Question 1

The functions f and g are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$. [2]

(b) Given that $(g \circ f)(a) = 27$, find the value of a. [2]

(c) Show that $(f \circ g)(x) = 2x + 6$. [2]

(d) Given that $(f \circ g)(b) = 44$, find the value of b. [2]

a) Sub $(4x - 10)$ into $g(x)$.

$(g \circ f)(x) = \frac{(4x-10)+8}{2}$

$(g \circ f)(x) = \frac{4x-2}{2}$

$\boxed{(g \circ f)(x) = 2x - 1}$

The functions f and g are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$. $\boxed{(g \circ f)(x) = 2x - 1}$ [2]

(b) Given that $(g \circ f)(a) = 27$, find the value of a. [2]

(c) Show that $(f \circ g)(x) = 2x + 6$. [2]

(d) Given that $(f \circ g)(b) = 44$, find the value of b. [2]

b) Set $(g \circ f)(x) = 27$ and solve for x.

$2x - 1 = 27$

$x = 14$

$\boxed{\therefore a = 14}$

The functions f and g are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$. [2]

(b) Given that $(g \circ f)(a) = 27$, find the value of a. [2]

(c) Show that $(f \circ g)(x) = 2x + 6$. [2]

(d) Given that $(f \circ g)(b) = 44$, find the value of b. [2]

c) Sub $\frac{x+8}{2}$ into $f(x)$.

$(f \circ g)(x) = \cancel{4}^{2}\left(\frac{x+8}{\cancel{2}}\right) - 10$

$(f \circ g)(x) = 2(x+8) - 10$

$(f \circ g)(x) = 2x + 16 - 10$

$\boxed{(f \circ g)(x) = 2x + 6}$

The functions f and g are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$.

[2]

(b) Given that $(g \circ f)(a) = 27$, find the value of a.

[2]

(c) Show that $(f \circ g)(x) = 2x + 6$.

$\boxed{(f \circ g)(x) = 2x + 6}$

[2]

(d) Given that $(f \circ g)(b) = 44$, find the value of b.

[2]

d) Set $(f \circ g)(x) = 44$ and solve for x.

$2x + 6 = 44$

$x = 19$

$\boxed{\therefore b = 19}$

Question 2

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 \qquad x \in R$$

$$g(x) = 4x - 3 \qquad x \in R$$

(a) Write down the range of $f(x)$.

[1]

a) $x^2 \geq 0$ ALWAYS POSITIVE

RANGE = OUTPUT VALUES

$\boxed{f(x) \geq 0}$

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

[4]

(c) Solve the equation $f(x) = g(x)$.

[2]

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 \qquad x \in R$$

$$g(x) = 4x - 3 \qquad x \in R$$

(a) Write down the range of $f(x)$. [1]

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

COMPOSITE FUNCTIONS

[4]

(c) Solve the equation $f(x) = g(x)$. [2]

b)

i) $f \circ g(x)$

SUBSTITUTE $g(x)$ INTO $f(x)$

$(4x-3)^2$

$\boxed{f \circ g(x) = 16x^2 - 24x + 9}$

ii) $g \circ f(x)$

SUBSTITUTE $f(x)$ INTO $g(x)$

$4(x^2) - 3$

$\boxed{g \circ f(x) = 4x^2 - 3}$

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = x^2 \qquad x \in R$$

$$g(x) = 4x - 3 \qquad x \in R$$

(a) Write down the range of $f(x)$. [1]

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

[4]

(c) Solve the equation $f(x) = g(x)$. [2]

c)

