



# EXAM PAPERS PRACTICE

GCSE OCR Math J560

Fibonacci & Geometric

Answers

*"We will help you to  
achieve A Star "*



### Answer 1

(b) Show that the 5th term of S is  $7 + 5\sqrt{2}$

$$\begin{array}{cccccc} n: & 1 & 2 & 3 & 4 & 5 \\ n^{\text{th}} \text{ Term:} & \sqrt{2}-1 & 1 & \sqrt{2}+1 & & \end{array}$$

$\swarrow \quad \searrow$   
 $\times(\sqrt{2}+1) \quad \times(\sqrt{2}+1)$

$$\begin{aligned} 4^{\text{th}} \text{ Term} &= (\sqrt{2}+1)(\sqrt{2}+1) \\ &= 2 + \sqrt{2} + \sqrt{2} + 1 \\ &= 3 + 2\sqrt{2} \end{aligned}$$

Common  
RATIO  
 $= \sqrt{2}+1$

$$\begin{aligned} 5^{\text{th}} \text{ Term} &= (3+2\sqrt{2})(\sqrt{2}+1) \\ &= 3\sqrt{2} + 3 + 4 + 2\sqrt{2} \\ &= \underline{\underline{7 + 5\sqrt{2}}} \end{aligned}$$

### Answer 2

The population of bacteria in flask A at the start of the 10th day is  $k$  times the population of bacteria in flask A at the start of the 6th day.

(b) Find the value of  $k$ .

$$\text{DAY } n = 1000 \times 1.5^{n-1}$$

$$\text{DAY } 10 = 1000 \times 1.5^9$$

$$\text{DAY } 6 = 1000 \times 1.5^5$$

$$\text{DAY } 10 = k \times \text{DAY } 6$$

$$k = \frac{\text{DAY } 10}{\text{DAY } 6}$$

$$k = \frac{1000 \times 1.5^9}{1000 \times 1.5^5}$$

$$k = \underline{\underline{5.0625}}$$



**Answer 3**

Here are the first six terms of a Fibonacci sequence.

1	2	3	4	5	6
1	1	2	3	5	8

The rule to continue a Fibonacci sequence is,

the next term in the sequence is the sum of the two previous terms.

(a) Find the 9th term of this sequence.

$$7^{\text{th}} \text{ term} = 5 + 8 = 13$$

$$8^{\text{th}} \text{ term} = 8 + 13 = 21$$

$$9^{\text{th}} \text{ term} = 13 + 21 = \underline{\underline{34}}$$

**Answer 4**

Given that the 3rd term is 7 and the 6th term is 29,

(c) find the value of  $a$  and the value of  $b$ .

$$3^{\text{rd}} \text{ term: } a + b = 7 \quad \text{--- ①}$$

$$6^{\text{th}} \text{ term: } 3a + 5b = 29 \quad \text{--- ②}$$

$$\text{①} \times 3 : \quad 3a + 3b = 21 \quad \text{--- ③}$$

$$\text{②} - \text{③} : \quad \frac{2b}{2} = \frac{8}{2}$$

$$\underline{\underline{b = 4}}$$

$$\rightarrow \text{①} \quad a + 4 - 4 = 7 - 4$$

$$\underline{\underline{a = 3}}$$

SOLVE SIMULTANEOUS  
EQUATIONS USING  
ELIMINATION METHOD



Answer 5

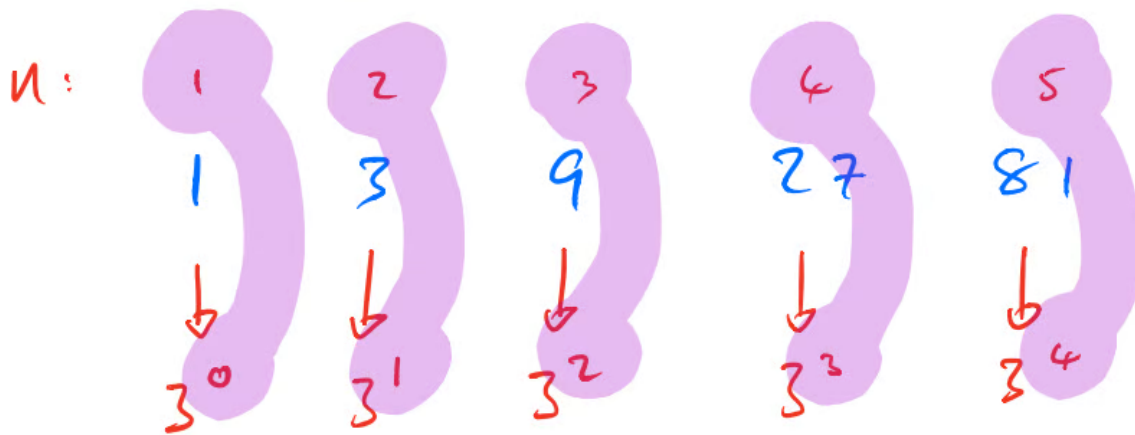
(b) 1, 3, 9, 27, 81, ...

