EXAM PAPERS PRACTICE

Mark Scheme

Factor theorem

Name: _____

Class: _____

Date: _____

Time: **125 min.**

Marks: **105 marks**

Comments:

1.

(a)

Answer	Mark	Comments
$(p(-2) =)$ $(-2)^3 + a(-2)^2 - (-2) - 21 = -7$ or $-8 + 4a + 2 - 21 = -7$	M1	oe. Uses Remainder Theorem with $x = -2$ to form equation in a, simplified or unsimplified. Allow one error.
$a = 5$	A1	CSO. Be convinced. Must follow from completely correct working.

(b)

$(p(-3) =) (-3)^3 + 5(-3)^2 - (-3) - 21$	M1	p(−3) attempted. Must use Factor Theorem.
$(p(-3) =) -27 + 45 + 3 - 21 = 0$	A1	CSO. Correctly shows p(−3) = 0.

(c)	Quadratic factor $(x^2 + 2x - 7)$	M1	Attempt to express numerator as product of $x + 3$ and a quadratic factor. Allow one error in quadratic factor. Can use inspection, long division or compare coefficients.
	$(x + 3)(x^2 + 2x - 7)$	A1	Numerator correctly expressed as product of $x + 3$ and a quadratic factor. May be implied by further correct working.
	$x^{\frac{3}{2}} + bx^{\frac{1}{2}} + dx^{-\frac{1}{2}}$ or $x\sqrt{x} + b\sqrt{x} + \frac{d}{\sqrt{x}}$	M1	Attempt to write their quotient as the sum of powers of x. $x + 3$ cancelled and division by $\sqrt{x}$ attempted. Allow one error in dividing quadratic term by $\sqrt{x}$. Allow sum of powers of x given in surd form with final term $\frac{d}{\sqrt{x}}$. Sum must be of three powers of x. ft if numerator was expressed in the form $(x + 3)(x^2 + bx + d)$
	$x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 7x^{-\frac{1}{2}}$ or $x\sqrt{x} + 2\sqrt{x} - \frac{7}{\sqrt{x}}$	A1	Correct expression for quotient as the sum of powers of x. Allow answer given in surd form.
	$\frac{2}{5} \times x^{\frac{5}{2}} + 2 \times \frac{2}{3} \times x^{\frac{3}{2}} - 7 \times 2 \times x^{\frac{1}{2}} + c$ or $\frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} - 14x^{\frac{1}{2}} + c$ or $\frac{2}{5}x^2\sqrt{x} + \frac{4}{3}x\sqrt{x} - 14\sqrt{x} + c$	B2ft	Correct answer simplified or unsimplified. Can be given in index or surd form. ft their integral provided previous M1 scored. B2 for fully correct answer. B1B0 for two terms correct including signs Condone omission of + c.
			Total 10 marks

2.

	Answer	Mark	Comments
(a)	$p^2 + bp + c = p^2 + 3mp + 2n$ or $p^2 + bp + c = 0$ and $p^2 - 3mp + 2n = 0$	B1	Application of the Factor Theorem to show either two correct quadratic expressions equated, or two correct equations in p set to equal zero
	$p(b + 3m) = 2n - c$	M1	Correct manipulation of their $p^2 + bp + c = p^2 + 3mp + 2n$ isolating p on one side and factorised
	$p = \frac{2n - c}{b + 3m}$	A1	CSO. Dependent upon first two marks awarded
(b)	No real roots $\Rightarrow b^2 - 4ac < 0$	B1	Condition for no real roots stated or used
	$(3k + 1)^2 - 4(3(k + 3)) < 0$ or $9k^2 + 6k + 1 - 12k - 36 < 0$	M1	Correct inequality unsimplified. Condone +36. Must include < 0
	$9k^2 - 6k - 35 < 0$	A1	Correct inequality with collected terms
	$(3k + 5)(3k - 7)$	M1	Correct factorisation of the quadratic expression or correct unsimplified quadratic equation formula $k = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 9 \times (-35)}}{2 \times 9}$
	$k = \frac{-5}{3}$ and $k = \frac{7}{3}$	A1	oe. For correct critical values. Fractions must be fully simplified, or decimal equivalents must be exact.
	$\frac{-5}{3} < k < \frac{7}{3}$	A1	oe. For correct inequality. Fractions must be fully simplified, or decimal equivalents must be exact.
			Total 9 marks