$x^2 = 4x - 3$

$x^2 - 4x + 3 = 0$

FACTORISE OR USE CALCULATOR

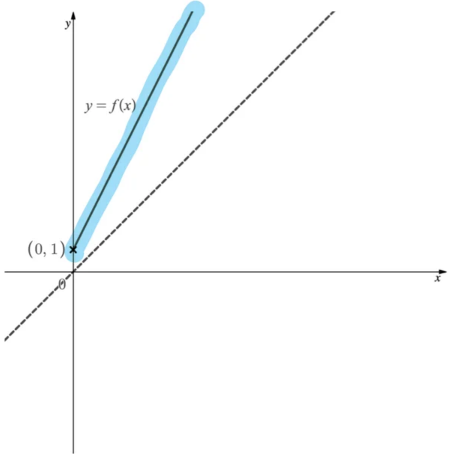
$(x-1)(x-3) = 0$

$\boxed{x = 1 \qquad x = 3}$

EXAM PAPERS PRACTICE

Question 3

The graph of $y = \mathrm{f}(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $\mathrm{f}(x)$.
(ii) Given that the point $(1, 1)$ lies on the dotted line, write down the equation of the line.

[3]

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

[2]

a) i) DOMAIN INPUT VALUES

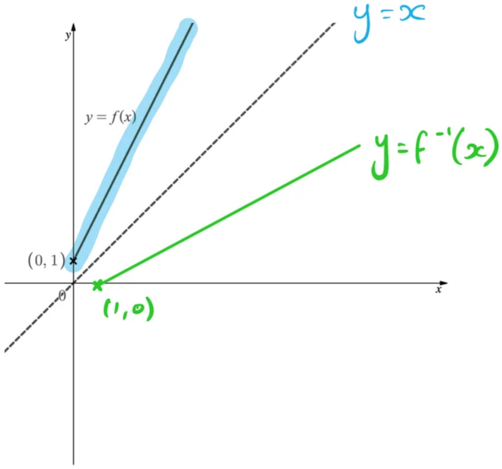
$\boxed{x \geq 0 \quad x \in \mathbb{R}}$

RANGE OUTPUT VALUES

$\boxed{f(x) \geq 1}$

ii) $\boxed{y = x}$

The graph of $y = \mathrm{f}(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $\mathrm{f}(x)$.
(ii) Given that the point $(1, 1)$ lies on the dotted line, write down the equation of the line.

[3]

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

[2]

b) INVERSE FUNCTION

REFLECTION IN $y = x$

Question 4

The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

(a) Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

[2]

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2]

(c) The domain of $f(x)$ is changed to $x > 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

[4]

a) $f(x) = \frac{x^2}{x^2} + \frac{1}{x^2}$

$= \boxed{1 + \frac{1}{x^2}}$

The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

(a) Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

[2]

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2]

(c) The domain of $f(x)$ is changed to $x > 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

[4]

b)

THE FUNCTION $f(x)$ IS MANY TO ONE SO INVERSE (ONE TO MANY) DOES NOT EXIST

FUNCTIONS MUST BE ONE TO ONE FOR INVERSE TO EXIST SO WE CAN CHANGE DOMAIN TO

$x > 0$ OR $x < 0$

The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \qquad x \in \mathbb{R},\ x \neq 0$$

(a) Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

[2]

(b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2]

(c) The domain of $f(x)$ is changed to $x > 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

[4]

c) LET $y = f(x)$

$y = 1 + \frac{1}{x^2}$

REARRANGE

$y - 1 = \frac{1}{x^2}$

$x^2(y-1) = 1$

$x^2 = \frac{1}{y-1}$

$x = \sqrt{\frac{1}{y-1}}$ OR $x = \frac{1}{\sqrt{y-1}}$

$\boxed{f^{-1}(x) = \sqrt{\frac{1}{x-1}} \text{ OR } \frac{1}{\sqrt{x-1}}}$

DOMAIN $x > 1$ RANGE $f(x) > 0$

Question 5

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = \frac{1}{2}(4x - 3) \qquad x \in \mathbb{R}$$

$$g(x) = 0.5x + 0.75 \qquad x \in \mathbb{R}$$

(a) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

[3]

(b) Write down $f^{-1}(x)$ and state its domain and range.

