

IB Maths: AA HL

Functions Toolkit

Topic Questions

These practice questions can be used by students and teachers and is Suitable for IB Maths AA HL Topic Questions

Course	IB Maths
Section	2. Functions
Topic	2.4 Functions Toolkit
Difficulty	Medium

Level: IB Maths

Subject: IB Maths AA HL

Board: IB Maths

Topic: Functions Toolkit

Question 1

The functions f and g are defined such that $f(x) = 4x - 10$ and $g(x) = \frac{x+8}{2}$.

(a) Show that $(g \circ f)(x) = 2x - 1$.

[2 marks]

(b) Given that $(g \circ f)(a) = 27$, find the value of a .

[2 marks]

(c) Show that $(f \circ g)(x) = 2x + 6$.

[2 marks]

(d) Given that $(f \circ g)(b) = 44$, find the value of b .

[2 marks]

Question 2

The functions $f(x)$ and $g(x)$ are defined as follows

$$\begin{array}{ll} f(x) = x^2 & x \in \mathbb{R} \\ g(x) = 4x - 3 & x \in \mathbb{R} \end{array}$$

(a) Write down the range of $f(x)$.

[1 mark]

(b) Find

(i) $(f \circ g)(x)$

(ii) $(g \circ f)(x)$

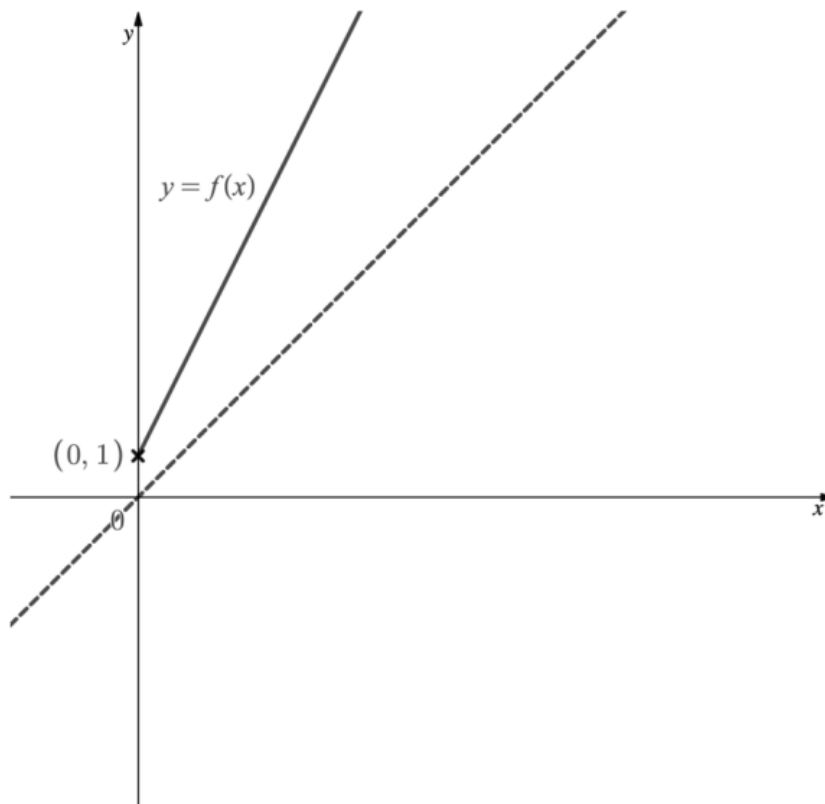
[4 marks]

(c) Solve the equation $f(x) = g(x)$.

[2 marks]

Question 3

The graph of $y = f(x)$ is shown below.



- (a) (i) Use the graph to write down the domain and range of $f(x)$.
(ii) Given that the point $(1, 1)$ lies on the dotted line, write down the equation of the line.

[3 marks]

- (b) On the diagram above sketch the graph of $y = f^{-1}(x)$.

[2 marks]

Question 4

The function $f(x)$ is defined as

$$f: x \mapsto \frac{x^2+1}{x^2} \quad x \in \mathbb{R}, x \neq 0$$

- (a) Show that $f(x)$ can be written in the form

$$f: x \mapsto 1 + \frac{1}{x^2}$$

[2 marks]

- (b) Explain why the inverse of $f(x)$ does not exist and suggest an adaption to its domain so the inverse does exist.

[2 marks]

- (c) The domain of $f(x)$ is changed to $x > 0$.
Find an expression for $f^{-1}(x)$ and state its domain and range.