$+2$     $+6$     $+18$     $+54$

$+4$     $+12$     $+36$

NON LINEAR / QUADRATIC

THINK POWERS - OF 3



Power is 1 LESS THAN  $n$

$n^{\text{th}} \text{ TERM} = \underline{\underline{3^{n-1}}}$



Answer 6

(b) Write down the  $n$ th term of sequence A.

Two MORE THAN  $n$

Answer(b) .....  $n+2$  .....

Answer 7

(d) (i) Find the  $n$ th term of sequence C in its simplest form.

$$\begin{aligned} \frac{2a}{2} = \frac{2}{2} \text{ so } \underline{a=1} \\ 3 \times 1 + b = 0 \text{ so } \underline{b=-3} \\ 1 - 3 + c = -3 \text{ so } \underline{c=-1} \end{aligned} \quad \left| \begin{array}{l} n^{\text{th}} \text{ TERM} \\ = n^2 - 3n - 1 \end{array} \right.$$

(ii) Find the 8th term of sequence C.

$$\begin{aligned} 8^{\text{th}} \text{ term} &= 8^2 - 3 \times 8 - 1 \\ &= \underline{39} \end{aligned}$$

QUADRATIC SEQUENCES

$$n^{\text{th}} \text{ TERM} = an^2 + bn + c$$

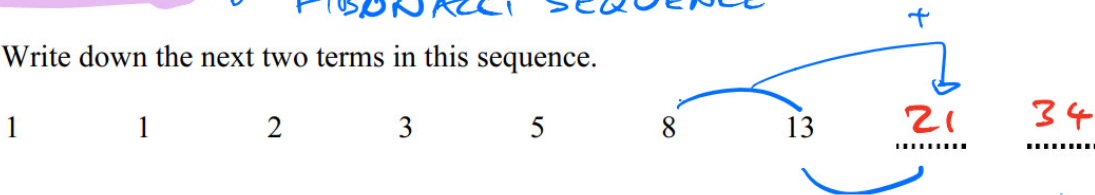
$n=1$	$n=2$	$n=3$
$a+b+c$	$4a+2b+c$	$9a+3b+c$
	$\uparrow$	$\uparrow$
	$3a+b$	$5a+b$
		$\uparrow$
		$2a$

COMPARE HIGHLIGHTED TERMS WITH DIFFERENCES IN YOUR SEQUENCE TO FIND  $a, b$  AND  $c$

Answer 8

In all the following sequences, after the first two terms, the rule is to add the previous two terms to find the next term. → "FIBONACCI SEQUENCE"

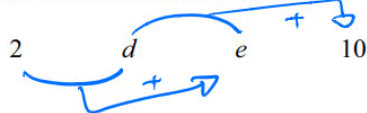
(a) Write down the next two terms in this sequence.





Answer 9

(c) (i) Find the value of  $d$  and the value of  $e$ .



$$2 + d = e \quad \text{--- (A)}$$

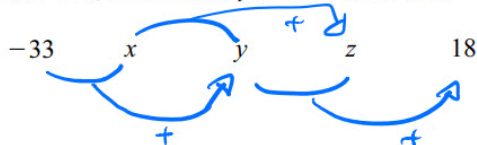
$$d + e = 10 \quad \text{--- (B)}$$

$$\begin{aligned} \text{(A)} \rightarrow \text{(B)} \quad d + (2 + d) &= 10 \\ \frac{2d}{2} &= \frac{8}{2} \\ \underline{d} &= \underline{4} \end{aligned}$$

$$\begin{aligned} \rightarrow \text{(A)} \quad 2 + 4 &= e \\ \underline{6} &= \underline{e} \end{aligned}$$

Answer(c)(i)  $d = \underline{4}$   
 $e = \underline{6}$

(ii) Find the value of  $x$ , the value of  $y$  and the value of  $z$ .



$$-33 + x = y \quad \text{--- (1)}$$

$$x + y = z \quad \text{--- (2)}$$

$$y + z = 18 \quad \text{--- (3)}$$

$$\text{(2)} - \text{(1)} \quad \begin{array}{r} y + 33 = z - y \\ \underline{y + y} \end{array}$$

$$2y + 33 = z$$

$$\begin{aligned} \rightarrow \text{(3)} \quad y + 2y + 33 &= 18 \\ \frac{3y}{3} &= \frac{-15}{3} \end{aligned}$$

$$\begin{aligned} \underline{y} &= \underline{-5} \\ \text{So } z &= 2 \times (-5) + 33 \\ \underline{z} &= \underline{23} \end{aligned}$$

$$\begin{aligned} \rightarrow \text{(2)} \quad x - 5 &= 23 \\ \underline{x} &= \underline{28} \end{aligned}$$



**Answer 10**

(b) the  $n$ th term.