[3]

a)

i) $f \circ g(x)$ SUBSTITUTE $g(x)$ INTO $f(x)$

$\frac{1}{2}\left(4\left(\frac{1}{2}x + \frac{3}{4}\right) - 3\right)$

$\cancel{\frac{1}{2}}\left(\cancel{2}x \cancel{+3} \cancel{-3}\right)$

$\boxed{f \circ g(x) = x}$

ii) $g \circ f(x)$ SUBSTITUTE $f(x)$ INTO $g(x)$

$\frac{1}{2}\left(\frac{1}{2}(4x - 3)\right) + \frac{3}{4}$

$\frac{1}{\cancel{4}}(\cancel{4}x - 3) + \frac{3}{4}$

$x - \cancel{\frac{3}{4}} + \cancel{\frac{3}{4}}$

$\boxed{g \circ f(x) = x}$

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = \frac{1}{2}(4x - 3) \qquad x \in \mathbb{R}$$

$$g(x) = 0.5x + 0.75 \qquad x \in \mathbb{R}$$

(a) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

$\boxed{f \circ g(x) = x \quad g \circ f(x) = x}$

[3]

(b) Write down $f^{-1}(x)$ and state its domain and range.

[3]

b) IF $f \circ g(x) = g \circ f(x) = x$

$f(x)$ AND $g(x)$ ARE INVERSE FUNCTIONS OF EACHOTHER

$f^{-1}(x) = 0.5x + 0.75$

DOMAIN $x \in \mathbb{R}$

RANGE $x \in \mathbb{R}$

Question 6

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20.$

(a) Find the value of $f\left(\frac{5}{2}\right)$.

[1]

(b) Write down the range of $f(x)$.

[2]

(c) Find the inverse function $f^{-1}(x)$.

[2]

(d) Write down the range of the inverse function.

[1]

a) Sub $x = \frac{5}{2}$ into $f(x)$.

$f\left(\frac{5}{2}\right) = 54\left(\frac{5}{2}\right) - 13$

$\boxed{f\left(\frac{5}{2}\right) = 122}$

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20.$

(a) Find the value of $f\left(\frac{5}{2}\right)$.

[1]

(b) Write down the range of $f(x)$.

[2]

(c) Find the inverse function $f^{-1}(x)$.

[2]

(d) Write down the range of the inverse function.

[1]

b) Use the domain of $f(x)$ to find its range.

$f(-2) = 54(-2) - 13$

$f(-2) = -121$

$f(20) = 54(20) - 13$

$f(20) = 1067$

$\boxed{\text{Range is } \{y \mid -121 < y < 1067\}}$

EXAM PAPERS PRACTICE

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20$.

(a) Find the value of $f\left(\frac{5}{2}\right)$.

[1]

(b) Write down the range of $f(x)$.

[2]

(c) Find the inverse function $f^{-1}(x)$.

[2]

(d) Write down the range of the inverse function.

[1]

c) Set $y = f(x)$.

$y = 54x - 13$

Swap x and y.

$x = 54y - 13$

Rearrange into $y = mx + c$.

$54y - 13 = x$

$54y = x + 13$

$y = \frac{x + 13}{54}$

$y = \frac{1}{54}x + \frac{13}{54}$

$\boxed{f^{-1}(x) = \frac{1}{54}x + \frac{13}{54}}$

A function is defined by $f(x) = 54x - 13, \quad -2 < x < 20$.

(a) Find the value of $f\left(\frac{5}{2}\right)$.

[1]

(b) Write down the range of $f(x)$.

[2]

(c) Find the inverse function $f^{-1}(x)$.

[2]

(d) Write down the range of the inverse function.

[1]

d) The domain of $f(x)$ is the range of $f^{-1}(x)$.

$\boxed{\text{Range is } \{y \mid -2 < y < 20\}}$

Question 7

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \leq x \leq 3$.

(a) Find

(i) $f(2)$

(ii) x when $f(x) = 15$.

[2]

(b) Find the range of $f(x)$.

[3]

(c) Write down the domain of the inverse function.

[1]

a) i) Sub $x = 2$ into $f(x)$.