[4 marks]

Question 5

The functions $f(x)$ and $g(x)$ are defined as follows

$$f(x) = \frac{1}{2}(4x - 3) \quad x \in \mathbb{R}$$

$$g(x) = 0.5x + 0.75 \quad x \in \mathbb{R}$$

- (a) Find
- (i) $fg(x)$
 - (ii) $gf(x)$

[3 marks]

- (b) Write down $f^{-1}(x)$ and state its domain and range.

[3 marks]

Question 6

A function is defined by $f(x) = 54x - 13$, $-2 < x < 20$.

- (a) Find the value of $f\left(\frac{5}{2}\right)$.

[1 mark]

- (b) Write down the range of $f(x)$.

[2 marks]

(c) Find the inverse function $f^{-1}(x)$.

[2 marks]

(d) Write down the range of the inverse function.

[1 mark]

Question 7

Consider the function $f(x) = -6x - 3$. The domain of $f(x)$ is $-5 \leq x \leq 3$.

(a) Find

(i) $f(2)$

(ii) x when $f(x) = 15$.

[2 marks]

(b) Find the range of $f(x)$.

[3 marks]

(c) Write down the domain of the inverse function.

[1 mark]

Question 8

Let $f(x) = -\frac{3}{x-3}$, for $x \neq 3$.

(a) For the graph of f , find:

- (i) the x – intercept
- (ii) the y – intercept
- (iii) the range of f .

[4 marks]

(b) Find the value of $f^{-1}(-1)$.

[2 marks]

(c) Given that $g(x) = f(x + 3) + 1$, find the domain and range of g .

[2 marks]

Question 9

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 3x^2 + 10x + 7$ and $g(x) = x + d$, where $d \in \mathbb{R}$.

(a) Find the range of f .

[2 marks]

(b) Given that $(g \circ f)(x)$ is always positive for all x , determine the set of possible values for d .

[4 marks]

Question 10

Let $f(x) = \frac{2x-5}{x+8}$, where $x \neq -8, x \in \mathbb{R}$.

(a) Write down

(i) the value of a

(ii) the range of f .

[2 marks]

(b) For the graph of f , find the equations of all the asymptotes.

[1 mark]

(c) Find $f^{-1}(x)$.

[2 marks]

(d) For the graph of f^{-1} , find the equation of

- (i) the horizontal asymptote
- (ii) the vertical asymptote.

[2 marks]

Question 11

Let $f(x) = 2x + 1$ for $x \in \mathbb{R}$.

(a) Write down an expression for the inverse function $f^{-1}(x)$.

[2 marks]

Consider another function $g(x) = \frac{1}{2}(x - 1)^2 + \frac{3}{2}$ for $x \geq k$, where k is an integer to be found.

(b) Given that the graph of g has an inverse, find the value of k .

[3 marks]

(c) Sketch the graphs of f and g , for the domain found in part (b), on the same set of axes, along with their inverses.

[4 marks]

Question 12

Consider the function f defined by $f(x) = 2x^3 + 3x^2 - 36x + 7$, $x \in \mathbb{R}$.

- (a) Sketch the graph of f . Clearly label the points where the graph intersects the axes, along with any points that are local maxima or minima.

[2 marks]

Let the function g be defined by $g(x) = 2x^3 + 3x^2 - 36x + 7$, $x \leq p$.

- (b) Given that g has an inverse:

- (i) Find the largest possible value of p
- (ii) Find the domain of g^{-1} for the value of p identified in part (b)(i)
- (iii) Find the value of $g^{-1}(0)$.

[3 marks]

Let the function h be defined by $h(x) = 2x^3 + 3x^2 - 36x + 7$, $x \geq q$.

- (c) Given that h has an inverse:

- (i) Find the smallest possible value of q
- (ii) Find the domain of h^{-1} for the value of q identified in part (c)(i)
- (iii) Find the value of $h^{-1}(0)$.

[3 marks]

Question 13

A function f is called a self-inverse function if $f^{-1}(x) = f(x)$ for all values of x in the domain.

Let $f(x) = \frac{\pi^2}{x}$, where $x \neq 0, x \in \mathbb{R}$.

(a) Show that $f(x)$ is a self-inverse function.

[2 marks]

Let $g(x) = \frac{-x-2}{5x+1}$, where $x \neq p, x \in \mathbb{R}$.

(b) Find the value of p .

[1 mark]

(c) Show that $g(x)$ is a self-inverse function.

[3 marks]