5xy + y = -x - 2$

$5xy + x = -y - 2$

$x(5y+1) = -y - 2$

$x = \frac{-y-2}{5y+1}$

$\boxed{g^{-1}(x) = \frac{-x-2}{5x+1} \quad \therefore g(x) \text{ is a self inverse function}}$

## Question 14

Consider the function $f$ defined by $f(x) = 2x^3 + 3x^2 - 36x + 7,\ x \in \mathbb{R}$.

(a) Sketch the graph of $f$. Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima.

[2]

Let the function $g$ be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7,\ x \le p$.

(b) Given that $g$ has an inverse:

(i) Find the largest possible value of $p$

(ii) Find the domain of $g^{-1}$ for the value of $p$ identified in part (b)(i)

(iii) Find the value of $g^{-1}(0)$.

[3]

Let the function $h$ be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7,\ x \ge q$.

(c) Given that $h$ has an inverse:

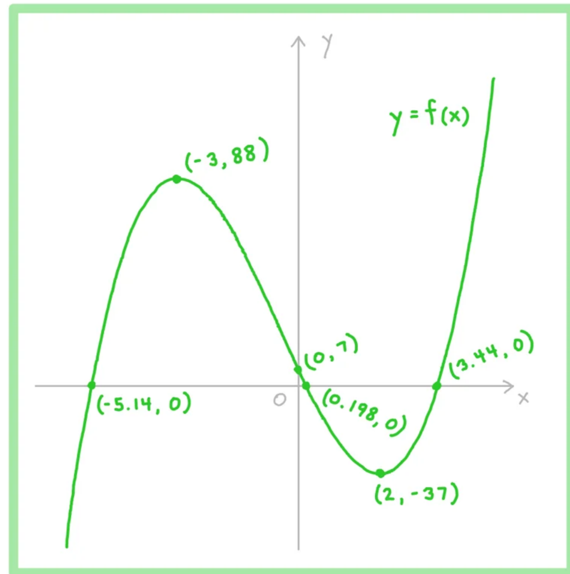
(i) Find the smallest possible value of $q$

(ii) Find the domain of $h^{-1}$ for the value of $q$ identified in part (c)(i)

(iii) Find the value of $h^{-1}(0)$.

[3]

a) Your GDC can help with this :




Non-exact values are rounded to 3 s.f.

Consider the function $f$ defined by $f(x) = 2x^3 + 3x^2 - 36x + 7,\ x \in \mathbb{R}$.

(a) Sketch the graph of $f$. Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima.

[2]

Let the function $g$ be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7,\ x \le p$.

(b) Given that $g$ has an inverse:

(i) Find the largest possible value of $p$

(ii) Find the domain of $g^{-1}$ for the value of $p$ identified in part (b)(i)

(iii) Find the value of $g^{-1}(0)$.

[3]

Let the function $h$ be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7,\ x \ge q$.

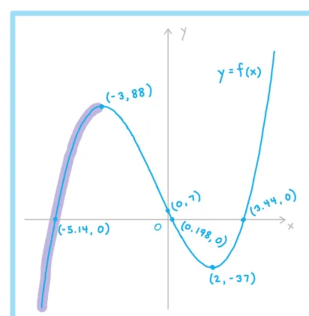
(c) Given that $h$ has an inverse:

(i) Find the smallest possible value of $q$

(ii) Find the domain of $h^{-1}$ for the value of $q$ identified in part (c)(i)

(iii) Find the value of $h^{-1}(0)$.

[3]




b) (i) For g to be one-to-one (and so have an inverse), its domain cannot cross over the local max at (-3, 88).

$\boxed{p = -3}$

(ii) The domain of $g^{-1}$ is the range of g.

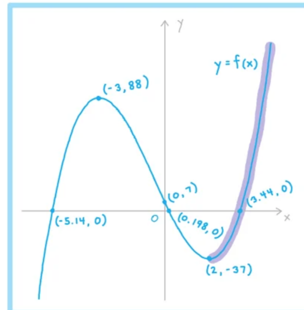
$\boxed{x \le 88}$

(iii) $g(-5.14) = 0 \implies$ $\boxed{g^{-1}(0) = -5.14 \text{ (3 s.f.)}}$

Consider the function $f$ defined by $f(x) = 2x^3 + 3x^2 - 36x + 7,\ x \in \mathbb{R}$.

(a) Sketch the graph of $f$. Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima.

[2]




Let the function $g$ be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7,\ x \leq p$.

(b) Given that $g$ has an inverse:

(i) Find the largest possible value of $p$

(ii) Find the domain of $g^{-1}$ for the value of $p$ identified in part (b)(i)

(iii) Find the value of $g^{-1}(0)$.

[3]

Let the function $h$ be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7,\ x \geq q$.

(c) Given that $h$ has an inverse:

(i) Find the smallest possible value of $q$

(ii) Find the domain of $h^{-1}$ for the value of $q$ identified in part (c)(i)

(iii) Find the value of $h^{-1}(0)$.

[3]

c) (i) For $h$ to be one-to-one (and so have an inverse), its domain cannot cross over the local min at $(2, -37)$.

$\boxed{q = 2}$

(ii) The domain of $h^{-1}$ is the range of $h$.

$\boxed{x \geq -37}$

(iii) $h(3.44) = 0 \implies \boxed{h^{-1}(0) = 3.44 \text{ (3 s.f.)}}$