$f(2) = -6(2) - 3$

$\boxed{f(2) = -15}$

ii) Set $f(x) = 15$ and rearrange for x.

$f(x) = 15$

$-6x - 3 = 15$) $+3$

$-6x = 18$) $\div(-6)$

$\boxed{x = -3}$

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \leq x \leq 3$.

(a) Find

(i) $f(2)$

(ii) x when $f(x) = 15$.

[2]

(b) Find the range of $f(x)$.

[3]

(c) Write down the domain of the inverse function.

[1]

b) Use the domain of $f(x)$ to find its range.

$f(-5) = -6(-5) - 3$

$f(-5) = 27$

$f(3) = -6(3) - 3$

$f(3) = -21$

$\boxed{\text{Range is } \{y \mid -21 \leq y \leq 27\}}$

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \leq x \leq 3$.

(a) Find

(i) $f(2)$

(ii) x when $f(x) = 15$.

[2]

(b) Find the range of $f(x)$. $\boxed{\text{Range is } \{y \mid -21 \leq y \leq 27\}}$

[3]

(c) Write down the domain of the inverse function.

[1]

c) The range of $f(x)$ is the domain of $f^{-1}(x)$.

$\boxed{\text{Domain is } \{x \mid -21 \leq x \leq 27\}}$

Question 8

Let $f(x) = -\frac{3}{x-3}$, for $x \neq 3$.

(a) For the graph of f, find:

(i) the x-intercept

(ii) the y-intercept

(iii) the range of f.

[4]

(b) Find the value of $f^{-1}(-1)$.

[2]

(c) Given that $g(x) = f(x+3) + 1$, find the domain and range of g.

[2]

a) i) x-intercepts occur when $f(x) = 0$.

$0 = -\frac{3}{x-3}$ $(f(x) \neq 0)$

$\boxed{\text{No solutions, } f(x) \text{ does not cross the } x\text{-axis.}}$

ii) y-intercepts occur when $x = 0$.

$f(0) = -\frac{3}{(0)-3}$

$f(0) = 1$

$\boxed{y\text{-intercept at } (0, 1).}$

iii) $\boxed{\text{Range} = (-\infty, 0) \cup (0, \infty)}$

$(f(x) \neq 0)$

Let $f(x) = -\frac{3}{x-3}$, for $x \neq 3$.

(a) For the graph of f, find:

(i) the x-intercept

(ii) the y-intercept

(iii) the range of f.

[4]

(b) Find the value of $f^{-1}(-1)$.

[2]

(c) Given that $g(x) = f(x+3) + 1$, find the domain and range of g.

[2]

b) Find $f^{-1}(x)$.

$y = f(x)$

$y = -\frac{3}{x-3}$

swap x and y

$x = -\frac{3}{y-3}$

rearrange

$y = -\frac{3}{x} + 3$

$\therefore f^{-1}(x) = -\frac{3}{x} + 3$

Sub $x = -1$ into $f^{-1}(x)$.

$f^{-1}(-1) = -\frac{3}{(-1)} + 3$

$f^{-1}(-1) = 3 + 3$

$\boxed{f^{-1}(-1) = 6}$

Let $f(x) = -\frac{3}{x-3}$, for $x \neq 3$.

(a) For the graph of f, find:

(i) the x-intercept

(ii) the y-intercept

(iii) the range of f.

[4]

(b) Find the value of $f^{-1}(-1)$.

[2]

(c) Given that $g(x) = f(x+3) + 1$, find the domain and range of g.

[2]

c) $g(x) = f(x+3) + 1$

Sub in $(x+3)$ for x and $+1$.

$g(x) = -\frac{3}{(x\cancel{+3})\cancel{-3}} + 1$

$g(x) = -\frac{3}{x} + 1$

For $g(x)$, $x \neq 0$.