SQUARE OF ONE MORE THAN  $n$

Answer (b).....  $(n+1)^2$ ..... [1]

**Answer 11**

S is a geometric sequence.  $\rightarrow$  Common RATIO BETWEEN TERMS

(a) Given that  $(\sqrt{x} - 1)$ , 1 and  $(\sqrt{x} + 1)$  are the first three terms of S, find the value of  $x$ .  
You must show all your working.

Common RATIO:  $\frac{\text{2nd TERM}}{\text{1st TERM}} = \frac{1}{\sqrt{x} - 1}$  } EQUATE  
Also  $\frac{\text{3rd TERM}}{\text{2nd TERM}} = \frac{\sqrt{x} + 1}{1}$

So  $(\sqrt{x} - 1) \times \frac{(\sqrt{x} + 1)}{1} = \frac{1}{\sqrt{x} - 1} \times (\sqrt{x} - 1)$

$(\sqrt{x} - 1)(\sqrt{x} + 1) = 1$

F O I L  
 $x + \sqrt{x} - \sqrt{x} - 1 = 1$

$\rightarrow$   $x = 2$



**Answer 12**

Louis and Robert are investigating the growth in the population of a type of bacteria.

They have two flasks A and B.

At the start of day 1, there are 1000 bacteria in flask A.

The population of bacteria grows exponentially at the rate of 50% per day.

EACH DAY WE MULTIPLY BY A CONSTANT

- INCREASE BY 50% PER DAY

(a) Show that the population of bacteria in flask A at the start of each day forms a geometric progression.

DAY 1    DAY 2    DAY 3  
1000    1500    2250  
    ↗    ↗  
    x1.5    x1.5

PERCENTAGE INCREASES (THE BEST WAY!)  
TO INCREASE BY, SAY, 3%. 50%.  
THINK: WE WANT 103%. 150%. SO WE  
MULTIPLY BY  $\frac{103}{100}$  (=1.03)     $\frac{150}{100} = 1.50$

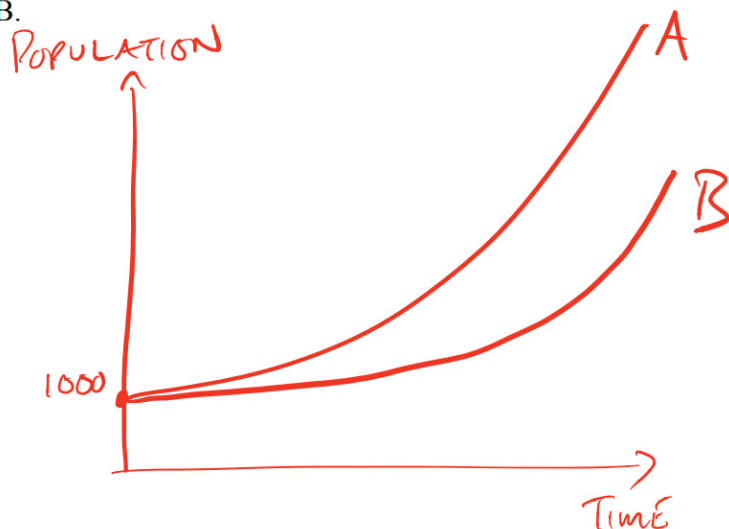
THIS IS A GEOMETRIC PROGRESSION.

**Answer 13**

At the start of day 1 there are 1000 bacteria in flask B.

The population of bacteria in flask B grows exponentially at the rate of 30% per day.

(c) Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.







**Answer 14**

The first three terms of a different Fibonacci sequence are

$$\begin{array}{ccc} 1 & 2 & 3 \\ a & b & a+b \end{array}$$

(b) Show that the 6th term of this sequence is  $3a + 5b$

$$4^{\text{th}} \text{ Term} = b + a + b = a + 2b$$

$$5^{\text{th}} \text{ Term} = a + b + a + 2b = 2a + 3b$$

$$6^{\text{th}} \text{ Term} = a + 2b + 2a + 3b = \underline{\underline{3a + 5b}}$$

**Answer 15**

Find the  $n$ th term of each of these sequences.

(a) 16, 19, 22, 25, 28, ...

LINEAR

$n^{\text{th}} \text{ Term} = 3n + c$

$$\underline{n=1} : \quad 16 = 3 \times 1 + c$$
$$\quad \quad \quad -3 \quad \quad -3$$

$$13 = c$$

$$n^{\text{th}} \text{ Term} = \underline{\underline{3n + 13}}$$