$\boxed{\text{Domain} = (-\infty, 0) \cup (0, \infty)}$

$\lim_{x \to \pm\infty} g(x) = -\frac{3}{(\pm\infty)} + 1$

$\lim_{x \to \pm\infty} g(x) = 0 + 1$

$\boxed{\text{Range} = (-\infty, 1) \cup (1, \infty)}$

Question 9

The functions f and g are defined for $x \in R$ by $f(x) = 3x^2 + 10x + 7$ and $g(x) = x + d$, where $d \in R$.

(a) Find the range of f.

[2]

(b) Given that $(g \circ f)(x)$ is always positive for all x, determine the set of possible values for d.

[4]

a) The graph of f is a parabola.

Axis of symmetry

$x = -\frac{b}{2a}$ (in formula booklet)

Sub $b = 10$ and $a = 3$ into formula.

$x = -\frac{\cancel{10}^{5}}{\cancel{2}(3)}$ $\therefore x = -\frac{5}{3}$

Sub $x = -\frac{5}{3}$ into $f(x)$.

$f(-\frac{5}{3}) = 3(-\frac{5}{3})^2 + 10(-\frac{5}{3}) + 7$

$f(-\frac{5}{3}) = 3(\frac{25}{9}) - \frac{50}{3} + 7$

$f(-\frac{5}{3}) = \frac{75}{9} - \frac{150}{9} + \frac{63}{9}$

$f(-\frac{5}{3}) = -\frac{\cancel{12}^{4}}{\cancel{9}_{3}} = -\frac{4}{3}$

$\boxed{\text{Range is } \{y \mid y \geq -\frac{4}{3}\}}$

The functions f and g are defined for $x \in R$ by $f(x) = 3x^2 + 10x + 7$ and $g(x) = x + d$, where $d \in R$.

(a) Find the range of f.

[2]

(b) Given that $(g \circ f)(x)$ is always positive for all x, determine the set of possible values for d.

[4]

b) $(g \cdot f)(x) = 3x^2 + 10x + 7 + d$

$(g \cdot f)(x)$ is a quadratic equation, with

$a = 3 \quad b = 10 \quad c = 7 + d$

Discriminant formula.

$\Delta = b^2 - 4ac$ (in formula booklet)

$\Delta = (10)^2 - 4(3)(7 + d)$

$\Delta = 100 - (84 + 12d)$

$\Delta = 16 - 12d$

$(g \cdot f)(x)$ is positive for all x when $\Delta < 0$.

$16 - 12d < 0$

$\boxed{d > \frac{4}{3}}$

Question 10

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a$, $x \in \mathbb{R}$.

(a) Write down

(i) the value of a

(ii) the range of f.

[2]

(b) For the graph of f, find the equations of all the asymptotes.

[1]

(c) Find $f^{-1}(x)$.

[2]

(d) For the graph of f^{-1}, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote.

[2]

a) 'a' is the value of x that makes the denominator equal zero.

i) $\boxed{a = -8}$

ii) As x tends towards $\pm\infty$, $f(x)$ tends towards 2.

$\boxed{\text{Range}: \{f(x) : f(x) \in R \mid f(x) \neq 2\}}$

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

(i) the value of a

(ii) the range of f.

[2]

(b) For the graph of f, find the equations of all the asymptotes.

[1]

(c) Find $f^{-1}(x)$.

[2]

(d) For the graph of f^{-1}, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote.

[2]

b) Vertical asymptote is the value of x that makes the denominator equal zero.

$$\boxed{x = -8}$$

Horizontal asymptote is the value $f(x)$ tends towards as x tends towards $\pm\infty$.

$$\boxed{y = 2}$$

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

(i) the value of a

(ii) the range of f.

[2]

(b) For the graph of f, find the equations of all the asymptotes.

[1]

(c) Find $f^{-1}(x)$.

[2]

(d) For the graph of f^{-1}, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote.

[2]

c) Let $y = f(x)$ and rearrange for x.

$$y = \frac{2x-5}{x+8}$$

$\times(x+8)$

$$xy + 8y = 2x - 5$$

$-2x - 8y$

$$xy - 2x = -5 - 8y$$

factorise

$$x(y-2) = -5 - 8y$$

$\div(y-2)$ and multiply by $\left(\frac{-1}{-1}\right)$ for a more appropriate form

$$x = \frac{-5-8y}{y-2} = \frac{8y+5}{2-y}$$

$$\boxed{f^{-1}(x) = \frac{8x+5}{2-x}}$$

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq a, x \in \mathbb{R}$.

(a) Write down

(i) the value of a

(ii) the range of f.

[2]

(b) For the graph of f, find the equations of all the asymptotes.

[1]

(c) Find $f^{-1}(x)$.

[2]

(d) For the graph of f^{-1}, find the equation of

(i) the horizontal asymptote

(ii) the vertical asymptote.

[2]

d) The asymptotes of f^{-1} are the same as f, but with the x and y switched.

$\boxed{x = 2}$ $\boxed{y = -8}$

Question 11

Let $f(x) = 2x + 1$ for $x \in \mathbb{R}$.

(a) Write down an expression for the inverse function $f^{-1}(x)$.

[2]

Consider another function $g(x) = \frac{1}{2}(x-1)^2 + \frac{3}{2}$ for $x \geq k$, where k is an integer to be found.

(b) Given that the graph of g has an inverse, find the value of k.

[3]

(c) Sketch the graphs of f and g, for the domain found in part (b), on the same set of axes, along with their inverses.

[4]

(a) LET $y = f(x)$

REARRANGE

$y = 2x + 1$

$y - 1 = 2x$

$\frac{y-1}{2} = x$

$x = \frac{1}{2}y - \frac{1}{2}$

$\boxed{f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}}$

Let $f(x) = 2x + 1$ for $x \in \mathbb{R}$.

(a) Write down an expression for the inverse function $f^{-1}(x)$.

[2]

Consider another function $g(x) = \frac{1}{2}(x-1)^2 + \frac{3}{2}$ for $x \geq k$, where k is an integer to be found.

(b) Given that the graph of g has an inverse, find the value of k.

[3]

(c) Sketch the graphs of f and g, for the domain found in part (b), on the same set of axes, along with their inverses.

[4]

(b) HAS INVERSE MEANS ONE $\Leftrightarrow$ ONE FUNCTION

FIND INVERSE $g^{-1}(x)$

LET $y = g(x)$ $\qquad y = \frac{1}{2}(x-1)^2 + \frac{3}{2}$

REARRANGE $\qquad 2y = (x-1)^2 + 3$

$$2y - 3 = (x-1)^2$$

$$\sqrt{2y-3} = x - 1$$

$$\sqrt{2y-3} + 1 = x$$

$$g^{-1}(x) = \sqrt{2x-3} + 1$$

ROOT CANNOT BE NEGATIVE

$$2x - 3 > 0$$

$$2x > 3$$

$$x > \frac{3}{2}$$

K IS AN INTEGER SO $k \geqslant 2$

$$\boxed{k \geqslant 2}$$

Let $f(x) = 2x + 1$ for $x \in \mathbb{R}$.

(a) Write down an expression for the inverse function $f^{-1}(x)$.

[2]

Consider another function $g(x) = \frac{1}{2}(x-1)^2 + \frac{3}{2}$ for $x \geq k$, where k is an integer to be found.

(b) Given that the graph of g has an inverse, find the value of k.

$\boxed{k \geqslant 2}$

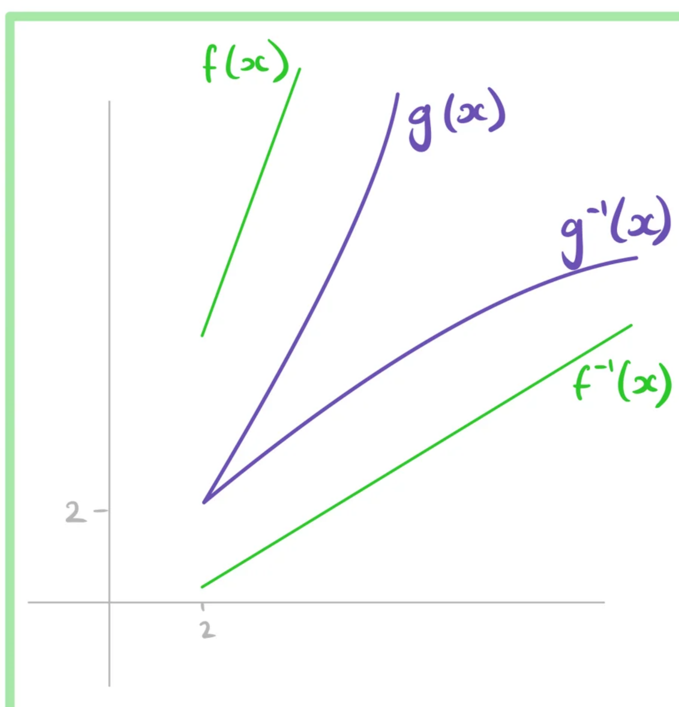
[3]

(c) Sketch the graphs of f and g, for the domain found in part (b), on the same set of axes, along with their inverses.

$x \geqslant 2$

[4]

(c) USE GDC TO SKETCH FOR $x \geqslant 2$



Question 12

Consider the function f defined by $f(x) = 2x^3 + 3x^2 - 36x + 7,\ x \in \mathbb{R}$.

(a) Sketch the graph of f. Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima. [2]

Let the function g be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7,\ x \leq p$.

(b) Given that g has an inverse:

(i) Find the largest possible value of p

(ii) Find the domain of g^{-1} for the value of p identified in part (b)(i)

(iii) Find the value of $g^{-1}(0)$. [3]

Let the function h be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7,\ x \geq q$.

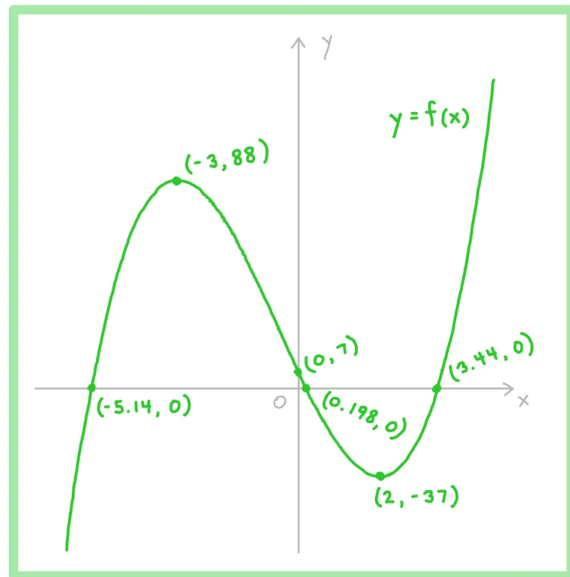
(c) Given that h has an inverse:

(i) Find the smallest possible value of q

(ii) Find the domain of h^{-1} for the value of q identified in part (c)(i)

(iii) Find the value of $h^{-1}(0)$. [3]

a) Your GDC can help with this :



Non-exact values are rounded to 3 s.f.

Consider the function f defined by $f(x) = 2x^3 + 3x^2 - 36x + 7,\ x \in \mathbb{R}$.

(a) Sketch the graph of f. Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima. [2]

Let the function g be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7,\ x \leq p$.

(b) Given that g has an inverse:

(i) Find the largest possible value of p

(ii) Find the domain of g^{-1} for the value of p identified in part (b)(i)

(iii) Find the value of $g^{-1}(0)$. [3]

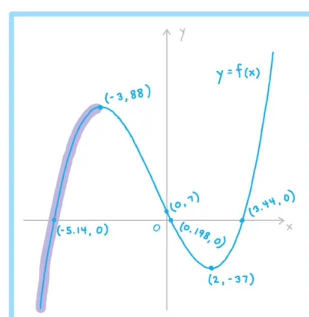
Let the function h be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7,\ x \geq q$.

(c) Given that h has an inverse:

(i) Find the smallest possible value of q

(ii) Find the domain of h^{-1} for the value of q identified in part (c)(i)

(iii) Find the value of $h^{-1}(0)$. [3]



b) (i) For g to be one-to-one (and so have an inverse), its domain cannot cross over the local max at (-3, 88).

$\boxed{p = -3}$

(ii) The domain of g^{-1} is the range of g.

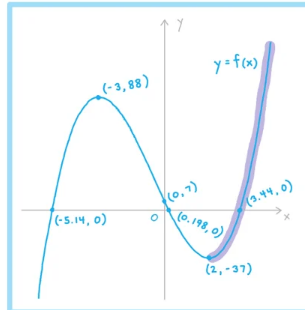
$\boxed{x \leq 88}$

(iii) $g(-5.14) = 0 \implies \boxed{g^{-1}(0) = -5.14 \text{ (3 s.f.)}}$

Consider the function f defined by $f(x) = 2x^3 + 3x^2 - 36x + 7,\ x \in \mathbb{R}$.

(a) Sketch the graph of f. Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima.

[2]



Let the function g be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7,\ x \le p$.

(b) Given that g has an inverse:

(i) Find the largest possible value of p

(ii) Find the domain of g^{-1} for the value of p identified in part (b)(i)

(iii) Find the value of $g^{-1}(0)$.

[3]

Let the function h be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7,\ x \ge q$.

(c) Given that h has an inverse:

(i) Find the smallest possible value of q

(ii) Find the domain of h^{-1} for the value of q identified in part (c)(i)

(iii) Find the value of $h^{-1}(0)$.

[3]

c) (i) For h to be one-to-one (and so have an inverse), its domain cannot cross over the local min at $(2, -37)$.

$\boxed{q = 2}$

(ii) The domain of h^{-1} is the range of h.

$\boxed{x \ge -37}$

(iii) $h(3.44) = 0 \implies \boxed{h^{-1}(0) = 3.44 \text{ (3 s.f.)}}$

Question 13

A function f is called a self-inverse function if $f^{-1}(x) = f(x)$ for all values of x in the domain.

Let $f(x) = \frac{\pi^2}{x}$, where $x \ne 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function.

[2]

a) Let $y = f(x)$ and rearrange for x.

$y = \frac{\pi^2}{x} \longrightarrow x = \frac{\pi^2}{y} \quad \therefore f^{-1}(x) = \frac{\pi^2}{x}$

$\boxed{\therefore f(x) \text{ is a self inverse function}}$

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \ne p, x \in \mathbb{R}$.

(b) Find the value of p.

[1]

(c) Show that $g(x)$ is a self-inverse function.

[4]

A function f is called a self-inverse function if $f^{-1}(x) = f(x)$ for all values of x in the domain.

Let $f(x) = \frac{\pi^2}{x}$, where $x \neq 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function.

[2]

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \neq p, x \in \mathbb{R}$.

(b) Find the value of p.

[1]

(c) Show that $g(x)$ is a self-inverse function.

[4]

b) The value of p is equal to the value of x that makes the denominator equal zero.

$$\boxed{p = -\frac{1}{5}}$$

A function f is called a self-inverse function if $f^{-1}(x) = f(x)$ for all values of x in the domain.

Let $f(x) = \frac{\pi^2}{x}$, where $x \neq 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function.

[2]

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \neq p, x \in \mathbb{R}$.

(b) Find the value of p.

[1]

(c) Show that $g(x)$ is a self-inverse function.

[4]

c) Let $y = g(x)$ and rearrange for x.

$$y = \frac{-x-2}{5x+1}$$

$$5xy + y = -x - 2$$

$$5xy + x = -y - 2$$

$$x(5y+1) = -y - 2$$

$$x = \frac{-y-2}{5y+1}$$

$$\boxed{g^{-1}(x) = \frac{-x-2}{5x+1} \quad \therefore g(x) \text{ is a self inverse function}}$$